3.1 Berechnen Sie die folgenden Integrale

(a) $\int \frac{1}{3-2x}\,dx$

(b) $\int \frac{2x-1}{\sqrt{x^2-x}}\,dx$

(c) $\int x^2 \cdot \sin(x)\,dx$

(d) $\int \frac{1}{x^2-16}\,dx$

Substituieren $u = 3-2x$

$u = 3-2x, \quad \frac{du}{dx} = -2$

$dx = \frac{du}{-2}$

a) $$\int \frac{1}{3-2x}\,dx = \frac{1}{u}\cdot dx$$

$$= \int \frac{1}{u}\,\frac{du}{-2} = \int \frac{1}{u}\left(-\frac{1}{2}\right)du = -\frac{1}{2}\int \frac{1}{u}\,du$$

$$= -\frac{1}{2}\ln(u) = \frac{\ln(3-2x)}{2} + C$$

b) $$\int \frac{2x-1}{\sqrt{x^2-x}}\,dx$$

$u = x^2 - x \quad \frac{du}{dx} = 2x-1$

$dx = \frac{du}{2x-1}$

$$= \int \frac{2x-1}{\sqrt{u}}\,dx = \int \frac{\cancel{2x-1}}{\sqrt{u}}\,\frac{du}{\cancel{2x-1}}$$

$$= \int \frac{1}{\sqrt{u}}\,du = \int u^{-\frac{1}{2}}\,du = \frac{u^{n+1}}{n+1}$$

$$= \frac{u^{\frac{1}{2}}}{\frac{1}{2}} = 2\sqrt{u} = 2\sqrt{x^2-x} + C$$

c) $\int x^2 \cdot \sin(x)$

$$= -x^2\cos(x) + 2x\sin(x) + 2\cos(x)$$

V	D	I
+	x^2	$\sin(x)$
-	$2x$	$-\cos(x)$
+	2	$-\sin(x)$
-	0	$\cos(x)$

d) $\int \frac{1}{x^2-16} = \frac{1}{(x-4)\cdot(x+4)} = \frac{A}{x+4} + \frac{B}{x-4}$

$$= \frac{A(x-4) + B(x+4)}{x^2-16}$$

$$1 = A(x-4) + B(x+4)$$

$x=-4 \qquad 1 = -8A \rightarrow A = -\frac{1}{8}$

$x = 4 \qquad 1 = 8B \Rightarrow B = \frac{1}{8}$

$$\int \frac{1}{x^2-16} = -\frac{1}{8}\int \frac{1}{x+4} + \frac{1}{8}\int \frac{1}{x-4} = -\frac{1}{8}\ln(x+4) + \frac{1}{8}\ln(x-4)$$

$$= \frac{1}{8}\ln\left(\frac{x-4}{x+4}\right) + C$$