The Runge-Kutta-Method for a pendulum

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Pendulum

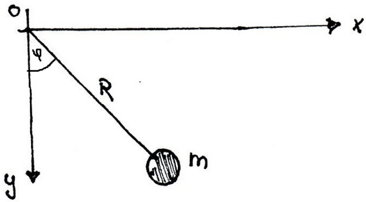

Figure: Pendulum

$$\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} R\sin\phi \\ R\cos\phi \end{pmatrix} \qquad L = T - V = \frac{1}{2} m R^2 \dot{\phi}^2 + mgR\cos\phi$$

$$\frac{d}{dt}\frac{dL}{d\dot{\phi}} - \frac{dL}{d\phi} = 0 \qquad \ddot{\phi} = -\frac{g}{R}\sin\phi$$

Pendulum - Approximation

$$\sin(\phi) \approx \phi$$

$$\ddot{\phi} = -\frac{g}{R}\phi$$

$$\boxed{\phi(t) = \phi_0 \cos(\omega t)}$$

The Runge-Kutta-Method

- method to solve initial value problems by approximation at discrete positions
- multistep procedure to reduce the variances
- most common: 4-step method

The Runge-Kutta-Nyström-Method

$$
\begin{aligned}
k_1 &= \frac{h}{2} f(x_i, y_i, y_i') \\
k_2 &= \frac{h}{2} f(x_i + \frac{h}{2}, y_i + \frac{h}{2} y_i' + \frac{h}{4} k_1, y_i' + k_1) \\
k_3 &= \frac{h}{2} f(x_i + \frac{h}{2}, y_i + \frac{h}{2} y_i' + \frac{h}{4} k_2, y_i' + k_2) \\
k_4 &= \frac{h}{2} f(x_i + h, y_i + h y_i' + h k_3, y_i' + 2k_3)
\end{aligned}
$$

$$
\begin{aligned}
y_{i+1} &= y_i + h y_i' + \frac{h}{3}(k_1 + k_2 + k_3) \\
y_{i+1} &= y_i' + \frac{1}{3}(k_1 + 2k_2 + 2k_3 + k_4)
\end{aligned}
$$

Our Program

Results

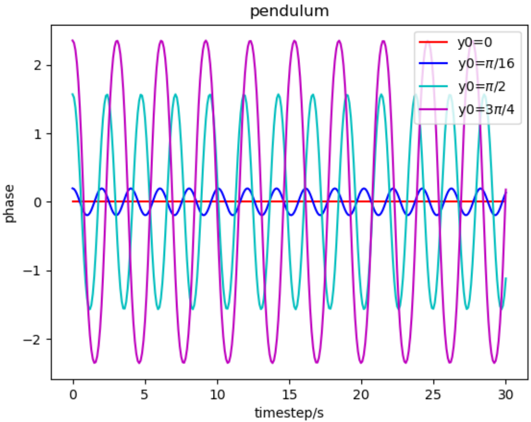

Figure: Oszillation for various initial angles

Results

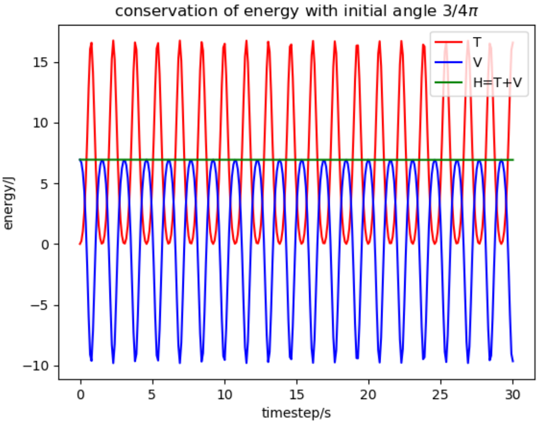

Figure: Conservation of energy

Results

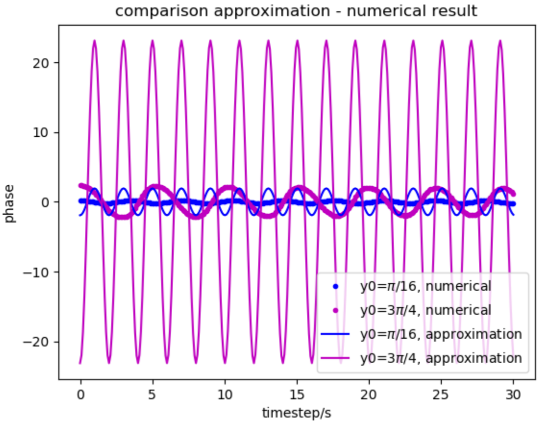

Figure: Comparison between Runge-Kutta-Method and approximation for small angles

Sources

- http://www.uni-magdeburg.de/ifme/l-numerik/mnmm-2-kapitel
- https://de.wikipedia.org/wiki/Runge-Kutta-Verfahren