Scaling Efficiency: Optimizing Energy Expenditure in Rock Climbing

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***Abstract*—Rock climbing is the art of balancing. A climber must balance a logical approach, mapping out a potential route before starting the route, with a process of trial and error. They must balance the battle between hold selection and muscle fatigue. In essence, to be a successful climber, one must optimize their approach to a climb. Thus, in this study we aim to apply a common optimization technique, the hill climbing algorithm, in order to simulate the selection of an optimal route up a particular climb. This is accomplished first by the generation of a mock rock face that is constrained by the mock climber's choice of difficulty rating using the Yosemite decimal scale, and whose properties highlight energy cost per one by one foot regions. This generated terrain was optimized by the application of a modified hill climbing algorithm that found the local minimum surrounding the climber's current position, thus determining their next optimal movement. This determination was in part constrained by the prioritizing of more optimal and safe climbing movements, which are any upward moves, as well as constrained by the goal of minimizing the energy cost of the climber's route. By formulating this problem, we seek to understand the interplay between energy expenditure, climbing difficulty, and optimal path selection on a rock face.**

***Index Terms*—Keywords: Optimization, Hill climbing Algorithm, Rock Climbing, YDS, Energy Expenditure**

I. Introduction

A. Introduction to Rock Climbing

Rock climbing to its core is pushing limits, overcoming exhaustion and injury in order to achieve what one once thought was out of reach. It presents both a mental and physical challenge, as one must strategically plan their next move while battling fatigue. At its core, rock climbing involves ascending natural rock formations or artificial climbing walls using a variety of techniques and equipment. The five types of climbing are bouldering, top-roping, lead, tradition, and sport climbing. Bouldering is essentially short climbs where a climber does not use a rope and routes are usually below 50 feet tall. Top-Rope climbing is the more widely pictured climbing method as it involves placing an anchor at the top of the route in order to allow a climber to use a rope during there ascension. Lead climbing is where a climber laying their own anchors or protection during the climb. Traditional climbing (Trad climbing) is similar to lead climbing however it is usually done in large crevices or cracks in a rock face. Finally sport climbing is also similar to lead climbing, however, there are pre-existing bolts in the wall the climber just needs to reach and anchor into them [1]. On a mountain, rock faces will often be assigned both a climbing method and a difficulty rating as well as a name; this information can be found online on sources such as the Mountain Project. Outdoor rock in the United States are assigned a difficulty rating based on the Yosemite Decimal Scale as shown in Table 1 below.

Climbing Level 1	Climbing Ratings
Beginner	5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8, 5.9
Intermediate	5.10a, 5.10b, 5.10c, 5.10d, 5.11a, 5.11b, 5.11c, 5.11d
Pro	5.12a, 5.12b, 5.12c, 5.12d, 5.13a, 5.13b, 5.13c, 5.13d
Advanced	5.14a, 5.14b, 5.14c, 5.14d, 5.15a, 5.15b, 5.15c

TABLE I
THE YOSEMITE DECIMAL SYSTEM (YDS) A RATING SYSTEM USED TO CLASSIFY THE DIFFICULTY OF A CLIMB, AND FOUR WIDELY RECOGNIZED CLIMBING LEVELS ASSIGNED TO SPECIFIC RANGES.

Note that a climber will often refer to a climb rated at a difficulty they have yet to achieve a 'project' climb meaning they will often be repetitively climbing the route over the following weeks. Rock climbing is about growth, one must learn how to adapt to new hold combinations and movements as well as build strength. It is not uncommon for a climber to be forced to bail out of a climb due to being stuck in one place on a wall for too long and thus losing strength. Thus, rock climbing is about optimizing your route to conserve energy.

B. Hill Climbing Optimization Algorithm

Mathematically, optimization is finding the best vector, or best value. This could refer to a best value for an entire collection of data (looking in the past), or an estimate of the best value (interpreting the future). Imagine you have a bunch of choices, and you want to pick the one that gives you the most benefit or the least trouble. Optimization is used everywhere in today's society: auto-adjusting brightness levels, data on world health, and planning student grants and funding.

A simple yet prevalent optimization technique used in computer science, and machine learning is the hill climbing algorithm. The basic and core mathematical model of the hill climbing algorithm is the gradient method. The gradient method is a mathematical technique defined by the equation below:

$$\boldsymbol{x}^t = \boldsymbol{x}^{t-1} - \gamma_t \nabla f\left(\boldsymbol{x}^{t-1}\right), \quad t = 1, 2, 3, \ldots \tag{1}$$

where $x_0 \in R^n$ is the initialization and $\gamma_t > 0$is called the step-size parameter [2]. The gradient method is futuristic looking, meaning that it does not know what the entire data set looks like, only the current and previous step. It will calculate what the next step should be based on the current step and a vector with a defined magnitude and direction.

The hill climbing algorithm, based on the gradient method, starts at a random point and then moves in the direction that improves the solution according to its calculation. Figuratively, it is like climbing a hill blindly, always choosing the path that seems to lead upwards. However, it can get stuck on local maxima, as it only considers nearby options. Despite its simplicity, hill climbing algorithm is often used as a starting point for more complex optimization techniques and serves as a basic introduction to the concept of optimization in computer science and problem-solving.

C. Study Goals

In this study, we aim to investigate the optimization of energy efficiency in rock climbing while imposing a constraint on the climbing difficulty. By formulating this problem, we seek to understand the interplay between energy expenditure, climbing difficulty, and optimal path selection on a rock face through the application of the hill climbing algorithm.

II. Methods

In this report we will simulate four mock climbers terrains and optimal routes for a top rope climb that is rated using the Yosemite Decimal System, as defined in Table 1.

A. Digital Simulation

For the purpose of this study, a two-step process was implemented using MATLAB in order to simulate a potential rock face and then apply the hill climbing algorithm. Prior to the simulation of any data, one must generate variables that can be used to integrate rock climbing into the realm of mathematics. This was done through two four-by-eight matrices, one containing the ratings from the Yosemite Decimal Scale divided into rows based on climbing level and the other containing the energy cost in kilo-calories (kcal) corresponding to YDS grading scale ratings. Note that the energy cost matrices were created arbitrarily where cost increases by one kcal across the columns and ten kcal down the rows as seen below.

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 \\ 21 & 22 & 23 & 24 & 25 & 26 & 27 & 28 \\ 31 & 32 & 33 & 34 & 35 & 36 & 37 & NaN \end{bmatrix}$$

For optimal usage, a Climbing Difficulty Scale Class was created in order to store climbing levels and their corresponding ratings and energy costs. This class was used to make data sets for beginners, intermediates, pros, advanced, and all levels.

Additionally, prior to the simulation, four mock climbers were randomly generated and the corresponding data was stored in the mock climber class. A mock climber consists of a climber's name, climbing skill level, and project climb. For this study, the climbing skill level was assigned as all four levels were modeled but the project climb rating was randomly assigned based on climbing skill level. Note that, the name is not relevant but for the humor of the coder. The four mock climbers used in this report can be found in the table below:

Climber	Climbing Skill Level	Rating of Climber Project Climb
Miranda	Beginner	5.8
Mischa	Intermediate	5.10d
Julia	Pro	5.12b
George	Advanced	5.14c

TABLE II
THE FOUR RANDOMLY GENERATED MOCK CLIMBERS AND THEIR CORRESPONDING CLIMBING LEVEL AND PROJECT CLIMB RATING USED IN THIS STUDY.

The first part of our simulation design is the creation of a mock rock face that fits the dimensional and difficulty rating constraints. The dimensions used for the rock face modeled in this study were twenty feet in width and two hundred feet in height. These measurements were arbitrarily generated, as rock climbing route dimensions are nonuniform; however, top rope climbs are often between one hundred to three hundred feet. Furthermore, this study aims to determine the optimal route for a climber to minimize energy usage during a project climb, and as such, the terrain simulated must have a difficulty rating that matches that of the climber's project climb rating. Note that the step size for our simulation is one foot, and thus, each one-foot by one-foot square is assigned an energy cost within the simulation. The energy cost to squares was assigned through a moderately random process where energy cost was constrained to two standard deviations away from the project climb's energy cost. The summation of the squares used to simulate the terrain will have an average energy cost close to the energy cost of the project climbing rating with an acceptable range of error of plus or minus one kcal.

After simulating the mock terrain for a climber, a modified version of the hill climbing algorithm is applied to the data to determine an optimal route. For this report, we predefined a starting point for the mock climber at [0,10], thus making the climber start at the midpoint of the lowest height of the terrain. Our hill climbing algorithm runs until it reaches the highest elevation of the rock face, and accepts a position of the mock climber, starting at the initial point, in order to finds its neighboring holds. The neighboring holds are represented by the adjacent one-by-one foot squares (to the left, right, up, up-right, and up-left) around the current position. Note that the design of the study did not allow for the mock climber to down climb due to the significant energy cost this requires, as well as the potential for this to cause loop errors. After neighboring routes were found, their potential cost was determined by obtaining the energy cost assigned to the portion of terrain and the energy cost of the movement trajectory. An ideal movement trajectory in climbing is perfectly upwards; however, any upward motion is more ideal than left or right movements. To account for this, the energy cost of a movement based solely on the domain was divided by trajectory cost to decrease the cost of more ideal movements. Trajectory cost was partially computed using trigonometry, where left and right movements were one-foot motions, and thus their trajectory cost was 1. Furthermore, using this principle the cost of diagonal motions was $\sqrt{2}$., however, upward motion had the cost of 2 which does not follow trigonometry by highlights its importance. The scaled energy cost for each neighboring route was then compared and the optimal movement, the movement which minimized energy cost, was selected. This loop of determining the minimum was repeated until the climber reached an elevation of 200 feet.

B. Mathematical Modeling

The simulation of terrain can not be efficiently done mathematically due to the necessity for random generation under difficulty rating constraints; however the process of our modified hill climbing algorithm can be completed through the repetitive application of the gradient descent method on data points. As mentioned in the introduction the gradient descent method is the equation below:

$$\boldsymbol{x}^t = \boldsymbol{x}^{t-1} - \gamma_t \nabla f\left(\boldsymbol{x}^{t-1}\right), \quad t = 1, 2, 3, \ldots \qquad (2)$$

where $x_0 \in R^n$ is the initialization and $\gamma_t > 0$is called the step-size parameter [2]. The step-size parameter of this simulation is 1 foot, and the initialization is the starting point. Based on the design of the code, the gradient descent method would be applied to the 5 data points that represent the neighboring hold to the mock climbers current position. These data points could be modeled by a corresponding equation which could thus be applied to the gradient descent method.

III. Results and Discussion

A. Terrain Simulation

All four of the climbing skill levels were tested using four mock climbers, and therefore, four different route terrains were generated for this report based on the climbers project climb's rating using the mechanisms mentioned above, thus producing the data listed in the tables below. The data was divided into two tables due to the formatting of the report. Note that energy cost comes from the pre-assigned energy cost per rating described in the method.

Climber	Project Rating	Ideal EC (kcal)
Miranda	7	7
Mischa	5.10d	14
Julia	5.12b	22
George	5.14c	33

TABLE III

The four mock climbers and there corresponding project climb rating used and its ideal Energy Cost (EC) for the terrain generation.

Climber	Average EC(kcal)	Average EC Rounded (kcal)
Miranda	6.825	7
Mischa	13.8993	14
Julia	21.769	22
George	31.9337	32

TABLE IV

The four mock climbers and there corresponding average energy cost (EC) both rounded and normal regarding their terrain generated.

Furthermore, the terrain data can be visual seen in Figures 1, 2, 3, and, 4. In these figures, it is important to note that the x-axis is the width of the route, y-axis is the height of the route, and the z-axis is the cost of each one by one square. The cost of the squares are also shown through the color assignment with lower energy costs in blue and high energy cost in yellow.

B. Optimization Application

The optimal routes we determined using our hill climbing algorithm were designed to limit energy usage thus decreasing the total energy cost of the climb for the mock climber, and it was successful as seen through the data in the table below.

Climber	Ideal Total EC (kcal)	Total EC (kcal)	Total EC with Movement(kcal)
Miranda	1393	1329	679.4117
Mischa	2786	2537	1345.1087
Julia	4378	3974	2098.2163
Geoerge	6567	5880	3084.9747

TABLE V

The four mock climbers, their corresponding ideal total energy cost, the energy cost of the optimal route without accounting for movement trajectory, and the energy cost of the optimal route with movement trajectory accounted.

The ideal total energy cost seen in the table above was generate using the project climb's rating's energy cost seen in Table 3 and multiplying it be the number of movements made during the climb, which was one hundred and ninety nine. Note that this is simulating a climb that consists of holds that are all rated to the project rating.

Overall, the resulting optimal route that was generated can be seen in figures 5, 6, 7, and 8. The figures are a two dimensional rendition of the three dimension terrain plots mentioned above, and as such, the color of the terrain representing the energy cost of the region, the x-axis is width, and y-axis is height. Note that these images are subplots; this was done to highlight the terrain and the optimal route since the size of the line generated caused some loss in the ability to visualize the energy cost per the one foot by one foot regions.

Additionally, analysis was preformed in order to visualize the cost per movement in regards to the one hundred and ninety nine movements that make up the mock climbers route. This analysis was done using Figures 9, 10, 11, and 12. These figures highlight graphs where movement is on the x-axis and the y-axis is the energy cost (without accounting for movement trajectory) in kcal. Furthermore, a line of regression was added to the plot in order to highlight the average cost per movement of the optimal route for the mock climber.

IV. Conclusion

A. Terrain Simulation

It is important to note that our terrain-building mechanism was successfully able to generate terrains that were in the 1kcal error range for averaged terrain energy cost as seen in Tables 3 and 4. Thus, this highlights how we were able to create a modeled terrain that allowed our data to match our goal of

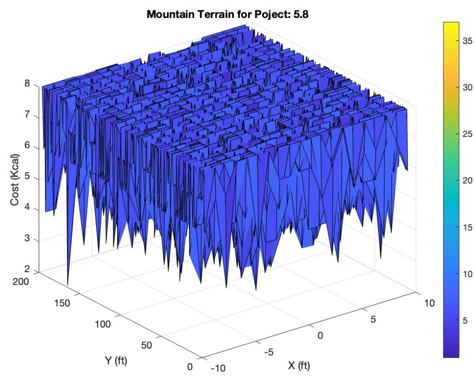

Fig. 1. The Terrain generated for the Mock Climber, Miranda, for a project climb rated 5.8.

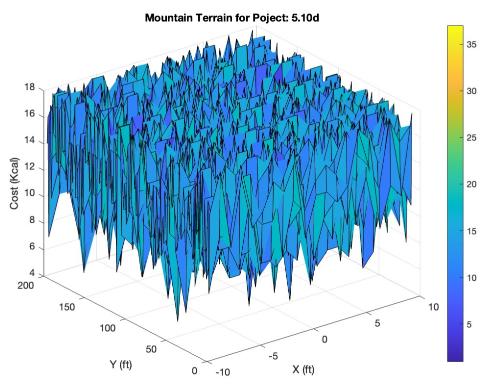

Fig. 2. The Terrain generated for the Mock Climber, Mischa, for a project climb rated 5.10d.

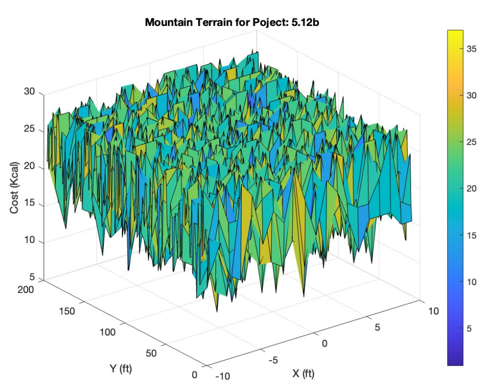

Fig. 3. The Terrain generated for the Mock Climber, Julia, for a poject climb rated 5.12b.

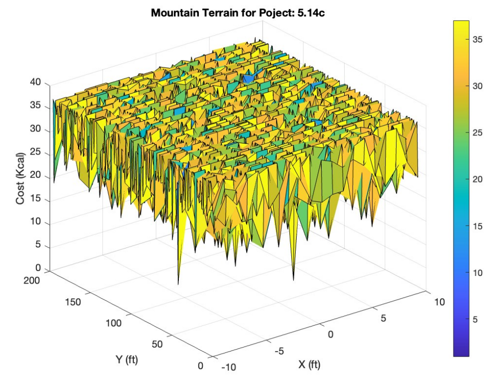

Fig. 4. The Terrain generated for the Mock Climber, George, for a poject climb rated 5.14c.

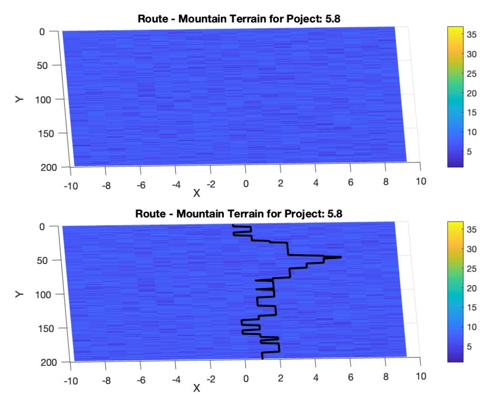

Fig. 5. The optimal route generated for the Mock Climber, Miranda, for a project climb rated 5.8.

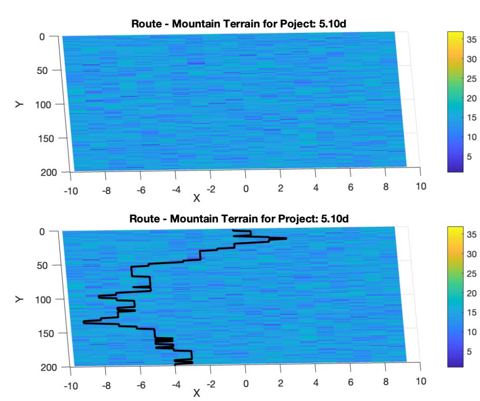

Fig. 6. The optimal route generated for the Mock Climber, Mischa, for a project climb rated 5.10d.

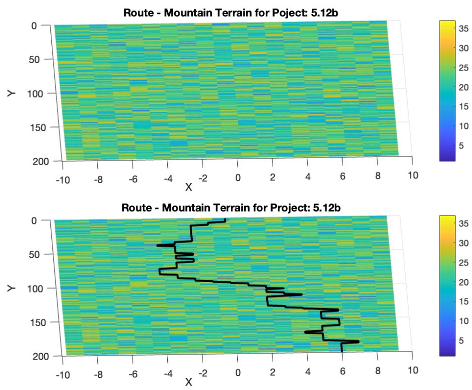

Fig. 7. The optimal route generated for the Mock Climber, Julia, for a project climb rated 5.12b.

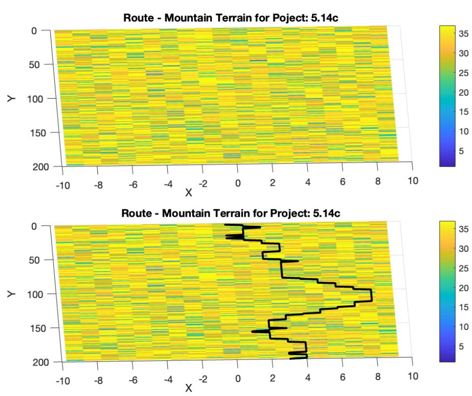

Fig. 8. The optimal route generated for the Mock Climber, Julia, for a project climb rated 5.14c.

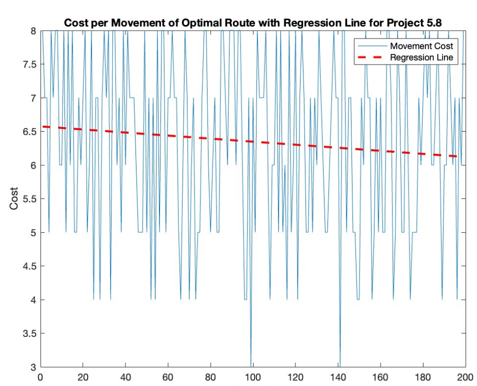

Fig. 9. The energy cost (kcal) per movement for the Mock Climber, Miranda, for a project climb rated 5.8.

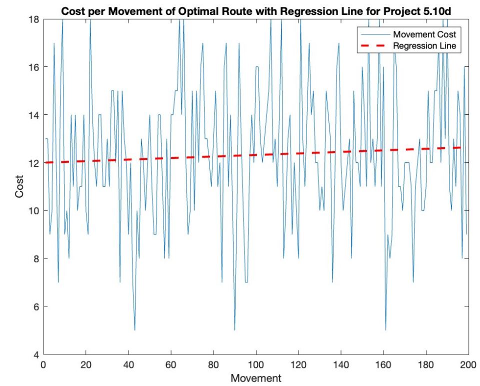

Fig. 10. The energy cost (kcal) per movement for the Mock Climber, Mischa, for a project climb rated 5.10d.

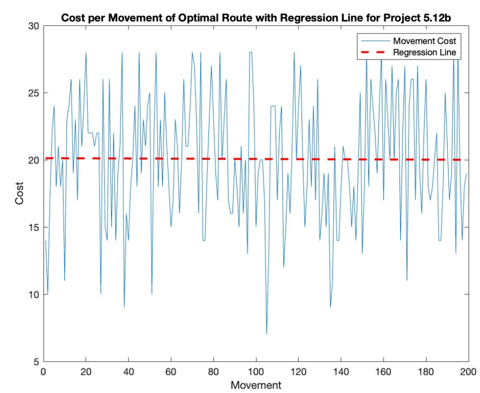

Fig. 11. The energy cost (kcal) per movement for the Mock Climber, Julia, for a project climb rated 5.12b.

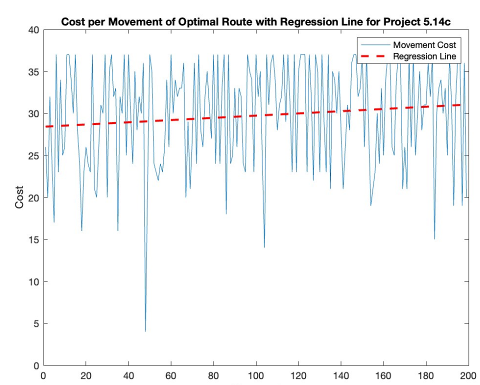

Fig. 12. The energy cost (kcal) per movement for the Mock Climber, George, for a project climb rated 5.14c.

maintaining a range of difficulty based on a project climb's rating. The slight deviation from the zero error rate between expected and generated average terrain energy cost seen for the fourth climber, George, can be accounted for based on the slightly smaller range of potential values in the advanced range of ratings.

Furthermore, as seen in Figures 1, 2, 3, and, 4, the terrains have a range of rating assignments throughout the z dimension. The range of rating used during random assignment are more clear on figures 3 and 4 as both these image contain green, blue, and yellow regions.

B. Optimization Application

Through the comparison of the ideal total energy cost to the total energy cost of the optimal route, not accounting for trajectory, one will see that our program was successfully able to find and select holds that have a minimal rating as seen in Table Five. Furthermore, this is highlight even more extensively in figures 9, 10, 11, and 12. As one can see the line of regression for each of these four figures are all essentially straight lines that average to values that are below the project climb's ratings energy cost. This highlights how on average a movement was made that was less than the expected energy cost thus making it a more ideal choice.

Referencing Table 5 once again, one should note that plotting and analysis was done using the energy cost determined solely by the terrain; however, movements decisions were made using the energy cost that accounts for the cost of the movement's trajectory. Thus, we found it necessary to express its average total energy cost as well and they are also significantly below the ideal total energy cost and that of the energy cost based on terrain. This highlights how the program is able to successfully select upward movements more often then left or right movements, which demonstrates our goal of modeling real world climbing choices. This can also be seen in figures 5, 6, 7, and 8 as in each of these figures diagonal and upwards movements seem more prevalent when compared to left and right movements. Due to the dimensions of the graph and the step size of the algorithm, these images were generated to highlight the optimal routes path since it is hard to visually see if the correct movement choices were made. Overall, based on the analysis preformed, we determined that our simulation was able to successfully meet this study's objectives and minimize the energy usage of a climber as they attempt a project climb, while retaining their constraints of prioritizing the more optimal upwards movements, and maintaining their desired level of difficulty.

C. Simulation Challenge

Outdoor rock climbing is inherently arbitrary and unregulated, and thus performing a simulation of a climber's ascension up a rock face has several issues. One of the hurdles we ran into while designing and implementing this study was that there exist no data regarding the energy cost of rock climbing as it relates to difficulty ratings. Therefore, we were forced to arbitrarily determine a energy cost and thus we are unable to definitively say that our simulation models the real world application in terms of exact total energy cost. Furthermore, the most prevalent issues we ran into was the modeling of a rock face using MATLAB. A rock climbing route is not constrained to any set dimensions and as such a general estimate had to be made based on limited data and personal experience. However, this does not impact the results of our data in the long run. We choose to model rock climbing holds using one foot by one foot squares and assign energy cost and rating to each square. After much trial and error this was determined to be the most ideal method of simulation for the time being. The rating of climbing routes is unregulated and arbitrary because ratings are assigned by the first person who completes the route, and as such the climb may be assigned a higher or lower ranking than what it technically should be based on the averages climbers height or physical strength and abilities. As such our simulation successfully generates a terrain but it does not account for hidden variables that exist in real world situations.

D. Future Applications

The digital modeling done within this study is the first steps toward a comprehensive application of optimization to rock climbing, in order to minimize the energy cost and determine an optimal route. In the future, we would like to redesign our application of our hill climbing method in order to allow the consideration of three or more movements in advance. A skilled climber looks beyond just the next hold when figuring out what direction they would like to take, because sometimes one should take a less optimal movement in order to get to an easier part of the rock face. In the realm of mathematics, this would be adapting our program in the hopes of being able to get past local minimums in order to find the global minimum. This is a common problem that the hill climbing algorithm and more broadly the gradient descent method runs into as mentioned in the introduction, however, there are combinations of other optimization techniques that can be manipulated in order to overcome this.

Additionally, another improvement we hope to make in the future is the consideration of rope tension and climbing safety in terms of the movement trajectory and its affect on energy cost. Currently, we use trajectory to highlight optimal movements. However, these are constant values and they do not account for the impact of the rope tension on the climbers abilities as they get closer to the anchor. Note, that safe movements for climbers have an almost triangular range up the route as a result of the anchored rope. Therefore, the higher the climber ascends the closer to center they should move. If one is too far out of ideal range they will have a large amount of excess rope and thus if they were to fall they would drop and swing dramatically, increasing the chance that they or their belayer would get injured. Thus, in order to account for this safety risk and the difficulty of moving against the tension of a top rope, we would like to make trajectory cost a changing variable that accounts for elevation changes and distance from center.

E. Final Statement

This study explored the optimization of energy expenditure in rock climbing through the application of a modified hill climbing algorithm while adhering to specific difficulty and movement constraints. Through digital simulation and mathematical modeling, we generated terrains representing climbing routes of varying difficulty levels and successfully identified the most energy-efficient paths for climbers. However, similar to other real world modeling there are many hidden variables that we have discovered along the way and as such there is significantly room for our project and simulation to grow and adapt.

References

[1] Griffith, Katie. ”Getting Started Rock Climbing”. *https://www.rei.com/learn/expert-advice/getting-started-rock-climbing.html*. Accessed 1 May 2024.

[2] Kamilov, U.S. ”ESE 415 – OPTIMIZATION NOTES BY U. S. KAMILOV”. Accessed 1 May 2024.