2.3 Berechnen Sie die partiellen Ableitungen 1. und 2. Ordnung.

(a) $u(x,y) = \frac{2x^2+7x^2y}{3xy}$ (b) $v(x,y) = \frac{2xy}{(x^2+y^2)^2}$

a) $u(x,y) = \frac{2x^2+7x^2y}{3xy} = \frac{\cancel{x}(2x+7xy)}{3\cancel{x}y} = \frac{2x}{3y} + \frac{7\cancel{xy}}{3\cancel{y}} = \frac{2x}{3y} + \frac{7x}{3}$

$u_x(x,y) = \frac{2\cdot 3y - 2x\cdot 0}{9y^2} + \frac{7\cdot 3 + 7x\cdot 0}{9}$

$= \frac{6y}{9y^2} + \frac{21}{9} = \frac{2}{3y} + \frac{7}{3} = \frac{2}{3y} + \frac{7y}{3y} = \frac{2+7y}{3y}$

$u_{xx}(x,y)$ 0, da kein x mehr vorhanden

$u_{xy}(x,y) = \frac{0\cdot 3y - 2\cdot 3}{9y^2} + \frac{7\cdot 3y - 7y\cdot 3}{9y^2} = \frac{6}{9y^2} + \frac{21y-21y}{9y^2}$

$= \frac{6}{9y^2} + \frac{0}{9y^2}$

$= \frac{6}{9y^2}$

$u_y(x,y) = \frac{0\cdot 3y - 2x\cdot 3}{9y^2} + \frac{0\cdot 3 - 7x\cdot 0}{9y^2}$

$= \frac{6x}{9y^2} + 0 = -\frac{2x}{3y^2}$

$u_{yy}(x,y) = \frac{0\cdot 3y^2 - (-2x)\cdot 6y}{9y^4} = \frac{12x\cancel{y}}{9y^4} = \frac{4x}{3y^3}$

$$u_{yx}(x,y) = \frac{-2 \cdot 3y^2 - (-2x) \cdot 0}{9y^4} = \frac{-6y^2}{9y^4} = -\frac{2}{3y^2}$$

b) $$v(x,y) = \frac{2xy}{(x^2+y^2)^2} = 2xy \cdot (x^2+y^2)^{-2}$$

$$v_x(x,y) = 2y \cdot ((x^2+y^2)^{-2}) + 2xy \cdot (-2 \cdot (x^2+y^2)^{-1})$$;

$$= \frac{2y}{(x^2+y^2)^2} + \frac{2xy}{-2(x^2+y^2)} = \frac{2y}{(x^2+y^2)^2} + \frac{4x^2y^2}{4(x^2+y^2)^2}$$

$$= \frac{2y + 4x^2y^2}{+5(x^2+y^2)^2} \longrightarrow 2y + 4x^2y^2 \cdot (+5(x^2+y^2)^{-2})$$

$$v_{xx} = 8xy^2 \cdot (+5(x^2+y^2)^{-2}) + 2y + 4x^2y^2 \cdot (+10(x^2+y^2)^{-1})$$

$$= \frac{8xy^2}{5(x^2+y^2)^2} + \frac{2y+4x^2y^2}{-10(x^2+y^2)} = \frac{8x\cancel{y}^2 + (2y+4x^2y^2)^2}{105(x^2+y^2)^2}$$

$$v_{yy}(x,y) = 2 + 8x^2y \cdot (-3(x^2+y^2)^{-2}) + 2y + 4x^2y^2 \cdot (-6(x^2+y^2)^{-1})$$

$$= \frac{2+8x^2y}{-3(x^2+y^2)^2} + \frac{2y+4x^2y^2}{6(x^2+y^2)}$$

$$v_y(x,y) = 2x \cdot (x^2+y^2)^{-2} + 2xy \cdot (-2 \cdot (x^2+y^2)^{-1})$$

$$= \frac{2x}{(x^2+y^2)^2} + \frac{2xy}{-2 \cdot (x^2+y^2)} = \frac{2x}{(x^2+y^2)^2} + \frac{4x^2y^2}{4 \cdot (x^2+y^2)^2}$$

$$= \frac{2x + 4x^2+y^2}{5 \cdot (x^2+y^2)^2}$$

$$V_{yy}(x,y) = 0 + 8x^2y \cdot (5(x^2+y^2)^{-2}) + 2x + 4x^2y^2 \cdot (-10(x^2+y^2)^{-1})$$

$$= \frac{8x^2y}{5(x^2+y^2)} + \frac{2x+4x^2y^2}{-10(x^2+y^2)} = \frac{8x^2y + (2x+4x^2y^2)^2}{105(x^2+y^2)}$$

$$V_{yx}=(x,y) = 2x + 8x\cancel{y}^2 \cdot (5(x^2+y^2)^{-2}) + 2x + \cancel{4x}^2y^2 \cdot (-10(x^2+y^2)^{-1})$$

$$= \frac{2x+8xy^2}{5(x^2+y^2)} + \frac{2x+4x^2y^2}{-10(x^2+y^2)}$$

spezielle Funktionen	Allgemeine Regel
$(x^n)' = n \cdot x^{n-1}$	$(k \cdot f)' = k \cdot f'$
$(e^x)' = e^x$	$(f \pm g)' = f' \pm g'$
$(\ln)' = \frac{1}{x}$	$(f \cdot g)' = f' \cdot g + f \cdot g'$
$(\sin(x))' = \cos(x)$	$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$
$(\cos(x))' = -\sin(x)$	$(f(g))' = f'(g) \cdot g'$
$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$	
$(\tan(x))' = \frac{1}{\cos^2(x)}$	
$= 1+\tan^2(x)$	

$(a^x)' = a^x \cdot \ln(a)$