

The Runge-Kutta-Method for a pendulum

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- 1 Pendulum
 - Pendulum - Approximation
- 2 The Runge-Kutta-Method
 - The Runge-Kutta-Nyström-Method
- 3 Our Program
- 4 Results
- 5 Sources

Pendulum

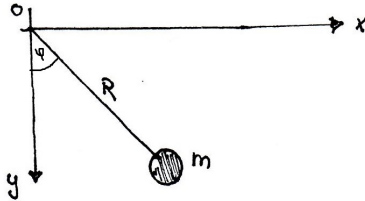


Abbildung: Pendulum

$$\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} R \sin \phi \\ R \cos \phi \end{pmatrix} \quad L = T - V = \frac{1}{2} m R^2 \dot{\phi}^2 + mgR \cos \phi$$

$$\frac{d}{dt} \frac{dL}{d\dot{\phi}} - \frac{dL}{d\phi} = 0 \quad \ddot{\phi} = -\frac{g}{R} \sin \phi$$

Pendulum - Approximation

$$\sin(\phi) \approx \phi$$

$$\ddot{\phi} = -\frac{g}{R}\phi$$

$$\phi(t) = \phi_0 \cos(\omega t)$$

The Runge-Kutta-Method

- method to solve initial value problems by approximation at discrete positions
- multistep procedure to reduce the variances
- most common: 4-step method

The Runge-Kutta-Nyström-Method

$$k_1 = \frac{h}{2} f(x_i, y_i, y'_i)$$

$$k_2 = \frac{h}{2} f\left(x_i + \frac{h}{2}, y_i + \frac{h}{2} y'_i + \frac{h}{4} k_1, y'_i + k_1\right)$$

$$k_3 = \frac{h}{2} f\left(x_i + \frac{h}{2}, y_i + \frac{h}{2} y'_i + \frac{h}{4} k_2, y'_i + k_2\right)$$

$$k_4 = \frac{h}{2} f(x_i + h, y_i + h y'_i + h k_3, y'_i + 2k_3)$$

$$y_{i+1} = y_i + h y'_i + \frac{h}{3} (k_1 + k_2 + k_3)$$

$$y'_{i+1} = y'_i + \frac{1}{3} (k_1 + 2k_2 + 2k_3 + k_4)$$

Our Program

Results

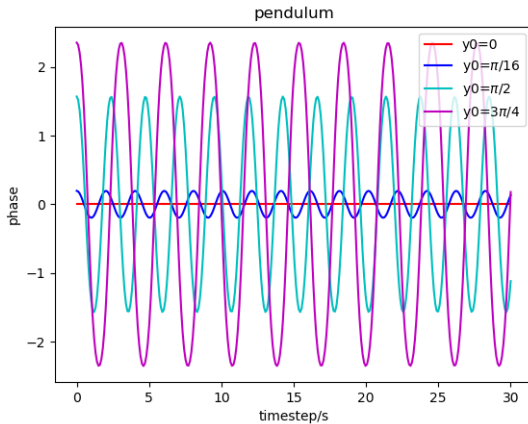


Abbildung: Oscillation for various initial angles

Results

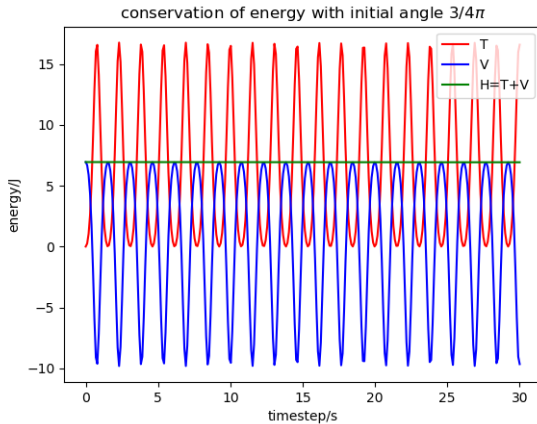


Abbildung: Conservation of energy

Thank you for your attention

Sources

- <http://www.uni-magdeburg.de/ifme/I-numerik/mnmm-2-kapitel>
- <https://de.wikipedia.org/wiki/Runge-Kutta-Verfahren>