Tutorial - 4

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Ans 1 =

$T(n) = 3T(n/2) + n^2$

$T(n) = a\,T\left(\frac{n}{b}\right) + f(n)$

$a = 3,\ b = 2$

$c = \log_2 3 = 1.58$

$n^c = n^{1.58}$

$f(n) = n^2$

By case 3: $f(n) > n^c$

$T(n) = \Theta(f(n)) = \Theta(n^2)$

Ans 2 =

$T(n) = 4T(n/2) + n^2$

$T(n) = aT(n/b) + f(n)$

$a = 4,\ b = 2$

$c = \log_2 4 = 2$

$n^c = n^2$

$f(n) = n^2$

By case 2: $f(n) = n^c$

$T(n) = \Theta(n^c \log n) = \Theta(n^2 \log n)$

3. $T(n) = T(n/2) + 2^n$

$T(n) = a\,T(n/b) + f(n)$

$a = 1,\ b = 2$

$c = \log_2 1 = 0$

$n^c = n^0 = 1,\ f(n) = 2^n$

By Case : $f(n) > n^c$

$T(n) = \Theta(f(n)) = \Theta(2^n)$

4. $T(n) = 2^n T(n/2) + n^n$

$T(n) = a\,T(n/b) + f(n)$

$a = 2^n,\ b = 2$

$c = \log_2 2^n = n$

$n^c = n^n,\ f(n) = n^n$

$f(n) = n^c$

$T(n) = \Theta(n^c \log n)$

$T(n) = \Theta(n^n \log n)$

5. $T(n) = 16\,T(n/4) + n$

$T(n) = a\,T(n/b) + f(n)$

$a = 16,\ b = 4$

$c = \log_4 16 = 2$

$n^c = n^2,\ f(n) = n$

$f(n) < n^c$

$T(n) = \Theta(n^c)$

$T(n) = \Theta(n^2)$

6. $T(n) = 2T(n/2) + n \log n$

$T(n) = aT(n/b) + f(n)$

$a = 2, \; b = 2$

$c = \log_2 2 = 1$

$n^c = n \; , \; f(n) = n \log n$

$f(n) > n^c$

$T(n) = \Theta(f(n))$

$T(n) = \Theta(n \log n)$

7. $T(n) = 2T(n/2) + n/\log n$

$a = 2, \; b = 2$

$c = \log_2 2 = 1$

$n^c = n^1 \; , \; f(n) = n/\log n$

$n^c > f(n)$

$T(n) = \Theta(n^c)$

$T(n) = \Theta(n)$

8. $T(n) = 2T(n/4) + n^{0.5}$

$T(n) = aT(n/b) + f(n)$

$a = 2, \; b = 4$

$c = \log_4 2 = 0.5$

$n^c = n^{0.5}$, $f(n) = n^{0.51}$

$f(n) > n^c$

$T(n) = \Theta(f(n))$

$T(n) = \Theta(n^{0.51})$

9. $T(n) = 0.5\,T(n/2) + 1/n$

$T(n) = aT(n/b) + f(n)$

$a = 0.5, \quad b = 2$

$c = \log_2 0.5 = -1$

$n^c = n^{-1} = 1/n, \quad f(n) = 1/n$

$f(n) = n^c$

$T(n) = \Theta(n^c \log n)$

$= \Theta(\log n / n)$

10. $T(n) = 16\,T(n/4) + n!$

$T(n) = aT(n/b) + f(n)$

$a = 16, \quad b = 4$

$c = \log_4 16 = 2$

$n^c = n^2, \quad f(n) = n!$

$f(n) > n^c$

$T(n) = \Theta(f(n))$

$T(n) = \Theta(n!)$

$T(n) = 4T(n/2) + \log n$

11.

$a = 4, \quad b = 2$

$c = \log_2 4 = 2$

$n^c = n^2, \quad f(n) = \log n$

$f(n) < n^c$

$T(n) = \Theta(n^c)$

$T(n) = \Theta(n^2)$

12. $T(n) = \text{sqrt}(n)\, T(n/2) + \log n$

$T(n) = aT(n/b) + f(n)$

$a = n^{1/2}, \quad b = 2, \quad f(n) = \log n$

$c = \log_2 n^{1/2} = \frac{1}{2} \log n$

$n^c = n^{\frac{1}{2}\log n}$

$f(n) < n^c$

$\therefore T(n) = \Theta(n^c)$

13. $T(n) = 3T(n/2) + n$

$a = 3, \quad b = 2$

$c = \log_2 3 = 1.58$

$n^c = n^{1.58}, \quad f(n) = n$

$f(n) < n^c$

$T(n) = \Theta(n^c)$

$T(n) = \Theta(n^{1.58})$

14. $T(n) = 3T(n/3) + \text{sqrt}(n)$

$a = 3, b = 3$

$c = \log_3 3 = 1$

$n^c = n^1$, $f(n) = n^{1/2}$

$f(n) < n^c$

$T(n) = \Theta(n^c)$

$T(n) = \Theta(n)$

15. $T(n) = 4T(n/2) + cn$

$a = 4, b = 2$

$c = \log_2 4 = 2$

$n^c = n^2$, $f(n) = cn = n$

$f(n) < n^c$

$T(n) = \Theta(n^c)$

$T(n) = \Theta(n^2)$

16. $T(n) = 3T(n/4) + n \log n$

$a = 3, b = 4$

$c = \log_4 3 = 0.79$

$n^c = n^{0.79}$, $f(n) = n \log n$

$f(n) > n^c$

$T(n) = \Theta(f(n))$

$T(n) = \Theta(n \log n)$

17. $T(n) = 3T(n/3) + n/2$

$a = 3, \quad b = 3$

$c = \log_3 3 = 1$

$n^c = n^1 \quad f(n) = n/2$

$f(n) < n^c$

$T(n) = \Theta(n^c)$

$T(n) = \Theta(n)$

18. $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, \quad b = 3$

$c = \log_3 6 = 1.63$

$n^c = n^{1.63}, \quad f(n) = n^2 \log n$

$f(n) > n^c$

$T(n) = \Theta(f(n))$

$T(n) = \Theta(n^2 \log n)$

19. $T(n) = 4T(n/2) + n/\log n$

$a = 4, \quad b = 2, \quad c = \log_2 4 = 2$

$n^c = n^2, \quad f(n) = n/\log n$

$f(n) < n^c$

$T(n) = \Theta(n^c) = \Theta(n^2)$

20. $T(n) = 64\,T(n/8) - n^2 \log n$

$a = 64,\ b = 8,\ c = \log_8 64 = 2$

$n^c = n^2,\ f(n) = -n^2 \log n = n^2 \log n^{-1}$

$= n^2 \log 1/n$

$f(n) > n^c$

$T(n) = \Theta(f(n)) = \Theta(n^2 \log 1/n)$

21. $T(n) = 7\,T(n/3) + n^2$

$a = 7,\ b = 3,\ c = \log_3 7 = 1.77$

$n^c = n^{1.77},\ f(n) = n^2$

$f(n) > n^c$

$T(n) = \Theta(f(n)) = \Theta(n^2)$

22. $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1,\ b = 2,\ c = \log_2 1 = 0$

$n^c = n^0 = 1,\ f(n) = n(2 - \cos n)$

$f(n) > n^c$

$T(n) = \Theta(f(n))$

$T(n) = \Theta(n(2 - \cos n))$