How does the capacitance of a series RLC circuit affect its resonant frequency?

1 Introduction

RLC stands for the electrical components that are connected in the electrical circuit; that is Resistors(L), Capacitors(C) and Inductors(L). Thanks to the features of capacitors and inductors, these circuits display a very useful property called **Resonance**. Electrical Resonance is a natural phenomenon in which the response of the circuit peaks at a particular frequency[1]. The frequency refers to the number of voltage cycles per second measured in Hertz(Hz) and at the **natural frequency** of an RLC circuit, the current peaks. This peak in current at particular frequencies corresponds to a net zero voltage across the capacitor and inductor and these properties have proven useful in telecommunications. All telecommunication devices make use of the resonance property of RLC circuits. Radio antenna receivers induce alternating voltages with frequencies equal to that of the radio wave and when a certain frequency channel is needed, there needs to be a way of filtering this certain range of frequencies from the whole spectrum of radio waves. RLC circuits behave as Band pass filters[2] which are the solution to this problem as they allow for the tuning of any electrical device to a specific frequency range. Figure 1 shows a common **ideal** current - angular frequency relationship through a series RLC circuit.

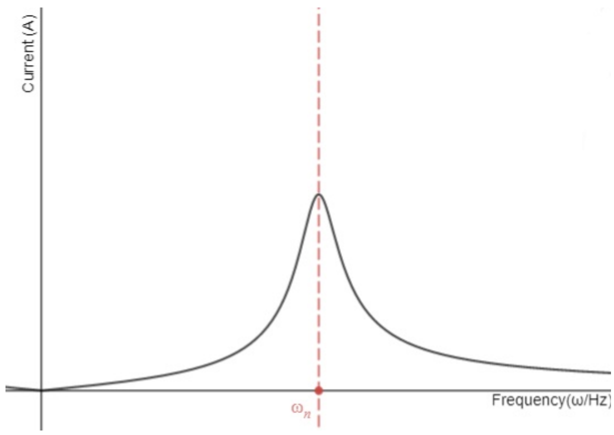

Figure 1: Example of a typical Current-Frequency diagram for an RLC circuit

From this graph, it is very clear to see how an RLC circuit can help in filtering out specific frequencies. The spike in current is due to a steep decline in the overall **Impedance** of the circuit and so a higher magnitude of current flows at the particular natural frequency, ω_n. Frequencies outside the narrow range are filtered out as their current is suppressed.

Having an individual RLC circuit with its own unique resonant frequency for every specific frequency channel would be very inconvenient and so the ways in which the circuit can be modified to change its natural frequency must be investigated. The major contributing factors of the resonant frequency are the Capacitance and Inductance of the circuit. Radios actually use variable capacitors to vary the resonant frequency of the RLC and to tune the circuit to the desired frequency.

This investigation tests how the capacitance of a series RLC circuit affect its resonant frequency. Figure 2 shows the circuit diagram of a series RLC.

[1] *Series Resonance in a Series RLC Resonant Circuit*. May 2020. URL: `https://www.electronics-tutorials.ws/accircuits/series-resonance.html`.

[2] *RLC circuit*. Dec. 2020. URL: `https://en.wikipedia.org/wiki/RLC_circuit#:~:text=Radio%20receivers%20and%20television%20sets,filter%20or%20high-pass%20filter.`.

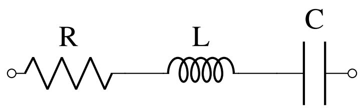

Figure 2: Series RLC circuit

2 Background science

An inductor is an electrical component which stores electrical potential energy in a magnetic field[3]. The simplest inductor is a solenoid, an insulated coil of wire. The current is proportional to the strength of the magnetic field induced due to the magnetic field (measured in Tesla, T). This result can be derived directly from Ampere's Law (4th Maxwell equation)[4]. Lenz's law describes the direction of the induced voltage across a coil of wire due to the changing magnetic field through the coil. A change in the magnetic field strength induces a voltage across the coil such that the current due to this voltage forms a magnetic field that opposes the initial change.[5]
Using the results from Ampere's law, the following relation between the magnetic field strength B and current I can be stated:

$$B \propto I \tag{1}$$

By Lenz's law and Faraday's law:

$$V \propto -\frac{d\phi}{dt} \tag{2}$$

where ϕ is the magnetic flux equivalent to :

$$\phi = BA \tag{3}$$

From (2) and (3) and taking the area A to be a constant, the voltage can be rewritten as:

$$V \propto -A\frac{dB}{dt} \tag{4}$$

From substituting (1) into (4), the relation between the voltage across an inductor and the current is established:

$$V \propto \frac{dI}{dt} \tag{5}$$

where V is the self induced emf by the inductor.
The constant of proportionality between these two properties is called the Inductance, also referred to as self inductance and denoted by L. The unit for inductance is the Henry(H)

Similarly, the voltage current relationship for capacitors can also be shown using the capacitance equation. Capacitors[6] store electrical potential in an electric field by maintaining a potential difference across its terminals. Capacitance is the ratio between the charge on one plate and the voltage across the capacitor. The units for capacitance are Farads(F):

$$C = \frac{q}{V} \tag{6}$$

Making V the subject of this equation and differentiating with respect to time yields:

$$\frac{dV}{dt} = \frac{1}{C}I \tag{7}$$

$$I = C\frac{dV}{dt} \tag{8}$$

The current I is the derivative of the charge with respect to time.

[3] Rick Grigalunas. *What Is An Inductor? - ES Components: A Franchised Distributor and Manufacturer*. July 2019. URL: https://www.escomponents.com/blog/2019/7/31/what-is-an-inductor.
[4] R Nave. URL: http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/amplaw.html#:~:text=The%20magnetic%20field%20in%20space,which%20serves%20as%20its%20source..
[5] Michael Bowen-Jones and David Homer. *IB physics*. Oxford University Press, 2014.
[6] Michael Bowen-Jones and David Homer. *IB physics*. Oxford University Press, 2014.

2.1 Reactance of Inductors and Capacitors

The Reactance is the opposition to the flow of current due to the inductive and capacitive effect[7]. It is calculated by the same procedure of resistance which is taking the ratios of the amplitude of the voltage and current and has the same units as resistance, Ohms Ω. Reactance is denoted by the letter X[8]:

$$X = \frac{|V|}{|I|} \tag{9}$$

From (8):

$$I = C\frac{dV}{dt} \tag{10}$$

As a sinusoidal ac supply is being used, the current I with amplitude I_0 is defined as:

$$I = I_0 sin(\omega t) \tag{11}$$

Substituting (11) into (10) yields

$$C\frac{dV}{dt} = I_0 sin(\omega t) \tag{12}$$

$$\frac{dV}{dt} = \frac{I_0}{C} sin(\omega t) \tag{13}$$

$$V = \frac{I_0}{C}\int_{t_0}^{t} sin(\omega t)dt \tag{14}$$

$$V = -\frac{I_0}{\omega C} cos(\omega t) \tag{15}$$

and therefore

$$\frac{V(t)}{I(t)} = -\frac{I_0 cos(\omega t)}{I_0 sin(\omega t) \cdot \omega C} \tag{16}$$

The amplitudes of cosine and sine are 1 and hence the reactance is:

$$X_C = -\frac{1}{\omega C} \tag{17}$$

The reactance for an inductor can be similarly calculated. L is the constant of proportionality in (5) and so

$$V = L\frac{dI}{dt} \tag{18}$$

By letting $I = I_0 sin(\omega t)$ again, the reactance of an inductor can be derived:

$$V = L\frac{d}{dt}\left(I_0 sin(\omega t)\right) \tag{19}$$

$$V = I_0 \omega L cos(\omega t) \tag{20}$$

Taking the amplitudes of the current and voltage yields

$$\frac{V}{I} = \omega L \frac{I_0 cos(\omega t)}{I_0 sin(\omega t)} \tag{21}$$

$$X_L = \omega L \tag{22}$$

[7] *Electrical reactance*. Jan. 2021. URL: https://en.wikipedia.org/wiki/Electrical_reactance#:~:text=In%20electric%20and%20electronic%20systems,for%20the%20same%20voltage%20applied..

[8] *AC Inductance and Inductive Reactance in an AC Circuit*. Apr. 2020. URL: https://www.electronics-tutorials.ws/accircuits/ac-inductance.html.

2.2 Impedance[9]

The resonant frequency of a series RLC circuit is identified by a spike in the circuit current and drop in the impedance of a circuit. The impedance is the circuit's overall opposition to current by considering the opposition of current due to the reactance and resistance. Impedance is a complex quantity denoted by Z where

$$Z = R + X\mathrm{i} \tag{23}$$

where X is the total reactance, R is the resistance

Impedance is analogous to resistance for AC circuits where $\mid Z \mid$ is the ratio of the voltage and current across the circuit while the $arg(Z)$ is equivalent to the phase change between the voltage and current in the circuit. Calculating $\mid Z \mid$ in terms of the angular frequency from (17), (22) and some value for resistance R leads to:

$$\begin{aligned} \mid Z \mid &= \mid R + (X_C + X_L)\mathrm{i} \mid && (24)\\ &= \sqrt{R^2 + (X_C + X_L)^2} && (25)\\ &= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} && (26) \end{aligned}$$

Using :

$$I = \frac{V}{\mid Z \mid} \tag{27}$$

where V is the voltage output
I is the current observed
The equation for the current in terms of the angular frequency can be determined: So the current is:

$$I = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \tag{28}$$

The maximum point of this function can be determined by differentiating it wrt ω and equating the derivative to be 0. Using the chain rule and differentiation properties, the following derivative can be evaluated:

$$\frac{dI}{d\omega} = -\frac{1}{2}\frac{V}{\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)^{\frac{3}{2}}} \cdot 2\left(\omega L - \frac{1}{\omega C}\right) \cdot \left(L + \frac{1}{\omega^2 C}\right) \tag{29}$$

By inspection and the fact that ω^2, C and L can only take positive values, (29) can only be made 0 by equating the middle bracket to 0:

$$0 = \omega L - \frac{1}{\omega C} \tag{30}$$

Making ω the subject of this yields:

$$\omega = \frac{1}{\sqrt{LC}} \tag{31}$$

ω, the angular frequency can be expressed in terms of the frequency f by

$$\omega = 2\pi f \tag{32}$$

Substituting this into (32) and replacing f with the natural frequency f_0 yields:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \tag{33}$$

[9] *Impedance and Reactance.* URL: https://electronicsclub.info/impedance.htm#:~:text=Impedance%20(symbol%20Z)%20is%20a,is%20measured%20in%20ohms%20(%20)..

2.3 Phase difference between the voltage and current[10]

The reason behind the reactance of capacitors and inductors being opposite in sign has to do with the phase change between the voltage and current. From **2.1** for an a.c. sinusoidal supply represented by $I_0 sin(\omega t)$, the voltage across the capacitor is (15) :

$$V_C = -\frac{I_0}{\omega C} cos(\omega t)$$

Considering that:

$$sin\left(\theta - \frac{\pi}{2}\right) = -cos(\theta) \tag{34}$$

A sine function is a $\frac{\pi}{2}$ radians translation of the negative cosine function in the positive x direction meaning the current leads the voltage by $\frac{\pi}{2}$ radians.

The voltage across the Inductor will be

$$V_L = I_0 \omega L cos(\omega t)$$

As:

$$sin\left(\theta + \frac{\pi}{2}\right) = cos(\theta) \tag{35}$$

In an inductor, the current lags the voltage by $\frac{\pi}{2}$ radians.

Figure 3 shows this:

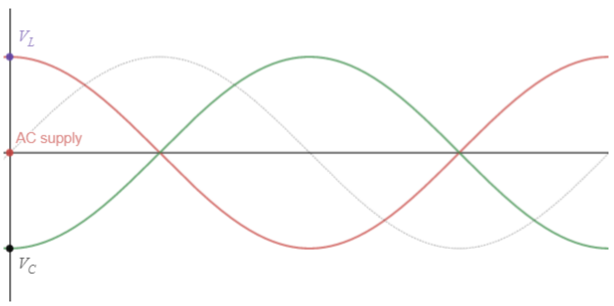

Figure 3: The voltage cycles across the inductor and capacitor for an AC sinusoidal supply

Noticing that the voltages across the inductor and capacitor are always π radians out phase and therefore opposite in direction, a voltage of zero across the two components is justified.

2.4 Hypothesis

As the capacitance of the circuit increases, the resonant frequency should decrease. A higher capacitance means that the capacitor can hold onto more charge for the same voltage and so the current being supplied will also increase proportionally to the increase in capacitance and therefore there will be a decrease in the reactance. This will mean that the reactance of the inductor and capacitor will not be equal and overall voltage across the two components will not be 0. The decrease in reactance of the capacitor means that in order for the reactance of the two components to be equal again, the reactance of the inductor must also decrease. The resonant frequency will therefore decrease so that the reactance of the inductor also decreases (Due to $X_L \propto \omega$). This decrease in frequency will also lead to the capacitive reactance increasing ($X_C \propto \frac{1}{\omega}$) and getting closer to the reactance of the inductor. Decreasing the angular frequency up to the point of maximum current will bring the circuit to its resonant frequency. Also using (33), it is apparent that ideally, increasing the capacitance should decrease the natural frequency, f_0, where $f_0 \propto \frac{1}{\sqrt{C}}$.

This investigation explores how in the real world, the capacitance affects the resonant frequency of a series RLC circuit and how closely this is linked to the theoretical relationship.

[10] *Series RLC Circuit and RLC Series Circuit Analysis*. Apr. 2019. URL: https://www.electronics-tutorials.ws/accircuits/series-circuit.html.

3 Methodology

Initially, an ammeter was going to be connected in series to measure the current against the frequency. The frequency at which the current was maximum would be recorded as the resonant frequency. The **preliminary experiment** was conducted in order to test the efficacy of the method and equipment. This showed the ac ammeters had too low a resolution to calculate the small currents oscillating through the circuit. Therefore, the method changed to measure the voltage across the inductor and capacitor to find the natural frequency. At resonant frequency, the voltage across the inductor and capacitor is 0 volts. Using an oscilloscope, the small voltages across the two components could be measured and the point of minimum voltage could be used.

3.1 Variables

Independent variable : The capacitance of the circuit will be the independent variable and will be changed by physically switching capacitors after each trial. The capacitance values are 1μF,2.2μF, 4.7μF, 10μF, 22μF. This range of capacitance is chosen as it should provide a sufficient range of frequencies from 339Hz to 1591Hz and the difference in theoretical resonant frequencies of each circuit aren't smaller than the instrumental error of the AC signal generator.

Dependent variable: The frequency at which the voltage across the inductor and capacitor is zero using an ac signal generator.

Control variable: For this investigation, the control variables will be factors that affect the resonant frequency of the circuit. If not controlled, the results will be invalid.

Control variable	Why it must be controlled	How it will be controlled.
Inductor	An inductance change will also change the resonant frequency as the reactance changes.	By using the same inductor throughout the investigation.
Temperature of the electrical components in particular inductor and capacitor	Electrolytic capacitors are composed of an insulating thin sheet between the two conducting plates. Temperature will affect the dielectric properties of the capacitor and hence the capacitance.	The resistor generates a large amount of heat especially at resonance frequency so the circuit will be opened for 5 minutes after each reading to allow the resistor to cool down.
Surroundings of the circuit	An external magnetic field in relative motion to the inductor will induce an unwanted potential difference and current across the inductor interfering with the net voltage across the capacitor and inductor therefore increasing the reactance.	It will be insured that the circuit is isolated and no effective magnetic field is present in a close proximity to the circuit e.g.mobile phones will be kept far away.

3.2 Equipment

1μF,2.2μF, 4.7μF, 10μF, 22μF ($\pm 20\%$) capacitors	
10mH ($\pm 30\%$) inductor	
Resistor of a 10 ($\pm 0.5\Omega$) resistance	
AC function generator (± 10,± 100 Hz)	
Oscilloscope (± 1mV)	
Crocodile clips	copper wire
BNC to Banana plug adaptor	

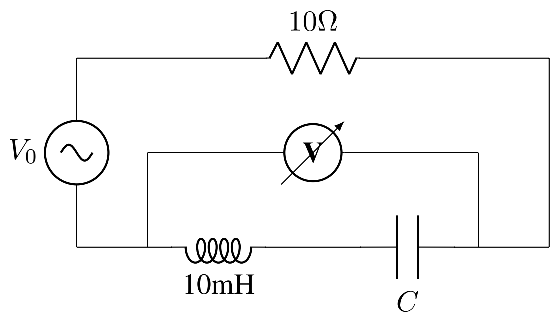

Figure 4: Circuit diagram of the RLC used

3.3 Apparatus

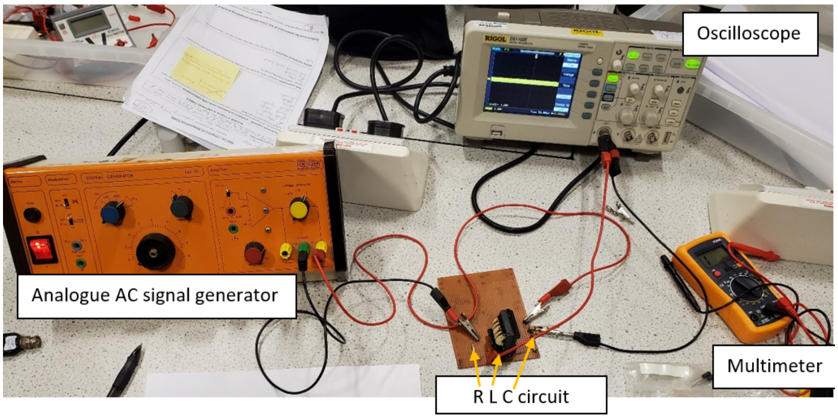

Figure 5: Annotated diagram of setup

3.4 Method

1. The circuit was set up according to Figure 4 with the resistor, inductor, 1μF ($\pm$ 20%) capacitor in series and oscilloscope parallel to the inductor and capacitor. The junctions were soldered and oscilloscope connected using crocodile clips.
2. To connect the oscilloscope, a BNC - banana plug adaptor was used. The BNC connector end was inserted in channel 1 of the oscilloscope and the two wires connected across the inductor and capacitor were connected to the the banana plug end.
3. Pressing the automatic button displayed the voltage signal and reading that needed to be observed. The principal axis of the signal was centered using the vertical position knob. The adjustment of the view window also has to be made so that the wave function's peak to trough can be seen.
4. The frequency was taken to its minimum possible value and for this function generator, this was 0.1Hz.
5. Then, the black frequency knob was rotated clockwise which increased the frequency of the voltage. This decreased the voltage measured across the inductor and capacitor. Initially, the decrease in voltage will be slow.
6. Initially,the black knob was rotated quickly and when the frequency knob reached its maximum at 0.1,1,10,100Hz or 1kHz, rotating the blue knob by 1 position higher increased the order of magnitude of the frequency. The rotation slowed down as the oscilloscope reading approaches to 0 volts. The resonant frequency is the point at which rotating the black knob in either direction (increasing or decreasing the frequency) increases the voltage.
7. Carefully, open the circuit allowing the components to cool to their original temperature for 5 minutes.
8. Repeat steps 2 to 7 with the same capacitor 5 times.
9. Replace the Capacitor with a capacitor of different capacitance and repeat the experiment.
10. For each, capacitance, an average value for the resonant frequency must be taken and then the graph of the frequency against the capacitance must be plotted.

3.5 Risk assessment

Risk	Why it is a risk	How to minimize the risk
High voltages and electrocution risk	When the circuit is opened, and there was current flowing in the circuit, there will be a very large potential difference across the the inductor. In order to maintain the current, a large voltage across the inductor will be induced. This is called inductive kickback.	Wearing rubber gloves and using insulated wire will prevent any risks of electrocution
Heat of the resistor and risks of burns	The resistor opposes the flow of current by and dissipates electrical potential energy as thermal energy, heating up in the process. At resonant frequency, the power output of the resistor will be at its maximum	Again, rubber is a great thermal insulator and therefore will prevent any burns

There are no environmental or ethical risks in conducting this investigation.

4 Results

Capacitance (μF$\pm$20%)	Resonant frequency ($\pm$10,$\pm$100Hz)				
	Trial 1	Trial 2	Trial 3	Trial 4	Average
1.0	2000	1600	1400	1600	1650
2.2	1000	1200	1000	1000	1050
4.7	820	820	800	800	810
10.0	540	560	560	540	550
22.0	280	300	280	280	285

Table 1: Raw data with the average calculated

The absolute error calculations for the resonant frequency is half the range of the values::

$$\Delta f = \frac{f_{\text{max}} - f_{\text{min}}}{2} \tag{36}$$

For the resonant frequency of the RLC with a 1μF capacitor, the absolute uncertainty would be calculated as:

$$\Delta f = \frac{2000 - 1400}{2} = \pm 300\text{Hz}$$

The actual frequency for the 1μF capacitor is 1650Hz and is not rounded to the same place value to the error, the rounded value becomes

$$1700 \pm 300\text{Hz}$$

The uncertainties in the capacitance are calculated by multiplying the capacitance by the instrumental error of the capacitor(20%) e.g. for the 4.7μF capacitor

$$4.7\mu\text{F} \times 0.2 = 0.94\mu\text{F} \rightarrow \pm 0.9\mu\text{F}(1\text{sf})$$

Capacitance (μF)	Δ Capacitance ($\pm\mu$F)	f (Hz)	Δ f ($\pm$Hz)
1.0	0.2	1700	300
2.2	0.4	1100	100
4.7	0.9	810	10
10.0	2.0	550	10
22.0	4.0	290	10

Table 2: Table of processed raw data with uncertainties

4.1 Graphs

Figure 6 shows the graph with the propagated errors for the capacitance and frequency. The uncertainties in the resonant frequency for the last 3 capacitors is too small to display on the graph.

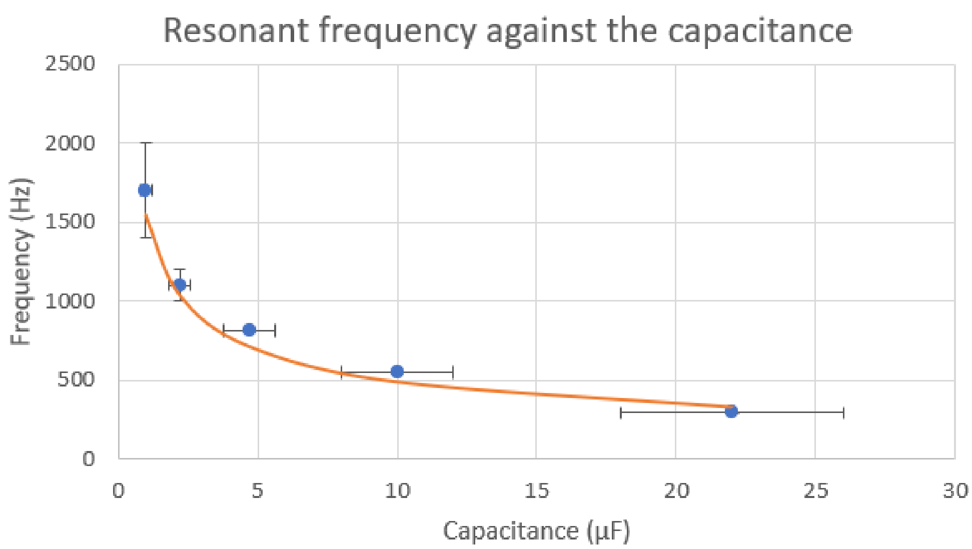

Figure 6: Graph of the resonant frequency against the capacitance

The equation for the natural frequency f is :

$$f = \frac{1}{2\pi\sqrt{LC}}$$
$$f = \left(\frac{1}{2\pi\sqrt{L}}\right)\frac{1}{\sqrt{C}}$$

The errors in the y axis remains unchanged. New errors are propagated for the x axis for the linearised graph. Using the properties for uncertainties, the percentage error in $C^{-\frac{1}{2}}$ can be calculated

$$\frac{\Delta C^{-\frac{1}{2}}}{C^{-\frac{1}{2}}} = \frac{1}{2} \times \frac{\Delta C}{C}$$

where $\frac{\Delta C^{-\frac{1}{2}}}{C^{-\frac{1}{2}}}$ is 20% for all the capacitors. And so the absolute uncertainty in $C^{-\frac{1}{2}}$ becomes

$$\Delta C^{-\frac{1}{2}} = \frac{1}{2} \times 20\% \times C^{-\frac{1}{2}}$$

Here is an example with the 4.7μF capacitor:

$$\Delta C^{-\frac{1}{2}} = \frac{1}{2} \times 20\% \times (4.7 \times 10^{-6})^{-\frac{1}{2}} = 46.1265 \rightarrow \pm 50\ (\mathrm{F})^{-\frac{1}{2}}\ (1\mathrm{sf})$$

Capacitance,C μF)	$\mathrm{C}^{-\frac{1}{2}}$ ($\mathrm{F}^{-\frac{1}{2}}$)	ΔC ($\pm\mathrm{F}^{-\frac{1}{2}}$)
1	1000	100
2.2	670	70
4.7	460	50
10	320	30
22	210	20

Table 3: Table of the capacitance uncertainties for the linearised graph

Theoretically, plotting the resonant frequency against $\frac{1}{\sqrt{C}}$ yields a linear relation as shown in Figure 7:

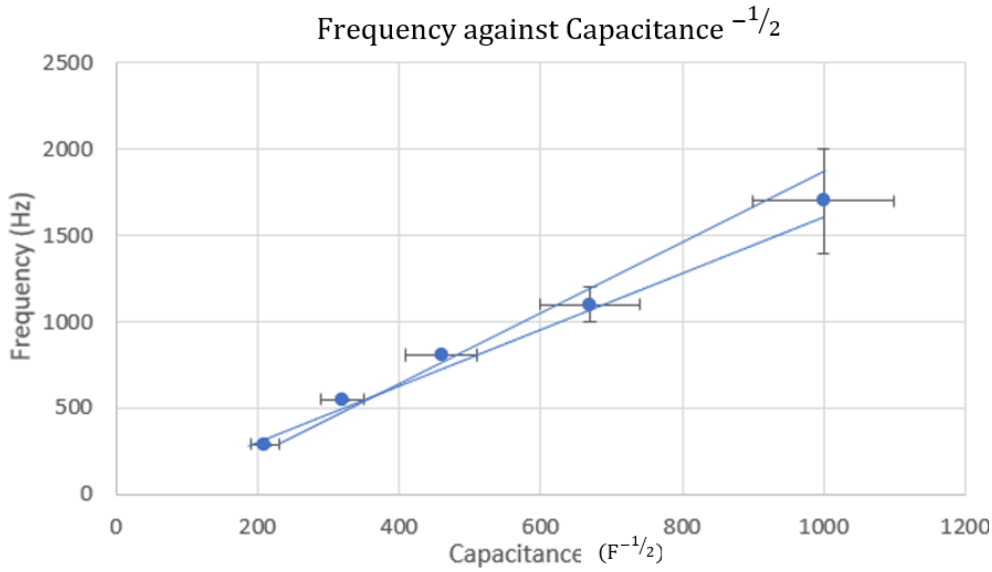

Figure 7: Graph showing the linear relationship between the resonant frequency and $\frac{1}{\sqrt{C}}$

Two lines with maximised and minimised slopes can be drawn through all the points with the maximum and minimum gradients m as:

$$m_{\text{min}} : 1.63,\ m_{\text{max}} : 2.07$$
$$m : 1.8 \pm 0.2$$

The y intercept c should ideally be 0 due to the direct proportionality however:

$$c_{\text{min}} : -185.6\text{Hz}, c_{\text{max}} : -18.8\text{Hz}$$
$$c : -100 \pm 80\text{Hz}$$

This indicates a systematic error.

5 Conclusion

This investigation has shown that an increase in the capacitance of an series RLC circuit leads to the decrease in the resonance frequency of the circuit. For example, increasing the capacitance of the capacitor from 4.7μF to 22μF decreases the natural frequency from 810Hz to 290Hz. Changing the x axis from C to $C^{-\frac{1}{2}}$ showed a linear relationship which satisfies the theoretically expected correlation. The linearised equation was:

$$f = 1.8(\pm 0.2)C^{-\frac{1}{2}} - 100(\pm 80)\text{Hz}$$

and the expected linearised curve has the equation

$$\begin{aligned} L &= 10\text{mH} \pm 30\% \\ \frac{1}{2\pi\sqrt{L}} &= 1.59\text{H}^{-\frac{1}{2}} \pm 15\% \\ &= 1.6 \pm 0.2\text{H}^{-\frac{1}{2}} \\ f &= 1.6(\pm 0.2)C^{-\frac{1}{2}} \end{aligned}$$

The two gradients overlap, showing that the experiment results were very accurate in showing the exact relationship that the capacitance and resonant frequency obey. The y intercept is evidence for a large systematic error where there is some error in the method or apparatus that is consistently being repeated.

In terms of precision, the only data value that had an abnormally large lack of precision was the resonant frequency for the 1μF capacitor where the frequency was 1700 ±300Hz meaning there was a percentage uncertainty of 17.6%, this is due to the instrumental error of the oscilloscope.

Figure 8 compares the values obtained for the resonant frequency and the literature values. The vertical error bars in the literature values are as a result of the uncertainty in the inductor and capacitor.

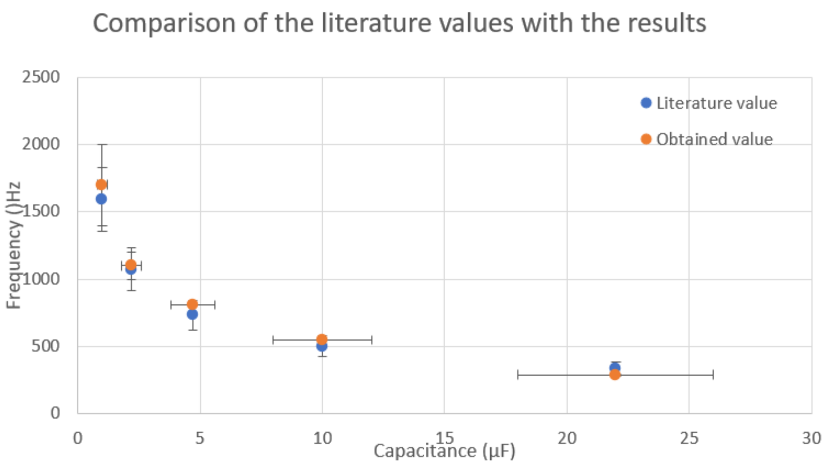

Figure 8: Graph comparing this investigation's results and the literature value

Clearly, all the values apart from the frequencies for the 22μF capacitor have a consistent overestimation of the values. The 22μF capacitor was underestimated most likely due to a random error.

6 Evaluation

The final 22μF capacitor reading is not anomalous however contains a random error as it is the only underestimate. The systematic error in the results does not affect the accuracy in measuring the correlation however the specific results contained inaccuracies. There are multiple reasons for this:

- Theoretically at resonance, the voltage across the inductor and capacitor is 0. However, the inductor, capacitor, wiring and solder have resistance therefore the voltage cannot be and wasn't 0. The minimum value voltage being reached was approximately 300mV for every value and made judging the minimum point by eye difficult increasing the scope for random error. On the oscilloscope, the voltage remained at a minimum for a narrow range of frequencies. To minimise this error, the highest frequency at which the voltage was its minimum was recorded. This may have caused the systematic error and overestimation of resonant frequencies. Decrease the bandwidth of the circuit will minimise this error. This is done by decreasing the resistance in the circuit and means the voltage has a steeper dip and makes recognising this downward spike in the voltage easier. **Further work** shows the theoretical background for this.

- The instrumental error of the analogue AC signal generator meant that the values that have been obtained for the resonant frequency are to the nearest 200Hz and 20Hz, as there were 5 graduations for every kHz or 100Hz. A digital function generator has a much higher resolution e.g. 1Hz resolution as compared to 200Hz meaning the accuracy of the resonant frequency would be far greater.

- The tolerance or the uncertainty of 30% on the 10mH inductor was extremely large. As the uncertainty of inductors decreases, their prices increases. However using an inductor with a much lower error will verify the results obtained.

- Another source of error is the leakage inductance[11]. This is where the imperfect winding of the inductor used leads to the magnetic field not changing proportionately with the current and therefore the voltage across the inductor not behaving ideally. Ways to prevent this would be to recoil the inductor so that the coil windings do not overlap and are evenly spaced.

There were certain strengths in the investigation. Most error bars on the graph were small meaning there was no scope for misinterpreting the relation that the resonant frequency and capacitance have e.g. no linear graph could be drawn. The experiment kept the temperature constant by having a 5 minute break between each reading. This proved useful as the resistor got extremely hot at resonance and heated its surroundings. At one point, the resistor overloaded and was replaced.
Also, the relationship between how the resonant frequency changes with capacitance is answered effectively. In fact, the average gradient, m, which was obtained through linearizing the Capacitance-Frequency leads to an accurate prediction of the inductance of the circuit:

$$m = \frac{1}{2\pi\sqrt{L}} \tag{37}$$

rearranging leads to:

$$\begin{aligned} L &= \frac{1}{4\pi^2 m^2} \\ m &= 1.8 \pm 0.2 \\ L &= 0.0078 \pm \Delta L \text{ H} \end{aligned}$$

As:

$$\begin{aligned} \frac{\Delta L}{L} &= 2\frac{\Delta m}{m} \\ \Delta L &= 2L\frac{\Delta m}{m} \\ \frac{\Delta m}{m} &= \frac{0.2}{1.8} \end{aligned}$$

From this, the inductance can be calculated:

$$\begin{aligned} L &= 0.008 \pm 0.002\text{H} \\ &= 8 \pm 2 \text{ mH} \end{aligned}$$

[11] *Voltech*. URL: `https://www.voltech.com/Articles/104-105/1_What_is_Leakage_Inductance`.

The true inductance lies within the range of the calculated inductance which shows how in the real world, the relationship between the capacitance and resonant frequency holds. This relationship is crucial in telecommunication as the way in which the capacitance must be varied to control the resonant frequency needs to be known in order to tune any circuit.

6.1 Further Work

Following on from this investigation, testing the quality of filtering could also be useful. The bandwidth, measured in Hz, is the frequency range at which the power output is at least half of the maximum power output. Lower the bandwidth, the narrower the Current - frequency peak and finer the tuning. To carry out this investigation, the current needs to be measured against frequency. An ammeter needs to be placed in series with the series RLC and the continuous graph of the current against the frequency must be plotted. Then, the $0.7071I_{peak}$ mark must be made on the y axis, where I_{peak} is the maximum current, from which two corresponding frequencies are obtained. The difference between these two frequencies gives the bandwidth[12]. A potential investigation could be, how does the resistance affect the bandwidth of a series RLC. The theoretical background of this involves using the current - angular frequency graph (28). To calculate the two end points of the bandwidth on the current- angular frequency graph, ω_0 and ω_1

$$I_{peak} = \frac{V}{R}$$
$$I = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$
$$\frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V}{\sqrt{2}R}$$

Now making ω the subject

$$0 = \omega^2 L \pm R\omega - \frac{1}{C}$$

Using the quadratic formula

$$\omega = \frac{\pm R \pm \sqrt{R^2 + \frac{4L}{C}}}{2L}$$

ω_1 and ω_0 are the two boundaries of the bandwidth

$$\omega_1 = \frac{R + \sqrt{R^2 + \frac{4L}{C}}}{2L}$$
$$\omega_0 = \frac{-R + \sqrt{R^2 + \frac{4L}{C}}}{2L}$$

The bandwidth Δf becomes[13]

$$\Delta\omega = \omega_1 - \omega_0 = \frac{R}{L}$$
$$\Delta f = \frac{R}{2\pi L}$$

This theoretically yields a linear relationship between the bandwidth and resistance.

[12] *Series Resonance in a Series RLC Resonant Circuit.*
[13] *Series Resonance in a Series RLC Resonant Circuit.*

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