Quantum Game Theory: An Overview

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***Abstract*—Quantum Game Theory (QGT) is an intriguing intersection of quantum mechanics and game theory, offering insights into decision-making and strategy in quantum environments. With the rise in quantum technology it has gained much attention in recent times for its applications to economics, and political science. In the past twenty years many research efforts have been made to make use of entanglement and superposition properties of quantum mechanics on improving game theory strategy and get better results than classical game theory approaches. In this literature review we will dive deep into the prisoner's dilemma and provide a comparative analysis between classical approach and the improved quantum approach.**

Key Words: Quantum Games, Game Theory, Nash Equilibrium, Prisoner's Dilemma

I. Introduction

Game theory was originally developed by the Hungarian-born American mathematician John Von Neumann and his Princeton University colleague, the German-born American economist Oskar Morgenstern. Von Neumann, renowned as a physicist, also delivered lectures on Quantum Theory at Princeton, and was part of the Manhattan Project, whilst simultaneously developing game theory. This highlights the interdisciplinary link between game theory and quantum mechanics from the early days.(Brams and Davis).

Game Theory is a branch of applied mathematics, which explores how rational agents interact in a strategic manner to optimize their awards given certain rules. A solution to a game describes the optimal decisions of the players, considering their similar, opposed, or mixed interests. It predicts the outcomes resulting from these decisions. The Nash equilibrium (NE) holds paramount significance in game theory as it signifies a scenario where no player can enhance their payoff through a unilateral alteration of their strategy. A Pareto optimal outcome denotes a situation where no player can elevate their utility without causing a corresponding reduction in another player's utility. In a two-player zero-sum game, the interests of the players are in direct opposition, resulting in a cumulative payoff of zero for every potential game outcome. Within such games, a saddle point refers to an element within the payoff matrix for the row player that simultaneously represents the minimum value within its row and the maximum value within its column (Flitney and Abbott).

At its core, Quantum Games seek to extend the boundaries of conventional game theory by integrating quantum concepts to explore the strategic decision-making processes in quantum environments. In contrast to classical game theory, where players make decisions based on classical strategies and outcomes are determined by classical rules, Quantum Games delve into scenarios where players may employ quantum strategies and outcomes are influenced by the principles of quantum mechanics like superposition, entanglement.

Entanglement is a unique quantum phenomenon where the states of two or more particles become correlated, such that the state of one particle instantaneously influences the state of the other(s), regardless of the distance separating them. In Quantum Games, entanglement can introduce cooperative or adversarial relationships between players that transcend classical strategic interactions. Players may entangle their quantum states to cooperate more effectively. For example, in a quantum prisoner's dilemma game, entangled players may coordinate their strategies to achieve higher payoffs compared to classical strategies (Khan et al.).

Superposition is another fundamental quantum principle that allows particles to exist in multiple states simultaneously. In Quantum Games, superposition enables players to adopt multiple strategies simultaneously, leading to richer and more nuanced game play. Players can employ superposition to mix their strategies probabilistically. For instance, in a quantum version of rock-paper-scissors, a player could prepare their quantum state in a superposition of all three strategies, effectively playing all three moves at once (Khan et al.). Superposition introduces uncertainty into the game, as the outcome depends on the collapse of the quantum state upon measurement. This uncertainty can lead to strategic bluffing and counter-bluffing, adding depth to the decision-making process. Superposition can confer strategic advantages by enabling players to explore multiple strategies simultaneously, potentially outmaneuvering opponents who are limited to classical strategies.

II. Prisoner's Dilemma

A 2x2 games is a game where each player has two possible moves. This can be generalized to larger strategic spaces or a greater number of players. One classic example of a 2x2 game that has received significant attention is the Prisoner's Dilemma. It is a nonzero-sum game, implying that players can achieve additional benefits through cooperation.

A. In Classical Setting

Prisoner's Dilemma Setup:

A classic Prisoner's Dilemma as setup by Wang involves two players, labeled as Alice (A) and Bob (B). The players have two possible moves: cooperation (C) or defection (D). Based on the choices of both players, they will receive a certain payoff. The objective of each player is to maximize their individual payoff.

Payoff Matrix:

	B: Cooperate	B: Defect
A: Cooperate	$(3,3)$	$(0,5)$
A: Defect	$(5,0)$	$(1,1)$

- The rows represent Player A's choices (Cooperate or Defect).
- The columns represent Player B's choices (Cooperate or Defect).
- The payoffs are presented in the form (a,b), where a is the payoff for Player A, and b is the payoff for Player B.

Explanation:

If both players choose to Cooperate, they each receive a payoff of 3, resulting in a total payoff of 6. If Player A chooses to Cooperate while Player B chooses to Defect, Player A receives a payoff of 0 and Player B receives a payoff of 5. If Player A chooses to Defect while Player B chooses to Cooperate, Player A receives a payoff of 5 and Player B receives a payoff of 0. If both players choose to Defect, they each receive a payoff of 1, resulting in a total payoff of 2.

Analysis:

The game is symmetric, meaning the payoffs are the same regardless of which player takes which action. There exists a dominant strategy, which is to always defect. This strategy yields a better payoff regardless of the opponent's move. If the opponent cooperates, the defector gets a payoff of five instead of three, and if the opponent defects, the defector gets a payoff of one instead of zero. The NE in this game is when both players choose to defect (D, D). This outcome arises because both players independently choose their dominant strategy. However, this outcome is not the most favorable for the players overall. The Pareto optimal outcome, where both players cooperate (C, C), would result in a higher total payoff for both players (three each). However, in the absence of communication or negotiation, the players often end up in the NE of defection, leading to a sub-optimal outcome for both.

B. In Quantum Setting

Jens Eisert demonstrated that players facing the Prisoner's Dilemma can enhance their outcomes by utilizing quantum strategies Eisert et al. (1999). In this section, we'll outline his reasoning.

Let's imagine we give each player a quantum bit, or qubit. In the classic version of the game, where players can either cooperate or defect, these actions correspond to two fundamental states: cooperation (C) and defection (D). In the realm of quantum mechanics, these actions can be represented by basis vectors, denoted as $|\mathrm{C}\rangle$ and $|\mathrm{D}\rangle$, respectively.

Now, let's consider the state of the game. It can be described by combining the strategies of both players, A and B, using tensor product. This operation creates combinations of the strategies, resulting in four possible states: $|\mathrm{CC}\rangle$, $|\mathrm{CD}\rangle$, $|\mathrm{DC}\rangle$, and $|\mathrm{DD}\rangle$.

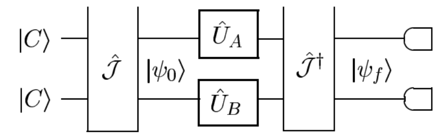

Fig. 1: Two player quantum game setting as shown in Eisert et al..

Let's envision the initiation of the game as a particular state, symbolized by $|\psi_0\rangle$. This initial state materializes through the application of a specialized mathematical operation known as a unitary operator, represented as $\mathcal{J}$, acting upon the state where both players cooperate, denoted as $|\mathrm{CC}\rangle$. Subsequently, each player engages by employing their own distinct unitary operators, denoted as $\hat{U}_{\mathrm{A}}$ and $\hat{U}_{\mathrm{B}}$ correspondingly. These operators encapsulate the strategic choices or actions undertaken by the players within the quantum framework. This configuration is depicted in Figure 1.

As per the formulation by Eisert, Wilkens, and Lewenstein Eisert et al. (1999), the strategy operator $\hat{U}$ can be expressed as a set of unitary matrices characterized by two parameters:

$$\hat{U}(\theta,\phi) = \begin{pmatrix} e^{i\phi}\cos\left(\frac{\theta}{2}\right) & \sin\left(\frac{\theta}{2}\right) \\ \cos\left(\frac{-\theta}{2}\right) & e^{-i\phi}\sin\left(\frac{\theta}{2}\right) \end{pmatrix},$$

where $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq \frac{\pi}{2}$. These parameters offer a means to characterize and vary the quantum strategies available to the players, allowing for a broad spectrum of potential moves within the game environment.

Next let's explore the strategies players can employ within setting as delineated by Eisert et al.. We'll start by considering all feasible strategies within a defined subset, $S_0 \equiv \{\hat{U}(\theta,0)|\theta \in [0,\pi]\}$.

Now, let's elucidate the two fundamental strategies: Cooperation ($\hat{C}$) and Defection ($\hat{D}$). The Cooperation strategy, denoted as $\hat{C}$, corresponds to a specific configuration of the unitary operator, $\hat{U}$, where $\theta = 0$. It essentially preserves the initial state unchanged.

$$\hat{C} \equiv \hat{U}(0,0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

On the other hand, the Defection strategy, $\hat{D}$, entails a spin flip operation, represented by $\theta = \pi$, leading to a specific matrix configuration reflecting the transition to a different quantum state.

$$\hat{D} \equiv \hat{U}(\pi,0) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Proceeding to the calculation of expected payoffs by authors let's delve into Player A's viewpoint. Their expected payoff, denoted as $\$_A$, is determined by various factors including the probabilities of different outcomes.

$$\$_A = rP_{CC} + pP_{DD} + tP_{DC} + sP_{CD},$$

These probabilities are contingent on the chosen strategies of both players. For instance, the probability of Player A choosing Cooperation ($p^{(c)}$) or Defection ($p^{(d)}$) can be expressed in terms of θ using trigonometric functions. Where $P_{CC} = p_A^{(C)} \times p_B^{(C)}$. The probability of the choice being cooperation c is:

$$p^{(c)} = \cos^2\left(\frac{\theta}{2}\right).$$

Similarly, the probability of the choice being defect $p(d)$ is:

$$p^{(d)} = 1 - \cos^2\left(\frac{\theta}{2}\right).$$

By introducing a pivotal variable, $\gamma \in [0, \frac{\pi}{2}]$, referred to as the entanglement of the model, the authors enables us to explore different levels of interaction between the players. In scenarios where the game is regarded as separable ($\gamma = 0$), Player A's optimal strategy tends towards Defection regardless of Player B's choices. This insight is depicted in Figure 2 of Eisert et al. (1999), illustrating that Player A's payoff is maximized when adopting the Defection strategy $\hat{D}$, regardless of Player B's actions.

Since this game is symmetric for both players, the strategy that optimizes Player B's payoff aligns with the classical paradigm, resulting in a preference for the Defection strategy $\hat{D}$. This mirrors conventional game theory but viewed through the lens of quantum mechanics.

Now, let's delve into a scenario where entanglement γ is introduced, specifically set at $\frac{\pi}{2}$. This introduces intriguing complexities. When Player B selects the Defect strategy $\hat{D}$, Player A's optimal response emerges as the $\hat{Q}$ strategy,

$$\hat{Q} \equiv \hat{U}(0, \frac{\pi}{2}) = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}.$$

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However, if Player B opts for Cooperation ($\hat{C}$), Player A's best course is to defect ($\hat{D}$). Consequently, a lack of a dominant strategy emerges for Player A, and this symmetry extends to Player B as well, disrupting the conventional Nash Equilibrium characterized by mutual defection.

Interestingly, a novel Nash Equilibrium arises for both players, converging on the adoption of the $\hat{Q}$ strategy. When Player B opts for $\hat{Q}$, Player A's anticipated payoff, denoted as $\$_A$, adheres to the inequality:

$$\$_A[\hat{U}(\theta, \phi), \hat{Q}] = \cos^2\left(\frac{\theta}{2}\right)\left(3\sin^2(\phi) + \cos^2(\phi)\right) \leq 3$$

This inequality holds true for all values of θ within the interval $[0, \pi]$ and ϕ within $[0, \frac{\pi}{2}]$. Furthermore, setting $\theta = 0$ and $\phi = \frac{\pi}{2}$ implies that if $\hat{Q}$ is chosen, the expected payoff for player A is maximized:

$$\$_A[\hat{Q}, \hat{Q}] = \$_A[\hat{U}(0, \pi/2), \hat{Q}]$$

$$= \cos^2(0)\left(3\sin^2\left(\frac{\pi}{2}\right) + \cos^2\left(\frac{\pi}{2}\right)\right) = 3 \geq \$_A[\hat{U}(\theta, \phi), \hat{Q}]$$

This equality holds for all θ within $[0, \pi]$ and ϕ within $[0, \frac{\pi}{2}]$.

Hence, it is evident that player A consistently benefits from selecting the $\hat{Q}$ strategy. Given the symmetric nature of the game for player B, an analogous rationale dictates their preference for $\hat{Q}$. This observation underscores a critical insight: the optimal strategy for a player lies in the adoption of $\hat{Q}$ when their counterpart also selects $\hat{Q}$. This strategic alignment ensures that neither player can gain further advantages by altering their course of action. Consequently, this alignment culminates in the establishment of a new Nash Equilibrium, wherein both players converge on $\hat{Q}$ as their preferred strategy Wang (2022).

This newly defined Nash Equilibrium marks a notable improvement in the expected payoff for both players. By harmonizing their strategies in this manner, players collectively enhance their outcomes within the context of the Prisoner's Dilemma. The employment of quantum principles proves instrumental in this endeavor, facilitating a strategic framework that optimizes player payoffs and promotes mutually beneficial outcomes. Thus, the application of quantum principles transcends classical game theoretic constraints in complex scenarios like the Prisoner's Dilemma as highlighted by Eisert et al.

III. QGT Application

Quantum Game Theory (QGT) harbors substantial potential for application across various domains, including economics, finance, and political science. By harnessing the principles of quantum mechanics to analyze strategic interactions, QGT offers novel insights and strategies that can revolutionize decision-making processes in these fields.

In economics, QGT provides a fresh perspective on traditional game theoretic models, enabling economists to explore complex scenarios with greater precision and depth. By considering quantum strategies and entanglement effects, economists can better understand phenomena such as market competition, bargaining, and cooperation. QGT can also shed light on issues related to resource allocation, pricing strategies, and strategic behavior in economic markets. Moreover, the application of QGT in economic modeling holds promise for optimizing economic policies and promoting welfare-enhancing outcomes.

In finance, QGT offers innovative approaches for modeling and analyzing financial markets and investment strategies. By incorporating quantum principles into financial models, researchers can develop more accurate risk assessment techniques, portfolio optimization methods, and trading strategies. QGT can also facilitate the exploration of complex financial derivatives and the dynamics of asset price movements. Additionally, the application of QGT in finance may lead to the development of quantum algorithms for high-frequency trading and algorithmic decision-making in financial markets.

In political science, QGT provides valuable tools for understanding strategic interactions among actors in political systems. By applying quantum concepts to political game theory, researchers can analyze issues such as conflict resolution, negotiation strategies, and coalition formation. QGT can also inform the study of voting behavior, electoral competition, and decision-making processes in governance structures. Furthermore, QGT may offer insights into the dynamics of international relations, including issues related to diplomacy, security, and cooperation among nations.

IV. Conclusion

In this review, we have discussed the groundbreaking work by Jens Eisert, Martin Wilkens, and Maciej Lewenstein (ESL), particularly their contributions to Quantum Game Theory (QGT) as applied to the classic game scenario of the Prisoner's Dilemma. Their seminal paper Eisert et al. (1999) provides a comprehensive analysis of how quantum strategies and entanglement can enhance decision-making and optimize player payoffs in strategic interactions.

Eisert, Wilkens, and Lewenstein (ESL) demonstrated that by leveraging quantum strategies and entanglement, players facing the Prisoner's Dilemma can achieve superior outcomes compared to classical game theory approaches. Through rigorous mathematical analysis, they showed how the introduction of entanglement, specifically setting it at $\frac{\pi}{2}$, gives rise to a new Nash Equilibrium centered around the adoption of the $\hat{Q}$ strategy. This strategic alignment ensures that both players can maximize their payoffs without the need for further strategic adjustments.

The work of ESL highlights the transformative potential of Quantum Game Theory (QGT) in reshaping our understanding of strategic decision-making in quantum environments. Their contributions have paved the way for further research and exploration in the field, offering valuable insights into the dynamics of quantum games and their applications in various domains.

V. Future Work

The field of Quantum Game Theory (QGT) presents numerous avenues for future exploration and research. Some potential areas for future work include:

Conducting experimental studies to validate the theoretical predictions of QGT in real-world scenarios. Experimental setups could involve quantum computers or quantum simulators to test the efficacy of quantum strategies in strategic interactions.

Extending QGT to analyze strategic interactions involving multiple players. Investigating the dynamics of multi-player quantum games and identifying optimal strategies for achieving desirable outcomes.

Exploring the development of quantum algorithms tailored for solving game theoretic problems efficiently. Leveraging the computational advantages of quantum computing to tackle complex strategic decision-making tasks.

Integrating QGT principles into the design of intelligent systems and algorithms. Developing quantum-inspired decision-making frameworks for enhancing the performance of AI agents in strategic environments.

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