Supplement For

Beauty of

VEDIC SPEED MATHEMATICS

(Journey from Limited Intelligence to Human Bio-Calculator)

.

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Paperback, eBook & **Video Course** on Vedic Speed Mathematics
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Note: We keep updating this file (adding new examples, new graphical representations etc.). Keep visiting for updates.

MULTIPLICATION

Multiplication using Base Method

Case 1: When both numbers (multiplicand and multiplier) are less than the working base:

Ex.1: 7×8	**Ex.2: 6×7**	**Ex.3: 94×96**	**Ex.4: 90×89**	**Ex.5: 997×993**	**Ex.6: 950×930**
B:10	**B:10**	**B:100**	**B:100**	**B:1000**	**B:1000**
7 -3 8 -2 ------ 5 | 6	6 -4 7 -3 -------- 3 | 12 3+1 | 2 4 | 2	94 -6 96 -4 --------- 90 | 24	90 -10 89 -11 ---------- 79 | 110 79+1 | 10 80 | 10	997 -3 993 -7 ---------- 990 | 021	950 -50 930 -70 ---------- 880 | 3500 880+3 | 500 883 | 500
56	**42**	**9024**	**8010**	**990021**	**883500**

Multiplication using Criss Cross Method

Case 1: Two Digit Numbers (2D×2D and 2D×1D) {D: Digit}

Graphical Representation:
Answer consists of three parts.

First Part:	**Second Part:**	**Third Part:**
a b c d	a b c d	a b c d
(a×c)	**(a×d + b×c)**	**(b×d)**

Case 2: Three Digit Numbers (3D×3D; 3D×2D and 3D×1D)

Answer consists of Five Parts.

First Part:	Second Part:	Third Part:	Fourth Part:	Fifth Part:
a b c d e f	a b c d e f	a b c d e f	a b c d e f	a b c d e f
(a×d)	**(a×e + b×d)**	**(a×f + b×e + c×d)**	**(b×f + c×e)**	**(c×f)**

CASE 3: (4×3; 4×3; 4×2; 4×1)

Answer consists of seven parts.

First Part:	Second Part:	Third Part:	Fourth Part:
a b c d e f g h	a b c d e f g h	a b c d e f g h	a b c d e f g h
(a×e)	**(a×f) + (b×e)**	**(a×g) + (b×f) + (c×e)**	**(a×h) + (b×g) + (c×f) + (d×e)**

Fifth Part:	Sixth Part:	Seventh Part:
a b c d e f g h	a b c d e f g h	a b c d e f g h
(b×h) + (c×g) + (d×f)	**(c×h) + (d×g)**	**(d×h)**

CASE 4: (5×5; 5×4; 5×3; 5×2; 5×1)

Answer consists of nine parts.

First Part:	**Second Part:**	**Third Part:**
a b c d e f g h i j	a b c d e f g h i j	a b c d e f g h i j
(a×f)	**(a×g)+(b×f)**	**(a×h)+(b×g)+(c×f)**

Fourth Part:	**Fifth Part:**	**Sixth Part:**
a b c d e f g h i j	a b c d e f g h i j	a b c d e f g h i j
(a×i)+(b×h)+ (c×g)+(d×f)	**(a×j)+(b×i)+(c×h)+ (d×g)+ (e×f)**	**(b×j)+(c×i)+ (d×h)+(e×g)**

Seventh Part:	**Eighth Part:**	**Nineth Part:**
a b c d e f g h i j	a b c d e f g h i j	a b c d e f g h i j
(c×j)+ (d×i)+ (e×h)	**(d×j)+ (e×i)**	**(e×j)**

CASE 5: (6×6; 6×5; 6×4; 6×3; 6×2; 6×1) (Do it Yourself)

Answer consists of eleven parts.

First Part:	**Second Part**	**Third Part:**	**Fourth Part**	**Fifth Part:**
a b c d e f g h i j k l	a b c d e f g h i j k l	a b c d e f g h i j k l	a b c d e f g h i j k l	a b c d e f g h i j k l

6th Part:	**7th Part:**	**8th Part:**	**9th Part:**	**10th Part:**	**11th Part**
a b c d e f g h i j k l	a b c d e f g h i j k l	a b c d e f g h i j k l	a b c d e f g h i j k l	a b c d e f g h i j k l	a b c d e f g h i j k l

DIVISION

Vinculum Number: A number that has atleast one vinculum digit is called vinculum number. Notation: Either dotted or dash above the number. **Ex.** $1\ddot{4}2$; $\ddot{9}641$;$2\bar{2}3$; $6\bar{2}3\bar{8}$; $8\bar{4}\bar{2}\bar{3}$; $\bar{8}$; $\bar{3}$

$1\bar{4}2$=100-40+2=62;
$\bar{9}641$=-9000+600+40+1=-8359
$2\bar{2}3$=200-20+3=183
$6\bar{2}3\bar{8}$=6000-200+30-8=5822
$8\bar{4}\bar{2}\bar{3}$=8000-400-20-3=7577
$\bar{8}$=-8
$\bar{3}$=-3

Division using Transpose and Apply

Ex.1:123÷11; Here Base=10 and Surplus=1; Negation of Surplus=-1= $\dddot{1}$

Step 1:	**Step 2:**	**Step 3:**	**Step 4:**	**Step 5:**
11) 1 2 ¦ 3 -1	11) 1 2 ¦ 3 -1 ------------ 1	11) 1 2 ¦ 3 -1 -1 ------------ 1	11) 1 2 ¦ 3 -1 -1 ------------ 1 1 ¦	11) 1 2 ¦ 3 -1 -1 ¦ -1 ------------- 1 1 ¦ 2

Answer: 11 (Q); 2 (R)

Ex.2: 1332÷114

B: 100; S: 14

114) 1 3 ¦ 3 2

$\dddot{\mathbf{1}}\ \dddot{\mathbf{4}}$ $\dddot{\mathbf{1}}$ ¦ $\dddot{\mathbf{4}}$

¦ $\dddot{\mathbf{2}}$ $\dddot{\mathbf{8}}$

----

1 2 ¦ $\dddot{\mathbf{3}}$ $\dddot{\mathbf{6}}$

(12-1) ¦ (114-36)

11 ¦ 78

Q: **11**; R: **78**

Division by FLAG (ध्वजांक) Method

Ex.2: Divide 949 by 22

Step 1	Step 2	Step 3	Answer
2^2 # $9_1 4 \mid 9$ 4 14-(2*4)=14-8=6 6÷2=3(Q) 0(R)	2^2 # $9_1 4 \mid_0 9$ 4 3 9-(2*3)=9-6=3 3÷2=1(Q) 1(R)	2^2 # $9_1 4 \mid_0 9_1$ 4 3|1	**43.1** (Upto 1 Decimal Point)

DECIMALS, FRACTIONS AND PERCENTAGES

Fractions: Is an expression that indicates the quotient of two quantities. **Ex.** $\boldsymbol{\frac{p}{q}}$**;** $\frac{1}{2}$; $\frac{1}{3}$; $\frac{42}{2.1}$; $\frac{445.61}{2.64}$; $\frac{785}{2}$; $\frac{36}{3}$; - $\frac{40}{20}$; - $\frac{1}{9}$ etc. Upper part is Numerator (N) and Lower part is Denominator. The Denominator can not be zero.

Basic Operations on Decimals

Addition:

Ex.1: 0.32+13.35+45.058+**696.368**+31.004

0.32 (320/1000);
13.35 (13350/1000);
45.058 (45058/1000);
696.368 (696368/1000);
31.004 (31004/1000)

$$\frac{320}{1000}+\frac{13350}{1000}+\frac{45058}{1000}+\frac{696368}{1000}+\frac{31004}{1000}=\frac{786100}{1000}=786.1$$

Ex.2: 143+365.9+**0.04**+**36.02**+**6986.36**+7469.3

143 (14300/100);
365.9 (36590/100);
0.04 (4/100);
36.02 (3602/100);
6986.36 (698636/100);
7469.3(746930/100)

$$\frac{14300}{100}+\frac{36590}{100}+\frac{4}{100}+\frac{3602}{100}+\frac{698636}{100}+\frac{746930}{100}=\frac{1500062}{100}$$
$$=15000.62$$

Subtraction:

Ex.1: 133.45-45.058-6.368-31.004

133.45 (133450/1000);

45.058 (45058/1000);

6.368 (6368/1000);

31.004 (31004/1000)

$$\frac{133450}{1000}-\frac{45058}{1000}-\frac{6368}{1000}-\frac{31004}{1000}=\frac{51020}{1000}=51.02$$

Ex.2: 143-65.9-0.004-36.002-6.986-7.3

143 (143000/1000);

65.9 (65900/1000);

0.004 (4/1000);

36.002 (36002/100);

6.986 (6986/1000);

7.3(7300/1000)

$$\frac{143000}{1000}-\frac{65900}{1000}-\frac{4}{1000}-\frac{36002}{1000}-\frac{6986}{1000}-\frac{7300}{1000}=\frac{26808}{1000}$$
$$=26.808$$

Multiplication:

Ex.1: **3.6×6.8**	**Ex.2:** **43.63×34.2**	**Ex.3:** **42.36×67.363**	**Ex.4:** **3.6×63.68×1.3**
$\frac{36}{10}\times\frac{68}{10}$ $=\frac{2448}{100}$ $=24.48$	$\frac{4363}{100}\times\frac{3420}{100}$ $=\frac{14921460}{10000}$ $=1492.146$	$\frac{42360}{1000}\times\frac{67363}{1000}$ $=\frac{2853496680}{1000000}$ $=2853.49668$	$\frac{360}{100}\times\frac{6368}{100}\times\frac{130}{100}$ $=\frac{298022400}{1000000}$ $=298.0224$
24.48	**1492.146**	**2853.49668**	**298.0224**

Basic Operations on Fractions:

Addition:

Case 1: When Denominators are same: Just add numerators; place denominator as it is and Reduce.

Case 2: When one Denominator is factor of another: Make same denominator by multiplying lower denominator and its numerator with the factor and add using case 1.

Case 3: Cross Product: Numerator: sum of cross products; Denominator: product of both denominators.

Ex.1:	**Ex.2:**	**Ex.3:**	**Ex.4:**
Case:**1**	Case:**2**	Case:**3**	Case:**3**
$\frac{68}{10}+\frac{36}{10}$ $\frac{68+36}{10}$ $\frac{104}{10}$ $\frac{52}{5}$	$\frac{150}{21}+\frac{1350}{63}$ $\frac{150\times 3}{21\times 3}+\frac{1350}{63}$ $\frac{450}{63}+\frac{1350}{63}$ $\frac{450+1350}{63}$ $\frac{1800}{63}=\frac{200}{7}$	$\frac{35}{16}+\frac{12}{20}$ $\frac{(35\times 20)\ +(16\times 12)}{(16\times 20)}$ $\frac{(700)\ +(192)}{(320)}$ $\frac{892}{320}$ $\frac{223}{80}$	$\frac{36}{10}+\frac{25}{12}+\frac{11}{14}$ $\frac{(36\times 12)\ +(10\times 25)}{(10\times 12)}+\frac{11}{14}$ $\frac{682}{120}+\frac{11}{14}$ $\frac{(682\times 14)\ +(120\times 11)}{(120\times 14)}$ $\frac{10868}{1680}=\frac{2717}{420}$
$\mathbf{\frac{52}{5}}$	$\mathbf{\frac{200}{7}}$	$\mathbf{\frac{223}{80}}$	$\mathbf{\frac{2717}{420}}$

Subtraction:

Case 1: When Denominators are same: Just subtract numerators; place denominator as it is and Reduce.

Case 2: When one Denominator is factor of another: Make same denominator by multiplying lower denominator and its numerator with the factor and subtract using case 1.

Case 3: Cross Product: Numerator: Difference of cross products; Denominator: product of both denominators.

Ex.1:	**Ex.2:**	**Ex.3:**	**Ex.4:**
Case:**1**	Case:**2**	Case:**3**	Case:**3**
$\frac{68}{10}-\frac{36}{10}$ $\frac{68-36}{10}$ $\frac{32}{10}$ $\frac{16}{5}$	$\frac{150}{21}-\frac{1350}{63}$ $\frac{150\times 3}{21\times 3}-\frac{1350}{63}$ $\frac{450}{63}-\frac{1350}{63}$ $\frac{450-1350}{63}$ $-\frac{900}{63}=-\frac{100}{7}$	$\frac{35}{16}-\frac{12}{20}$ $\frac{(35\times 20)\ -(16\times 12)}{(16\times 20)}$ $\frac{(700)\ -(192)}{(320)}$ $\frac{508}{320}=\frac{127}{80}$	$\frac{36}{10}-\frac{25}{12}-\frac{11}{14}$ $\frac{(36\times 12)\ -(10\times 25)}{(10\times 12)}-\frac{11}{14}$ $\frac{182}{120}-\frac{11}{14}$ $\frac{(182\times 14)\ -(120\times 11)}{(120\times 14)}$ $\frac{1228}{1680}=\frac{307}{420}$
$\mathbf{\frac{16}{5}}$	$-\mathbf{\frac{100}{7}}$	$\mathbf{\frac{127}{80}}$	$\mathbf{\frac{307}{420}}$

Multiplication:

Option 1: Multiply Numerator with numerator and denominator with denominator and REDUCE.

Option 2: Write factors and cancel common factors. Then Multiply Numerator with numerator and denominator with denominator and REDUCE.

Ex.1:	**Ex.2:**	**Ex.3:**
$\frac{36}{10}\times\frac{68}{10}$ $=\frac{2448}{100}$ $=\frac{612}{25}$	$\frac{84}{44}\times\frac{66}{42}$ $=\frac{21\times 4}{22\times 2}\times\frac{22\times 3}{21\times 2}$ $=\frac{1}{1}\times\frac{3}{1}=3$	$\frac{360}{210}\times\frac{630}{60}\times\frac{30}{90}$ $=\frac{60\times 6}{210}\times\frac{210\times 3}{60}\times\frac{30}{3\times 30}$ $=\frac{6}{1}\times\frac{1}{1}\times\frac{1}{1}=6$
$\mathbf{\frac{612}{25}}$	**3**	**6**

Division: Write the first set as it is; replace ÷ (division sign) by × (multiplication sign); Exchange Numerator and Denominator of second set; Perform Multiplication.

$$\frac{N1}{D1} \div \frac{N2}{D2} = \frac{N1}{D1} \times \frac{D2}{N2}$$

$$\frac{84}{44} \div \frac{42}{66} = \frac{84}{44} \times \frac{66}{42} = \frac{21 \times 4}{22 \times 2} \times \frac{22 \times 3}{21 \times 2} = \frac{4}{2} \times \frac{3}{2} = 3$$

Reciprocals

Ending in 9:

Ex. 1: $\mathbf{\frac{1}{19}}$?

A: Denominator is 19. Positive Osculator of 19 is 2. (Go through "Divisibility" Unit to understand osculator concept).

$$\frac{1}{19} \approx \frac{1}{20} = \frac{0.1}{2}$$

Now for $\frac{0.1}{2}$

Divisor	2	2	2	2	2	2	2	2	2	2	2	2	2
Dividend	0.1	1	10	-	5	12	6	3	11	15	17	18	9
Quotient	-	**0.**	**0**	**5**	**2**	**6**	**3**	**1**	**5**	**7**	**8**	**9**	**4**
Remainder	-	-	-	0	1	0	0	1	1	1	1	0	1

Divisor is 2 in all cases. Initial Dividend is 0.1. We can not divide 0.1 by 2 so in quotient we need to give decimal point. New Dividend is 1. Again we can not divide 1 by 2 so in quotient we need to add zero. New Dividend is 10. We divide 10 by 2. Q is 5 and R is 0. New Dividend is RQ (05). Remainder and then append Quotient. 5÷2; Q=2; R=1; Next Dividend=12 (∵R=1 & Q=2). Like this we go on dividing. Final answer: Quotient Row.

$$\frac{1}{19} = \mathbf{0.052631578} \ldots.$$

POLYNOMIALS

Multiplication using Criss Cross Method

Note: Read Multiplication using Criss Cross or Vertically & Crosswise Method from Multiplication Unit

Steps:

1. Write coefficients of given polynomials one below the other separated by space or vertical bar.
2. Multiply coefficients using formula (refer formula OR graphical representation).
3. Last part is constant. Go on incrementing powers of variable by 1 from right. Second last is x, then x^2, x^3, x^4, x^5 and so on.
   Note: Write coefficient as zero if any term is absent.

CASE 1: (2×2; 2×1)

First Part:	Second Part:	Third Part:
a b c d	a b c d	a b c d
(a×c)	**(a×d + b×c)**	**(b×d)**

CASE 2: (3×3; 3×2; 3×1)

First Part:	Second Part:	Third Part:	Fourth Part:	Fifth Part:
a b c d e f	a b c d e f	a b c d e f	a b c d e f	a b c d e f
(a×d)	**(a×e + b×d)**	**(a×f + b×e + c×d)**	**(b×f + c×e)**	**(c×f)**

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CASE 3: (4×3; 4×3; 4×2; 4×1)

First Part:	Second Part:	Third Part:	Fourth Part:
a b c d e f g h	a b c d e f g h	a b c d e f g h	a b c d e f g h
(a×e)	**(a×f) + (b×e)**	**(a×g) + (b×f) + (c×e)**	**(a×h) + (b×g) + (c×f) + (d×e)**

Fifth Part:	Sixth Part:	Seventh Part:
a b c d e f g h	a b c d e f g h	a b c d e f g h
(b×h) + (c×g) + (d×f)	**(c×h) + (d×g)**	**(d×h)**

Ex.1: $(x^3+5x^2+3x+2)\ (2x^3-4x^2-7x+3)$
1 5 3 2 2 -4 -7 3 (1×2) | (1×-4)+(5×2) | (1×-7)+(5×-4)+(3×2) | (1×3)+(5×-7)+(3×-4)+(2×2) | (5×3)+(3×-7)+(2×-4) | (3×3)+(2×-7) | (2×3) 2 | 6 | -21 | -40 | -14 | -5 | 6 $2x^6+6x^5-21x^4-40x^3-14x^2-5x+6$
$2x^6+6x^5-21x^4-40x^3-14x^2-5x+6$

CASE 4: (5×5; 5×4; 5×3; 5×2; 5×1)

First Part:	Second Part:	Third Part:
a b c d e f g h i j	a b c d e f g h i j	a b c d e f g h i j
(a×f)	**(a×g)+(b×f)**	**(a×h)+(b×g)+(c×f)**

Fourth Part:	Fifth Part:	Sixth Part:
a b c d e f g h i j	a b c d e f g h i j	a b c d e f g h i j
(a×i)+(b×h)+ (c×g)+(d×f)	**(a×j)+(b×i)+(c×h)+ (d×g)+ (e×f)**	**(b×j)+(c×i)+ (d×h)+(e×g)**

Seventh Part:	Eighth Part:	Nineth Part:
a b c d e f g h i j	a b c d e f g h i j	a b c d e f g h i j
(c×j)+ (d×i)+ (e×h)	**(d×j)+ (e×i)**	**(e×j)**

Ex.1: $(2x^4+3x^3+3x^2+2x+4)$ $(3x^4-2x^3+4x^2-7x-8)$ (Do it Yourself)
2 3 3 2 4 3 -2 4 -7 -8 | | | | | | | | |
$6x^8+5x^7+11x^6-2x^5-17x^4-45x^3-22x^2-44x-32$

CASE 5: (6×6; 6×5; 6×4; 6×3; 6×2; 6×1) (Do it Yourself)

First Part:	Second Part:	Third Part:	Fourth Part:	Fifth Part:
a b c d e f g h i j k l	a b c d e f g h i j k l	a b c d e f g h i j k l	a b c d e f g h i j k l	a b c d e f g h i j k l

Sixth Part:	Seventh P:	Eighth P:	Nineth P:	Tenth P:
a b c d e f g h i j k l	a b c d e f g h i j k l	a b c d e f g h i j k l	a b c d e f g h i j k l	a b c d e f g h i j k l

Ex.1: $(3x^5-2x^4-4x^3+2x^2-3x+3)\ (4x^5-6x^4+3x^3-2x^2+6x+2)$
3 -2 -4 2 -3 3 4 -6 3 -2 6 2 | | | | | | | | | | |
$12x^{10}-26x^9+5x^8+20x^7-14x^6+38x^5-59x^4+19x^3-20x^2+12x+6$

Division using Transpose and Apply

Ex.1: $(x^3+9x^3+20x+12) \div (x+1)$

$$\begin{array}{l|rrrr} \frac{(x+1)}{-1} & x^3 & +9x^3 & +20x & +12) \\ & & -1 & -8 & -12 \\ \hline & 1 & 8 & 12 & \mathbf{0} \end{array}$$

Q: $x^2+8x+12=(x+2)(x+6)$ **R:** 0

Q: $x^2+8x+12=(x+2)(x+6)$ **R:** 0

Ex.3: $(3x^4-2x^3+x^2-2x+3) \div (x^2-2x+6)$

$$\begin{array}{l|rrrrr} \frac{(x^2-2x+6)}{+2 \mid -6} & 3x^4 & -2x^3 & +x^2 & -2x & +3 \\ & & +6 & -18 & & \\ & & & 8 & -24 & \\ & & & & -18 & 54 \\ \hline & 3 & +4 & -9 & \mathbf{-44} & \mathbf{+57} \end{array}$$

Q: $3x^2+4x-9$ **R:** $-44x+57$

Q: $3x^2+4x-9$ **R:** $-44x+57$

Ex.4: $(2x^5+2x^4-x^3+x^2-2x+2) \div (x^2+3x-4)$

$$\begin{array}{l|rrrrrr} \frac{(x^2+3x-4)}{-3 \mid +4} & 2x^5 & +2x^4 & -x^3 & +x^2 & -2x & +2 \\ & & -6 & 8 & & & \\ & & & 12 & -16 & & \\ & & & & -57 & 76 & \\ & & & & & 216 & -288 \\ \hline & 2 & -4 & 19 & -72 & 290 & -286 \end{array}$$

Q: $2x^3-4x^2+19x-72$ **R:** $290x-286$

Q: $2x^3-4x^2+19x-72$ **R:** $290x-286$

SIMPLE EQUATIONS

Simple Algebraic Equations contains only one variable.

	TYPE-1	TYPE-2
General Form	px+q=rx+s	(x+p) (x+q) =(x+r) (x+s)
Value of x	$x = \frac{s-q}{p-r}$	if p×q=r×s then **x=0** else $x = \frac{rs-pq}{p+q-r-s}$
Ex.1	5x+3=3x+9 $x = \frac{9-3}{5-3}$ $= \frac{6}{2} = \mathbf{3}$	(x+4) (x+3) = (x+2) (x+6) p×q=r×s 4×3=2×6 => 12=12 ∴ **x=0**
Ex.2	3x-3=4x+7 $x = \frac{7-(-3)}{3-4} = \frac{10}{-1} = \mathbf{-10}$	(x+3) (x+5) = (x+2) (x+4) p×q≠r×s $x = \frac{2\times4-3\times5}{3+5-2-4}$ $= -\,\mathbf{\frac{7}{2}}$
Ex.3	6x+6=8x+8 $x = \frac{8-6}{6-8} = \frac{2}{-2} = \mathbf{-1}$ **OR** 6x+6=8x+8 6(x+1)=8(x+1) x+1=0 **x=-1**	(x-2) (x+3) = (x+4) (x-5) p×q≠r×s $x = \frac{4\times(-5)-(-2)\times3}{-2+3-4-(-5)}$ $= -\,\mathbf{7}$

	TYPE-3	**TYPE-4**
General Form	$\frac{px+q}{rx+s} = \frac{t}{u}$	$\frac{a}{x+p} + \frac{b}{x+q} = 0$
Value of x	$x = \frac{ts-uq}{up-tr}$	$x = -\frac{aq+bp}{a+b}$
Ex.1	$\frac{2x+3}{3x+5} = \frac{3}{2}$ $x = \frac{3\times5-2\times3}{2\times2-3\times3} = -\mathbf{\frac{9}{5}}$	$\frac{3}{x+2} + \frac{4}{x+3} = 0$ $x = -\frac{3\times3+4\times2}{3+4} = -\mathbf{\frac{17}{7}}$
Ex.2	$\frac{x-4}{2x+2} = \frac{7}{3}$ $x = \frac{7\times2-3\times(-4)}{3\times1-7\times2} = -\mathbf{\frac{26}{11}}$	$\frac{5}{x-3} + \frac{6}{x+4} = 0$ $x = -\frac{5\times4+6\times-3}{5+6} = -\mathbf{\frac{2}{11}}$
Ex.3	$\frac{3x-2}{x-3} = \frac{4}{3}$ $x = \frac{4(-3)-3(-2)}{3\times3-4\times1} = -\mathbf{\frac{6}{5}}$	$\frac{4}{x+2} + \frac{4}{x+3} = 0$ $x = -\frac{4\times3+4\times2}{4+4} = -\mathbf{\frac{5}{2}}$ **OR** x+2+x+3=0 2x+5=0 x= **- 5/2**

Solution using "If the Set is same, it is ZERO"

4.

A. If N1+N2=D1+D2 then

N1+N2=D1+D2=0

$$\frac{3x+5}{5x+4} = \frac{4x+6}{2x+7}$$

3x+5+4x+6=**7x+11**

5x+4+2x+7=**7x+11**

∴ 7x+11=0

x = **-** $\mathbf{\frac{11}{7}}$

B. If **m**(N1+N2)=**n**(D1+D2) then

N1+N2=D1+D2=0 (**m,n**=common factors)

$$\frac{7x+2}{2x+1} = \frac{5x+6}{x+1}$$

7x+2+5x+6=**12x+8=4(3x+2)**

2x+1+x+1=**3x+2=1(3x+2)**

∴ 3x+2=0

x = $-\frac{2}{3}$

C. If **m**(N1-D1)=**n**(N2-D2) then

N1-D1=N2-D2=0 (**m,n**=common factors)

$$\frac{3x+5}{5x+4} = \frac{4x+6}{2x+7}$$

3x+5-5x-4= -2x+1 and 4x+6-2x-7= 2x-1 = -1 (-2x+1)

∴ -2x+1=0; 2x=1; x = $\frac{1}{2}$

QUADRATIC EQUATIONS

Solution using Calculus

This sutra tells that: Differentiation of expression is equal to square root of discriminant.

E=> $ax^2+bx+c = 0$ ----(Expression)
D(E) = 2ax+b ----(Differentiation of Expression)
Discriminant = b^2-4ac
Square Root of Discriminant = $\sqrt{b^2-4ac}$
∴ 2ax+b=$\sqrt{b^2-4ac}$
(Differentiation of expression = square root of discriminant.)

$$\therefore \ x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Ex.1: $x^2+7x+12=0$

using Calanā kalanābhyām (चलनकलनाभ्याम्)
a=1; b=7; c=12 $\sqrt{b^2-4ac} = \sqrt{7^2-4 \times 1 \times 12} = \sqrt{1} = \pm 1$ $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-7 \pm 1}{2 \times 1}$ x=(-7+1)/2 OR x=(-7-1)/2; x=-6/2 OR x=-8/2; x=-3 OR x=-4
x = -3 OR x = -4

Ex.2: $x^2+6x+8=0$	**Ex.3: $3x^2+12x+7=0$**	**Ex.4: $6x^2+8x+5=0$**
a=1; b=6; c=8 $\sqrt{b^2-4ac}$ $=\sqrt{6^2-4\times1\times8}$ $=\sqrt{4}=\pm2$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-6\pm2}{2\times1}$ x=(-6+2)/2 OR x=(-6-2)/2 x=-4/2 OR x=-8/2 x=-2 OR x=-4	**a=3; b=12; c=7** $\sqrt{b^2-4ac}$ $=\sqrt{12^2-4\times3\times7}$ $=\sqrt{60}$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-12\pm\sqrt{60}}{2\times3}$ $x=\frac{-12\pm2\sqrt{15}}{6}$ $x=\frac{2(-6\pm\sqrt{15})}{6}$ $x=\frac{(-6\pm\sqrt{15})}{3}$	**a=6; b=8; c=5** $\sqrt{b^2-4ac}$ $=\sqrt{8^2-4\times6\times5}$ $=\sqrt{-56}$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-8\pm\sqrt{-56}}{2\times6}$ $x=\frac{-8\pm\sqrt{4\times14\times i^2}}{12}$ $x=\frac{-8\pm2i\sqrt{14}}{12}$ $x=\frac{-4\pm i\sqrt{14}}{6}$
x = -2 OR x = -4	$\boldsymbol{x=\frac{(-6\pm\sqrt{15})}{3}}$	$\boldsymbol{x=\frac{-4\pm i\sqrt{14}}{6}}$

Reciprocals By Mere Observation {Sub Sutra 13: Vilokanam}

Given Expression:	**Convert it into:**	**Values:**
$\frac{x1}{x2}\pm\frac{x2}{x1}=\frac{a}{b}$	$\frac{x1}{x2}\pm\frac{x2}{x1}=\frac{c}{d}\pm\frac{d}{c}$	$\frac{x1}{x2}=\frac{c}{d}\ or\ \pm\frac{d}{c}$

TYPE-1 (Plus)

Ex.1	**Ex.2**	**Ex.3**	**Ex.4**
$\boldsymbol{x+\frac{1}{x}=\frac{10}{3}}$ $\frac{x}{1}+\frac{1}{x}=\frac{10}{3}$ $\frac{x}{1}+\frac{1}{x}=\frac{3}{1}+\frac{1}{3}$ $\frac{x}{1}=\frac{3}{1}\ OR\ \frac{1}{3}$	$\boldsymbol{x+\frac{1}{x}=\frac{65}{8}}$ $\frac{x}{1}+\frac{1}{x}=\frac{65}{8}$ $\frac{x}{1}+\frac{1}{x}=\frac{8}{1}+\frac{1}{8}$ $\frac{x}{1}=\frac{8}{1}\ OR\ \frac{1}{8}$	$\boldsymbol{\frac{4x+3}{7x+1}+\frac{7x+1}{4x+3}=\frac{73}{24}}$ $\frac{73}{24}=\frac{3}{8}+\frac{8}{3}$ $\frac{4x+3}{7x+1}=\frac{3}{8}OR\frac{8}{3}$ Solve for x	$\boldsymbol{\frac{5x+2}{x-3}+\frac{x-3}{5x+2}=\frac{74}{35}}$ $\frac{74}{35}=\frac{5}{7}+\frac{7}{5}$ $\frac{5x+2}{x-3}=\frac{5}{7}OR\frac{7}{5}$ Solve for x
3 OR 1/3	**8 OR 1/8**	**-21/11 OR 1/44**	**-29/30 OR -31/18**

TYPE-2 (Minus):

Ex.5	Ex.6	Ex.7	Ex.8
$x-\frac{1}{x}=\frac{7}{12}$ $\frac{x}{1}-\frac{1}{x}=\frac{7}{12}$ $\frac{x}{1}-\frac{1}{x}=\frac{4}{3}-\frac{3}{4}$ $\frac{x}{1}=\frac{4}{3}\ OR-\frac{3}{4}$	$x-\frac{1}{x}=\frac{11}{30}$ $\frac{x}{1}-\frac{1}{x}=\frac{11}{30}$ $\frac{x}{1}-\frac{1}{x}=\frac{6}{5}-\frac{5}{6}$ $\frac{x}{1}=\frac{6}{5}\ OR-\frac{5}{6}$	$\frac{4x-3}{7x-1}-\frac{7x-1}{4x-3}=\frac{73}{24}$ $\frac{73}{24}=\frac{3}{8}-\frac{8}{3}$ $\frac{4x-3}{7x-1}=\frac{3}{8}\ OR\ \frac{8}{3}$ Solve for x	$\frac{5x-2}{x-3}-\frac{x-3}{5x-2}=\frac{21}{10}$ $\frac{21}{10}=\frac{5}{2}-\frac{2}{5}$ $\frac{5x-2}{x-3}=\frac{5}{2}\ OR-\frac{2}{5}$ Solve for x
4/3 OR -3/4	**6/5 OR -5/6**	**-21/11 OR 1/44**	**-11/5 OR 16/27**

Solution using "If the Set is same, it is ZERO"

First Factor:

If **m**(N1+N2)=**n**(D1+D2) then

N1+N2=D1+D2=0 (**m,n**=common factors)

Second Factor:

If **m**(N1-D1)=**n**(N2-D2) then

N1-D1=N2-D2=0 (**m,n**=common factors)

Ex.1	Ex.2
$\frac{3x+5}{5x+4}=\frac{4x+6}{2x+7}$ N1+N2=3x+5+4x+6=7x+11 D1+D2=5x+4+2x+7=7x+11 N1-D1=3x+5-5x-4=-2x+1 N2-D2=4x+6-2x-7=2x-1=-1(-2x+1) First Factor: 7x+11=0; 7x=-11; **x=-11/7** Second Factor: -2x+1=0; 2x=1; **x=1/2**	$\frac{2x+6}{4x+3}=\frac{6x+3}{4x+6}$ N1+N2=2x+6+6x+3=8x+9 D1+D2=4x+3+4x+6=8x+9 N1-D1=2x+6-4x-3=-2x+3 N2-D2=6x+3-4x-6=2x-3=-1 (-2x+3) First Factor: 8x+9=0; 8x=-9; **x=-9/8** Second Factor: -2x+3=0; 2x=3; **x=3/2**
x = -11/7 OR x = 1/2	**x = -9/8 OR x = 3/2**

CUBIC EQUATIONS

CASE 3:

$$\frac{1}{D1} + \frac{1}{D2} = \frac{1}{D3} + \frac{1}{D4}$$

if D1+D2=D3+D4
First Value: D1+D2=0
Second Value: D1-D2= D3-D4
Third Value: D1-D2= D4-D3

$\boldsymbol{Ex}$: $\frac{1}{4x+2} + \frac{1}{7x-6} = \frac{1}{5x+4} + \frac{1}{6x-8}$

D1=4x+2; D2=7x-6;
D3=5x+4; D4=6x-8
D1+D2=11x-4; D3+D4=11x-4;
D1-D2=-3x+8; D3-D4=-x+12 D4-D3=x-12

First: 11x-4=0; x=**4/11;**
Second: -3x+8=-x+12; x=**-2**
Third: -3x+8=x-12; x=**5**

SIMULTANEOUS EQUATIONS

Solution using Criss Cross Method

General Form: $a_1x+b_1y=c_1$ and $a_2x+b_2y=c_2$; Values are

$$x = \frac{b_1c_2 - b_2c_1}{a_2b_1 - a_1b_2} \quad y = \frac{a_2c_1 - a_1c_2}{a_2b_1 - a_1b_2}$$

$a_1x+b_1y=c_1$	$a_1x+b_1y=c_1$	$a_1x+b_1y=c_1$
$a_2x+b_2y=c_2$	$a_2x+b_2y=c_2$	$a_2x+b_2y=c_2$

EX.1:	EX.2:	EX.3:
$3x+7y=27$ – (I) $5x+2y=16$ –(II) $\boldsymbol{x} = \frac{7\times16-27\times2}{5\times7-3\times2}$ $= \frac{112-54}{35-6} = \frac{58}{29} = \mathbf{2}$ $\boldsymbol{y} = \frac{5\times27-3\times16}{5\times7-3\times2}$ $= \frac{135-48}{35-6} = \frac{87}{29} = \mathbf{3}$	$2x+y=7$ – (I) $3x-y=8$ –(II) $\boldsymbol{x} = \frac{1\times8-(-1)\times7}{3\times1-2\times-1}$ $= \frac{8+7}{3+2} = \frac{15}{5} = \mathbf{3}$ $\boldsymbol{y} = \frac{3\times7-2\times8}{3\times1-2\times-1}$ $= \frac{21-16}{3+2} = \frac{5}{5} = \mathbf{1}$	$-2x+y=1$ – (I) $-3x+2y=5$ –(II) $\boldsymbol{x} = \frac{1\times5-2\times1}{-3\times1-(-2)\times2}$ $= \frac{5-2}{-3+4} = \frac{3}{1} = \mathbf{3}$ $\boldsymbol{y} = \frac{-3\times1-(-2)\times5}{-3\times1-(-2)\times2}$ $= \frac{-3+10}{-3+4} = \frac{7}{1} = \mathbf{7}$
x=2 and y=3	**x=3 and y=1**	**x=3 and y=7**

INTRODUCTION TO VEDIC MATHEMATICS

Sutras: Meaning and Usage

Sutra 3: Ūrdhva – tiryagbhyām (ऊर्ध्वतिर्यग्भ्याम)

Meaning: Vertically and Crosswise.

Usage: This is used to to multiply given numbers vertically and crosswise.

$$\begin{array}{r} ab \\ \times \ \ cd \\ \hline \end{array}$$

= (a×c) | (a×d + b×c) | (b×d)

$$\begin{array}{r} abc \\ \times \ \ def \\ \hline \end{array}$$

= (a×d) | (a×e + b×d) | (a×f + b×e + c×d) | (b×f + c×e) | (c×f)

Ex: 25×67

First Part:	**Second Part:**	**Third Part:**
a b 2 5 c d 6 7	a b 2 5 c d 6 7	a b 2 5 c d 6 7
(a×c) **(2×6=12)**	**(a×d + b×c)** **(2×7+5×6=44)**	**(b×d)** **(5×7=35)**

12|44|35

12|44+3|5

12|47|5

12+4|7|5

16|7|5

1675

Applications: to find Product of given numbers and polynomials.
Refer Units: Multiplication and Polynomials.

Sutra 5: Sūnyam Samyasamuccaye (शून्यं साम्यसमुच्चये)

Meaning: If the Samuccay (समुच्चय or समूह or Set) is same, it is ZERO.

Usage:

CASE 3: If the numerical numerators of two fractions are same, then the sum of denominators is ZERO.

Ex. $\frac{3}{4x+5} + \frac{3}{7x-3} = 0$

Here 3=3; so 4x+5+7x-3=0;

11x+2=0; 11x=-2; x=-2/11.

Ex. $\frac{-7}{x^2+6} + \frac{-7}{6x+3} = 0$

Here -7=-7; so x^2+6+6x+3=0;

x^2+6x+9=0; $(x+3)^2$=0; x= -3.

Ex. $\frac{16}{3x+5} - \frac{16}{8x-12} = 0$

16=16; so 3x+5-(8x-12)=0; 3x+5-8x+12=0; -5x+17=0; x= 17/5.

Note: Here sign is negative, so we have subtracted (D1-D2)

CASE 4:

A. If N1+N2=D1+D2 then N1+N2=D1+D2=0

$$\frac{3x+5}{5x+4} = \frac{4x+6}{2x+7}$$

3x+5+4x+6=**7x+11**; 5x+4+2x+7=**7x+11**

∴ 7x+11=0; x = **-** $\mathbf{\frac{11}{7}}$

B. If **m**(N1+N2)=**n**(D1+D2) then N1+N2=D1+D2=0
(**m,n**=common factors)

$$\frac{7x+2}{2x+1}=\frac{5x+6}{x+1}$$

7x+2+5x+6=**12x+8=4(3x+2)**

2x+1+x+1=**3x+2=1(3x+2)**

∴ 3x+2=0

x = $-\frac{2}{3}$

C: If **m**(N1-D1)=**n**(N2-D2) then N1-D1=N2-D2=0 (**m,n**=common factors)

$$\frac{3x+5}{5x+4}=\frac{4x+6}{2x+7}$$

3x+5-5x-4= -2x+1 and 4x+6-2x-7= 2x-1 = -1 (-2x+1)

∴ -2x+1=0; 2x=1; x = $\frac{1}{2}$

Sutra 13: Sopantyadvayamantyam (सोपान्त्यद्वयमन्त्यम्)

Meaning: The ultimate and twice the penultimate

Usage:

$$\frac{1}{AB}+\frac{1}{AC}=\frac{1}{AD}+\frac{1}{BC}$$

If A, B, C and D are in Arithmetic Progression then D+2C=0

$$Ex.\frac{1}{(x+3)(x+4)}+\frac{1}{(x+3)(x+5)}=\frac{1}{(x+3)(x+6)}+\frac{1}{(x+4)(x+5)}$$

A=(x+3);

B=(x+4);

C=(x+5);

D=(x+6);

Here A, B, C & D are in Arithmetic Progression;

So, D+2C=0;

(x+6)+2(x+5)=0;
x+6+2x+10=0; 3x+16=0; **x=-16/3.**

Sub Sutras: Meaning and Usage

Sub Sutra 9: Antyayoreva (अन्त्ययोरेव)

Meaning: Only the Last Terms
Usage: This sutra is used to solve certain equations of the type: After ignoring Constants; Numerator and the Denominator of one side are in the ratio of Numerator and Denominator of the other side.

$$if\ \frac{m(N1)+c1}{m(D1)+c2}=\frac{N2}{D2}\ where\ N1=N2\ and\ D1=D2\ then\ \frac{N2}{D2}=\frac{c1}{c2}$$

m is common factor; c1 and c2 are constants.

$$\boldsymbol{Ex}.\frac{x^2+x+2}{x^2+2x+5}=\frac{x+1}{x+2}$$

$$=>\ \frac{x(x+1)+2}{x(x+2)+5}=\frac{x+1}{x+2}\ then\ \frac{x+1}{x+2}=\frac{2}{5};x=-\frac{1}{3}$$

Sub Sutra 12: Vilokanam (विलोकनं)

Meaning: By Mere Observation
Usage: This sutra is used in solving quadratic and simultaneous equations. Refer Unit: Quadratic Equations.
Ex: $x+\frac{1}{x}=\frac{10}{3}$; By Vilokanam (विलोकनं) we say that x=3.
