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Outline

Analytical Derivation

Larmor formula

The Larmor formula with $c = 1$ reads

$$\boxed{\frac{q^2}{6\pi}\gamma^6\left(\|\dot{\vec{\beta}}\| - \|\vec{\beta}\times\dot{\vec{\beta}}\|^2\right)}$$

Larmor formula

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For a constant perpendicular magnetic field, this reduces to

$$\boxed{P = \frac{q^2\|\vec{\beta}\|^4\ \gamma^4}{6\pi r^2}}$$

Larmor formula (cont'd)

We rearrange:

$$\begin{aligned} E_{\text{kin}} &= (\gamma - 1)mc^2 \\ \iff \gamma &= \frac{E_{\text{kin}}}{mc^2} + 1 \end{aligned} \tag{1}$$

$$\text{where } \gamma = \frac{1}{\sqrt{1 - \|\vec{\beta}\|^2}} \iff \|\vec{\beta}\| = \frac{\sqrt{\gamma^2 - 1}}{\gamma} \tag{2}$$

We can write down the ODE representing the loss of energy as follows:

$$\frac{\mathrm{d}E_{\text{kin}}}{\mathrm{d}t} = -P(\gamma) = -\frac{q^2 \|\vec{\beta}\|^4 \; \gamma^4}{6\pi r^2} = -\frac{q^2 \gamma^4 \left(\frac{\sqrt{\gamma^2 - 1}}{\gamma} \right)^4}{6\pi r^2} \tag{3}$$

For a non- and -relativistic electron the circular orbit frequency read as [1] (eq 5.21)

$$\omega_g = \frac{q\boldsymbol{B}}{mc}, \qquad \omega_{g \text{ rel}} = \frac{\omega_g}{\gamma} \tag{4}$$

Relativistic approximation $v = \|c\vec{\beta}\| \approx 1$ leads to

$$\begin{aligned} r\omega_{g \text{ rel}} &= c \\ \iff r &= \frac{c}{\omega_{g \text{ rel}}} = \frac{c\gamma}{\omega_g} = \frac{mc^2\gamma}{q\boldsymbol{B}} \end{aligned} \tag{5}$$

ODE Formulation

Substituting (5) into (3) yields

$$P(\gamma) = \frac{q^2\gamma^4\left(\frac{\sqrt{\gamma^2-1}}{\gamma}\right)^4}{6\pi\frac{mc^2\gamma}{q\boldsymbol{B}}} = \frac{q^2\gamma^4\left(\frac{\sqrt{\gamma^2-1}}{\gamma}\right)^4}{6\pi\frac{mc^2\gamma}{q\boldsymbol{B}}} = \frac{q^3\boldsymbol{B}\gamma^3\left(\frac{\sqrt{\gamma^2-1}}{\gamma}\right)^4}{6\pi mc^2} \tag{6}$$

Substituting (1) into a chain-rule

$$\begin{aligned}\frac{\mathrm{d}E_{\text{kin}}}{\mathrm{d}t} &= \frac{\mathrm{d}E_{\text{kin}}}{\mathrm{d}\gamma}\frac{\mathrm{d}\gamma}{\mathrm{d}t} = mc^2\frac{\mathrm{d}\gamma}{\mathrm{d}t} \\ \Longleftrightarrow \frac{\mathrm{d}\gamma}{\mathrm{d}t} &= \frac{1}{mc^2}\frac{\mathrm{d}E_{\text{kin}}}{\mathrm{d}t}\end{aligned} \tag{7}$$

which yields the ODE

Important

$$\frac{\mathrm{d}\gamma}{\mathrm{d}t} = -\frac{1}{mc^2}P(\gamma) = -\frac{q^3\boldsymbol{B}\gamma^3\left(\frac{\sqrt{\gamma^2-1}}{\gamma}\right)^4}{6\pi m^2c^4} \tag{8}$$

This $\gamma(t)$ has a positive solution

$$\gamma(t) = \frac{\sqrt{\frac{a}{\gamma_0^2-1}+a+2t}}{\sqrt{\frac{a}{\gamma_0^2-1}+2t}} \quad \text{where } a = \frac{q^3\boldsymbol{B}}{6\pi m^2c^4}, \gamma_0 = \gamma(0) \tag{9}$$

This leads to a $\beta(t)$:

$$\beta(t) = \frac{\sqrt{\frac{\frac{q^3\boldsymbol{B}}{6\pi m^2c^4}}{\gamma_0{}^2-1}+2t}\sqrt{\frac{\frac{\frac{q^3\boldsymbol{B}}{6\pi m^2c^4}}{\gamma_0{}^2-1}+\frac{q^3\boldsymbol{B}}{6\pi m^2c^4}+2t}{\frac{\frac{q^3\boldsymbol{B}}{6\pi m^2c^4}}{\gamma_0{}^2-1}+2t}-1}}{\sqrt{\frac{\frac{q^3\boldsymbol{B}}{6\pi m^2c^4}}{\gamma_0{}^2-1}+\frac{q^3\boldsymbol{B}}{6\pi m^2c^4}+2t}} \tag{10}$$

Visualization

Correct orbit radius

The Lorentz transformation of E and B:

$$\boldsymbol{E}_{\text{bunch}} = \gamma(\boldsymbol{E}_{\text{lab}} + \boldsymbol{v} \times \boldsymbol{B}_{\text{lab}}) - (\gamma - 1)(\boldsymbol{E}_{\text{lab}} \cdot \hat{\boldsymbol{v}})\hat{\boldsymbol{v}}$$

$$\boldsymbol{B}_{\text{bunch}} = \gamma\left(\boldsymbol{B}_{\text{lab}} + \frac{\boldsymbol{v} \times \boldsymbol{E}_{\text{lab}}}{c^2}\right) - (\gamma - 1)(\boldsymbol{B}_{\text{lab}} \cdot \hat{\boldsymbol{v}})\hat{\boldsymbol{v}}$$

where $\hat{\boldsymbol{v}} = \frac{\boldsymbol{v}}{\|\boldsymbol{v}\|}$

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where $\hat{\boldsymbol{v}} = \frac{\boldsymbol{v}}{\|\boldsymbol{v}\|}$

Our case $\boldsymbol{E}_{\text{lab}} = \boldsymbol{B}_{\text{lab}} \cdot \hat{\boldsymbol{v}} = 0, c = 1$:

$$\boldsymbol{E}_{\text{bunch}} = \gamma\boldsymbol{\beta} \times \boldsymbol{B}_{\text{lab}}$$

$$\boldsymbol{B}_{\text{bunch}} = \gamma\boldsymbol{B}_{\text{lab}}$$

Section Two

how to do it

[3] and [5] and [4] but not [2] show: Howver: We want to solve the kAK-equation:

$$k\Delta A = K$$

Guarantee:

- Stability
- Performance
- Accuracy

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