Severity Distributions for GLMs: Gamma or Lognormal? Evidence from Monte Carlo Simulations

Luyang Fu, Ph.D, and
Richard B. Moncher, FCAS, MAAA

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Abstract

Insurance claim costs have been found in numerous studies to be positive and usually positively skewed with variances often proportional to the mean squared. In practice, the gamma and lognormal distributions are the ones with those desired properties most widely used. Most actuarial research in GLMs also report results from normal distributions as a comparison. In this study, we apply Monte Carlo simulation techniques to examine the unbiasedness and stability of the GLM classification relativities assuming gamma, lognormal, and normal distributions. We find that the gamma distribution provides better predictive accuracy and efficiency.

1. Introduction

Generalized Linear Models (GLMs) have been widely used in property-casualty ratemaking recently because they consider all rating factors simultaneously and adjust for interactions and correlations among them. Numerous studies, such as Brown (1988), Holler, Sommer, and Geoff (1999), Mildenhall (1999), and Murphy, Brockman, and Lee (2000) have shown that one-way analysis leads to systematic bias and that GLMs can be used to calculate classification relativities and to reduce estimation errors.

Traditional linear models assume independent and identical normally distributed residuals. Insurance data, such as losses and severities are positive and usually positively skewed. GLMs assume that data is sampled from an exponential family of distributions[1]. Many distributions in this family (e.g., gamma, inverse Gaussian, and negative binomial) are consistent with the nature of insurance data (positive and positively skewed). In practice, GLMs with gamma and normal distributions are usually used for severity relativity calculations. Most empirical analyses of GLMs in actuarial research report results from gamma and normal distributions.

Besides the gamma distribution, the lognormal is the other widely used distribution with the desired characteristics of insurance data. Mildenhall (1999) discussed that the logarithm of the response variable, the variance-stabilizing transformation, is often used

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[1] The exponential family of distributions has the probability density function for continuous variables or the probability function for discrete variables in the form of $f(x;\theta,\phi) = \exp\{\frac{x\theta - b(\theta)}{a(\phi)} + c(x,\phi)\}$. The lognormal, Pareto and Weibull distributions are not in the exponential family.

in linear models to improve normality. The log-transformed model also converts multiplicative models to linear ones. The underlying assumption of the log-transformed model is that the response variable follows a lognormal distribution.

McCullagh and Nelder (1989) contend that it is common for data "in the form of continuous measurements" to have variance positively correlated with the mean. So, the constant coefficient of variation (mean / standard deviation) is a more realistic assumption than constant variance. This property of constant coefficient of variation is found to be appropriate for insurance data by Murphy, Brockman, and Lee (2000) and Mildenhall (1999). The former study also shows that the correct selection of the non-constant variance function significantly improves the robustness of parameter estimates.

Both gamma and lognormal distributions have the property of constant coefficient of variation. A gamma distribution with parameters α and θ has the following density function:

$$G(x;\alpha,\theta) = \frac{x^{a-1}e^{-x/\theta}}{\Gamma(\alpha)\theta^{\alpha}}, \quad (1)$$

where $\Gamma(\alpha)$ is the gamma function. $G(x,\alpha,\theta)$ has mean $\mu = \alpha\theta$ and variance $\sigma^2 = \alpha\theta^2$. So, the gamma distribution has its variance proportional to its mean squared, i.e., $\sigma^2 = \frac{1}{\alpha}\mu^2$. The lognormal distribution with parameters M and Ω has density:

$$L(x,M,\Omega) = \frac{1}{\Omega x\sqrt{2\pi}}e^{-(\ln x - M)^2/2\Omega^2}. \quad (2)$$

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$L(x, M, \Omega)$ has mean $\mu = e^{M + \Omega^2 / 2}$ and variance $\sigma^2 = e^{2M + \Omega^2}(e^{\Omega^2} - 1)$. The variance of the lognormal distribution is also proportional to its mean squared, i.e.,[1,1]

$$\sigma^2 = (e^{\Omega^2} - 1)\mu^2.$$

In this study, Monte Carlo simulation analysis is applied to investigate the following questions:

1. Under what conditions are the assumed severity distributions important?

2. If the severity distribution is unknown or difficult to test, which of the gamma, lognormal or normal distribution assumptions yield the most robust result (i.e., minimized estimation bias and standard error)?

The individual losses are generated randomly from gamma and lognormal distributions. For each simulated dataset, three models are fit: GLM with a gamma distribution and log link; GLM with a normal distribution and log link; and GLM on the log-transformed severity with a normal distribution and identity link.

This paper assumes the reader is familiar with basic class ratemaking and the fundamentals of GLMs. It is organized as follows. Section 2 discusses the details of our Simulation Methodology, and Section 3 reviews Simulation Results. Section 4 outlines our Conclusions, and Section 5 provides ideas for Future Research.

2. Simulation Methodology

Simulation Assumptions

The simulation is numerically based on the predicted claim severity for private passenger

auto collision claims adjusted for severity trend. The data is from Mildenhall (1999) and

McCullagh and Nelder (1989)[2], and it includes thirty-two severity observations for two

classification variables: eight age groups and four types of vehicle-use (which are based

on 8,942 individual claims). In this study, the response variable is the average claim

severity and the simulation assumptions include the following:

1). The individual losses have a constant coefficient of variation. Let $L_{i,j}^k$ denote

the loss for claim k with age group i and vehicle-use group j. We assume that $L_{i,j}^k$ are

independently distributed with mean $E(L_{i,j}^k) = \mu_{i,j}$ and variance

$\text{var}(L_{i,j}^k) = \sigma_{i,j}^2 = (\mu_{i,j} * cv)^2$, where cv is the coefficient of variation. Mildenhall (1999)

and McCullagh and Nelder (1989) discuss that the assumption of constant variance is

unrealistic and the standard deviation of severity is more likely to be positively correlated

with the mean severity. Following their research, we assume a constant coefficient of

variation (rather than constant variance) in the simulation. The average claim severity

$$S_{i,j} = \frac{1}{n_{i,j}} \sum_{k=1}^{n_{i,j}} L_{i,j}^k \text{ , where } n_{i,j} \text{ is the total claims for age group i and vehicle-use group}$$

j, while $S_{i,j}$ has mean $\mu_{i,j}$ and variance $\dfrac{\sigma_{i,j}^2}{n_{i,j}}$.

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[2] We used this dataset, as it was used in these two authoritative studies.

2). The relationship between severities and rating variables is multiplicative. The mean severity equals:

$$\mu_{i,j} = Base * Rel_i * Rel_j, \quad (3)$$

where Rel_i is the severity relativity for age group i and Rel_j is the relativity for vehicle-use group j. This is equivalent to a generalized linear model with log link function:

$$\log(\mu_{i,j}) = Intercept + \sum_i a_i x_i + \sum_j b_j y_j, \quad (4)$$

where x_i is the dummy variable for age group i ($x_i = 1$ if age is i; 0 otherwise) and y_j is the dummy variable for vehicle-use group j; a_i is the GLM coefficient for x_i and b_j is the coefficient for y_j.

3). The "true" base severity and relativities Rel_i and Rel_j (or GLM coefficients a_i and b_j) distributions are known. Those "true" values are the predicted GLM values in Mildenhall (1999) based on the gamma distribution and log links[3]. The age group "60+" and vehicle-use group "for pleasure" are selected as the base[4], and the base severity is 195. Table 1.1 and Table 1.2 list the "true" relativities for age groups and vehicle-use groups, respectively. Table 1.3 shows the "true" severities and relativities for each combined age and vehicle-use group.

4). The coefficients of variation of the severity distribution are known. We set the coefficients of variation to be 1.0, 2.0 or 3.0[5].

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[3] The conclusions in this study are still valid if we use the predicted GLM severities from other distributions in Mildenhall (1999).
[4] The selection of the base does not affect the GLM results numerically.
[5] The data used by Mildenhall (1999) is average claim severity, and the individual claim information is not available. We calculated sixty-five severity coefficients of variation from the major coverages in fifteen states from our company data. The average of variation coefficients was 1.434, the minimum 0.529, the

Distribution Parameters

From the simulation assumptions, the losses $L_{i,j}^k$ follow distributions with means of $\mu_{i,j}$

and variances of $(\mu_{i,j} * cv)^2$. The parameters of the gamma and lognormal distributions

can be calculated based on the assumed mean and variance. The gamma distribution

$G(x, \alpha, \theta)$ has mean $\alpha\theta$ and variance $\alpha\theta^2$. This implies $L_{i,j}^k$ has gamma parameters of

$\alpha = \dfrac{1}{cv^2}$ and $\theta = \mu_{i,j} * cv^2$. Similarly, the lognormal distribution $L(x, M, \Omega)$ has mean

$= e^{M + \Omega^2/2}$ and variance $= e^{2M + \Omega^2}(e^{\Omega^2} - 1)$. This implies that $L_{i,j}^k$ has lognormal

parameters of $M = \log(\mu_{i,j}) - \log(1 + cv^2)/2$ and $\Omega^2 = \log(1 + cv^2)$. In other words,

$\log(L_{i,j}^k)$ is normally distributed with mean M and variance Ω^2.

Simulation Procedure

   1). For each coefficient of variation, the individual losses are generated based on

lognormal and gamma distributions. For each combined age group i and vehicle-use

group j, $n_{i,j}$ individual losses of $L_{i,j}^k$ are simulated. The average claim severity is

calculated as the mean of those individual losses. In total, 8,942 individual losses and

thirty-two claim severities $S_{i,j}$ are generated in each round of simulation.

   2). For each dataset, three models are fitted: GLM with gamma distribution and

log link; GLM with normal distribution and log link; and GLM on the log-transformed

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maximum 3 464, and the standard deviation 0.516. Fifty-seven of the sixty-five coefficients of variation
were within the interval [1.0, 3.0].

severities with normal distribution and identity link (the "log-transformed linear model" in Mildenhall 1999). In each regression, we calculate thirteen coefficients (one intercept, eight age groups with 60+ as zero, four vehicle-use with pleasure as zero), seven age relativities, and three vehicle-use relativities. We also calculate thirty-two predicted severities for each combined age and vehicle-use group. Following Mildenhall (1999), we use severity as the response variable and the claim frequencies as the weights in the linear regressions.

$$\log(S_{i,j}) = Intercept + \sum a_i x_i + \sum b_j y_j + \varepsilon_{i,j}, \quad (5)$$

If gamma or normal distributions are assumed, the severity based on a GLM with a log link is:

$$S_{i,j} = e^{int\,ercept} * e^{a_i + b_j}, \quad (6)$$

For lognormally distributed losses, Klugman, Panjer, and Willmot (1998) and Mildenhall (1999) show that the severity based on log-transformed regression is:

$$S_{i,j} = e^{intercept} * e^{a_i + b_j + \frac{\Omega_{i,j}^2}{2n_{i,j}}}, \quad (7)$$

where $\Omega_{i,j}^2$ is the variance of the logarithm of the individual loss. We designate $e^{\Omega_{i,j}^2 / 2n_{i,j}}$ as the volatility adjustment factor. In the numerical analysis, $\Omega_{i,j}^2$ is estimated by $\Omega_{i,j}^2 = \log(1 + cv^2)$ for the coefficients of variation 1.0, 2.0, and 3.0, respectively.

    3). Steps 1-2 are repeated one thousand times, so the sampling distributions of ten coefficients ($\hat{a}_i$ and $\hat{b}_j$) and thirty-two predicted severities ($\hat{S}_{i,j}$) are generated. The

mean and standard error of the coefficients and predicted severities are calculated based on the sampling distributions.

For each combination of the "true" severity distribution and the assumed distribution in the regressions, we evaluate the performance of the models by two criteria. Following Bailey (1963), we use the weighted absolute bias to measure the accuracy of the model. We also evaluate the model from an alternative perspective: the stability of the coefficients and predicted values, which is measured by the weighted standard error.

From the definition of unbiasedness, the mean estimate is equal to the true value, $E(\hat{S}_{i,j}) = \mu_{i,j}$. If the sampling mean of one thousand predicted $\hat{S}_{i,j}$ is equal to the "true" severity and the model is unbiased, then the estimation bias could be measured by $E(\hat{S}_{i,j}) - \mu_{i,j}$. Bailey (1963) suggests using the weighted absolute bias to measure the accuracy of the model:

$$wab = \frac{\sum w_{i,j} \mid E(\hat{S}_{i,j}) - \mu_{i,j} \mid}{\sum w_{i,j}}. \quad (8)$$

Besides unbiasedness, stability is the other important criteria to measure model performance. The standard error is a commonly used statistic for stability. Similar to (8), we use the weighted standard error to measure the stability of the model:

$$wse = \frac{\sum w_{i,j} \hat{\sigma}_{i,j}}{\sum w_{i,j}}, \quad (9)$$

where $\hat{\sigma}_{i,j}$ is the sampling standard deviation of one thousand $\hat{S}_{i,j}$.

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3. Simulation Results

Data Generated

We repeat the simulation 1,000 times, and 8,942 individual losses are generated in each
round of the simulation. As it is inefficient to list all individual losses for all combined
groups of age and vehicle-use, we only report the details of simulated average claim
severity for two classifications.

Classification I - Age 17-20 and Pleasure Use

Classification II - Age 40-49 and DTW Short (Short Drive to Work, less than 15 miles)

Classification I is used as an example of a small-sample classification (as it only has
twenty-one observations. Classification II includes 970 observations and is an example
of a large-sample classification. Tables 2.1 and 2.2 show statistical summaries of the
1,000 simulated severities for Classification I with coefficients of variation 1.0, 2.0, and
3.0 for the gamma and lognormal distributions, respectively. Tables 2.3 and 2.4 show
statistical summaries for Classification II. Figures 3.1, 3.3, and 3.5 report the scatter
plot, density plot, QQ plot, and histogram for gamma distributions with coefficients of
variation 1.0, 2.0, and 3.0 for Classification I, respectively. Figures 3.7, 3.9, 3.11 report
those plots for lognormal distributions, while Figures 3.2, 3.4, 3.6, 3.8, 3.10, and 3.12 are
the corresponding plots for Classification II.

From the simulations, the severity of Classification II is asymptotically normal because
of its sample size, even though the individual losses follow gamma or lognormal
distributions. The Q-Q plots are close to 45-degree straight lines; and the density

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function and histogram are close to symmetric. Tables 2.1-2.4 also show that

Classification II has much smaller standard deviation and skewness. On the other hand,

the severity of Classification I is positively skewed. The Q-Q plots are concave; and the

density function and histogram have longer tails on the right side. From Tables 2.1-2.4,

the larger the coefficient of variation, the more positively skewed the severity. The

severities of lognormal losses have larger skewness because the lognormal distribution

has longer right-side tails than the gamma distribution.

Regression Results

For each round of simulation, two datasets are generated based on gamma and lognormal

distributions. Six regressions are performed on these two datasets: G-G, G-L, G-N, L-G,

L-L, and L-N[6]. For each regression, the GLM coefficients $\hat{a}_i$ for each age group and $\hat{b}_j$

for each vehicle-use group, and predicted severity $\hat{S}_{i,j}$ are calculated. $e^{\hat{a}_i}$ is the

relativity for age group i, and $e^{\hat{b}_j}$ is the relativity for vehicle-use group j.

For log-transformed models, a volatility adjustment factor $e^{\Omega_{i,j}^2 / 2n_{i,j}}$ is applied to

reduce the estimation bias. The weighted absolute bias (*wab*) with and without the

adjustment is reported in Table 4.1. Without the adjustment, the overall *wab* s are 0.38,

1.50, and 2.66 for G-L models with cv = 1.0, 2.0, and 3.0. After the adjustment, *wabs*

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[6] G-G implies that the loss follows a Gamma distribution and a Gamma distribution is assumed in the regression; similarly, G-L implies the loss follows a Gamma distribution but a Lognormal is assumed in the regression, and G-N implies the loss follows a Gamma distribution but a Normal is assumed in the regression. The same logic applies for L-G, L-L, and L-N.

are reduced to 0.24, 0.85, and 1.81. On average, the *wab* reduction is 37%. Similarly, the *wab* reduction for L-L models is 35% on average.

Table 4.2 exhibits the *wabs* for the eight age groups with cv = 1.0, 2.0, and 3.0; and Tables 4.3-4.5 show the detailed information for predicted severities and biases for all thirty-two classifications. For the small-sample classifications, the prediction errors without adjustment could be very large. For example, the *wab* for age 17-20 is 39.5 for the G-L model with cv=3.0, and 22.1 for the L-L model if no adjustment is applied. After the adjustment, the *wabs* are reduced to 28.3 and 13.2, respectively. Without the adjustment, the *wabs* of the same models for age 40-49 are 1.25 and 1.83. After the adjustment, the weighted absolute biases are reduced slightly to 1.16 and 1.50, respectively. The volatility adjustment factor could reduce the biases of small-sample classifications significantly. In practice, log-transformed models are often applied. Without adjustment, the predicted severities (or relativities) are underestimated. In the case of G-L with cv=3.0, thirty-one of the thirty-two predicted severities are lower than the "true" relativities. Because the log-transformed model with adjustment is significantly better than the model without adjustment, only the former model is used in the following analysis.

For all the six models (G-G, G-L, G-N, L-G, L-L, and L-N), the weighted absolute biases and weighted standard errors of the predicted severities are used to measure the unbiasedness and stability of the models. The 95% confidence intervals are calculated based on the 2.5% quantile and 97.5% quantile of the sampling distributions. The *wab*

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and *wse* of the predicted severities for gamma and lognormal losses are reported in Table 5.1 and 5.2, respectively. The detailed information for the mean, bias, and standard deviation of the thirty-two predicted severities is shown in Tables 5.3, 5.5, 5.7, 5.9, 5.11, and 5.13 for G-G, G-L, G-N, L-G, L-L, and L-N, respectively. The corresponding confidence intervals are reported in Tables 5.4, 5.6, 5.8, 5.10, 5.12, and 5.14. Figures 6.1, 6.3, and 6.5 are the scatter and density plots of Classification I predicted severities for gamma losses with coefficient of variations 1.0, 2.0, and 3.0, respectively. Figures 6.2, 6.4, and 6.6 are the same plots for Classification II. Figures 6.7-6.12 show the corresponding plots for lognormal losses.

From Tables 5.3-5.14, the larger size the classification, the smaller bias and standard errors of predicted severities, and the more accurate the classification relativities. For example, the estimation biases of the G-G model with $cv=1.0$ are 1.65, 1.99, 2.43, and 2.77 for the four age group 17-20 classifications with vehicle-use pleasure, DTW short, DTW long, and business, respectively. The estimation biases for the four age group 40-49 classifications are much smaller (-0.12, 0.05, 0.04, -0.20). This is also true for the standard errors of the models.

When data is less volatile and the sample size of the classification is large enough, the predicted severity is asymptotically normal and the confidence interval is close to symmetric across the mean. On the other hand, when data is volatile and the sample size of the classification is small, the predicted severity is not symmetric across the mean. For example, the confidence interval of the predicted severity with coefficient of variation 1.0

for Classification II is (194.90, 214.72) based on the G-G model. It is symmetric across

the "true" mean of 204.54. The confidence interval of the predicted severity with

coefficient of variation 3.0 for Classification I is (119.40, 458.67) based on the G-G

model. It is far from symmetric with the true "mean" of 254.90. In practice, confidence

intervals are usually estimated by adding and subtracting two times the standard error to

the mean. Our study shows that this could be very wrong for the small classifications

(i.e., asymmetrical confidence intervals might be more appropriate).

Residual Diagnostics

To validate the distribution assumptions in the GLMs, "residual Q-Q" plots and

"residual-fitted value" plots are often used to examine the heteroscedasticity within the

error structure of the model (e.g., Holler, Sommer, and Geoff 1999 and Murphy,

Brockman, and Lee 2000). In contrast to traditional linear models, deviance and Pearson

residuals are applied[7]. To make the plots comparable, the residuals are standardized. If

the "Q-Q" plots of deviance residuals are nonlinear or the residuals are fanning inwards

or outwards (when plotted against the predicted values), the severity distribution

assumptions are inappropriate.

We repeat the simulation one thousand times. It is too voluminous to report the residuals

plots one thousand times for each model. So, we run an extra simulation independent of

the previous ones and show the "residual Q-Q" and "residual-fitted value" plots in

Figures 7.1-7.12 for each of the six models with $cv=1.0$, 2.0, and 3.0. If a gamma

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[7] For detailed explanations of deviance and Pearson residuals, please refer to McCullagh and Nelder
(1989). If a normal distribution is assumed in GLMs, deviance residuals are equal to Pearson residuals.

distribution is assumed, both Pearson and deviance residual plots are reported. If lognormal or normal distributions are assumed, only deviance residual plots are reported because Pearson and deviance residuals are identical.

From Figures 7.1-7.12, the "residual Q-Q" and "residual-fitted value" plots are similar for the gamma, lognormal, and normal models. It is difficult to examine the assumptions of severity distributions based on average severity data (summarized data). As discussed above, the distribution of average severity may be very different from the distribution of individual losses. When the sample size of the classification is large enough, the average severity is asymptotically normal no matter how the individual losses are distributed. The smoothing effect of summarized data makes the residual plots insensitive to the distributions assumed by GLMs.

If we run GLMs based on the individual losses of the same dataset (with 8,942 observations), residual plots are very sensitive to the distribution assumptions. Figures 7.13 and 7.14 report the "residual Q-Q" and "residual-fitted value" plots for gamma losses with $cv=1.0$. Figures 7.15 and 7.16 are plots for lognormal losses. It is clear that when the distribution assumptions are consistent with the "true" distribution, the Q-Q plots of deviation residuals are 45-degree straight lines and the residuals are randomly scattered across zero in the "residual-fitted value" plots[8]. Therefore, residual plots work well to examine the distribution assumptions on individual data, but not necessarily on summarized/average data.

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[8] For the lognormal models on individual data, the volatility adjustment factor is $e^{\Omega_{i,j}^2/2}$ because each observation represents one claim and has one as the weight.

4. Conclusions

Insurance data with continuous measurement (severities and pure premium) have been found in numerous studies to be non-normal: 1) positive and usually positively skewed; and 2) variances are proportionally correlated to the mean squared. In practice, gamma and lognormal are two widely-used distributions with those desired properties. Traditional linear models assume a normal distribution, and don't have those properties, though most GLM actuarial research also report results from normal distributions as a benchmark. In this study, we apply Monte Carlo simulation techniques to examine the gamma, lognormal, and normal distributions, and determine which one provides better estimation in terms of unbiasedness and stability.

The simulation is numerically based on the predicted claim severity for private passenger auto collision claims used by Mildenhall (1999) and McCullagh and Nelder (1989). In each round of 1,000 simulations, six datasets of individual losses are generated based on gamma and lognormal distributions with "true" (known) classification severities and coefficients of variation (1.0, 2.0, and 3.0). For each dataset, three models are fitted on the average severities: GLM with gamma distribution and log link; GLM with normal distribution and log link; and the GLM on the log-transformed severities with normal distribution and identity link.

Based on the simulation results, we find that:

1). When the gamma distribution is "true", the G-G model is dominant in both unbiasedness and stability (except the G-L model is slightly more stable).

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2). When the lognormal distribution is "true", the L-L model is dominant in terms of stability.

3). GLMs with a normal distribution never dominate based on any criteria, and they have the worst weighted standard error (*wse*).

4). GLMs with a gamma distribution are dominant in terms of unbiasedness, no matter whether the "true" distribution is gamma or lognormal.

5). Overall, GLMs with a gamma distribution perform slightly better than the log-transformed model and GLMs with a normal distribution. This result is consistent with the statistical research by Firth (1988).

6). When the data is not volatile, the distribution selection in GLMs may not be as important because all distribution assumptions yield small biases and standard errors.

7). When the log-transformed model is used, the classification relativities should be adjusted by a volatility-adjustment factor. Without the adjustment, the relativities are undervalued.

8). Residual plots may work well to examine the distribution assumptions on individual data, but not necessarily on summarized/average data.

5. Future Research

McCullagh and Nelder (1989) and Mildenhall (1999) discussed that insurance data is more likely to have a constant coefficient of variation rather than constant variance. However, it is possible that the variance increases with the mean but not proportional to the mean squared. Other distributions in the exponential family, such as the negative

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binomial and inverse Gaussian[9] have those properties. In future research, it might be interesting to generate losses based on negative binomial and inverse Gaussian distributions and also run GLMs assuming those distributions.

We investigated two classifications using private passenger auto severity data. However, simulation research on GLMs could be extended to other response variable (e.g., pure premium), other classifications (e.g., credit, territory), and other lines of business (e.g., homeowners, general liability, workers compensation).

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[9] Negative binomial distribution $f(x;\mu,k) = \dfrac{\Gamma(x+1/k)}{\Gamma(x+1)\Gamma(1/k)} \dfrac{(k\mu)^k}{(1+k\mu)^{1+1/k}}$ has variance $\mu + k\mu^2$.

Inverse Gaussian distribution $f(x;\mu,\sigma) = \dfrac{1}{\sqrt{2\pi x^3}\sigma} \exp\left(-\dfrac{1}{2x}\left(\dfrac{x-\mu}{\mu\sigma}\right)^2\right)$ has variance $\sigma^2\mu^3$.

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Appendix 1
"True" Relativities and Severities Assumed from Simulations

Table 1.1: "True" Relativities for Each Age Group

Age	Relativity	GLM Coefficient
17–20	1.307	0 268
21–24	1.301	0.263
25–29	1.206	0.187
30–34	1.156	0.145
35–39	0.931	-0.071
40–49	1.007	0.007
50–59	1.022	0.022
60+	Base	0.000

Table 1.2: "True" Relativities for Each Vehicle-Use Group

Vehicle Use	Relativity	GLM Coefficient
Business	1.644	0.497
DTW Long	1.264	0.234
DTW Short	1.042	0.041
Pleasure	Base	0.000

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Table 1.3: "True" Severities and Relativities for Thirty-Two Classifications

Age	Vehicle Use	# of Claims	Severity	Relativity
17–20	Business	5	419.07	2.149
17–20	DTW Long	23	322.17	1.652
17–20	DTW Short	40	265.56	1.362
17–20	Pleasure	21	254.90	1.307
21–24	Business	44	417.10	2.139
21–24	DTW Long	92	320.66	1.644
21–24	DTW Short	171	264 31	1.355
21–24	Pleasure	63	253.70	1.301
25–29	Business	129	386.66	1.983
25–29	DTW Long	318	297.26	1.524
25–29	DTW Short	343	245.02	1.257
25–29	Pleasure	140	235.19	1.206
30–34	Business	169	370.53	1.900
30–34	DTW Long	361	284 85	1.461
30–34	DTW Short	448	234 80	1.204
30–34	Pleasure	123	225.37	1.156
35–39	Business	166	298.35	1.530
35–39	DTW Long	381	229 37	1.176
35–39	DTW Short	479	189 06	0.970
35–39	Pleasure	151	181 47	0.931
40–49	Business	304	322.78	1.655
40–49	DTW Long	719	248.15	1.273
40–49	DTW Short	970	204.54	1.049
40–49	Pleasure	245	196.33	1.007
50–59	Business	162	327.72	1.681
50–59	DTW Long	504	251.95	1 292
50–59	DTW Short	859	207.67	1.065
50–59	Pleasure	266	199.34	1.022
60+	Business	96	320.60	1 644
60+	DTW Long	312	246.47	1.264
60+	DTW Short	578	203.16	1.042
60+	Pleasure	260	195.00	1.000

Appendix 2
Summaries of the Simulated Severities for Selected Classifications

Table 2.1: Statistical Summary of Simulated Gamma Severity for Age 17-20 and Pleasure Use with Variation Coefficients 1.0, 2.0, and 3.0

CV	Min	25% Q	Median	Mean	75% Q	Max	Stdev	Skewness
1.0	86.7	217.2	254.0	258.2	292.5	505.2	57.6	0.605
2.0	51.0	168.1	232.3	251 0	314.7	771.1	111 7	0.846
3.0	4.3	133.4	214.6	255 7	338 4	1,122 0	170.4	1.413

Table 2.2: Statistical Summary of Simulated Lognormal Severity for Age 17-20 and Pleasure Use with Coefficients of Variation 1.0, 2.0, and 3.0

CV	Min	25% Q	Median	Mean	75% Q	Max	Stdev	Skewness
1 0	128.2	215.5	247.5	254.5	284.3	557 6	54.1	0.772
2.0	71.73	181.6	230.6	252.3	291.9	1,097.0	109.6	2.112
3.0	57.95	157.3	220.5	258.1	301.3	4,219.0	200.2	9.162

Table 2.3: Statistical Summary of Simulated Gamma Severity for Age 40-49 and DTW Short Use with Coefficients of Variation 1.0, 2.0, and 3.0

CV	Min	25% Q	Median	Mean	75% Q	Max	Stdev	Skewness
1.0	186.4	200 0	204.5	204.7	209.2	227.9	6.7	0.122
2 0	156.0	193 9	203.0	203.7	213.1	247.5	13.9	0.190
3.0	156.8	190 0	204.0	204.8	217.7	269.3	20.1	0.298

Table 2.4: Statistical Summary of Simulated Lognormal Severity for Age 40-49 and DTW Short Use with Coefficients of Variation 1.0, 2.0, and 3.0

CV	Min	25% Q	Median	Mean	75% Q	Max	Stdev	Skewness
1.0	182.4	200.0	204.4	204.4	208.9	226.5	6.5	0.141
2.0	168.3	195.8	203.3	204.3	212.6	252 0	12 9	0.253
3.0	154.5	191 0	202.2	204.2	215.6	284 8	19 6	0.650

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Appendix 3
Plots of Simulated Severities for Selected Classifications

Figure 3.1: Plot Summary of Simulated Gamma Severity for Age 17-20 and Pleasure Use with Coefficient of Variation 1.0

simulated severity for age 17-20 for.pleasure   severity density for age 17-20 for pleasure

severity QQ Plot for age 17-20 for pleasure   severity histogram for age 17-20 for pleasure
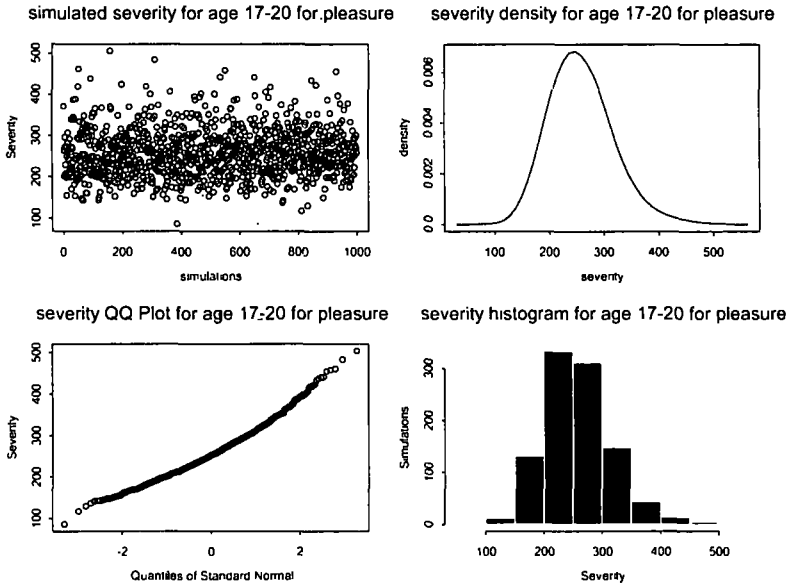
- The density is estimated by the non-parametric method from Silverman (1986).
- A 45-degree straight line in the Q-Q plot implies that the severity follows a normal distribution.

Figure 3.2: Plot Summary of Simulated Gamma Severity for Age 40-49 and DTW Short Use with Coefficient of Variation 1.0

simulated severity for age 40-49 for DTW Short   severity density for age 40-49 for DTW Short

severity QQ Plot for age 40-49 for DTW Short   severity histogram for age 40-49 for DTW Short

Figure 3.3: Plot Summary of Simulated Gamma Severity for Age 17-20 and Pleasure Use with Coefficient of Variation 2.0

simulated severity for age 17-20 for pleasure

severity density for age 17-20 for pleasure

severity QQ Plot for age 17-20 for pleasure

severity histogram for age 17-20 for pleasure

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Figure 3.4: Plot Summary of Simulated Gamma Severity for Age 40-49 and DTW Short Use with Coefficient of Variation 2.0

simulated severity for age 40-49 for DTW Short   severity density for age 40-49 for DTW Short

severity QQ Plot for age 40-49 for DTW Short   severity histogram for age 40-49 for DTW Short

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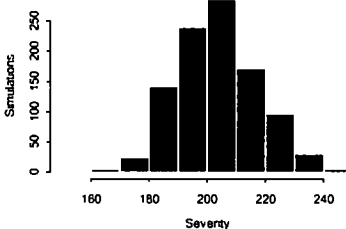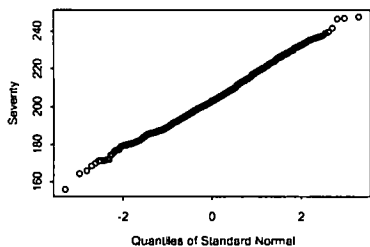
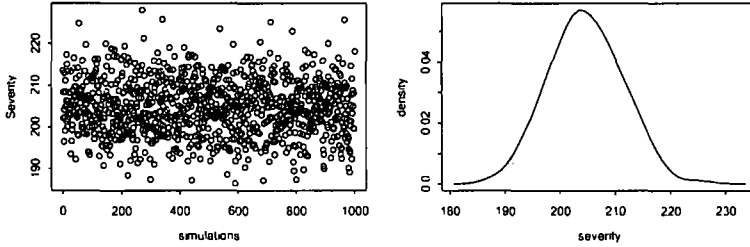**Figure 3.5: Plot Summary of Simulated Gamma Severity for Age 17-20 and Pleasure Use with Coefficient of Variation 3.0**

simulated severity for age 17-20 for pleasure
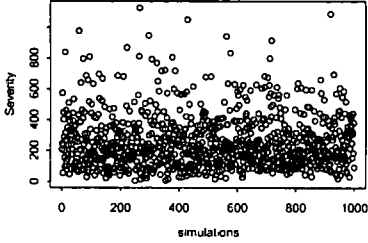
severity density for age 17-20 for pleasure
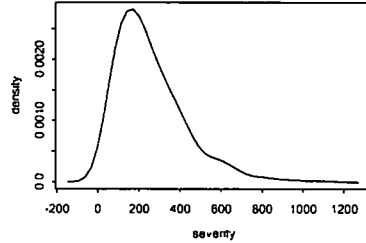
severity QQ Plot for age 17-20 for pleasure
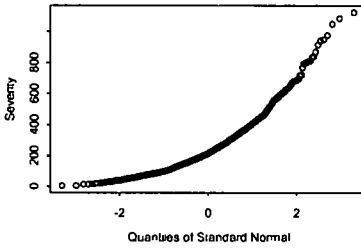
severity histogram for age 17-20 for pleasure
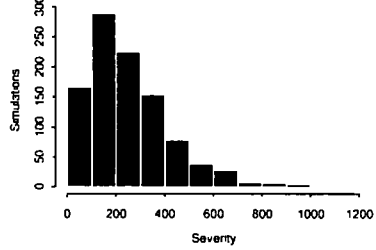
Figure 3.6: Plot Summary of Simulated Gamma Severity for Age 40-49 and DTW Short Use with Coefficient of Variation 3.0

simulated severity for age 40-49 for DTW Short   severity density for age 40-49 for DTW Short

severity QQ Plot for age 40-49 for DTW Short   severity histogram for age 40-49 for DTW Short

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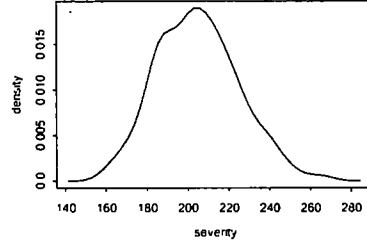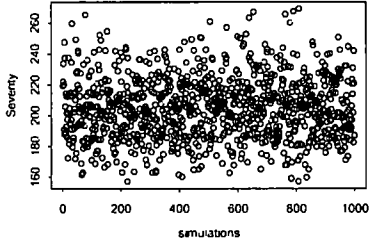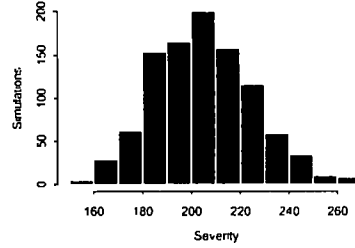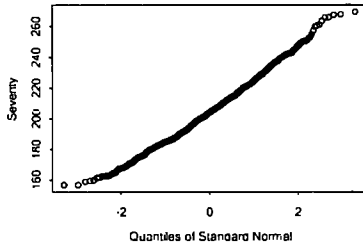
Figure 3.7: Plot Summary of Simulated Lognormal Severity for Age 17-20 and Pleasure Use with Coefficient of Variation 1.0

simulated severity for age 17-20 for pleasure

severity density for age 17-20 for pleasure

severity QQ Plot for age 17-20 for pleasure

severity histogram for age 17-20 for pleasure
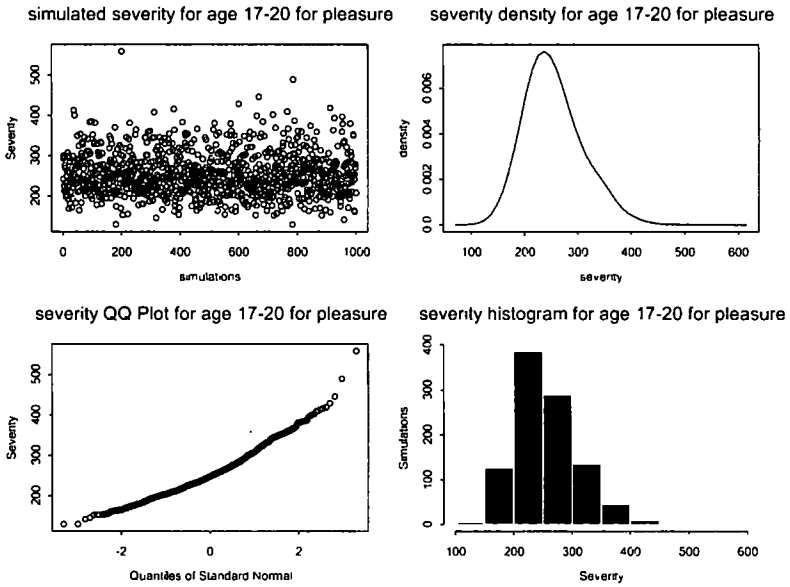
Figure 3.8: Plot Summary of Simulated Lognormal Severity for Age 40-49 and DTW Short Use with Coefficient of Variation 1.0

simulated severity for age 40-49 for DTW Short   severity density for age 40-49 for DTW Short

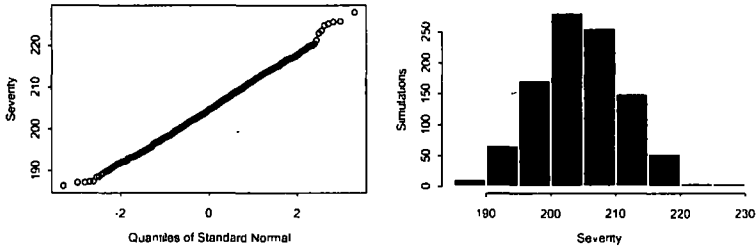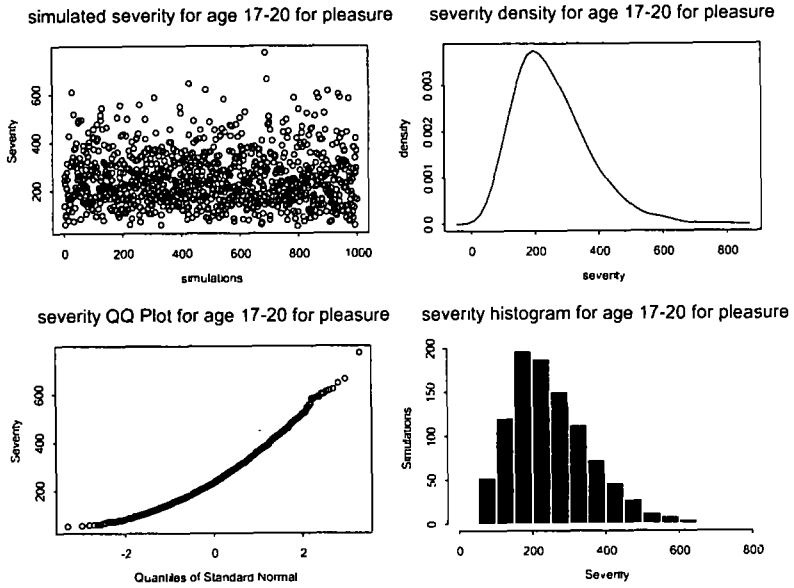severity QQ Plot for age 40-49 for DTW Short   severity histogram for age 40-49 for DTW Short

180
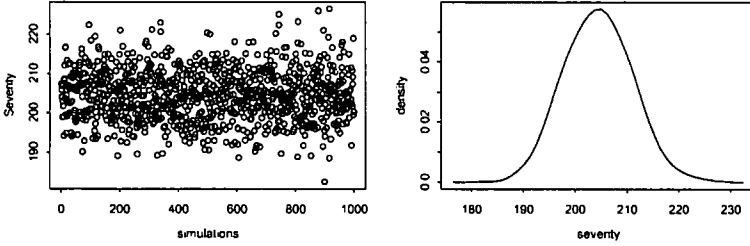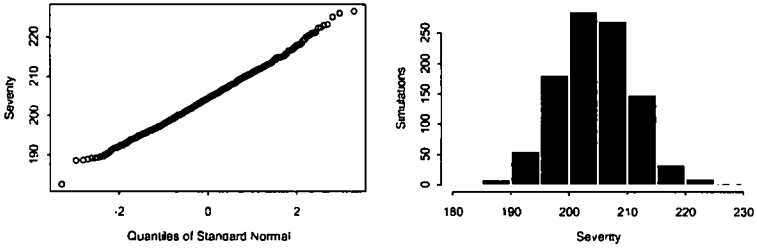
Figure 3.9: Plot Summary of Simulated Lognormal Severity for Age 17-20 and Pleasure Use with Coefficient of Variation 2.0

simulated severity for age 17-20 for pleasure

severity density for age 17-20 for pleasure

severity QQ Plot for age 17-20 for pleasure

severity histogram for age 17-20 for pleasure
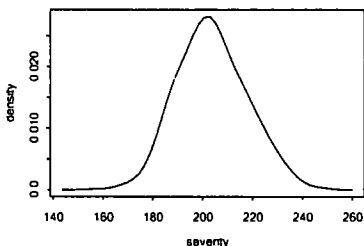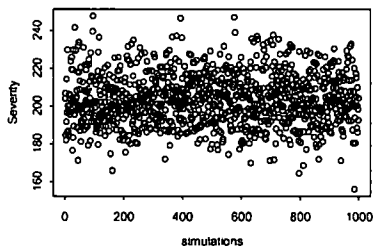
181
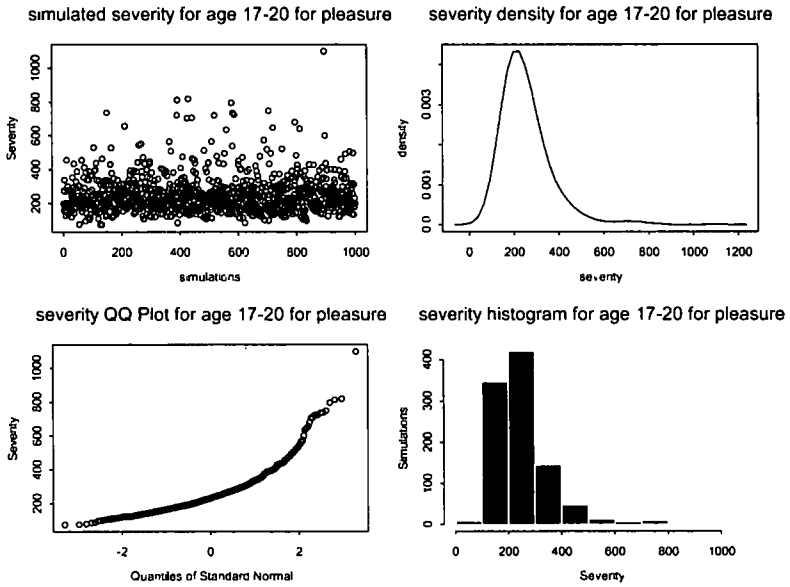
Figure 3.10: Plot Summary of Simulated Lognormal Severity for Age 40-49 and DTW Short Use with Coefficient of Variation 2.0

simulated severity for age 40-49 for DTW Short   severity density for age 40-49 for DTW Short
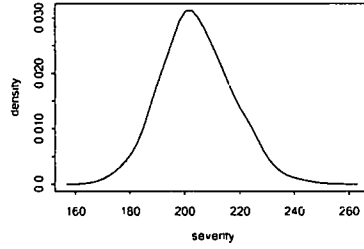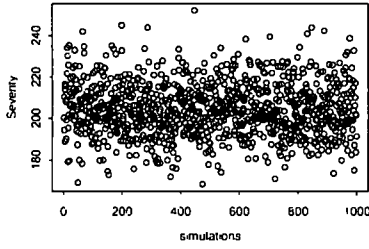
severity QQ Plot for age 40-49 for DTW Short   severity histogram for age 40-49 for DTW Short
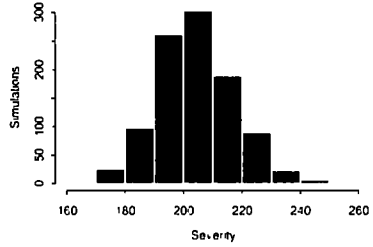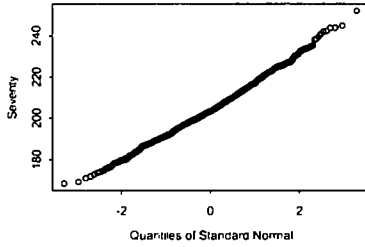
Figure 3.11: Plot Summary of Simulated Lognormal Severity for Age 17-20 and Pleasure Use with Coefficient of Variation 3.0

simulated severity for age 17-20 for pleasure

seventy density for age 17-20 for pleasure

severity QQ Plot for age 17-20 for pleasure

severity histogram for age 17-20 for pleasure
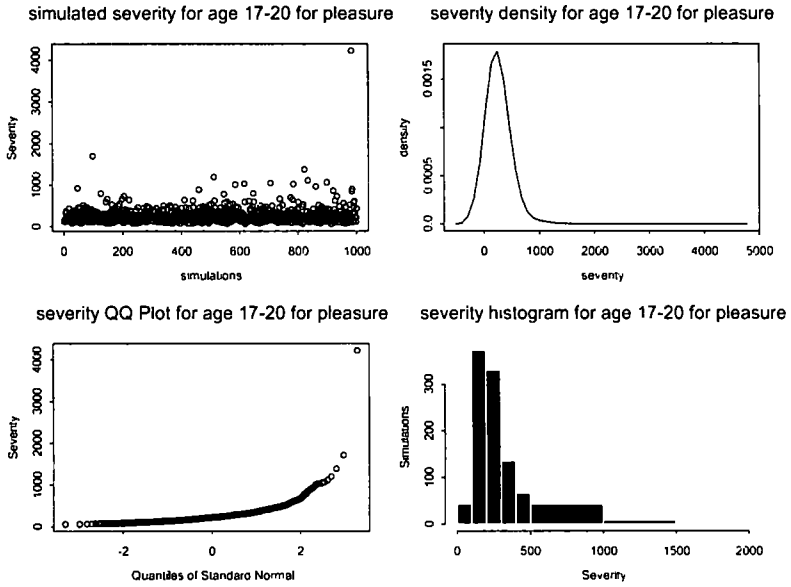
Figure 3.12: Plot Summary of Simulated Lognormal Severity for Age 40-49 and DTW Short Use with Coefficient of Variation 3.0

simulated severity for age 40-49 for DTW Short   severity density for age 40-49 for DTW Short

severity QQ Plot for age 40-49 for DTW Short   severity histogram for age 40-49 for DTW Short

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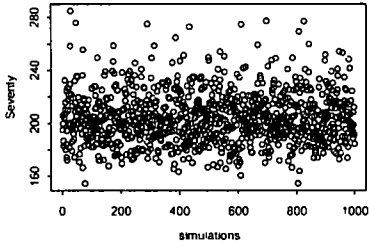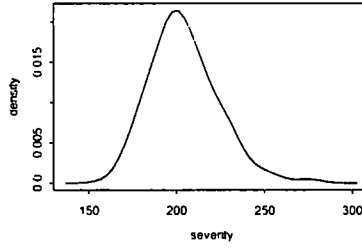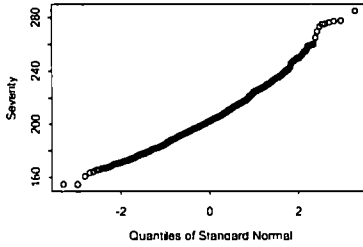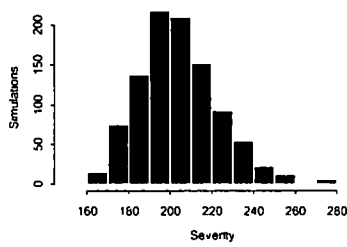
Appendix 4
Predicted Severities and Biases of Log-transformed Models with and without Adjustment

Table 4.1: Weighted Absolute Biases with and without Adjustment

Coefficient	wab			
Of Variation	G-L1	G-L2	L-L1	L-L2
1.0	0.380	0.240	0.278	0.202
2.0	1.500	0.852	1.539	0.844
3.0	2.661	1.808	2.359	1.589

- G-L1 is the log-transformed model on gamma losses without volatility adjustment;
- G-L2 is the log-transformed model on gamma losses adjusted by the volatility adjustment factors;
- L-L1 is the log-transformed model on lognormal losses without volatility adjustment;
- L-L2 is the log-transformed model on lognormal losses adjusted by the volatility adjustment factors.

Table 4.2: Weighted Absolute Biases with and without Adjustment for Age Groups

Age	Coefficient Of Variation	wab G-L1	G-L2	L-L1	L-L2
17-20	1 0	2.75	2.20	3.69	2 17
17-20	2.0	21.90	14.56	14.01	7.86
17-20	3.0	39.95	28.30	22.10	13.19
21-24	1.0	1.45	0 35	0.27	1.16
21-24	2.0	6.74	4.06	4.36	1.66
21-24	3.0	8.94	5.13	6.47	2.63
25-29	1.0	0.48	0.13	0.59	0.15
25-29	2.0	1.35	0.48	1 95	0.95
25-29	3.0	2.77	1 34	2 29	1.47
30-34	1.0	0 33	0.37	0.22	0.25
30-34	2.0	1.20	0.51	2.73	1.92
30-34	3.0	1.75	1.10	2.19	1.53
35-39	1.0	0.52	0.26	0 28	0.10
35-39	2 0	1.02	0.41	1.13	0.54
35-39	3.0	2.67	1.80	1 81	1.25
40-49	1.0	0.22	0 19	0.18	0.14
40-49	2.0	1.30	0.95	0.68	0.41
40-49	3.0	1 25	1.16	1.83	1.50
50-59	1.0	0.22	0.21	0.15	0.12
50-59	2 0	0.90	0.45	1.13	0 69
50-59	3 0	1.20	0.73	1.58	1 19
60+	1.0	0.23	0.15	0 21	0.10
60+	2.0	0.55	0.31	0 97	0.35
60+	3.0	3.49	2.62	2 53	1.65

Table 4.3: Predicted Severities and Biases of Log-transformed Models with and without Adjustment for Variation Coefficient 1.0

Age	Vehicle Use	Predicted Severity				Prediction Bias			
		G-L1	G-L2	L-L1	L-L2	G-L1	G-L2	L-L1	L-L2
17-20	Pleasure	252.00	256.20	251.36	255.55	-2.89	1.30	-3.53	0.65
17-20	DTW Short	263 26	265.55	262.24	264.52	-2.30	-0.01	-3.32	-1.04
17-20	DTW Long	319.26	324.11	318.28	323.11	-2.91	1.93	-3.89	0.94
17-20	Business	414.06	443.78	412.76	442.39	-5.00	24.71	-6 31	23.32
21-24	Pleasure	252.06	253.45	253 48	· 254.88	-1.64 ·	-0.25	-0 22	1.18
21-24	DTW Short	263.30	263.83	264 45	264.98	-1 01	-0.48	0.13	0.67
21-24	DTW Long	319.25	320.46	320.93	322.14	-1.41	-0.20	0.27	1.48
21-24	Business	414.12	417 40	416 20	419.49	-2.97	0.30	-0.90	2.39
25-29	Pleasure	234 43	235.01	234.52	235.10	-0.75	-0.17	-0.67	-0.08
25-29	DTW Short	244 89	245.14	244.65	244.90	-0.14	0.11	-0.37	-0.12
25-29	DTW Long	296.92	297.25	296.88	297.21	-0.33	-0.01	-0 37	-0 05
25-29	Business	385.16	386.20	385.05	386.09	-1.50	-0.46	-1 60	-0.57
30-34	Pleasure	225.05	225.68	225.05	225.69	-0.33	0.31	-0.32	0.32
30-34	DTW Short	235.07	235.25	234.77	234.96	0 27	0.45	-0.03	0.16
30-34	DTW Long	285.03	285 31	284.90	285.17	0.18	0.45	0.04	0.32
30-34	Business	369.71	370 47	369.48	370.24	-0.82	-0.06	-1.04	-0.28
35-39	Pleasure	180.75	181.16	181.10	181.52	-0.72	-0.31	-0.37	0.04
35-39	DTW Short	188.82	188.95	188.93	189.07	-0.25	-0.11	-0.13	0.00
35-39	DTW Long	228.95	229.16	229.27	229.48	-0.41	-0.21	-0 10	0.11
35-39	Business	296.97	297.59	297.34	297.96	-1.38	-0.76	-1.01	-0.39
40-49	Pleasure	195.90	196.18	196.03	196 31	-0.43	-0.15	-0.30	-0.02
40-49	DTW Short	204.63	204.70	204.51	204 58	0.08	0.16	-0.04	0.04
40-49	DTW Long	248.11	248.23	248.16	248.28	-0.03	0.09	0.02	0.14
40-49	Business	321.83	322.20	321.85	322.22	-0.95	-0.58	-0.93	-0.56
50-59	Pleasure	198.95	199.21	199.04	199.30	-0.39	-0.13	-0.29	-0.03
50-59	DTW Short	207.82	207.91	207 65	207.74	0.15	0.23	-0.02	0.06
50-59	DTW Long	252.01	252.18	252.00	252.17	0.06	0.23	0.05	0 23
50-59	Business	326.89	327.59	326.79	327.49	-0.84	-0 14	-0.93	-0 23
60+	Pleasure	194.47	194.73	194.63	194.89	-0.53	-0 27	-0.38	-0 12
60+	DTW Short	203.15	203.27	203.06	203.18	-0.01	0 11	-0.10	0 02
60+	DTW Long	246.34	246 61	246.42	246.70	-0.13	0.14	-0.05	0.23
60+	Business	319 52	320 67	319 59	320.75	-1.08	0.07	-1.00	0.15

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Table 4.4: Predicted Severities and Biases of Log-transformed Models with and without Adjustment for Variation Coefficient 2.0

	Vehicle	Predicted Severity				Prediction Bias			
Age	Use	G-L1	G-L2	L-L1	L-L2	G-L1	G-L2	L-L1	L-L2
17-20	Pleasure	234.75	243.92	242.18	251.64	-20.15	-10.98	-12.72	-3.26
17-20	DTW Short	245.84	250.83	252.99	258.13	-19.72	-14.73	-12.57	-7.43
17-20	DTW Long	297.79	308.39	306.59	317.51	-24.38	-13.78	-15.58	-4.66
17-20	Business	383.88	450.91	395.37	464.40	-35 19	31.84	-23.70	45.34
21-24	Pleasure	247.25	250.43	249.90	253.11	-6.45	-3.27	-3.80	-0.59
21-24	DTW Short	259.01	260.23	260.96	262.19	-5.30	-4.08	-3.36	-2.13
21-24	DTW Long	313.70	316 46	316.32	319.10	-6.96	-4.20	-4.33	-1 56
21-24	Business	404 78	412.25	407 99	415.52	-12.32	-4.85	-9.11	-1.58
25-29	Pleasure	233.53	234.87	233.61	234.96	-1.66	-0.31	-1.57	-0.23
25-29	DTW Short	244.62	245.20	243.96	244.53	-0 40	0.17	-1.06	-0.49
25-29	DTW Long	296.23	296.98	295.62	296.36	-1.03	-0.28	-1.64	-0.89
25-29	Business	382.31	384.71	381.16	383.55	-4.35	-1.95	-5.50	-3.11
30-34	Pleasure	223.92	225.39	223.19	224 65	-1.45	0.02	-2.18	-0.72
30-34	DTW Short	234.52	234.95	233.05	233.47	-0.28	0.15	-1.75	-1.33
30-34	DTW Long	283.95	284.58	282.43	283.06	-0.91	-0.27	-2.43	-1.80
30-34	Business	366.44	368.19	364 15	365.88	-4.08	-2.33	-6.38	-4.64
35-39	Pleasure	180.24	181.21	180 61	181.57	-1.23	-0.27	-0.87	0.10
35-39	DTW Short	188.75	189.07	188 56	188.88	-0.31	0.01	-0.50	-0.18
35-39	DTW Long	228.59	229.07	228.48	228.96	-0.78	-0.30	-0.89	-0.41
35-39	Business	294.94	296.38	294.63	296.07	-3.41	-1.97	-3.72	-2.29
40-49	Pleasure	194.79	195.43	195.77	196.42	-1.54	-0.90	-0.56	0.09
40-49	DTW Short	204.01	204.18	204.45	204.62	-0.54	-0.37	-0.10	0.07
40-49	DTW Long	247.06	247.34	247.76	248.04	-1.09	-0 81	-0.39	-0 11
40-49	Business	318.75	319.59	319.45	320.30	-4.03	-3.18	-3.32	-2.48
50-59	Pleasure	198.01	198.61	198.25	198.85	-1.33	-0.73	-1.09	-0.49
50-59	DTW Short	207.37	207.56	207.06	207 25	-0.31	-0.11	-0.62	-0.42
50-59	DTW Long	251.16	251.56	250.92	251 32	-0.78	-0.38	-1.03	-0.63
50-59	Business	324.08	325.69	323.51	325 12	-3.64	-2.03	-4 21	-2.60
60+	Pleasure	193.95	194.55	194.01	194 61	-1.06	-0.46	-0 99	-0.39
60+	DTW Short	203.15	203.43	202.64	202.92	-0.01	0.27	-0.52	-0.24
60+	DTW Long	246.10	246.73	245.59	246.23	-0.37	0.26	-0.88	-0.24
60+	Business	317.57	320.24	316.66	319.33	-3.03	-0 36	-3.94	-1.27

Table 4.5: Predicted Severities and Biases of Log-transformed Models with and without Adjustment for Variation Coefficient 3.0

Age	Vehicle Use	Predicted Severity				Prediction Bias			
		G-L1	G-L2	L-L1	L-L2	G-L1	G-L2	L-L1	L-L2
17-20	Pleasure	218.27	230 58	233.25	246.40	-36.62	-24.32	-21.65	-8.50
17-20	DTW Short	229.70	236 41	246.73	253.93	-35 86	-29.15	-18.84	-11 63
17-20	DTW Long	277.35	291.59	297.28	312.54	-44.82	-30.58	-24.89	-9.63
17-20	Business	354.90	446.80	381.85	480.72	-64.16	27.73	-37.22	61.65
21-24	Pleasure	245.26	249.79	246.14	250.68	-8.44	-3.91	-7.56	-3.02
21-24	DTW Short	257.67	259.41	260.50	262.26	-6.64	-4.90	-3.82	-2.06
21-24	DTW Long	311.91	315.84	313.76	317.71	-8.75	-4.82	-6.90	-2.95
21-24	Business	398.09	408.64	402 80	413.48	-19 01	-8.46	-14.30	-3.62
25-29	Pleasure	232.32	234 24	231.81	233.72	-2.87	-0.95	-3 38	-1.47
25-29	DTW Short	244.17	244 99	245.33	246.15	-0.86	-0.04	0.31	1.13
25-29	DTW Long	295.23	296.30	295.33	296.40	-2.02	-0.95	-1.92	-0.85
25-29	Business	377.08	380.46	379.37	382.77	-9.58	-6.20	-7.29	-3.89
30-34	Pleasure	223.42	225.52	222.07	224.16	-1.95	0.15	-3.30	-1.21
30-34	DTW Short	234.78	235 38	234.81	235.42	-0 02	0.58	0.01	0.62
30-34	DTW Long	283.96	284 86	282.80	283.71	-0.90	0.01	-2 05	-1.15
30-34	Business	362.53	365 01	363.04	365.53	-7 99	-5.51	-7.48	-5 00
35-39	Pleasure	178.68	180.04	178.71	180.08	-2.80	-1.43	-2.77	-1 40
35-39	DTW Short	187.86	188.32	188.98	189.44	-1.20	-0.75	-0.08	0.37
35-39	DTW Long	227.27	227.95	227.62	228.31	-2.10	-1.42	-1.74	-1.06
35-39	Business	290.28	292.30	292.26	294.29	-8.08	-6.06	-6.10	-4.06
40-49	Pleasure	195.13	196.05	193.36	194.27	-1.20	-0.28	-2.97	-2.06
40-49	DTW Short	205.16	205.40	204.50	204.74	0.62	0.86	-0.05	0.20
40-49	DTW Long	248.06	248.46	246.34	246.74	-0.09	0.31	-1.81	-1.41
40-49	Business	316 74	317.94	316 14	317.34	-6.04	-4 83	-6.64	-5 44
50-59	Pleasure	197.46	198.32	196.31	197.17	-1.87	-1.02	-3 02	-2.17
50-59	DTW Short	207 56	207.84	207.65	207.93	-0.11	0.17	-0.02	0.26
50-59	DTW Long	251.10	251.67	250.14	250.71	-0.85	-0.27	-1.81	-1.24
50-59	Business	320.78	323.07	321.03	323.32	-6.94	-4.65	-6.70	-4.41
60+	Pleasure	191.11	191.96	191.14	191.98	-3.89	-3.05	-3.87	-3.02
60+	DTW Short	200 84	201.25	202.26	202.66	-2.32	-1 92	-0.90	-0.50
60+	DTW Long	243 15	244.05	243 67	244.57	-3.32	-2.42	-2 80	-1.90
60+	Business	310 56	314.31	312.80	316.58	-10.04	-6.29	-7.80	-4.02

Appendix 5
Summary Tables of Predicted Severities

Table 5.1: Overall Unbiasedness and Stability of Predicted Severities for Gamma Loss

Coefficient	wab			wse		
Of Variation	G-G	G-L	G-N	G-G	G-L	G-N
1.0	0.180	0.240	0.221	8.170	8.177	8.568
2.0	0.475	0.852	0.509	16.498	16.514	17.239
3.0	0 860	1.808	1.139	25.223	25.097	26.986

G-G implies that the loss follows a Gamma distribution and a Gamma distribution is assumed in the regressions;
similarly, G-L implies the loss follows a Gamma distribution but a Lognormal is assumed in the regressions; and G-
N implies the loss follows a Gamma distribution but a Normal is assumed in the regressions.

Table 5.2: Overall Unbiasedness and Stability of Predicted Severities for Lognormal Loss

Coefficient	wab			wse		
Of Variation	L-G	L-L	L-N	L-G	L-L	L-N
1.0	0.151	0.202	0 175	8.309	8.284	8.754
2.0	0.498	0.844	0 604	16.426	16 113	17.721
3.0	0.720	1.589	1.006	24 328	23.214	27 608

Table 5.3: Summarized Statistics for Predicted Severities with Gamma Loss and Coefficient of Variation 1.0

Age	Vehicle Use	Mean			Bias			Standard Error		
		G-G	G-L	G-N	G-G	G-L	G-N	G-G	G-L	G-N
17-20	Pleasure	256 55	256 20	256.42	1.65	1 30	1.53	26.95	27.31	27.80
17-20	DTW Short	267.55	265.55	267.55	1.99	-0.01	1.99	27.64	27 83	28.72
17-20	DTW Long	324.60	324 11	324.69	2.43	1.93	2.52	34.06	34.55	35.56
17-20	Business	421.83	443 78	422.33	2.77	24.71	3.27	44.80	47.88	47.10
21-24	Pleasure	253.27	253 45	253.24	-0.43	-0.25	-0.46	14.48	14.52	15.38
21-24	DTW Short	264.12	263.83	264.18	-0 20	-0.48	-0 13	13.85	13.88	14.77
21-24	DTW Long	320.39	320.46	320 56	-0 27	-0.20	-0.10	16 88	16.93	18.17
21-24	Business	416 43	417.40	417.02	-0.67	0.30	-0 08	24 43	24.56	26.34
25-29	Pleasure	235.00	235.01	234.82	-0 18	-0.17	-0.36	9.61	9.63	9.91
25-29	DTW Short	245.07	245.14	244.98	0.04	0.11	-0.04	8.54	8.54	8.92
25-29	DTW Long	297.28	297 25	297.24	0.02	-0.01	-0.01	10.26	10.26	10.80
25-29	Business	386.39	386.20	386.70	-0.27	-0.46	0.04	16.69	16.66	17.83
30-34	Pleasure	225.54	225.68	225.49	0.16	0.31	0.12	9.06	9.08	9 49
30-34	DTW Short	235.18	235.25	235.22	0 38	0.45	0.42	7.46	7.47	7 88
30-34	DTW Long	285.30	285.31	285 42	0.45	0.45	0 56	9.46	9.46	10 02
30-34	Business	370.80	370.47	371.27	0.27	-0.06	0.75	14.94	14.94	15.71
35-39	Pleasure	181 13	181.16	180 94	-0 34	-0.31	-0.54	6 84	6.84	7.16
35-39	DTW Short	188.90	188.95	188.76	-0.17	-0.11	-0.30	5 98	5 98	6.32
35-39	DTW Long	229.16	229.16	229.04	-0.21	-0.21	-0.32	7 71	7.70	8.00
35-39	Business	297.83	297.59	297.93	-0.52	-0.76	-0.42	12.10	12.09	12.25
40-49	Pleasure	196.21	196.18	196.01	-0.12	-0 15	-0.32	6.80	6.80	7.21
40-49	DTW Short	204.60	204.70	204 47	0.05	0.16	-0.08	5 07	5.07	5.39
40-49	DTW Long	248.19	248.23	248.09	0.04	0.09	-0.06	6.27	6.27	6.51
40-49	Business	322.58	322.20	322.72	-0.20	-0 58	-0.06	11.22	11 22	11.58
50-59	Pleasure	199.30	199 21	199.24	-0.03	-0 13	-0.09	6.73	6.72	7.02
50-59	DTW Short	207.84	207 91	207.85	0.16	0 23	0.18	5.33	5.33	5 58
50-59	DTW Long	252.14	252 18	252.22	0.19	0 23	0.27	7.22	7.21	7.54
50-59	Business	327.71	327.59	328.10	-0.01	-0.14	0 38	12.37	12.37	12.90
60+	Pleasure	194.87	194.73	194.71	-0 13	-0.27	-0.29	7.08	7.08	7.38
60+	DTW Short	203.22	203.27	203.13	0.06	0.11	-0.03	6.03	6.03	6.32
60+	DTW Long	246 54	246.61	246.49	0.07	0.14	0.02	7.99	7.99	8.29
60+	Business	320.42	320 67	320.63	-0.18	0.07	0.03	12.61	12 64	13 08

Table 5.4: 95% Confidence Intervals for Predicted Severities with Gamma Loss and Coefficient of Variation 1.0

Age	Vehicle Use	G-G Lower	G-G Upper	G-L Lower	G-L Upper	G-N Lower	G-N Upper
17-20	Pleasure	207.63	310.60	207.59	311.14	206 28	310.82
17-20	DTW Short	216 34	322.12	214.26	321.27	215.28	322.54
17-20	DTW Long	263.80	392.67	263.01	392.03	261.02	394.83
17-20	Business	340.83	512.07	358.86	541.99	338.00	517 58
21-24	Pleasure	226.25	283 09	226.57	282.19	224.42	284.02
21-24	DTW Short	237.62	290.70	237 59	290.38	236.56	293 08
21-24	DTW Long	289.50	354.99	289.55	356.13	286.43	357.91
21-24	Business	372 10	468.31	372 53	470 09	369.86	473.94
25-29	Pleasure	216.42	253.50	216.61	253.21	216.42	253.86
25-29	DTW Short	229.15	261.89	229.24	262.06	228.08	262.99
25-29	DTW Long	277.55	315.84	277.48	315.65	276.57	317.05
25-29	Business	353.94	418.31	354.08	417.04	354 51	424.11
30-34	Pleasure	208.37	244.26	208.38	244.39	207.70	244.94
30-34	DTW Short	220.31	250.36	220.32	250.59	219.66	251.02
30-34	DTW Long	267.57	304.54	267.29	304.64	266.47	305.14
30-34	Business	343.29	400.92	343.27	400.73	340.73	402.13
35-39	Pleasure	168.49	194.75	168.73	194 88	167.68	194.87
35-39	DTW Short	177.86	201.01	177.93	200.92	176.73	200.90
35-39	DTW Long	214.26	244.34	214.21	244 50	213.75	245.04
35-39	Business	274 05	321.18	273.63	320.86	274.55	322 64
40-49	Pleasure	183.77	209.55	183.81	209.44	182.42	210.65
40-49	DTW Short	194.90	214.72	194.97	214.84	194.43	215.61
40-49	DTW Long	235.56	260.38	235.60	260.46	234.78	260.80
40-49	Business	302.55	344.78	301.93	344.46	301.67	346.83
50-59	Pleasure	186.45	212.74	186.43	212.74	185.40	212.99
50-59	DTW Short	197.56	218.44	197.69	218.57	197.29	219.28
50-59	DTW Long	237.88	265.95	238.07	266.05	237.88	267.72
50-59	Business	304.74	352.71	304.30	352.50	304.37	353.81
60+	Pleasure	180.54	208.23	180.54	208.05	180.35	208.72
60+	DTW Short	190.79	214.47	190.96	214.48	190.91	215.67
60+	DTW Long	230 42	262.11	230.51	262.10	230.67	263.05
60+	Business	296.82	345.96	297.28	346.23	296.32	346.93

Table 5.5: Summarized Statistics of Predicted Severities with Gamma Loss and Coefficient of Variation 2.0

Age	Vehicle Use	Mean			Bias			Standard Error		
		G-G	G-L	G-N	G-G	G-L	G-N	G-G	G-L	G-N
17-20	Pleasure	252.38	243.92	252.38	-2.52	-10.98	-2.52	55.66	55.58	57.27
17-20	DTW Short	262.56	250.83	262.73	-3.00	-14.73	-2.83	56.54	55.93	58.13
17-20	DTW Long	318.61	308.39	319.25	-3.56	-13.78	-2 92	69.08	69.36	71.04
17-20	Business	414.11	450.91	415.88	-4.95	31.84	-3.19	89.73	101.38	92.78
21-24	Pleasure	252.11	250.43	252.29	-1.59	-3.27	-1.41	29.49	29.59	31 10
21-24	DTW Short	262 37	260.23	262.75	-1.94	-4.08	-1 56	28.87	28.96	30.77
21-24	DTW Long	318.36	316.46.	319.33	-2.30	-4.20	-1.33	35.76	35.89	38.73
21-24	Business	414.18	412.25	416 32	-2.92	-4.85	-0.78	50.28	50 63	54.30
25-29	Pleasure	235.66	234.87	235.09	0.48	-0 31	-0.10	19 06	19.18	19.74
25-29	DTW Short	245.25	245.20	244.81	0 23	0.17	-0.21	17.30	17.39	18.03
25-29	DTW Long	297.54	296.98	297.42	0 28	-0.28	0.16	21 24	21.34	22 34
25-29	Business	387.17	384.71	387.86	0.51	-1.95	1.20	33.57	33.60	35.31
30-34	Pleasure	225 81	225.39	225.34	0.44	0.02	-0.03	17.85	17.92	18.71
30-34	DTW Short	234.96	234.95	234.61	0.17	0.15	-0.19	15.33	15 35	16.10
30-34	DTW Long	285.01	284.58	284.95	0.15	-0.27	0.10	18 13	18.07	18.98
30-34	Business	370.86	368.19	371.61	0.33	-2 33	1.09	29.81	29.59	31.50
35-39	Pleasure	181.70	181.21	181.16	0 23	-0.27	-0.31	14 54	14.65	15.20
35-39	DTW Short	189.05	189.07	188.61	-0.02	0.01	-0.45	12.32	12.33	13.20
35-39	DTW Long	229.36	229.07	229.13	0 00	-0.30	-0.24	15.45	15.42	16.25
35-39	Business	298 40	296.38	298.68	0.04	-1.97	0.33	24.05	23.92	24.61
40-49	Pleasure	195.92	195.43	195.58	-0.41	-0.90	-0.75	13.42	13.51	13.95
40-49	DTW Short	203.85	204.18	203.61	-0.69	-0.37	-0.93	10 52	10.54	10.84
40-49	DTW Long	247.32	247.34	247.37	-0.83	-0 81	-0.77	13 36	13.38	13.81
40-49	Business	321.73	319 59	322.46	-1 05	-3.18	-0.32	22.19	22.05	22.61
50-59	Pleasure	199.37	198 61	199.04	0 04	-0.73	-0.29	13.81	13.80	14.37
50-59	DTW Short	207.43	207 56	207.20	-0.25	-0.11	-0.47	10.65	10.67	11.02
50-59	DTW Long	251.70	251.56	251.78	-0.25	-0.38	-0.17	14.13	14.14	14.71
50-59	Business	327 46	325.69	328.25	-0.26	-2.03	0 53	23.59	23.50	24.38
60+	Pleasure	195 53	194.55	195.17	0.53	-0.46	0 17	13.93	13.93	14.57
60+	DTW Short	203.48	203.43	203 22	0.31	0.27	0.06	11.71	11.70	12.33
60+	DTW Long	246.95	246.73	246 99	0.48	0.26	0.52	16.01	16.02	16.92
60+	Business	321.30	320.24	322.03	0.70	-0 36	1.43	25 65	25.61	26.83

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Table 5.6: 95% Confidence Intervals of Predicted Severities with Gamma Loss and Coefficient of Variation 2.0

	Vehicle	G-G		G-L		G-N	
Age	Use	Lower	Upper	Lower	Upper	Lower	Upper
17-20	Pleasure	158.35	376.66	149.52	365 56	158.84	376.87
17-20	DTW Short	165.84	381.71	155.59	373.76	163.21	387.07
17-20	DTW Long	200.10	466.14	189.67	458.93	199 68	471.12
17-20	Business	255.66	598.37	277 67	657.76	257.72	616.11
21-24	Pleasure	200.06	312.91	198 77	312.26	195.56	318.09
21-24	DTW Short	209.62	323.74	206 82	320.18	208.76	328.66
21-24	DTW Long	252.26	395.51	250.10	395.99	248.29	400 67
21-24	Business	326 46	518.11	324.64	519 12	323.28	537 11
25-29	Pleasure	197 04	272 30	196.73	271.76	196.67	273.53
25-29	DTW Short	213.02	279 50	212.57	280.36	211 69	281.30
25-29	DTW Long	255.31	338 45	253.74	338.05	254 69	342.84
25-29	Business	324.38	454 40	322.35	452.27	321.71	457.47
30-34	Pleasure	193.78	264.05	193 12	262.77	191.06	265.62
30-34	DTW Short	205.47	264.82	205.22	265.26	203.59	265.85
30-34	DTW Long	251 10	323.21	250.81	322 41	248.71	323 42
30-34	Business	316 42	436.16	314.93	433 69	315.41	439 43
35-39	Pleasure	155 51	212.04	154.64	212.05	154.29	212.11
35-39	DTW Short	166.78	213 24	166.14	213.32	165.21	214.54
35-39	DTW Long	200.79	260 60	200.55	261.02	198 82	263.34
35-39	Business	253.47	345 96	250 98	344.36	254.36	347.61
40-49	Pleasure	171.25	222.39	170.62	222.44	170.23	223.89
40-49	DTW Short	183.16	225.39	183.49	225.63	182.62	225 13
40-49	DTW Long	222.45	274.25	222.16	274.26	223.13	275 54
40-49	Business	277.71	368.54	276.58	364.18	278.55	371.04
50-59	Pleasure	173 47	226.93	172.88	226.73	173.55	228.03
50-59	DTW Short	187.16	228.85	187.30	229 00	186.60	229.03
50-59	DTW Long	225.66	280.26	225.69	280.43	224 79	281.50
50-59	Business	280.90	371.99	279 61	369.27	282.24	375.68
60+	Pleasure	169.76	223.18	168.20	221.76	168.22	223.76
60+	DTW Short	181.97	228 44	181 85	228.06	180.28	228 05
60+	DTW Long	217 10	278.89	217.39	278.64	216.90	280 57
60+	Business	272.16	372.71	272 14	372.18	270.86	376 79

Table 5.7: Summarized Statistics of Predicted Severities with Gamma Loss and Coefficient of Variation 3.0

Age	Vehicle Use	Mean G-G	Mean G-L	Mean G-N	Bias G-G	Bias G-L	Bias G-N	Standard Error G-G	Standard Error G-L	Standard Error G-N
17-20	Pleasure	257.52	230.58	255.72	2.62	-24.32	0.83	86.21	82.61	91.05
17-20	DTW Short	267.20	236.41	265.82	1 64	-29.15	0.26	89.76	83.92	96.62
17-20	DTW Long	324.01	291.59	323.56	1.83	-30.58	1.39	107.64	102.76	117.81
17-20	Business	422 28	446.80	425.16	3.22	27.73	6 10	145.96	162.99	166.59
21-24	Pleasure	256 19	249.79	256.05	2.49	-3.91	2.35	46 74	46.19	49.70
21-24	DTW Short	265 13	259.41	265 23	0.82	-4.90	0.92	43 92	43.52	46.77
21-24	DTW Long	322.33	315.84	323.53	1.67	-4.82	2.87	55 62	55.06	60.23
21-24	Business	419.14	408.64	424.14	2.04	-8.46	7.04	77.39	76.22	87.22
25-29	Pleasure	237.39	234.24	236.73	2.20	-0.95	1.54	30.43	30.22	32.09
25-29	DTW Short	245.76	244.99	245.30	0.74	-0.04	0.28	26.57	26.55	28.07
25-29	DTW Long	298.44	296.30	298.80	1.18	-0.95	1.54	32.26	32.16	34.83
25-29	Business	388 38	380.46	391 89	1.72	-6 20	5.23	50.92	50.05	56.39
30-34	Pleasure	227.81	225 52	226.64	2.43	0.15	1.27	27 68	27.37	29.44
30-34	DTW Short	235.78	235.38	234.79	0 98	0.58	-0.01	22.94	22.97	24.44
30-34	DTW Long	286.39	284 86	285.99	1 54	0.01	1.14	28.60	28.59	30.52
30-34	Business	372.51	365.01	374.85	1 99	-5.51	4.33	45.04	44 69	49.22
35-39	Pleasure	181.93	180.04	180 93	0.46	-1.43	-0.54	21.04	20 82	22.97
35-39	DTW Short	188.41	188.32	187 49	-0.66	-0.75	-1.57	18.14	18.17	19.54
35-39	DTW Long	228 91	227.95	228 40	-0.46	-1.42	-0.97	23.28	23.21	24.45
35-39	Business	297.90	292 30	299.37	-0.46	-6 06	1.02	37.73	37.16	39.44
40-49	Pleasure	197.77	196 05	196.66	1.44	-0 28	0.33	19 95	19.84	21.73
40-49	DTW Short	204.82	205.40	203 80	0 27	0 86	-0 74	15.94	15 99	16.94
40-49	DTW Long	248.70	248.46	248.17	0.56	0.31	0 02	19.20	19.25	20.32
40-49	Business	323.57	317.94	325.23	0.79	-4.83	2 45	33.97	33 53	35.81
50-59	Pleasure	200.61	198.32	199.95	1 28	-1.02	0.61	20.93	20 78	22.21
50-59	DTW Short	207.71	207.84	207.21	0.04	0.17	-0.46	16.55	16.58	17.52
50-59	DTW Long	252.35	251.67	252.51	0.40	-0.27	0.56	21.62	21.72	23.34
50-59	Business	328.47	323.07	331.15	0.74	-4 65	3.43	38.02	37.68	41.24
60+	Pleasure	194.72	191.96	193.88	-0 29	-3.05	-1 12	22.21	21.99	23.43
60+	DTW Short	201.57	201.25	200.93	-1.59	-1.92	-2 23	17.96	17.94	19.47
60+	DTW Long	245 07	244.05	244.97	-1 40	-2.42	-1.50	25.02	24 97	26.55
60+	Business	318.89	314.31	321.13	-1.71	-6.29	0 53	40.26	40 16	43.00

Table 5.8: 95% Confidence Intervals of Predicted Severities with Gamma Loss and Coefficient of Variation 3.0

Age	Vehicle Use	G-G Lower	G-G Upper	G-L Lower	G-L Upper	G-N Lower	G-N Upper
17-20	Pleasure	119.40	458.67	100.30	417.64	116.72	464.45
17-20	DTW Short	123.57	463.79	104.96	429.05	121.19	476.23
17-20	DTW Long	152.25	553.60	129.10	522.80	147.50	588.54
17-20	Business	197.98	736.88	192.31	811.30	189 61	791.06
21-24	Pleasure	174 28	354.30	169.13	348.49	168.95	363.52
21-24	DTW Short	187.71	356.20	182.58	347.55	184.32	362.97
21-24	DTW Long	224.39	437.50	216.92	434 11	218.99	450.27
21-24	Business	286.22	582 62	274.97	572.10	282.43	609.41
25-29	Pleasure	182.60	302.19	178.15	297.26	178.27	301.54
25-29	DTW Short	198.08	302 01	198 39	300.56	195.33	301.90
25-29	DTW Long	238.87	369.25	237.79	364.67	235.15	371.60
25-29	Business	295.64	493.23	288 69	486.40	294 77	512.37
30-34	Pleasure	176.23	288.30	175.55	284.70	171.04	290.97
30-34	DTW Short	193.91	284.31	194.30	283.27	189.77	283.53
30-34	DTW Long	230.77	343.30	229.00	341.21	227.96	345.22
30-34	Business	287.59	462.08	281.84	456.24	284.66	482.26
35-39	Pleasure	143.20	227.33	142.39	224.37	138.70	228.38
35-39	DTW Short	153.81	224.66	154.40	225.26	150.07	225.74
35-39	DTW Long	187.28	276.19	186.76	274.11	183.66	277.67
35-39	Business	229.72	377.77	224.79	368.38	227.96	381.40
40-49	Pleasure	160.82	239 99	159.21	238.90	157.29	243.15
40-49	DTW Short	175.46	237 44	176.41	238.21	172.95	238 09
40-49	DTW Long	213.32	287 29	212.09	286.54	210.15	288.20
40-49	Business	261.98	390.65	255.71	386.11	262.30	398.89
50-59	Pleasure	160 21	245.48	159.82	242.52	156.29	245.10
50-59	DTW Short	177.04	241.75	177.40	242.20	174.27	242.14
50-59	DTW Long	213 72	299.75	212.50	299.24	209.77	301.62
50-59	Business	261.28	402.18	258.73	397.92	261.86	418.11
60+	Pleasure	156.33	241.67	154.37	239.29	150.36	243.77
60+	DTW Short	167.59	239.55	167.96	238.12	164.24	241.78
60+	DTW Long	201.44	297.13	200.01	295.99	196.45	298.56
60+	Business	251.86	403.89	247.03	397.12	249 41	419.10

Table 5.9: Summarized Statistics of Predicted Severities with Lognormal Loss and Coefficient of Variation 1.0

Age	Vehicle Use	Mean			Bias			Standard Error		
		L-G	L-L	L-N	L-G	L-L	L-N	L-G	L-L	L-N
17-20	Pleasure	255.65	255.55	255.69	0.76	0.65	0.79	28.25	27.67	29.69
17-20	DTW Short	266.26	264.52	266 36	0.70	-1.04	0.80	28.82	28.07	30.60
17-20	DTW Long	323.31	323 11	323.50	1.14	0.94	1.32	35.60	34.88	37.76
17-20	Business	420.12	442.39	420.79	1.06	23 32	1.73	47.25	48.78	50.10
21-24	Pleasure	254.78	254.88	254.83	1.08	1.18	1.13	14.83	14.70	15.76
21-24	DTW Short	265.36	264.98	265.44	1.05	0.67	1.12	14.34	14 19	15.29
21-24	DTW Long	322.19	322.14	322.36	1.53	1.48	1.70	18 02	17.82	19 46
21-24	Business	418.66	419.49	419.33	1.56	2 39	2.23	25.10	24.91	27 17
25-29	Pleasure	235.07	235 10	234.98	-0.11	-0.08	-0.21	10.06	10.04	10.37
25-29	DTW Short	244.82	244 90	244.75	-0.21	-0.12	-0.27	8.63	8.61	9.07
25-29	DTW Long	297.22	297.21	297.21	-0.04	-0.05	-0.05	10.69	10.68	11.30
25-29	Business	386.25	386.09	386.65	-0.40	-0.57	-0.01	17.15	17.07	18.17
30-34	Pleasure	225.53	225.69	225.39	0.16	0.32	0.02	9.19	9.18	9.70
30-34	DTW Short	234.88	234.96	234.75	0 08	0.16	-0.04	7.62	7.62	7.97
30-34	DTW Long	285.16	285.17	285.07	0 31	0.32	0.22	9.58	9.56	10.11
30-34	Business	370.56	370.24	370.82	0.03	-0.28	0.29	15.08	15.05	15.67
35-39	Pleasure	181.48	181 52	181.33	0 01	0.04	-0.14	7 01	6.98	7.47
35-39	DTW Short	189.01	189.07	188.87	-0.05	0.00	-0.19	5.91	5.91	6.27
35-39	DTW Long	229.47	229.48	229 35	0.10	0.11	-0.02	7.48	7 47	7.87
35-39	Business	298.19	297 96	298.33	-0.16	-0.39	-0.02	11.90	11.86	12.17
40-49	Pleasure	196.33	196.31	196.26	0.00	-0.02	-0.07	6.63	6.61	6.93
40-49	DTW Short	204.47	204.58	204.43	-0.07	0.04	-0.12	5.03	5.03	5.33
40-49	DTW Long	248.24	248.28	248.23	0.09	0.14	0.09	6.25	6.24	6.73
40-49	Business	322 59	322.22	322.91	-0.19	-0.56	0.13	11.32	11 27	11.64
50-59	Pleasure	199 40	199.30	199.32	0.06	-0.03	-0.02	6.87	6.84	7.14
50-59	DTW Short	207.68	207.74	207.62	0.00	0 06	-0 06	5.47	5.46	5.81
50-59	DTW Long	252.14	252 17	252.12	0.20	0.23	0.18	7.51	7.50	7.83
50-59	Business	327.62	327.49	327.93	-0.10	-0.23	0.21	11.49	11.45	11.90
60+	Pleasure	195.03	194.89	194.92	0.03	-0.12	-0.09	7.17	7.14	7.53
60+	DTW Short	203.14	203.18	203.05	-0.02	0.02	-0.11	6.46	6 45	6.87
60+	DTW Long	246.64	246 70	246.57	0.17	0.23	0.10	8.40	8 39	8.80
60+	Business	320.50	320.75	320 74	-0 10	0 15	0.14	13.31	13.31	13.70

Table 5.10: 95% Confidence Intervals of Predicted Severities with Lognormal Loss and Coefficient of Variation 1.0

	Vehicle	G-G		G-L		G-N	
Age	Use	Lower	Upper	Lower	Upper	Lower	Upper
17-20	Pleasure	205.05	316 70	203.40	307.55	203 19	318.52
17-20	DTW Short	214.43	328.01	211.71	320.61	211.73	330.81
17-20	DTW Long	259 17	401.24	256.36	390.40	256.56	402.51
17-20	Business	337.08	522.18	333.09	511.61	330.32	529.09
21-24	Pleasure	226.56	283.43	225.60	281.63	225.85	287.69
21-24	DTW Short	238.46	295.12	237.34	293.23	237.01	298 77
21-24	DTW Long	284 28	358 08	284.01	355.83	284.56	361.52
21-24	Business	370.62	469.04	368.95	464 63	368.65	479.95
25-29	Pleasure	216.26	256.18	216.02	255.22	216.23	256 25
25-29	DTW Short	228 67	262.28	228.51	262.00	228.41	263 20
25-29	DTW Long	276.69	317 48	276.13	317.43	276.64	319.12
25-29	Business	353.85	422.39	353.10	420.98	354 53	426.60
30-34	Pleasure	208 28	243.70	208.13	242.93	207.73	244.50
30-34	DTW Short	220 65	251.47	220.63	250.98	220.31	251 32
30-34	DTW Long	267.84	305.43	267 76	305.27	267.06	306.66
30-34	Business	341.59	400.71	340.58	399.32	341 81	404.39
35-39	Pleasure	168.71	195 98	168.23	195.07	167.11	195 74
35-39	DTW Short	177.91	200 87	177.83	200.70	176.99	200.95
35-39	DTW Long	215.74	245.05	215.66	245 02	214.24	245.06
35-39	Business	276.01	322.57	275.42	321.61	276.00	322.86
40-49	Pleasure	184.24	209.35	183.89	209.06	183.42	210.19
40-49	DTW Short	194.75	214.64	194 72	214 56	194.01	214.94
40-49	DTW Long	236.39	260.36	236 16	260 14	235.00	261.73
40-49	Business	301 33	344.57	300.79	343.99	301.76	345.62
50-59	Pleasure	185 96	212.58	185.85	212.37	185.65	212 70
50-59	DTW Short	197.23	219.06	197 24	218.95	196.83	219.79
50-59	DTW Long	238.18	267.51	238.03	267.33	238 02	268.39
50-59	Business	306.11	351 17	305.46	350.51	306 26	352 32
60+	Pleasure	180.79	209 77	180.52	209.25	179.41	209 82
60+	DTW Short	190.49	217.81	190.44	217.72	189.52	217.20
60+	DTW Long	230.66	263.46	230.34	263.25	229.77	264.51
60+	Business	295 57	348.05	294.77	347.68	295.51	348 73

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Table 5.11: Summarized Statistics of Predicted Severities with Lognormal Loss and Coefficient of Variation 2.0

Age	Vehicle Use	Mean L-G	Mean L-L	Mean L-N	Bias L-G	Bias L-L	Bias L-N	Standard Error L-G	Standard Error L-L	Standard Error L-N
17-20	Pleasure	255.85	251.64	256.33	0 95	-3 26	1.44	55.75	49.77	59.13
17-20	DTW Short	265.63	258.13	266.16	0.07	-7.43	0.60	57.32	50.41	61.16
17-20	DTW Long	322.42	317.51	323.67	0.25	-4.66	1.50	69.90	62.08	75.29
17-20	Business	418.84	464.40	421.82	-0.22	45.34	2.75	93.53	93.27	101.45
21-24	Pleasure	254.33	253.11	254.09	0.63	-0.59	0.39	28.83	27.88	31.54
21-24	DTW Short	263.96	262.19	263.61	-0.36	-2.13	-0.70	27.46	26 46	29.34
21-24	DTW Long	320.45	319.10	320.57	-0.21	-1.56	-0.09	34 77	33.68	37.51
21-24	Business	416 30	415.52	417.92	-0.80	-1.58	0.82	50 90	49.07	56.94
25-29	Pleasure	235.71	234.96	235.68	0.52	-0.23	0.50	19 58	19.15	20.76
25-29	DTW Short	244.64	244.53	244.60	-0 38	-0 49	-0.42	17.24	17 02	18.44
25-29	DTW Long	296.89	296.36	297.34	-0.37	-0 89	0.08	21.27	20.96	23.18
25-29	Business	385.55	383 55	387.36	-1.11	-3.11	0.70	33.33	32 45	36.85
30-34	Pleasure	225 01	224 65	224.82	-0.36	-0.72	-0.56	17.52	17.25	18.80
30-34	DTW Short	233.52	233.47	233.28	-1.28	-1.33	-1.51	14.69	14.54	15.80
30-34	DTW Long	283 42	283 06	283.59	-1.43	-1.80	-1.26	18.75	18.53	20.24
30-34	Business	368.05	365.88	369.46	-2.48	-4.64	-1.07	29.87	29.08	33.07
35-39	Pleasure	182.07	181.57	181.48	0.60	0 10	0 01	14.76	14.35	16.12
35-39	DTW Short	188.92	188.88	188.26	-0 14	-0 18	-0.80	12.04	11.86	13.02
35-39	DTW Long	229.27	228.96	228.81	-0 10	-0.41	-0.56	14.77	14.53	15 85
35-39	Business	297.76	296.07	298.05	-0.59	-2.29	-0.30	24.34	23.69	25.74
40-49	Pleasure	196.89	196.42	196.61	0.56	0.09	0 28	12 88	12.65	14.00
40-49	DTW Short	204.36	204.62	204.04	-0.19	0 07	-0 51	9.95	9.94	10.79
40-49	DTW Long	248.03	248.04	248.02	-0.12	-0.11	-0.13	12.98	12.92	13.83
40-49	Business	322.09	320 30	323 04	-0.68	-2.48	0.27	22 92	22.28	24.17
50-59	Pleasure	199.53	198.85	198 99	0.20	-0.49	-0.34	13.34	13.09	14 40
50-59	DTW Short	207.12	207.25	206 54	-0.55	-0.42	-1.14	11.03	11.01	11.86
50-59	DTW Long	251 38	251.32	251 07	-0.57	-0.63	-0.88	14.15	14.06	15.27
50-59	Business	326 42	325 12	327.02	-1.30	-2 60	-0 71	23.84	23 34	25.70
60+	Pleasure	195 55	194.61	195.32	0.54	-0.39	0.32	14.17	13.90	15.28
60+	DTW Short	202 99	202.92	202.74	-0.17	-0 24	-0 42	12.32	12.19	13.28
60+	DTW Long	246.40	246.23	246.49	-0 07	-0 24	0 02	16.13	15.91	17.45
60+	Business	319.99	319.33	321 15	-0 61	-1 27	0.55	26.10	25.31	29 18

Table 5.12: 95% Confidence Intervals of Predicted Severities with Lognormal Loss and Coefficient of Variation 2.0

Age	Vehicle Use	L-G Lower	L-G Upper	L-L Lower	L-L Upper	L-N Lower	L-N Upper
17-20	Pleasure	169.68	394.47	168.87	366.93	166.67	399.58
17-20	DTW Short	179 65	405.01	177.92	374.96	175.01	409 57
17-20	DTW Long	210.88	495.47	213.68	463.80	209.70	502 79
17-20	Business	271.22	635.59	304 54	663.21	266.23	670 44
21-24	Pleasure	205.17	312 46	204.53	309.27	201.69	318.60
21-24	DTW Short	216.70	323.67	215 35	321 28	213.56	324.67
21-24	DTW Long	258.44	393.94	258.94	391 20	259.45	398.67
21-24	Business	329.32	530.54	329.47	525.37	327.32	543 69
25-29	Pleasure	200 46	274.84	200.99	273.93	198.76	279.92
25-29	DTW Short	212.92	282 04	213 04	281.60	210.55	283.22
25-29	DTW Long	256.54	343.83	256.74	342.09	256.53	346.75
25-29	Business	328.91	455.32	327.13	451 43	325 82	469.24
30-34	Pleasure	193.74	263.39	193.06	263.06	190.05	264.27
30-34	DTW Short	207.16	264.35	207.40	264.12	205 53	265.69
30-34	DTW Long	250.27	322.50	250.03	321.97	248 98	325.70
30-34	Business	316.23	424 68	314.65	421.11	313.51	437.02
35-39	Pleasure	156.79	212 64	156.19	212.36	152.92	213.55
35-39	DTW Short	168.63	214.29	168.49	214.08	164.60	214.69
35-39	DTW Long	203.37	260.87	203.10	260.29	201.08	260.93
35-39	Business	254.97	347.13	253.51	343.22	253.65	350.60
40-49	Pleasure	174.01	222 20	173 79	221.63	172.78	225.16
40-49	DTW Short	185.78	224.90	185.79	224.75	184.21	226.20
40-49	DTW Long	221.97	274.58	222.26	274 31	220.83	276.09
40-49	Business	280.72	370.43	279.97	367.95	281.36	373.81
50-59	Pleasure	173 89	225.92	173.44	225.01	171.95	229 45
50-59	DTW Short	186.98	229.99	187.14	230.30	184.86	230 96
50-59	DTW Long	224.64	281.15	225 04	280.18	223.36	283.44
50-59	Business	283.77	376.14	282.83	372.19	281.38	382.39
60+	Pleasure	169.52	226.13	169.24	225.40	167.43	227.65
60+	DTW Short	180.88	229.26	181.06	228.47	180 05	229.66
60+	DTW Long	217.11	280.25	216.93	279.46	216 22	281.31
60+	Business	272.80	372.72	273.14	371.94	273 31	382.09

**Table 5.13: Summarized Statistics of Predicted Severities with Lognormal Loss and Coefficient of Variation 3.0**

Age	Vehicle Use	Mean L-G	Mean L-L	Mean L-N	Bias L-G	Bias L-L	Bias L-N	Standard Error L-G	Standard Error L-L	Standard Error L-N
17-20	Pleasure	257.16	246.40	258.06	2.26	-8.50	3.16	85.62	67.59	117.76
17-20	DTW Short	268.54	253.93	267.82	2.98	-11.63	2.26	85.90	67.80	89.92
17-20	DTW Long	324.71	312.54	325 60	2.54	-9.63	3.43	104.78	84.05	115.61
17-20	Business	422.41	480.72	425.05	3.34	61.65	5.98	140.16	134.16	154.98
21-24	Pleasure	254.82	250.68	254.43	1.12	-3.02	0.73	40.94	37.56	45.87
21-24	DTW Short	266.36	262.26	265.81	2.04	-2.06	1.50	40.30	37.18	45.00
21-24	DTW Long	321.82	317.71	322.71	1.16	-2.95	2.05	49.09	45.70	57.36
21-24	Business	418.49	413.48	421.22	1.40	-3.62	4 12	70.58	65.59	83 84
25-29	Pleasure	236.11	233 72	235.41	0.92	-1.47	0.23	27.95	26.32	31.21
25-29	DTW Short	246.79	246.15	245.89	1.77	1.13	0.86	25 80	24.91	28.01
25-29	DTW Long	298.07	296.40	298.13	0.81	-0.85	0.87	30.77	29.33	33.63
25-29	Business	387 89	382.77	389.66	1.23	-3.89	3.00	51.09	47 36	58.73
30-34	Pleasure	226.07	224.16	226.09	0.70	-1.21	0 72	27.30	25.28	33.06
30-34	DTW Short	236.03	235.42	235.68	1 23	0.62	0 88	22.03	21.28	24.80
30-34	DTW Long	285.20	283.71	285.96	0.35	-1.15	1.11	27.71	26.58	31 74
30-34	Business	370.86	365.53	373 41	0.33	-5 00	2.89	44.73	41.68	54.94
35-39	Pleasure	181.61	180.08	180 72	0.13	-1.40	-0.76	20.60	19.56	24.19
35-39	DTW Short	189.67	189.44	188.53	0.61	0.37	-0.53	16.98	16.57	19.27
35-39	DTW Long	229.19	228.31	228.69	-0.18	-1.06	-0.68	21.60	21.00	24.12
35-39	Business	298 06	294.29	298.47	-0 29	-4.06	0.12	35.52	33.60	39.75
40-49	Pleasure	195.69	194.27	195.05	-0 64	-2.06	-1.28	19.43	18.52	23.19
40-49	DTW Short	204.40	204.74	203.48	-0 14	0.20	-1.06	14.83	14.73	16.66
40-49	DTW Long	247.03	246.74	246.92	-1.12	-1.41	-1.23	19.79	19.41	22.24
40-49	Business	321.11	317.34	322.08	-1.67	-5 44	-0.70	33.26	31.46	37.36
50-59	Pleasure	199.01	197.17	198.29	-0.33	-2.17	-1.05	20.50	19.49	23.42
50-59	DTW Short	207.89	207.93	206.95	0.22	0.26	-0 72	16.55	16 31	18.01
50-59	DTW Long	251.24	250.71	251 12	-0.71	-1.24	-0 82	21.46	20.97	23.55
50-59	Business	326.62	323 32	327.89	-1.11	-4.41	0 17	35.76	33.63	44.49
60+	Pleasure	194.21	191.98	193.59	-0.79	-3.02	-1.42	20.49	19.59	23.44
60+	DTW Short	203.00	202 66	202 22	-0.16	-0 50	-0.94	18.29	17.80	20.91
60+	DTW Long	245.37	244.57	245.41	-1.10	-1.90	-1.06	23.93	23.06	27.15
60+	Business	319.04	316.58	320.35	-1.56	-4.02	-0.25	38.26	36.22	45.39

Table 5.14: 95% Confidence Intervals of Predicted Severities with Lognormal Loss and Coefficient of Variation 3.0

Age	Vehicle Use	L-G Lower	L-G Upper	L-L Lower	L-L Upper	L-N Lower	L-N Upper
17-20	Pleasure	150.84	453 79	147.45	408.89	145.28	467 57
17-20	DTW Short	161.08	473.55	153.81	411 66	159.79	483 70
17-20	DTW Long	189.15	565 01	185.34	501.03	183 93	584 01
17-20	Business	239 60	742.03	261.24	752.07	236 26	766 60
21-24	Pleasure	186.60	352 85	185.83	336.33	180 20	365 38
21-24	DTW Short	201 75	357 23	200.30	344.52	196.74	377.53
21-24	DTW Long	242 47	434.58	240.99	419.43	235.19	461.13
21-24	Business	307.98	586.35	306.03	559.09	300.17	627.96
25-29	Pleasure	190.82	297.56	189.29	288.25	185.34	302.97
25-29	DTW Short	205.21	303.08	204.96	300.56	199.10	304.64
25-29	DTW Long	246.31	367.25	245.78	360.33	243.93	376.80
25-29	Business	297.11	503.33	296.69	485.59	297.80	516.34
30-34	Pleasure	184.39	283.27	182.85	279.17	177.12	293.13
30-34	DTW Short	199 74	283.08	199.18	280.41	192.81	288.72
30-34	DTW Long	240 12	348.16	239.32	342.69	232 60	357.79
30-34	Business	293.64	465 80	292.36	452.40	291 62	488.31
35-39	Pleasure	145 54	224.85	145.63	220.97	141 15	231.34
35-39	DTW Short	160.64	225 26	160.97	224.35	156.05	227.58
35-39	DTW Long	193.77	276 10	192.86	273.97	189.01	279.85
35-39	Business	238.76	378.53	235.37	369.80	237.17	394.84
40-49	Pleasure	162.62	239.36	161 18	233.75	157.09	241.60
40-49	DTW Short	178.70	237.68	178 91	238.54	173.96	238.30
40-49	DTW Long	213 76	288.45	213.47	287.55	209.57	292.07
40-49	Business	265.59	391.26	262.73	385.95	262 02	400.44
50-59	Pleasure	163 04	243.77	162.93	238.02	161 17	244.78
50-59	DTW Short	180.15	245 63	180.36	243.67	175 68	244.27
50-59	DTW Long	211 62	297.42	211.79	295 67	207.37	298.99
50-59	Business	265.87	402 81	264.66	396 51	263.59	409 25
60+	Pleasure	158.82	239.61	158.17	234 44	153.71	243.81
60+	DTW Short	171.32	243.17	170 95	240.05	166.97	245 22
60+	DTW Long	206.10	300.94	205.74	296.58	198.58	308 97
60+	Business	257.01	404.61	254 96	394.06	251.04	420.91

Appendix 6
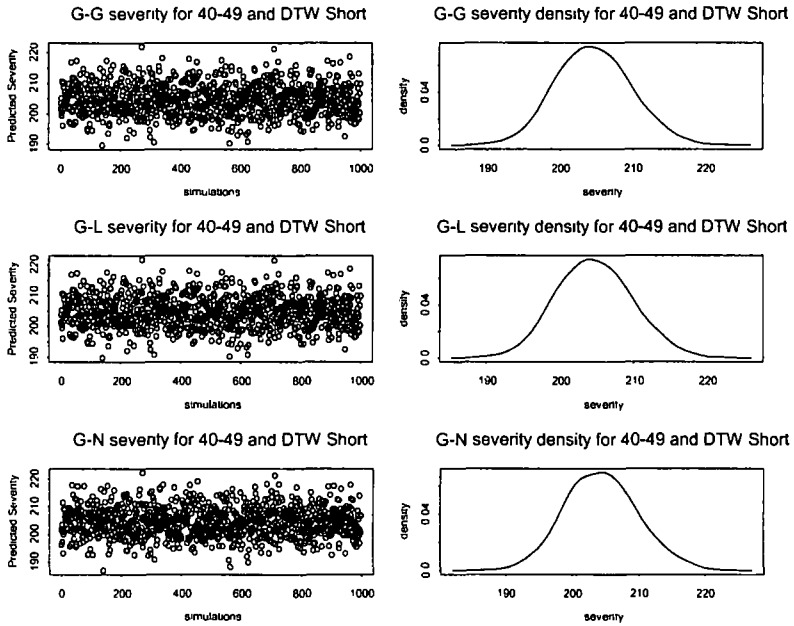## Plots of Predicted Severities for Selected Classifications

Figure 6.1: Scatter and Density Plots of Predicted Severities for Gamma Loss with Coefficient of Variation 1.0 for Age 17-20 and Pleasure Use
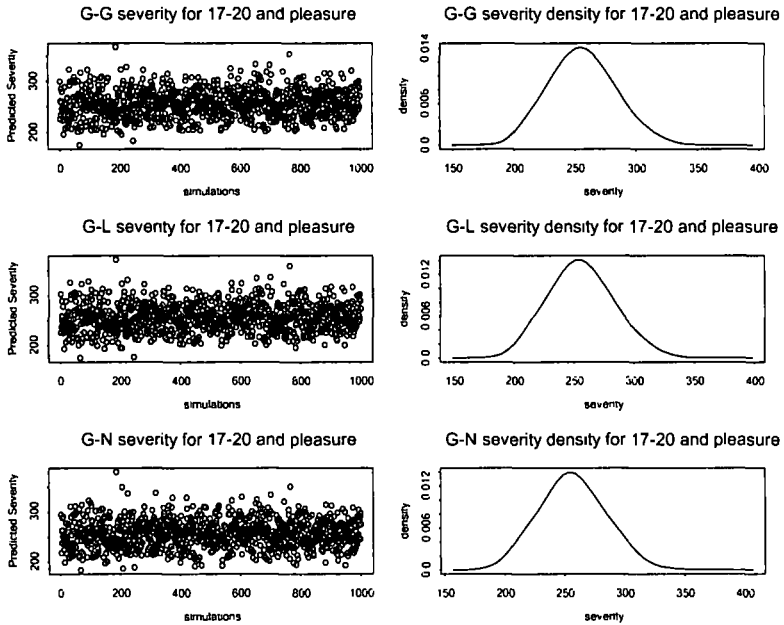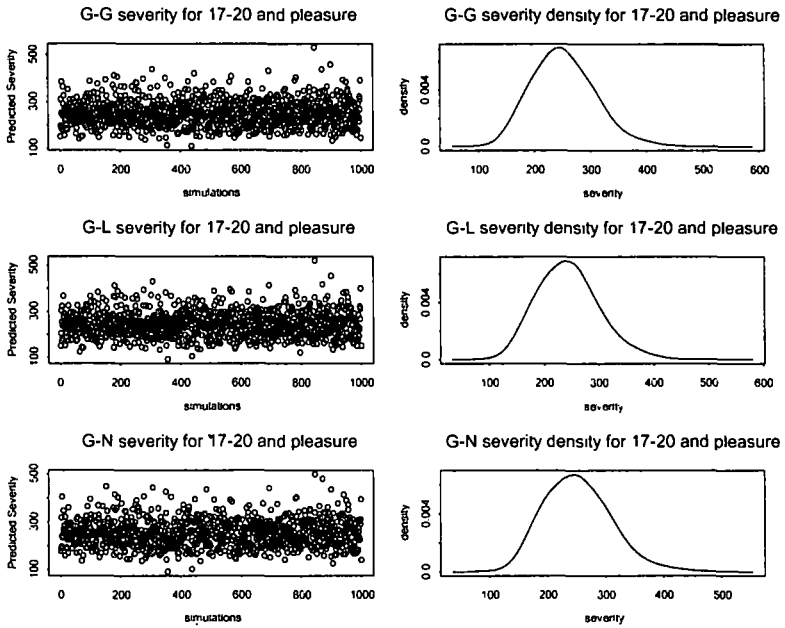

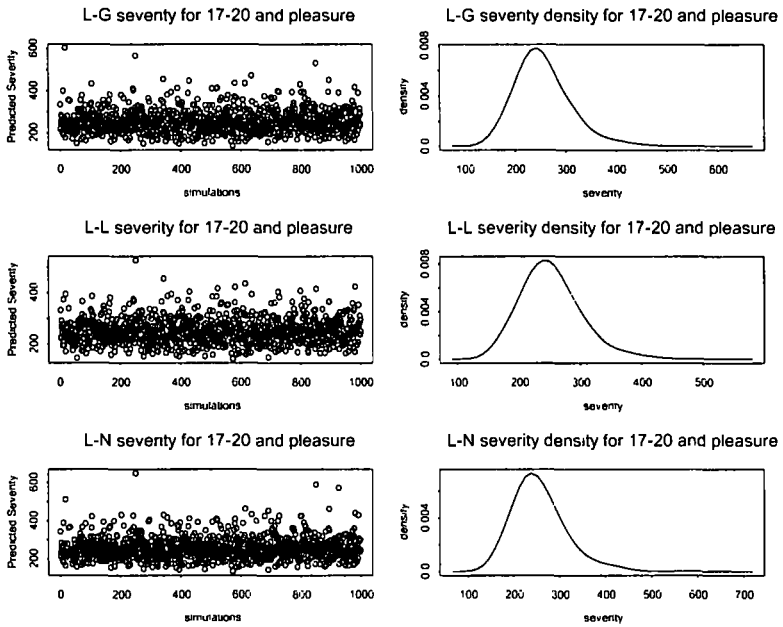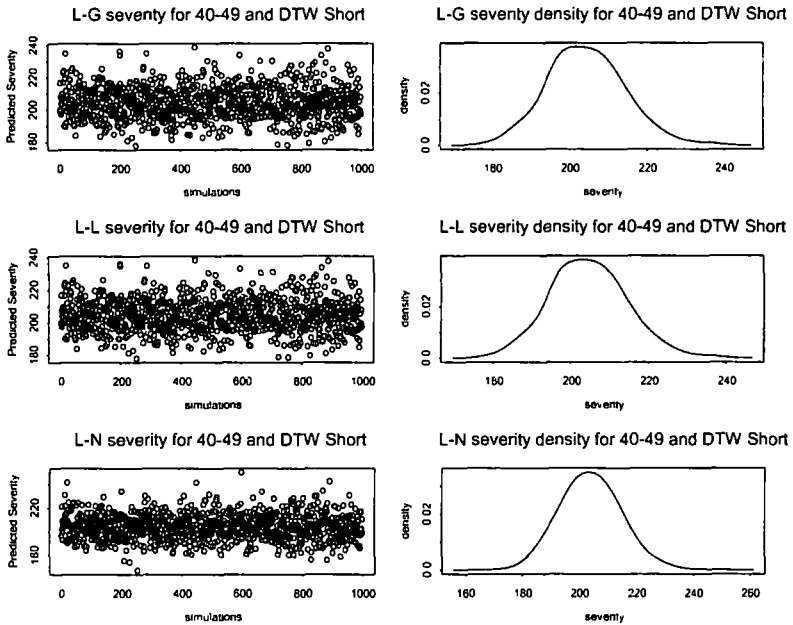
- G-G implies that the loss follows a Gamma distribution and a Gamma distribution is assumed in the regressions; similarly, G-L implies the loss follows a Gamma distribution but a Lognormal is assumed in the regressions; and G-N implies the loss follows a Gamma distribution but a Normal is assumed in the regressions.
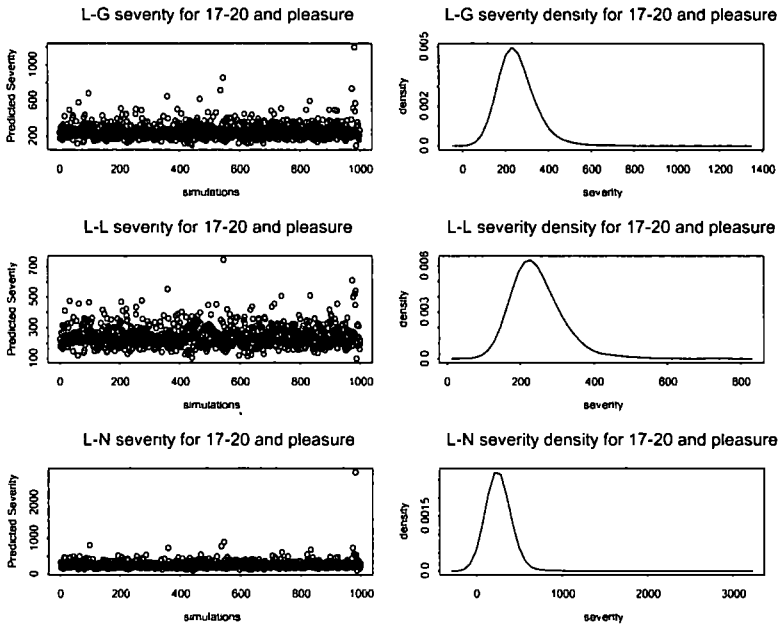- The density function is estimated by the non-parametric method from Silverman (1986).

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Figure 6.2: Scatter and Density Plots of Predicted Severities for Gamma Loss with Coefficient of Variation 1.0 for Age 40–49 and DTW Short Use

Figure 6.3: Scatter and Density Plots of Predicted Severities for Gamma Loss with Coefficient of Variation 2.0 for Age 17-20 and Pleasure Use

G-G severity for 17-20 and pleasure

G-G severity density for 17-20 and pleasure

G-L severity for 17-20 and pleasure

G-L severity density for 17-20 and pleasure

G-N severity for 17-20 and pleasure

G-N severity density for 17-20 and pleasure

Figure 6.4: Scatter and Density Plots of Predicted Severities for Gamma Loss with Coefficient of Variation 2.0 for Age 40-49 and DTW Short Use
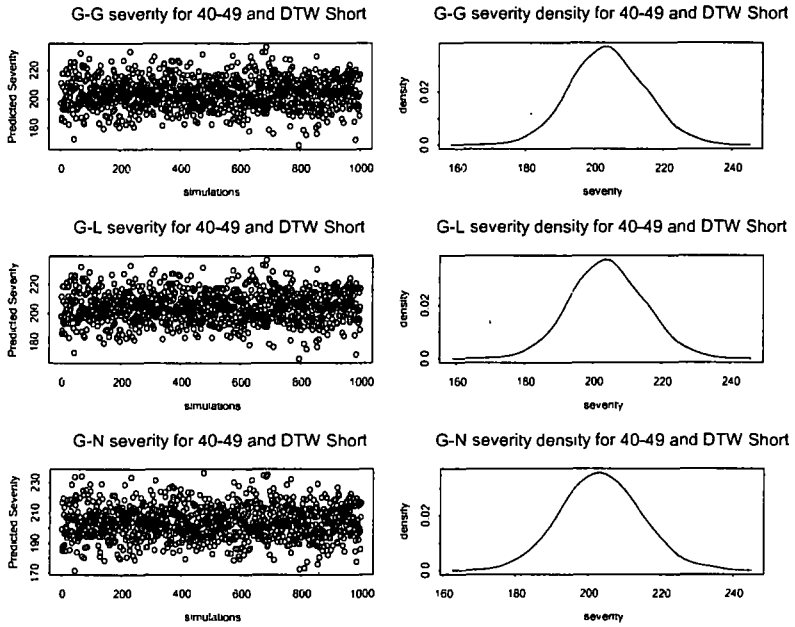
Figure 6.5: Scatter and Density Plots of Predicted Severities for Gamma Loss with Coefficient of Variation 3.0 for Age 17-20 and Pleasure Use
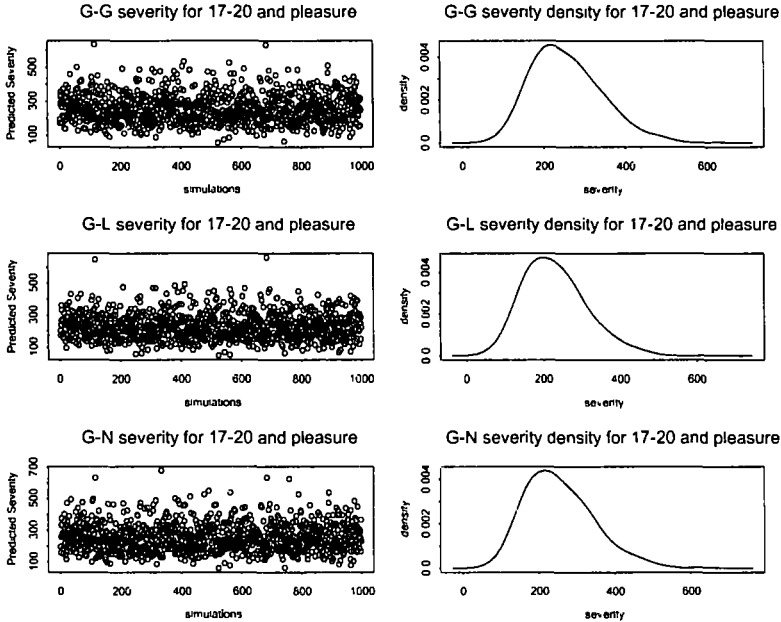
Figure 6.6: Scatter and Density Plots of Predicted Severities for Gamma Loss with Coefficient of Variation 3.0 for Age 40-49 and DTW Short Use

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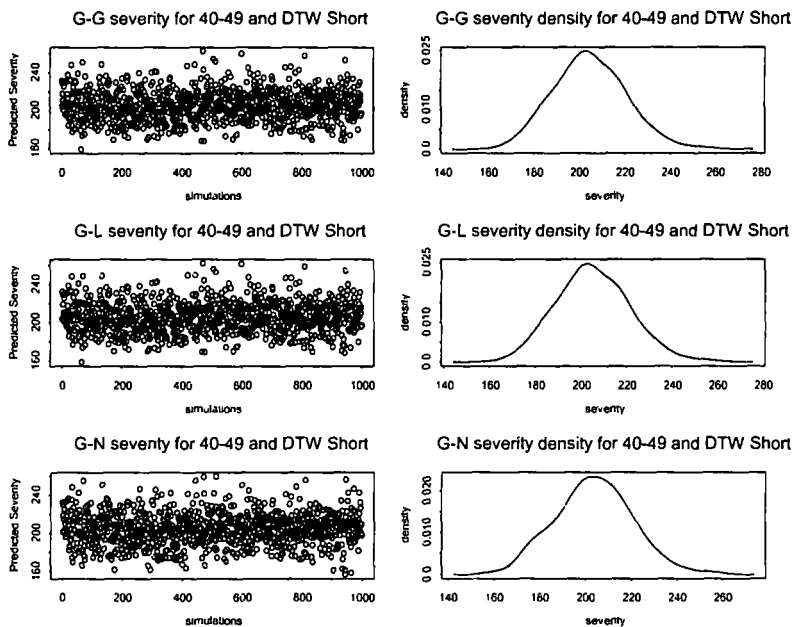
Figure 6.7: Scatter and Density Plots of Predicted Severities for Lognormal Loss with Coefficient of Variation 1.0 for Age 17-20 and Pleasure Use
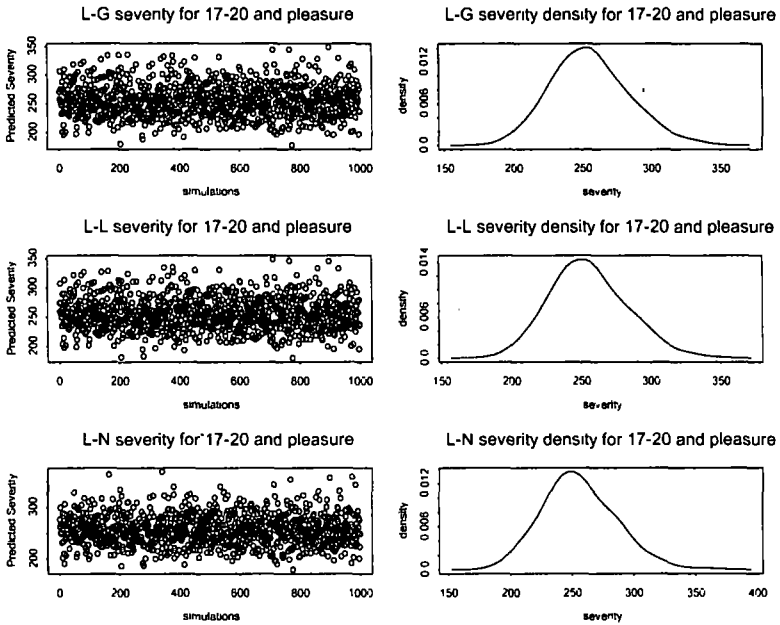
Figure 6.8: Scatter and Density Plots of Predicted Severities for Lognormal Loss with Coefficient of Variation 1.0 for Age 40-49 and DTW Short Use
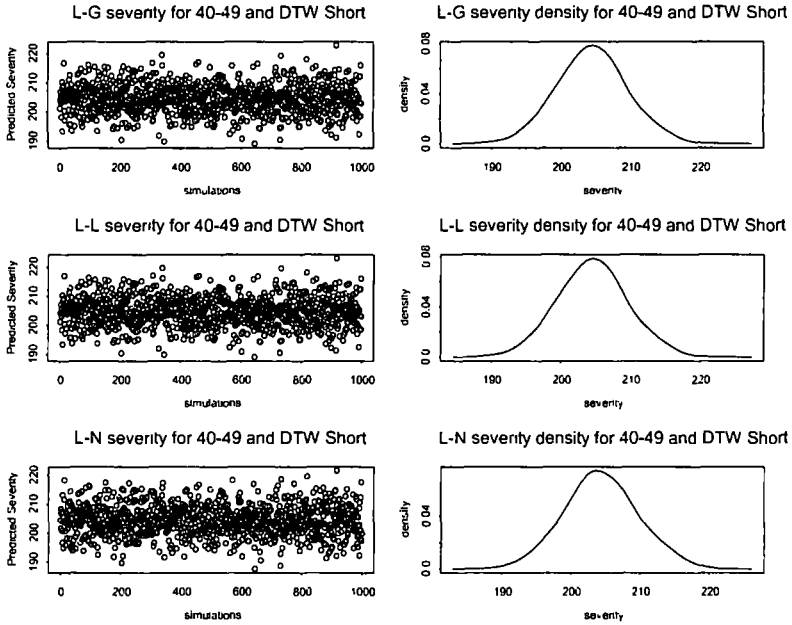
Figure 6.9: Scatter and Density Plots of Predicted Severities for Lognormal Loss with Coefficient of Variation 2.0 for Age 17-20 and Pleasure Use

Figure 6.10: Scatter and Density Plots of Predicted Severities for Lognormal Loss with Coefficient of Variation 2.0 for Age 40-49 and DTW Short Use

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Figure 6.11: Scatter and Density Plots of Predicted Severities for Lognormal Loss with Coefficient of Variation 3.0 for Age 17-20 and Pleasure Use

L-G severity for 17-20 and pleasure

L-G severity density for 17-20 and pleasure

L-L severity for 17-20 and pleasure

L-L severity density for 17-20 and pleasure

L-N severity for 17-20 and pleasure

L-N severity density for 17-20 and pleasure

Figure 6.12: Scatter and Density Plots of Predicted Severities for Lognormal Loss with Coefficient of Variation 3.0 for Age 40-49 and DTW Short Use

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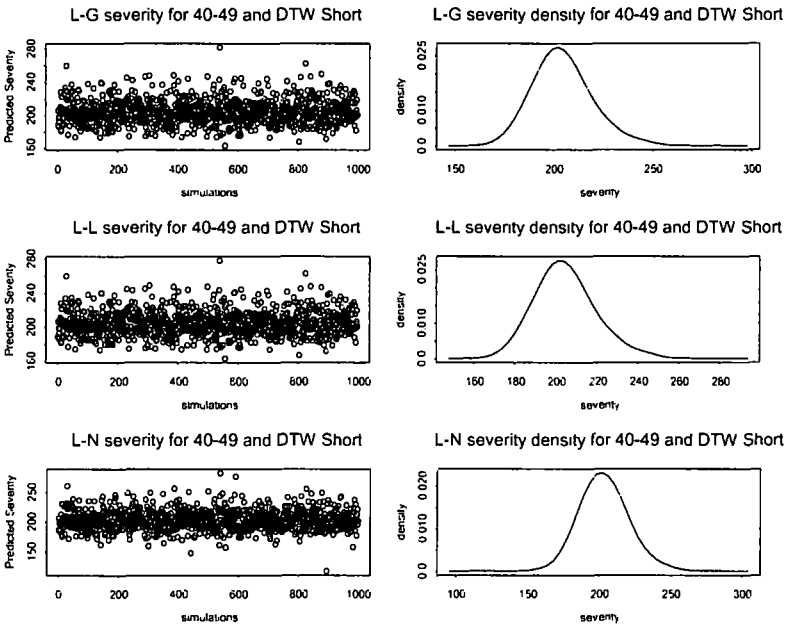
Appendix 7
Residual Plots for Regression Diagnostics

Figure 7.1: QQ Plots of Standardized Residuals for Gamma Loss with Coefficient of Variation 1.0
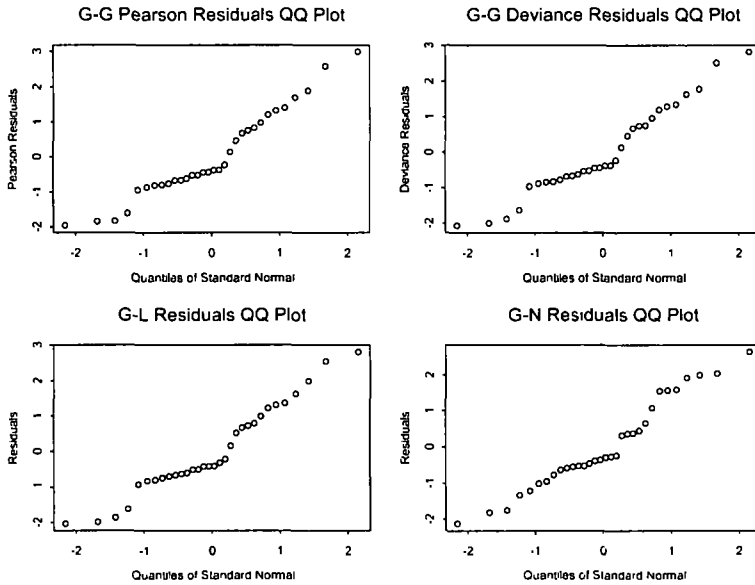
G-G implies that the loss follows a Gamma distribution and a Gamma distribution is assumed in the regressions; similarly, G-L implies the loss follows a Gamma distribution but a Lognormal is assumed in the regressions, and G-N implies the loss follows a Gamma distribution but a Normal is assumed in the regressions.

Figure 7.2: Plots of Predicted Severities vs Standardized Residuals for Gamma Loss with Coefficient of Variation 1.0

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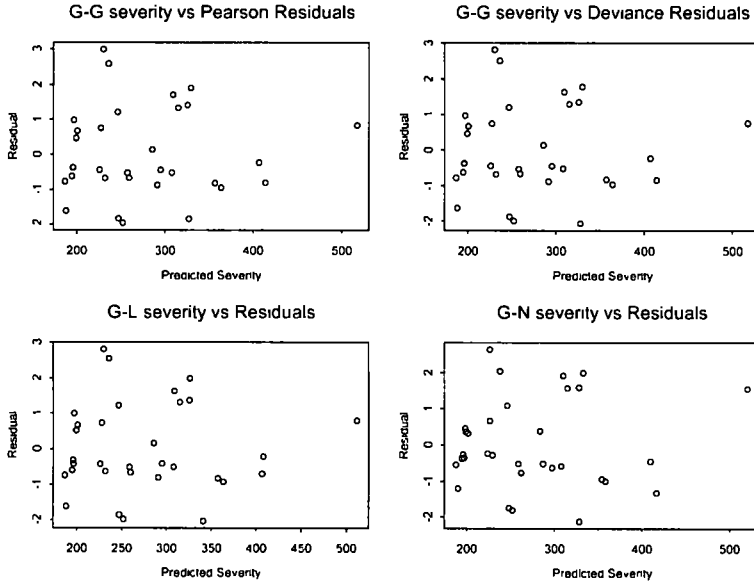
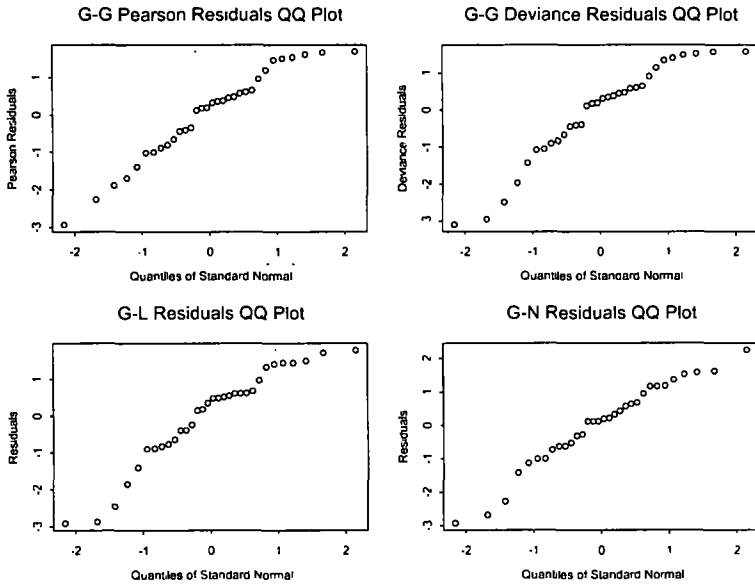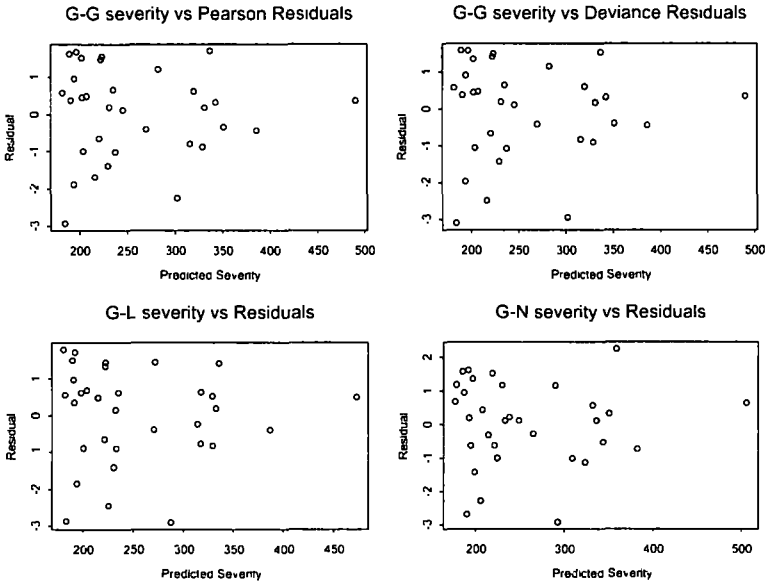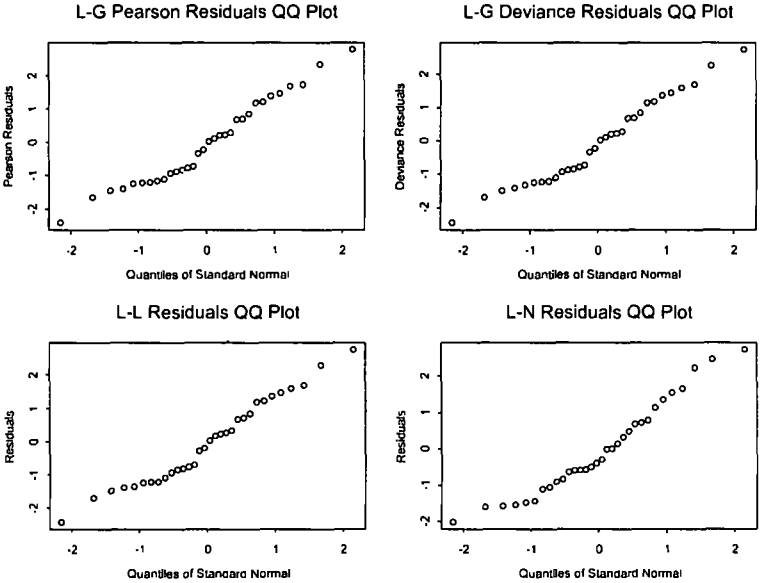**Figure 7.3: QQ Plots of Standardized Residuals for Gamma Loss with Coefficient of Variation 2.0**

G-G Pearson Residuals QQ Plot

G-G Deviance Residuals QQ Plot

G-L Residuals QQ Plot

G-N Residuals QQ Plot
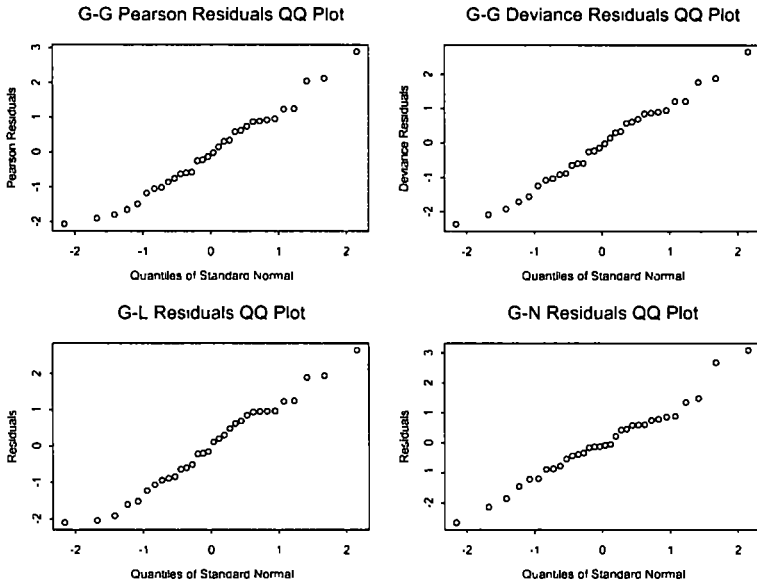
Figure 7.4: Plots of Predicted Severities vs Standardized Residuals for Gamma Loss with Coefficient of Variation 2.0

G-G severity vs Pearson Residuals

G-G severity vs Deviance Residuals

G-L severity vs Residuals

G-N severity vs Residuals
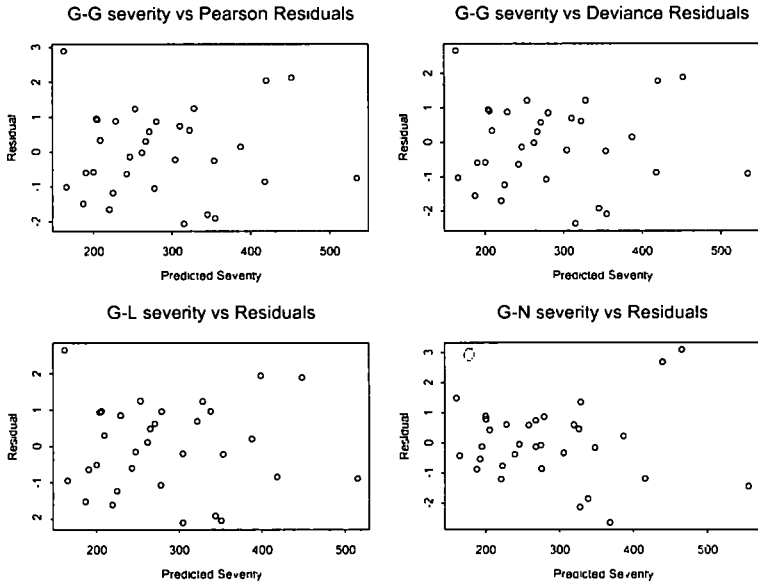
Figure 7.5: QQ Plots of Standardized Residuals for Gamma Loss with Coefficient of Variation 3.0

G-G Pearson Residuals QQ Plot

G-G Deviance Residuals QQ Plot

G-L Residuals QQ Plot

G-N Residuals QQ Plot

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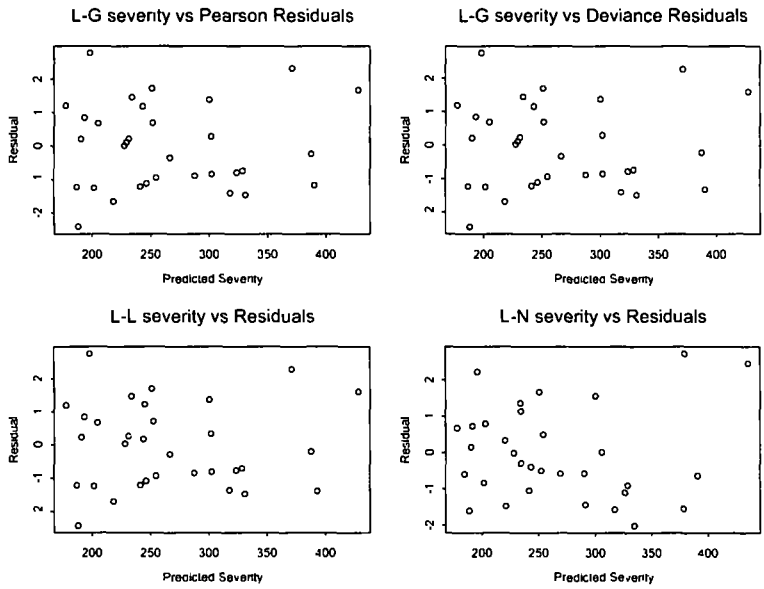**Figure 7.6: Plots of Predicted Severities vs Standardized Residuals for Gamma Loss with Coefficient of Variation 3.0**

G-G severity vs Pearson Residuals

G-G severity vs Deviance Residuals

G-L severity vs Residuals

G-N severity vs Residuals

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Figure 7.7: QQ Plots of Standardized Residuals for Lognormal Loss with Coefficient of Variation 1.0

L-G Pearson Residuals QQ Plot

L-G Deviance Residuals QQ Plot

L-L Residuals QQ Plot

L-N Residuals QQ Plot

Figure 7.8: Plots of Predicted Severities vs Standardized Residuals for Lognormal Loss with Coefficient of Variation 1.0

L-G severity vs Pearson Residuals

L-G severity vs Deviance Residuals

L-L severity vs Residuals

L-N severity vs Residuals

Figure 7.9: QQ Plots of Standardized Residuals for Lognormal Loss with Coefficient of Variation 2.0
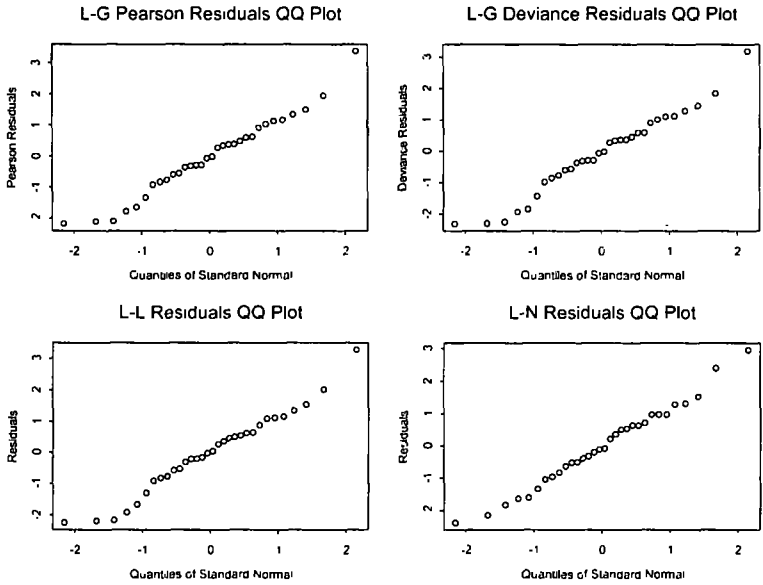
Figure 7.10: Plots of Predicted Severities vs Standardized Residuals for Lognormal Loss with Coefficient of Variation 2.0
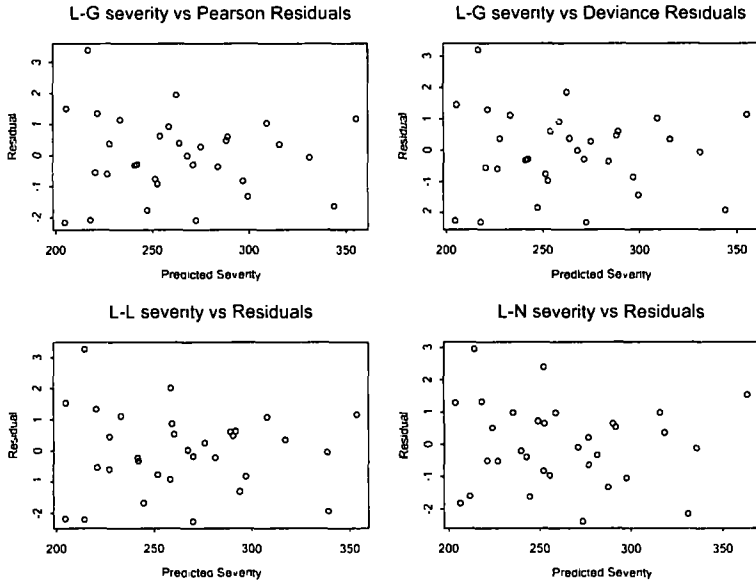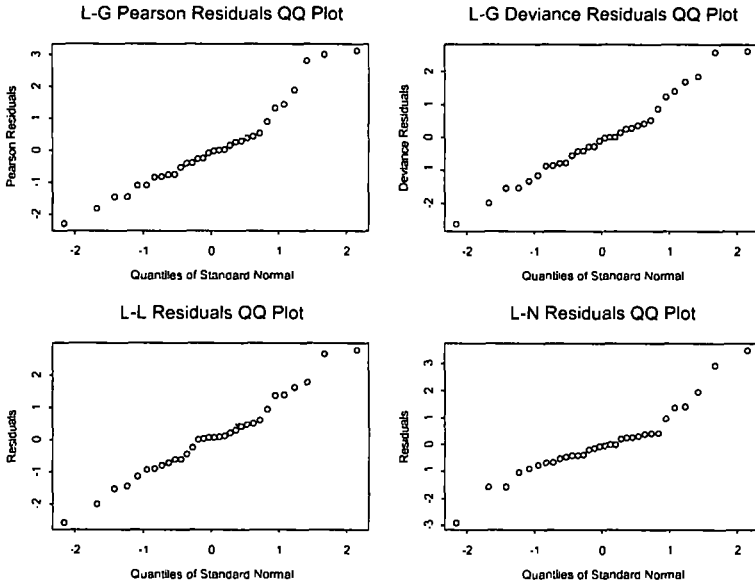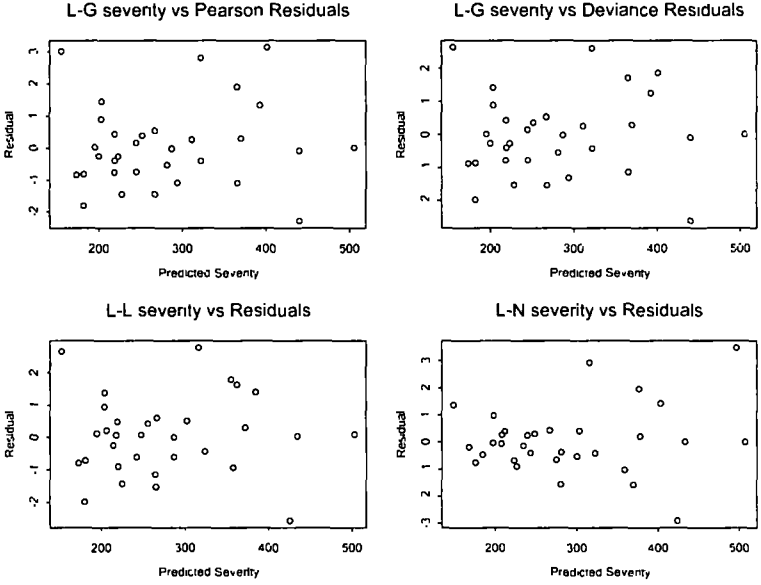

L-G severity vs Pearson Residuals

L-G severity vs Deviance Residuals

L-L severity vs Residuals

L-N severity vs Residuals

Figure 7.11: QQ Plots of Standardized Residuals for Lognormal Loss with Coefficient of Variation 3.0

L-G Pearson Residuals QQ Plot

L-G Deviance Residuals QQ Plot

L-L Residuals QQ Plot

L-N Residuals QQ Plot

Figure 7.12: Plots of Predicted Severities vs Standardized Residuals for Lognormal Loss with Coefficient of Variation 3.0

L-G severity vs Pearson Residuals

L-G severity vs Deviance Residuals

L-L severity vs Residuals

L-N severity vs Residuals

Figure 7.13: QQ Plots of Standardized Residuals for Gamma Loss with Coefficient of Variation 1.0 based on Individual Data

G-G Pearson Residuals QQ Plot

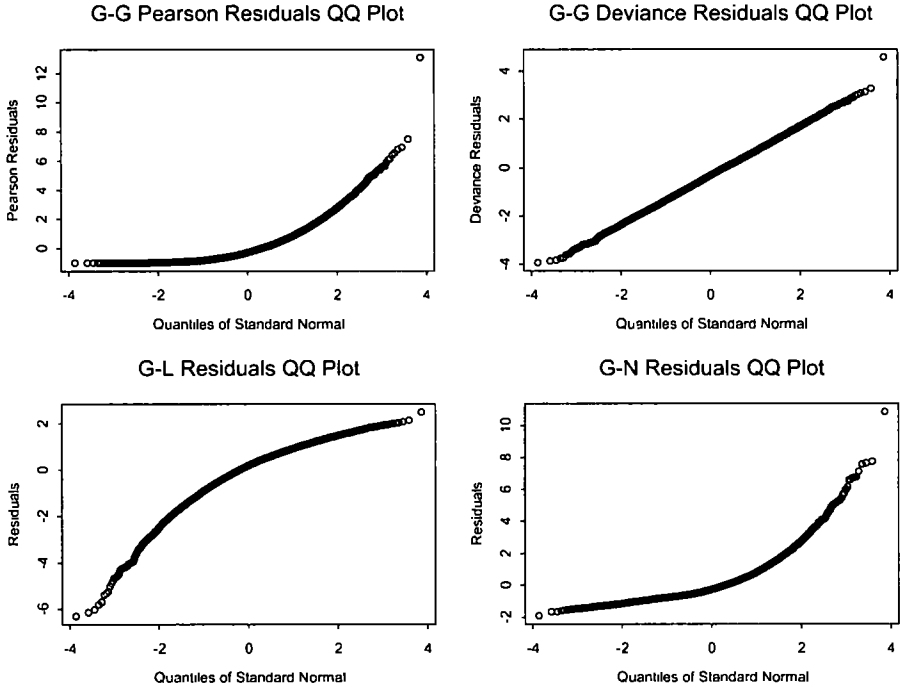G-G Deviance Residuals QQ Plot

G-L Residuals QQ Plot

G-N Residuals QQ Plot

Figure 7.14: Plots of Predicted Severities vs Standardized Residuals for Gamma Loss with Coefficient of Variation 1.0 based on Individual Data
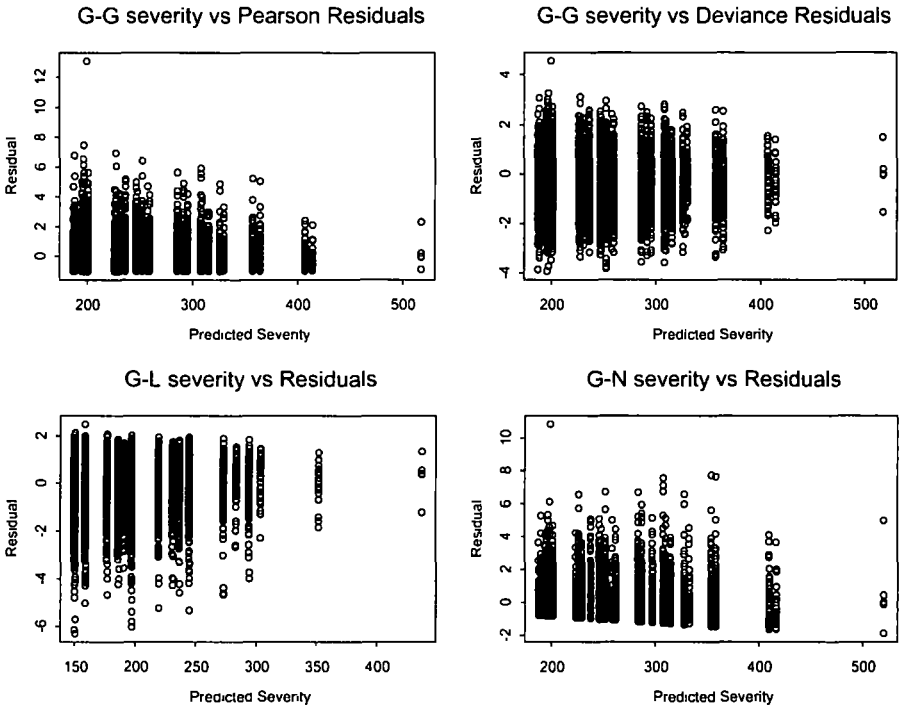

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Figure 7.15: QQ Plots of Standardized Residuals for Lognormal Loss with Coefficient of Variation 1.0 based on Individual Data

L-G Pearson Residuals QQ Plot

L-G Deviance Residuals QQ Plot

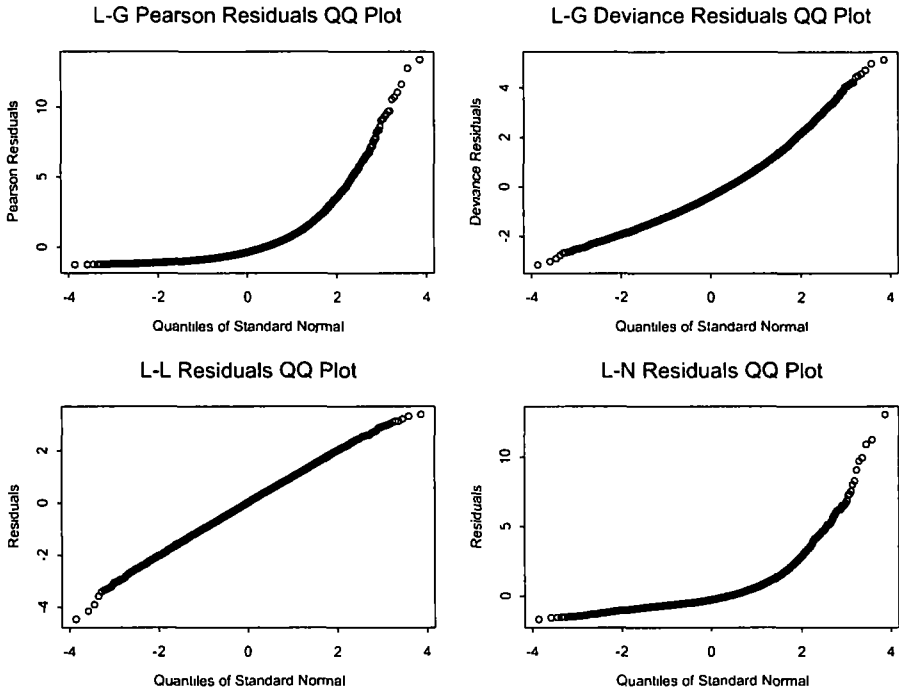L-L Residuals QQ Plot

L-N Residuals QQ Plot

Figure 7.16: Plots of Predicted Severities vs Standardized Residuals for Lognormal Loss with Coefficient of Variation 1.0 based on Individual Data

L-G severity vs Pearson Residuals

L-G severity vs Deviance Residuals

L-L severity vs Residuals

L-N severity vs Residuals