Transformer-based model compression

December 12, 2023

Plan

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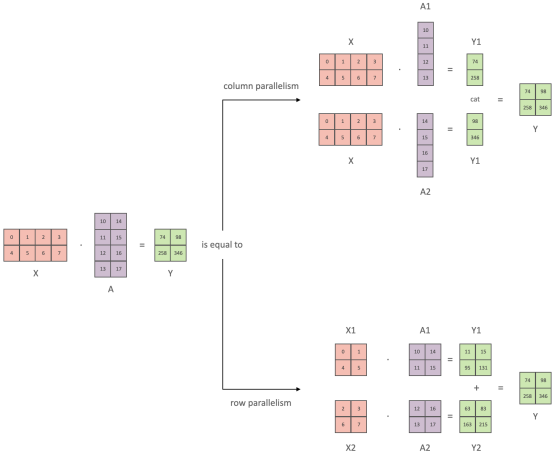
Training Parallelism

- **Data parrallelism** - pieces of a given batch are placed on a different GPU cards
- **Tensor parrallelism** - pieces of a model (blocks, layers, parts of the layers) are placed on a different GPU cards

Data Parallelism

- It creates and dispatches copies of the model, one copy per each accelerator.
- It shards the data to the n devices. If full batch has size B, now size is $\frac{B}{n}$.
- It finally aggregates all results together in the backpropagation step, so resulting gradient in module is average over n devices.

Tensor parrallelism



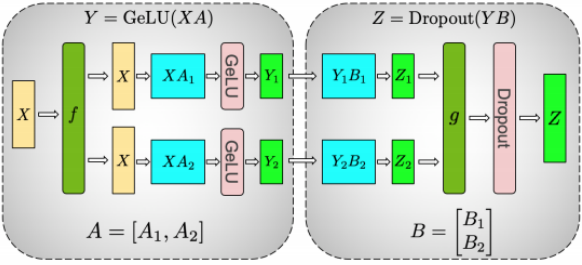
Different ways of splitting the matrix between several GPUs

Tensor parrallelism

A column-wise splitting provides matrix multiplications XA_1 through XA_n in parallel, then we will end up with N output vectors $Y_1, \ldots, Y_n$ which can be fed into GeLU independently

$$[Y_1, Y_2] = [GeLU(XA_1), GelU(XA_2)]$$

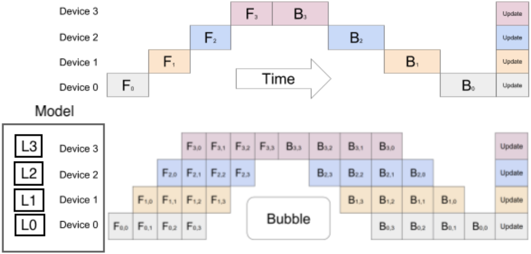
Using this principle, we can update an MLP of arbitrary depth, without the need for any synchronization between GPUs until the very end [1]:



(a) MLP

[1] Megatron

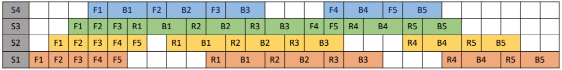
Pipelining



Top: *The naive model parallelism strategy leads to severe underutilization due to the sequential nature of the network. Only one accelerator is active at a time.* ***Bottom:*** *GPipe divides the input mini-batch into smaller micro-batches, enabling different accelerators to work on separate micro-batches at the same time.*

Pipelining

Interleaved pipelining aims to reduce "bubble" size.



(a) Varuna Schedule

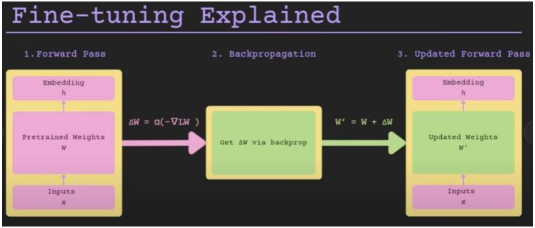
(b) Gpipe Schedule

Varuna [2] model scheduler. F - forward pass, B- backward pass, R - recomputation. Varuna recomputes activations by re-running the forward computation, sinse activations take lot of of memory (checkpointing).

[2] https://arxiv.org/pdf/2111.04007.pdf

LORA:Low-rank adaptation of large language models [3]

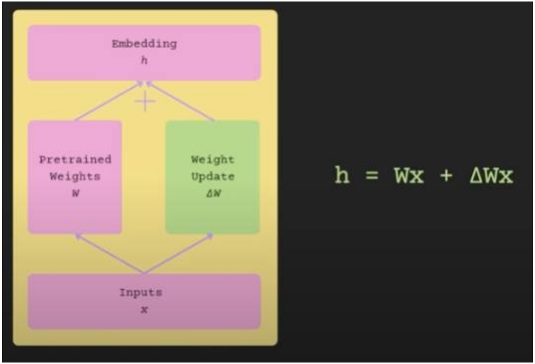
Fine-tuning: if the model has 100 billion trained parameters, storing all of them in memory becomes a significant bottleneck.



[3] https://arxiv.org/pdf/2106.09685.pdf

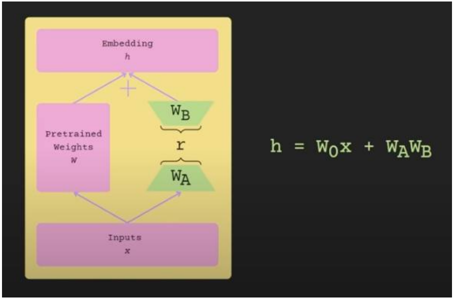
LORA:Low-rank adaptation of large language models

Thus, the pre-trained weights remain static and we only manipulate the delta weights. This is a crucial aspect of the algorithm.



LORA:Low-rank adaptation of large language models

The low-rank approximation is generated by using a technique called singular value decomposition (SVD). SVD decomposes the base model into a set of rank-1 matrices. These rank-1 matrices are then combined to form the target model.



LORA

Results on GLUE (Roberta)

Model & Method	# Trainable Parameters	MNLI	SST-2	MRPC	CoLA	QNLI	QQP	RTE	STS-B	Avg.
RoB_{base} (FT)*	125.0M	**87.6**	94.8	90.2	**63.6**	92.8	**91.9**	78.7	91.2	86.4
RoB_{base} (BitFit)*	0.1M	84.7	93.7	**92.7**	62.0	91.8	84.0	81.5	90.8	85.2
RoB_{base} (Adpt^{D})*	0.3M	$87.1_{\pm.0}$	$94.2_{\pm.1}$	$88.5_{\pm1.1}$	$60.8_{\pm.4}$	$93.1_{\pm.1}$	$90.2_{\pm.0}$	$71.5_{\pm2.7}$	$89.7_{\pm.3}$	84.4
RoB_{base} (Adpt^{D})*	0.9M	$87.3_{\pm.1}$	$94.7_{\pm.3}$	$88.4_{\pm.1}$	$62.6_{\pm.9}$	$93.0_{\pm.2}$	$90.6_{\pm.0}$	$75.9_{\pm2.2}$	$90.3_{\pm.1}$	85.4
RoB_{base} (LoRA)	0.3M	$87.5_{\pm.3}$	$\mathbf{95.1}_{\pm.2}$	$89.7_{\pm.7}$	$63.4_{\pm1.2}$	$\mathbf{93.3}_{\pm.3}$	$90.8_{\pm.1}$	$\mathbf{86.6}_{\pm.7}$	$\mathbf{91.5}_{\pm.2}$	**87.2**

SVD and TTM layers

- Singular Value Decomposition (SVD)
- Tensor Train Matrix Decomposition (TTM)

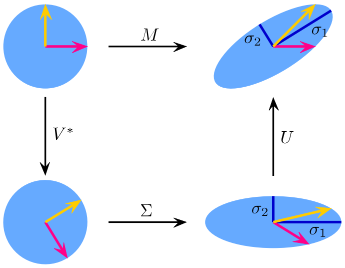
Methods

- **SVD** - FC layers are replaced with their SVD representations
- **FWSVD** - FC layers are replaced with their SVD representations reweighted with Fisher Information matrix [**?**]
- **TTM** - FC layers are replaced with their TTM representations
- **FWTTM** - FC layers are replaced with their TTM representations reweighted with Fisher Information matrix

Singular Value Decomposition (SVD)

- The Singular Value Decomposition (SVD) of a matrix $\mathbf{M} \in \mathcal{R}^{m \times n}$ is a factorization $\mathbf{M} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^*$ where $\mathbf{U} \in \mathcal{R}^{m \times m}$, $\boldsymbol{\Sigma} \in \mathcal{R}^{m \times n}$ - is a diagonal matrix, $\mathbf{V} \in \mathcal{R}^{n \times n}$.
- $\tilde{\mathbf{M}}$ is a **truncated approximation M** if $\tilde{\mathbf{M}} = \mathbf{U}\tilde{\Sigma}\mathbf{V}^*$, where $\tilde{\boldsymbol{\Sigma}}$ is a $\boldsymbol{\Sigma}$ a non-zero only the r largest singular values.

Singular Value Decomposition (SVD)



$$M = U \cdot \Sigma \cdot V^*$$

Figure: The interpretation of SVD. The operator of transformation **M** is decomposed into rotation **U**, scaling **Σ** and rotation back $\mathbf{V}^{\mathbf{T}}$.

Three factor matrices can be treated as rotation, scaling and rotation back operations. We translate **M** into new space, truncate the elements with minot weights, and then transfer the Matrix back.

Layer based on Singular Value Decomposition (SVD)

Layer weight matrix $\mathbf{W} \in \mathbb{R}^{D_{in} \times D_{out}}$

- After the SVD decomposition $\mathbf{W} = \mathbf{U\Sigma V^T}$ we have two sequentual linear layer with weights

$$\mathbf{W_2} = \mathbf{U_r}\sqrt{\mathbf{\Sigma_r}}, \tag{1}$$
$$\mathbf{W_1} = \sqrt{\mathbf{\Sigma_r}}\mathbf{V_r^T}. \tag{2}$$

- The compression rate is

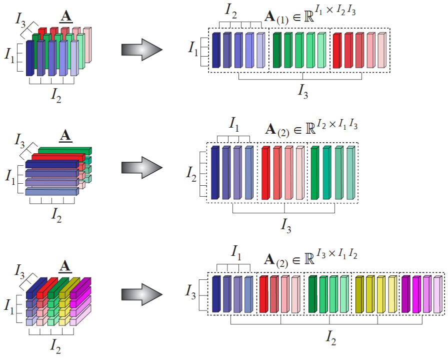
$$c_{\text{rate}} = \frac{r \times (D_{\text{in}} + D_{\text{out}})}{D_{\text{in}} \times D_{\text{out}}}$$

Tensor symbols and notations.

Tensor - the multidimentional array.

Notation/Symbol	Meaning
$\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \ldots I_N}$	N^{th} order tensor of size $I_1 \times I_2 \times \ldots I_N$
$x_{i_1,i_2,\ldots,i_n} = \mathcal{X}(i_1, i_2, \ldots, i_n)$	$(i_1, i_2, \ldots t_N)^{th}$ entry of the tensor $\mathcal{X}$
$\underline{\mathcal{G}}, \mathcal{G}^{(k)}, \mathcal{G}^k$	Core tensors in Tucker and Tensor Train decompositions
$\mathbf{A}, \mathbf{b}$	Matrix and vector
$\mathcal{X}_{[\mathrm{n}]}$	Mode-n matricization (unfolding). This operation represents a tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times \cdots \times I_n \times \cdots \times I_N}$ in a matrix $\mathcal{X}_{[\mathrm{n}]} \in \mathbb{R}^{I_n \times I_1 \cdots \cdots I_N}$

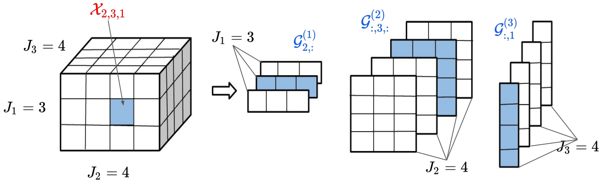
Tensor symbols and notations: Tensor Unfolding



Tensor Train (TT) Format

$$\mathcal{X}(i_1 \dots i_N) = \underbrace{\mathcal{G}^{(1)}[i_1, :]}_{1 \times R_1} \underbrace{\mathcal{G}^{(2)}[:, i_2, :]}_{R_1 \times R_2} \dots \underbrace{\mathcal{G}^{(N-1)}[:, i_{N-1}, :]}_{R_{N-2} \times R_{N-1}} \underbrace{\mathcal{G}^{(N)}[:, i_N]}_{R_{N-1} \times 1}$$

$\mathcal{G}_k \in \mathbb{R}^{R_k \times I_k \times R_{k+1}}$, $k = \overline{1, N}$, are 3-dimentional *core tensors (cores)* of TT decomposition. The $R_1, \dots, R_k$ are called *TT-ranks*



The example of computation tensor elements via it's TT representation

Algorithm for TT compression

Algorithm 2 TT-SVD [Oseledets, 2011b]

Input: d-dimentional tensor $\mathcal{A}$;

Output: Cores $\mathcal{G}^1, \ldots, \mathcal{G}^d$ of TT decomposition of $\mathcal{A}$

Temporary tensor $\mathcal{C} = \mathcal{A}, r_0 = 1$

for $k = 1$ to $d - 1$ **do**

$\mathbf{C}$:= reshape($\mathcal{C}$, $[r_{k-1}n_k, \frac{\text{prod(C.shape)}}{r_{k-1}n_k}]$

r_k-truncated SVD: $\mathbf{C} \approx \mathbf{U\Sigma_{r_k}V^T}$

$\mathcal{G}^k$:= reshape($\mathbf{U}$, $[r_{k-1}, n_k, r_k]$)

$\mathbf{C}$:= $\Sigma_{r_k} V^T$

end for

$\mathcal{G}^k = \mathbf{C}$

Tensor Train Matrix (TTM) format

- Tensor is now a N-dimetional objects over 2-dimentional matrices or $2N$-dimetional objects over 1-dimentional points.
- Unique index i turns into an index tuple (i, j).
- Now we have N core tensors with dimention 4 (not 3).

$$\mathcal{X}\left((i_1, j_1)\ldots(i_N, j_N)\right) = \underbrace{\mathcal{G}^{(1)}[i_1, j_1, :]}_{1\times R_1}\underbrace{\mathcal{G}^{(2)}[:, i_2, j_2, :]}_{R_1\times R_2}\ldots$$

$$\ldots\underbrace{\mathcal{G}^{(N-1)}[:, i_{N-1}, j_{N-1}, :]}_{R_{N-2}\times R_{N-1}}\underbrace{\mathcal{G}^{(N)}[:, i_N], j_N}_{R_{N-1}\times 1}$$

$\mathcal{G}_k \in \mathbb{R}^{R_k\times I_k\times J_k\times R_{k+1}}$, $k = \overline{1, N}$

Represent matrix of the FC layer as a Tensor Train Matrix

Matrix of FC layer $\mathbf{W} \in \mathbb{R}^{D_{in} \times D_{\text{out}}}$, where $D_{\text{in}} = \prod_{m=1}^{M} I_m$ and $D_{\text{out}} = \prod_{m=1}^{M} J_m$

- Reshape $\mathbf{W}$ into $2N$-dimetional objects. If we want 4 cores, $N = 4$:

$$\mathcal{B} = \mathcal{W}.\text{reshape}\left(I_1, J_1, \ldots, I_N, J_N\right)$$

- Permute axis of object so that the required axes from the tuple of indices are adjacent:

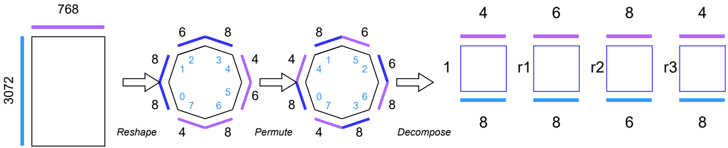
$$\mathcal{C} = \mathcal{B}.\text{permute}\left(1, N+1, 2, N+2, \ldots, N, 2N\right)$$

- Apply TT-SVD on C .
- Than compression rate (acoording to parameters number):

$$c_{\text{rate}} = \frac{R(I_1 J_1 + I_M J_M) + R^2 \sum_{m=2}^{M-1} I_m J_m}{\prod_{m=1}^{M} I_m J_m}$$

Linear Layer as TTM

We reshape a layer weights matrix to an N-dimensional object and represent it as a TT product. The key idea is to create the maximum possible number of kernels of the minimum size - in this case, we can get the best compression.



Issue in Signal Propagation

If we represent the FC layer as a sequence of $\mathcal{G}$ cores, we assume Forward pass as a contraction process between input activations **X** and all these cores.

Opt-Einsum library generates a string-type expression, which:

1. defines the shapes of input and resulting tensors (e.g. "ikl,lkj $\rightarrow$ ij")
2. defines contraction schedule (e.g., firstly along axis 'l' and then along axis 'k').

The contraction between a set of multidimensional objects may not be memory-optimal.

Layer	TTM-16	Fully-Connected
Backprop Strategy	Torch Autodiff	Torch Autodiff
Single Layer, Batch 16	1100 MB	395 Mb

Issue in Signal Propagation

Forward Pass

- **Einsum** is a torch.opt.einsum, contaction scheduler optimized by the time
- **Custom Einsum** we set fixed contraction scheduler by ourself, multiplying the cores sequentially

Backward Pass

A gradient computation might be considered as a tensor contraction:

$$\frac{\partial \mathcal{L}}{\partial \mathcal{G}_m} = \frac{\partial \mathcal{L}}{\partial \mathbf{W}} \frac{\partial \mathbf{W}}{\partial \mathcal{G}_m} = \mathbf{X}^T \frac{\partial \mathcal{L}}{\partial \mathcal{Y}} \frac{\partial \mathbf{W}}{\partial \mathcal{G}_m}.$$

- **Full Einsum** for all $\frac{\partial \mathcal{L}}{\partial \mathcal{G}_m}$ we can share intermediate results for contractions steps.
- **Full Matrix** tensors $\mathcal{X}$ and $\frac{\partial \mathcal{L}}{\partial \mathcal{Y}}$ are convolved along batch axis, and the rest schedule is further optimized

Optimization strategy in signal propagation

Optimization in Forward/Backward Strategies

Forward	Backward	Memory, Mb	Time, ms
Einsum	Torch Autodiff	1008	23.6
Einsum	Full Einsum	192	55.7
Einsum	**Full Matrix**	**192**	**17.5**
Custom Einsum	Torch Autodiff	2544	58.4
Custom Einsum	Full Einsum	192	84
Custom Einsum	Full Matrix	192	125

BERT and BART Compression Using SVD/TTM Decomposition

- Decompose the FC layer inside BERT and BART for cores with sizes $[1 \times 8 \times 6 \times R]$, $[10 \times 4 \times 4 \times R]$, $[R \times 4 \times 8 \times R]$, $[R \times 8 \times 4 \times 1]$.

BERT			BART		
C. Rate	SVD	TTM	C. Rate	SVD	TTM
53 mln	6	10	83 mln	10	10
69 mln	183	60	102 mln	210	64
102 mln	534	110	125 mln	460	96

Ranks for different compression approaches.

- $I_k \cdot J_k$ are equal or approximately equal to $(I \cdot J)^{1/N}$

Adding Fisher Information: SVD

How add information about task objective into decompostion algorithm? **Fisher information** is a way of measuring the amount of information that an observable random variable X carries about an unknown parameter θ of a distribution that models X.

We estimate Fisher information as follows:

$$\mathcal{I}_W = \mathbb{E}\left[\frac{\partial}{\partial \omega} \log p\left(\mathcal{D}|\omega\right)^2\right] \approx \frac{1}{|\mathcal{D}|} \sum_{d_i=1}^{|\mathcal{D}|} \left(\frac{\partial}{\partial \omega} \mathcal{L}(d_i;\omega)\right)^2,$$

where $\mathcal{D}$ is observable dataset

$$\hat{W} = \mathcal{I}_W W = USV^T$$
$$\hat{U} = \mathcal{I}_W^{-1} U,$$

where W - matrix of layer weights, $\tilde{\mathcal{I}}_W$ - matrix of the Fisher information of the same size

Adding Fisher Information:TTM

- Start by calculating the Fisher matrix $\mathcal{I}_W$ using the original layer weight matrix W.
- Transform $\mathcal{I}_W$ similarly as done with W to derive the "Fisher tensor" represented as $\hat{\mathcal{I}}_W$.
- In every SVD phase within the TTM process, the Fisher matrix is employed exactly like in the FWSVD method.

Results for BERT

Natural Language Understanding (**GLUE Benchmark**: language acceptance (CoLA), sentiment analysis (SST-2), paraphrasing (MRPC, QQP), semantic similarity (STS-B), and natural language inference (MNLI, QNLI, RTE, and WNLI))

Method	C. Rate	AVG	STSB	CoLA	MNLI	MRCP	QNLI	QQP	RTE	SST2	WNLI
Full	100 %	<u>0.79</u>	<u>0.88</u>	<u>0.57</u>	<u>0.84</u>	<u>0.90</u>	<u>0.91</u>	0.87	<u>0.67</u>	<u>0.92</u>	0.54
DistilBERT	61 %	0.76	0.87	0.51	0.82	0.87	0.89	<u>**0.88**</u>	0.59	0.91	0.48
FP16 eval.	100%†	0.78	0.88	0.55	0.83	0.88	0.90	<u>**0.88**</u>	0.67	0.91	0.48
Block Pruning (75%)	61%	0.72	0.85	0.24	0.83	0.83	0.86	0.87	0.52	0.88	0.56
SVD		0.37	0.24	0.00	0.36	0.20	0.50	**0.47**	0.48	0.52	0.51
FWSVD	3*49 %	0.38	0.25	0.00	0.33	**0.39**	0.50	0.40	0.49	0.51	<u>**0.56**</u>
TTM		0.40	0.38	0.00	0.37	0.20	0.53	0.42	**0.50**	0.69	0.51
FWTTM		**0.44**	**0.58**	0.01	**0.37**	0.26	0.56	0.42	**0.50**	**0.70**	0.51
SVD		0.45	0.63	0.01	0.36	0.22	0.51	0.54	0.54	0.78	0.48
FWSVD	3*63 %	**0.55**	0.54	**0.07**	**0.52**	**0.55**	0.62	**0.70**	**0.58**	**0.79**	**0.55**
TTM		0.44	0.65	0.01	0.40	0.16	0.54	0.52	0.48	0.74	0.48
FWTTM		0.47	**0.71**	0.00	0.43	0.18	**0.64**	0.56	0.47	0.72	0.49
SVD		0.70	0.81	0.26	0.82	0.69	0.88	0.87	0.53	0.90	0.53
FWSVD	3*95 %	**0.78**	<u>**0.88**</u>	0.55	**0.84**	0.87	**0.90**	<u>**0.88**</u>	0.64	<u>**0.92**</u>	**0.55**
TTM		0.76	0.87	0.52	0.79	0.86	0.87	0.86	0.65	0.91	0.48
FWTTM		0.77	<u>**0.88**</u>	**0.56**	0.83	**0.88**	**0.90**	<u>**0.88**</u>	**0.66**	<u>**0.92**</u>	0.46

Results for BART: style transfer

Dataset Paradetox:

Phrase	now i feel like an a*s
Paraphrase	now i feel like worthless

STA (style transfer accuracy), **SIM** (similarity), and **FL** (fluency of generated text)

Method	C. Rate	STA	SIM	FL	J
`bart-base`	100%	0.89	0.60	0.82	0.44
FP16 eval.	100%†	0.89	0.60	0.82	0.44
Block Pruning (65%)	74%	0.82	0.60	0.73	0.36
SVD	60%	0.75	**0.59**	0.65	0.28
FWSVD	60%	**0.78**	**0.59**	0.68	**0.30**
TTM	60%	0.74	0.58	0.64	0.27
FWTTM	60%	0.77	**0.59**	**0.69**	**0.30**
SVD	74%	0.82	0.60	0.77	0.38
FWSVD	74%	**0.87**	0.61	**0.80**	**0.42**
TTM	74%	0.82	0.61	0.75	0.37
FWTTM	74%	0.84	**0.62**	0.76	0.38
SVD	90%	0.86	0.61	0.81	**0.43**
FWSVD	90%	**0.87**	0.61	0.81	**0.43**
TTM	90%	0.86	0.61	0.80	0.41
FWTTM	90%	0.86	**0.62**	**0.82**	**0.43**

TTM Layers in GPT-2, From Scratch

Language modelling task on a GPT medium

Model	Training	Validation	Number of parameters	% of classic GPT-2 size	Perplexity
GPT-2 med	Webtext	Wikitext-103	354 823 168	100	20.56
GPT-2 TTM-72	Openwebtext	Wikitext-103	218 303 488	61	30.85
GPT-2 SVD-50	Openwebtext	Wikitext-103	220 920 832	62	55.46
Distill GPT-2	Openwebtext	Wikitext-103	81 912 576	23	51.45
OPT 350m	Openwebtext + BookCorpus + Pile Pile	Wikitext-103	331 196 416	93	24.75

Text summarization on CNN/Daily Mail

Model	ROUGE-1	ROUGE-2	ROUGE-L
GPT-2 med	20.5	4.6	10.2
OPT-350m	15.9	3.7	11.5
GPT-2 TTM-72	20.1	4.1	9.9
GPT-2 SVD-50	18.1	2.3	11.3
DistilGPT	12.7	2.3	7.5

Memory needed for training

Memory, needed to provide one forward-backward operation on a NVIDIA A40. The batch size is equal to 1.

BERT

Memory (MB) / Layers	Regular FC	TTM-10	SVD-6
Model + input	418.7	204.6	204.9
After Forward	1069.5	840.1	851.9
After Backward	869.3	433.6	429.9
Peak usage	1101.1	901.8	865.7

Avaraged inference time and power consumption

The **Eco2AI** library for tracking CO2 emissions monitors the energy consumption of GPU devices.

- estimate GPU and CPU energy usage
- multiply it to emission intensity coefficient
- convert to equivalent CO2 emissions

BERT

FC Layers	Regular FC	TTM-10	SVD-6
Inference time (ms)	37.8	63.8	27.6
Power (10^{-5} kWh)	1.7	2.11	1.4

Thank you for your attention =)