Navigation Control For

AUTONOMOUS ROVER

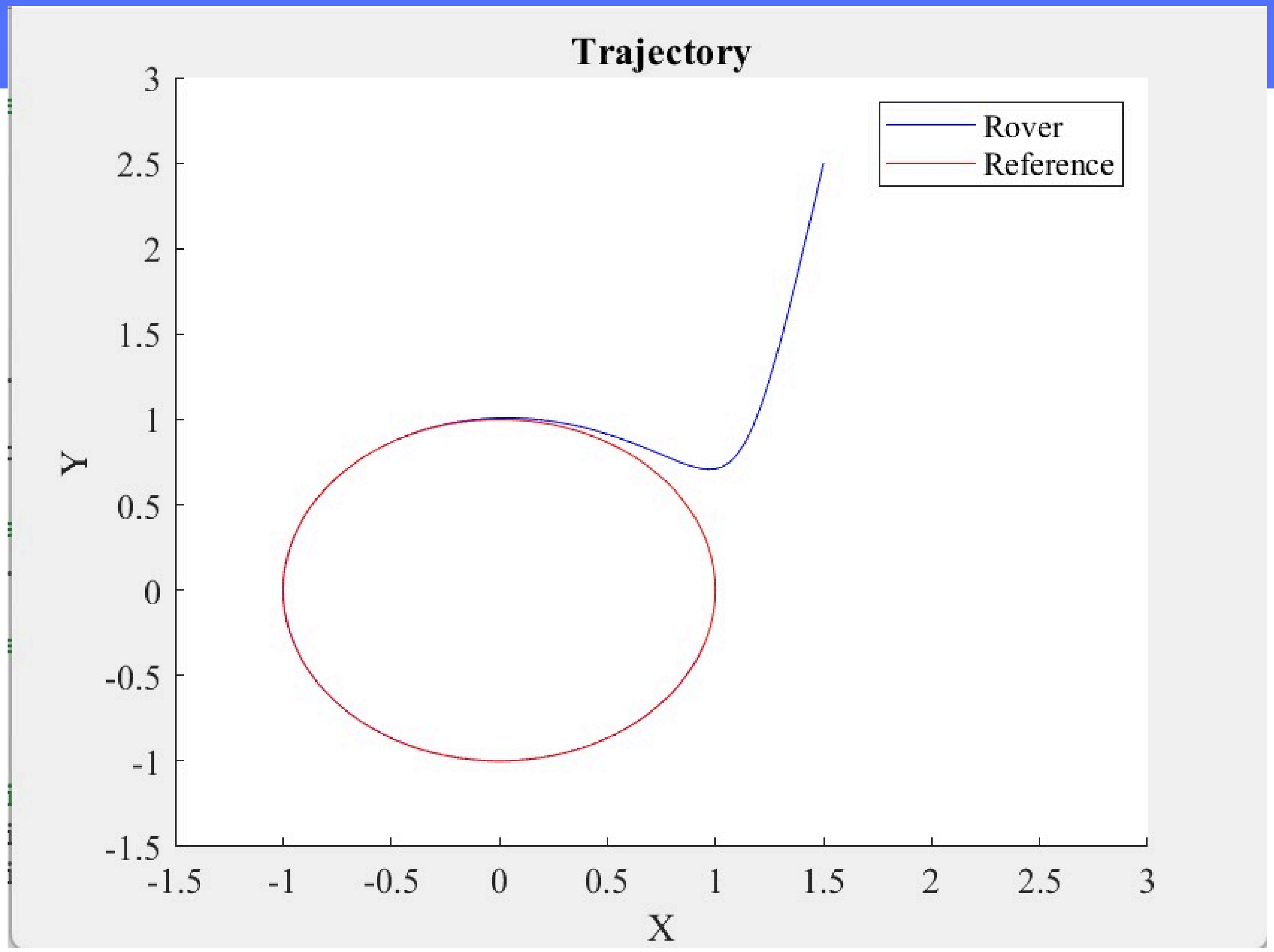
Designing a State-Feedback Controller for a Trajectory Following Rover

Group members :

Aditya A. Agrawal (210057)
Anunay Shukla (210169)
Divyanshi Bansal (210361)
M R Rakshith (210566)
Shraman Das (210996)

INDEX

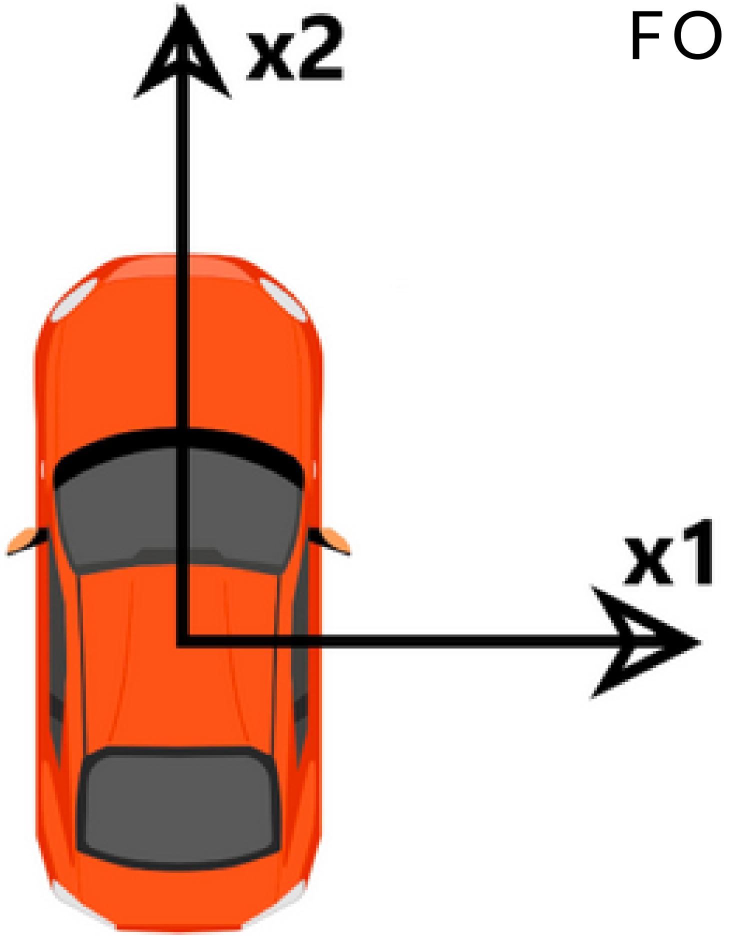
- DYNAMICS OF THE SYSTEM
- STATE SPACE MODEL
- OBJECTIVE OF DESIGN
- DESIGN
- RESULT
- JUSTIFICATION



DYNAMICS OF THE SYSTEM

- LET THERE BE A GLOBAL FIXED COORDINATE SYSTEM WITH AXES X_1 AND X_2. THE POSITION OF THE CAR AND THE REFERENCE SIGNAL ARE MEASURED WITH RESPECT TO THIS GLOBAL AXIS SYSTEM.

- THE CONTROL SIGNALS APPLIED TO THE CAR U_1 AND U_2 ARE VELOCITY ALONG THE X_1 AND X_2 DIRECTION.



FOLLOWING ARE THE EQUATIONS OF THE DYNAMICS OF THE SYSTEM:

$$\frac{dX_1}{dt} = U_1 \quad ; \quad \frac{dX_2}{dt} = U_2$$

LET (X_{i1}, X_{i2}) BE THE INITIAL POSITION OF THE CAR AND (X_{f1}, X_{f2}) BE THE FINAL POSITION OF THE CAR AFTER TIME dt

$$dX_1 = X_{f1} - X_{i1} \quad ; \quad dX_2 = X_{f2} - X_{i2}$$

$$X_{f1} = X_{i1} + U_1 * dt \quad ; \quad X_{f2} = X_{i2} + U_2 * dt$$

STATE-SPACE MODEL

- LET OUR CURRENT STATE BE X, OUTPUT BE Y, FORCED CONTROL INPUT BE U WHERE, X IS THE CURRENT POSITION OF THE CAR WITH RESPECT TO THE GLOBAL FIXED FRAME, Y BE THE FINAL POSITION OF THE CAR WITH RESPECT TO THE GLOBAL FIXED FRAME AND U BE THE VELOCITY APPLIED WITH RESPECT TO THE GLOBAL FIXED FRAME. THEN IN A DISCRETE STATE SPACE MODEL:

$$\frac{dX(t)}{dt} = I.U(t) \quad ; \quad Y(t) = I.U(t)+(dt).I.U(t)$$

SINCE $dt \rightarrow 0$, THIS REDUCES OUR SECOND EQUATION TO : $Y(t) = I.U(t)$

THUS, OUR STATE SPACE MODEL IS AS FOLLOWS :

$$\frac{dX(t)}{dt} = A.X(t) + B.U(t)$$

$$Y(t) = C.X(t) + D.U(t)$$

$$A = \begin{bmatrix} 0 \ , & 0 \\ 0 \ , & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \ , & 0 \\ 0 \ , & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 \ , & 0 \\ 0 \ , & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \ , & 0 \\ 0 \ , & 0 \end{bmatrix}$$

OBJECTIVE OF THE CONTROLLER

- THIS IS A TRACKING PROBLEM SINCE THE OUTPUT HAS TO TRACK THE REFERENCE SIGNAL INPUT BY US INTO THE PROGRAM. WE ARE GIVING 3 REFERENCE INPUTS WHICH ARE AS FOLLOWS:

  1. A circular path
  2. A sinusoidal wave
  3. Waypoints in 2D space

$U(t) = -K.X(t) + N.r(t)$ ➡ $\frac{dX(t)}{dt} = A_{cl}.X(t) + B.N.r(t)$ where $A_{cl} = (A-B.K)$

WE HAVE TO DESIGN OUR SYSTEM FOR STABILIZING THE FOLLOWING SYSTEM :

$\frac{dX(t)}{dt} = A_{cl}.X(t)$, BY PLACING IT'S EIGENVALUES WITH NEGATIVE REAL PART

WE PLACE OUR POLES IN [-20,-20]

THE MATRIX USED FOR REFERENCE SIGNAL IS: $N = -C.(A-B.K).B^{-1}$

DESIGN OF THE LUENBERGER OBSERVER

- LET OUR ESTIMATED STATE BE X_e AND THE ACTUAL STATE BE X. WE WILL THUS HAVE THE FOLLOWING SYSTEM DYNAMICS:

$$\frac{dX_e(t)}{dt} = (A-L.C).\ X_e(t) + B.U + L.y$$

WHERE L IS THE OBSERVER GAIN

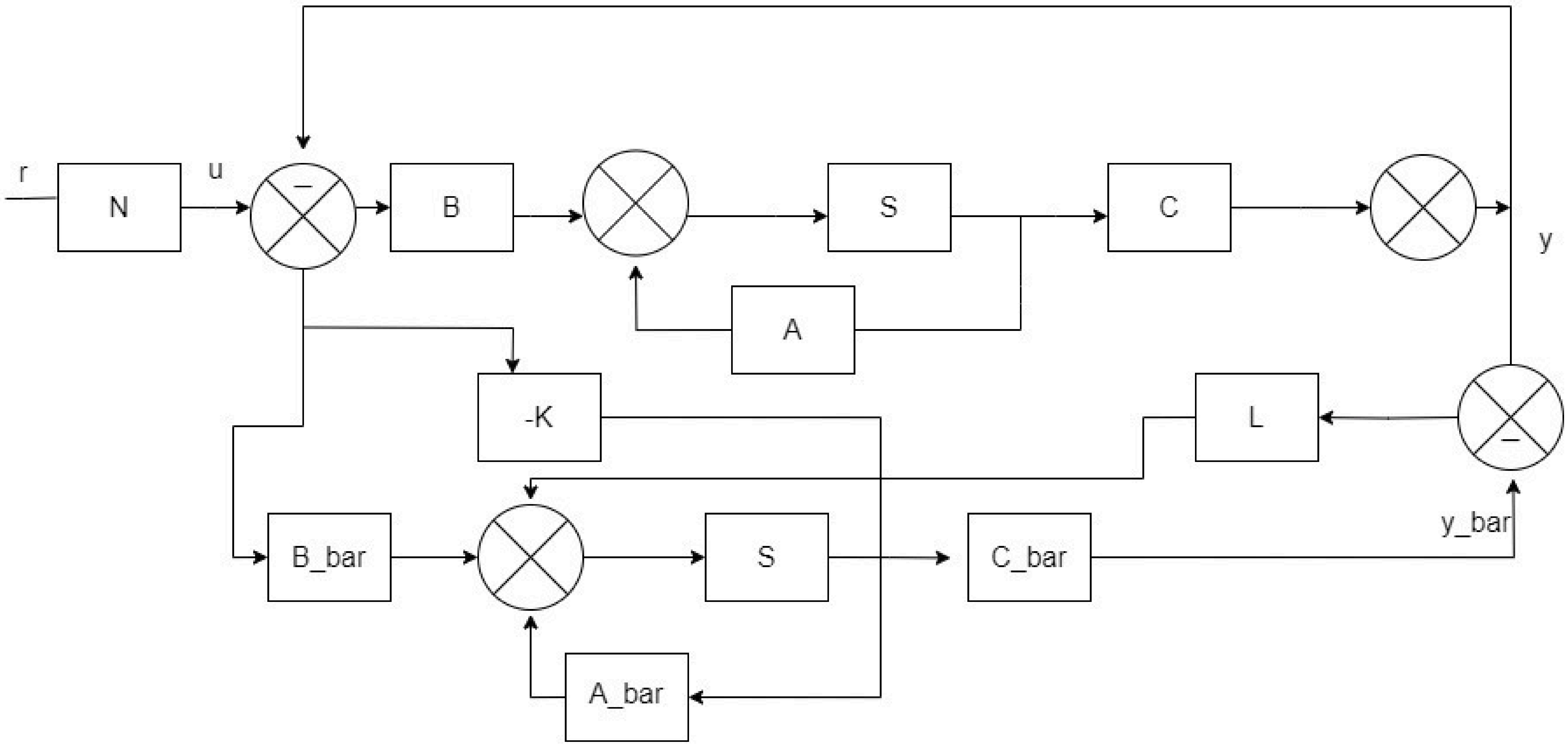
$$\frac{dX(t)}{dt} = (A-L.C).X(t)\ ;\ y = C.X(t)$$

- OUR ERROR TERM IS AS FOLLOWS :

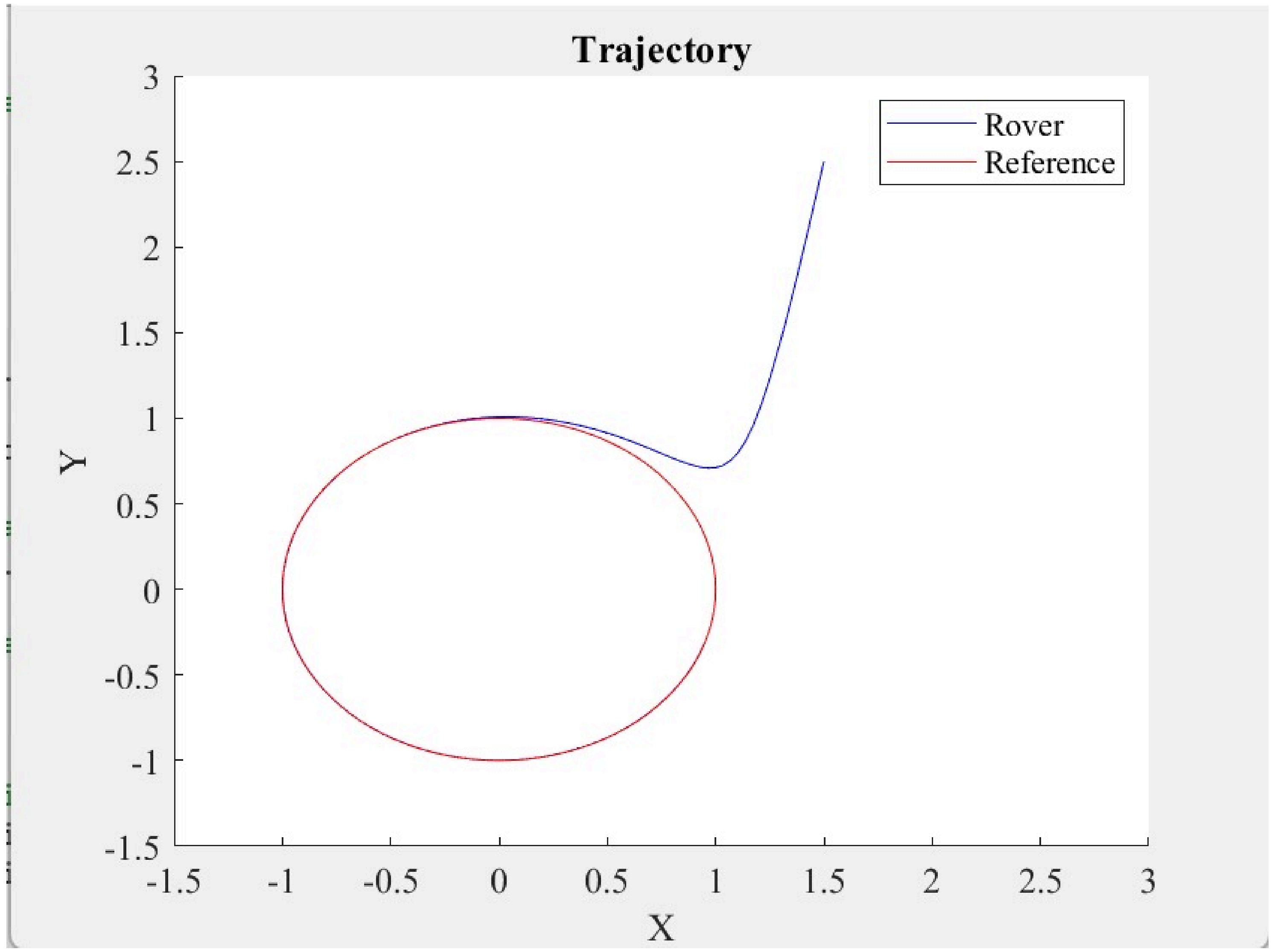
$$\frac{de}{dt} = \frac{d(X(t) - X_e(t))}{dt} = (A-L.C).e$$

TO ENSURE THAT THE ERROR TERM DECAYS TO ZERO ASYMPTOTICALLY, WE HAVE TO PLACE THE EIGENVALUES OF THE MATRIX (A - L.C) IN THE OPEN LEFT HALF PLANE. WE PLACE IT AT [-100, -100]

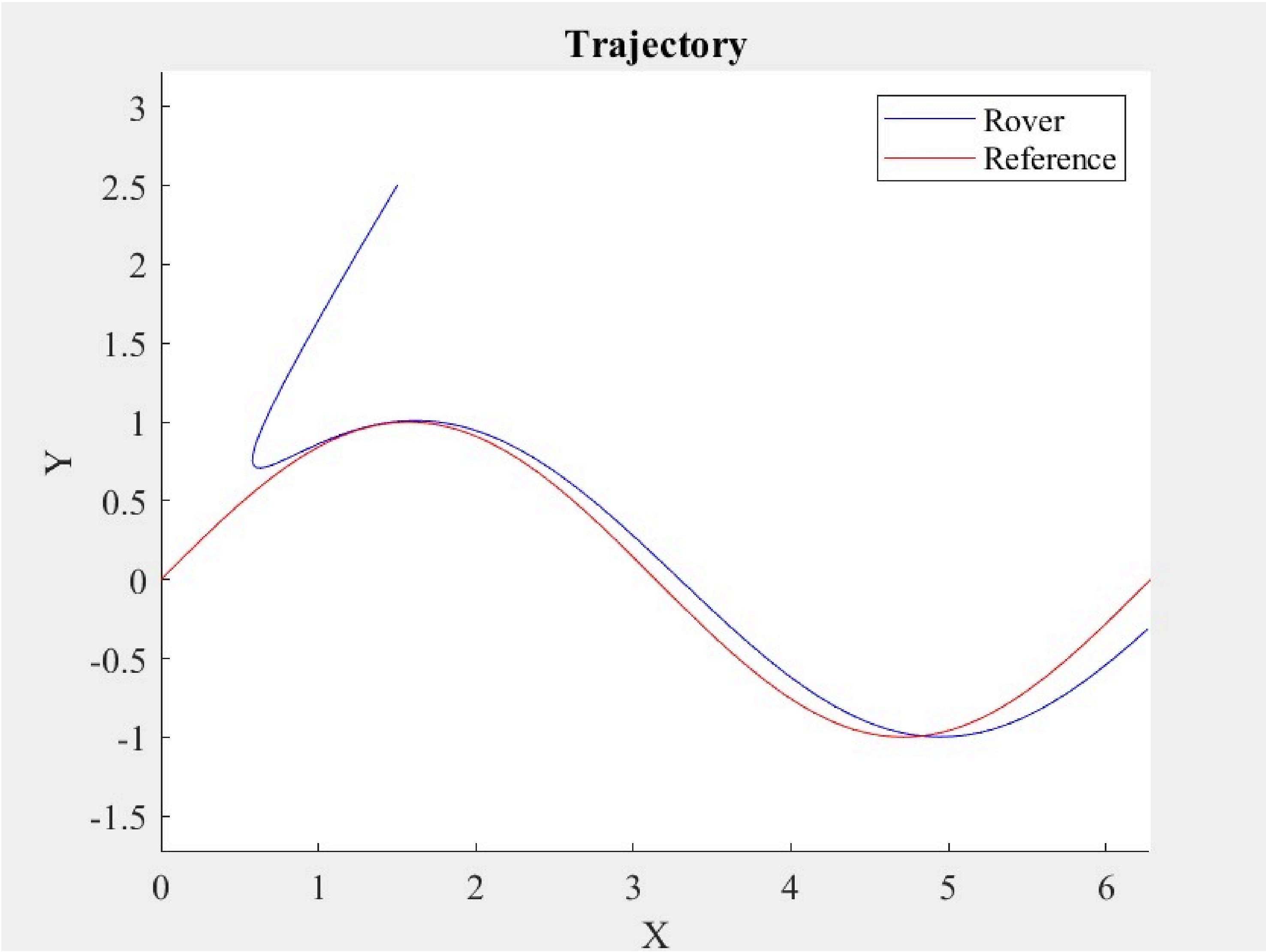
DESIGN OF THE CONTROLLER



RESULTS OBTAINED

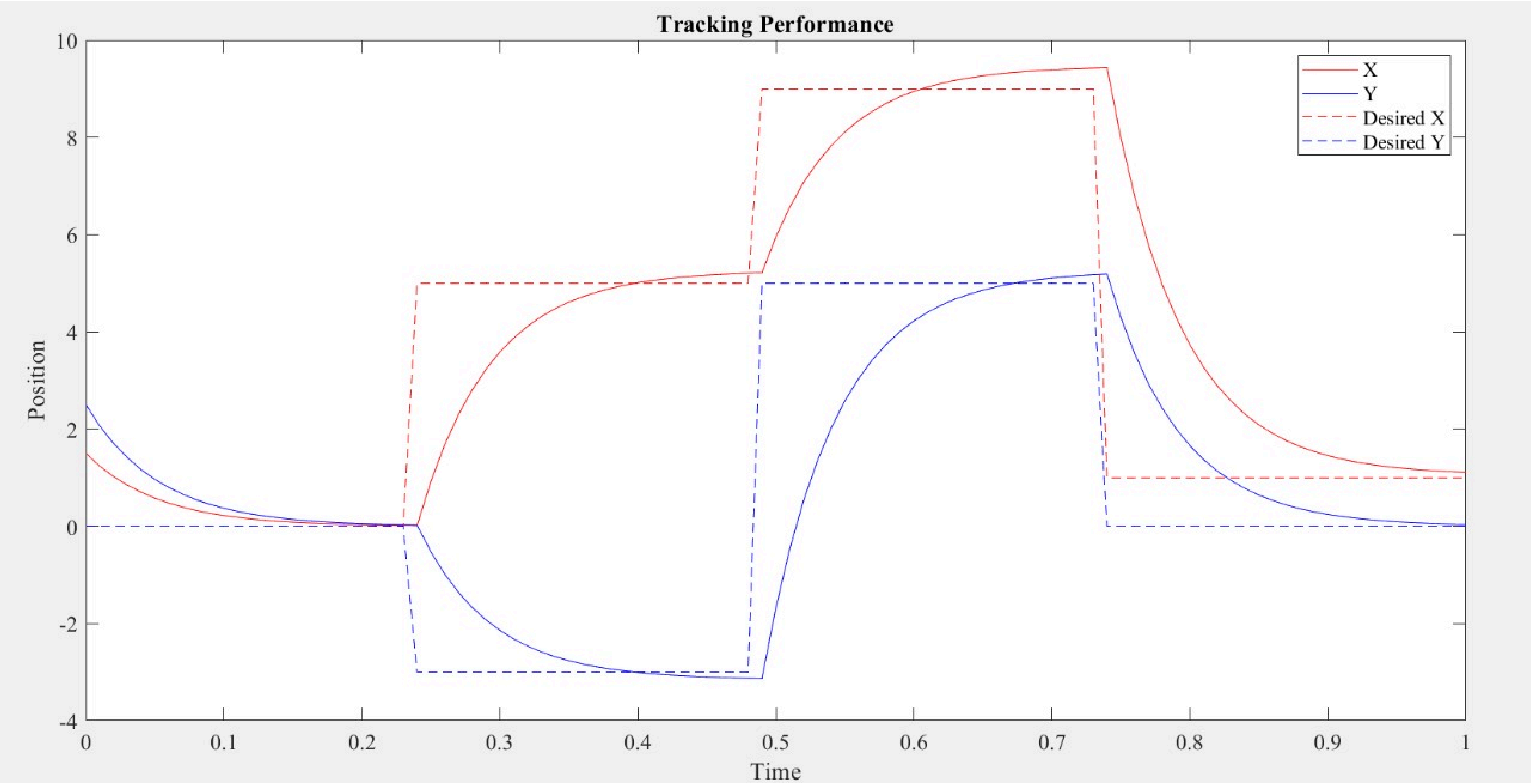


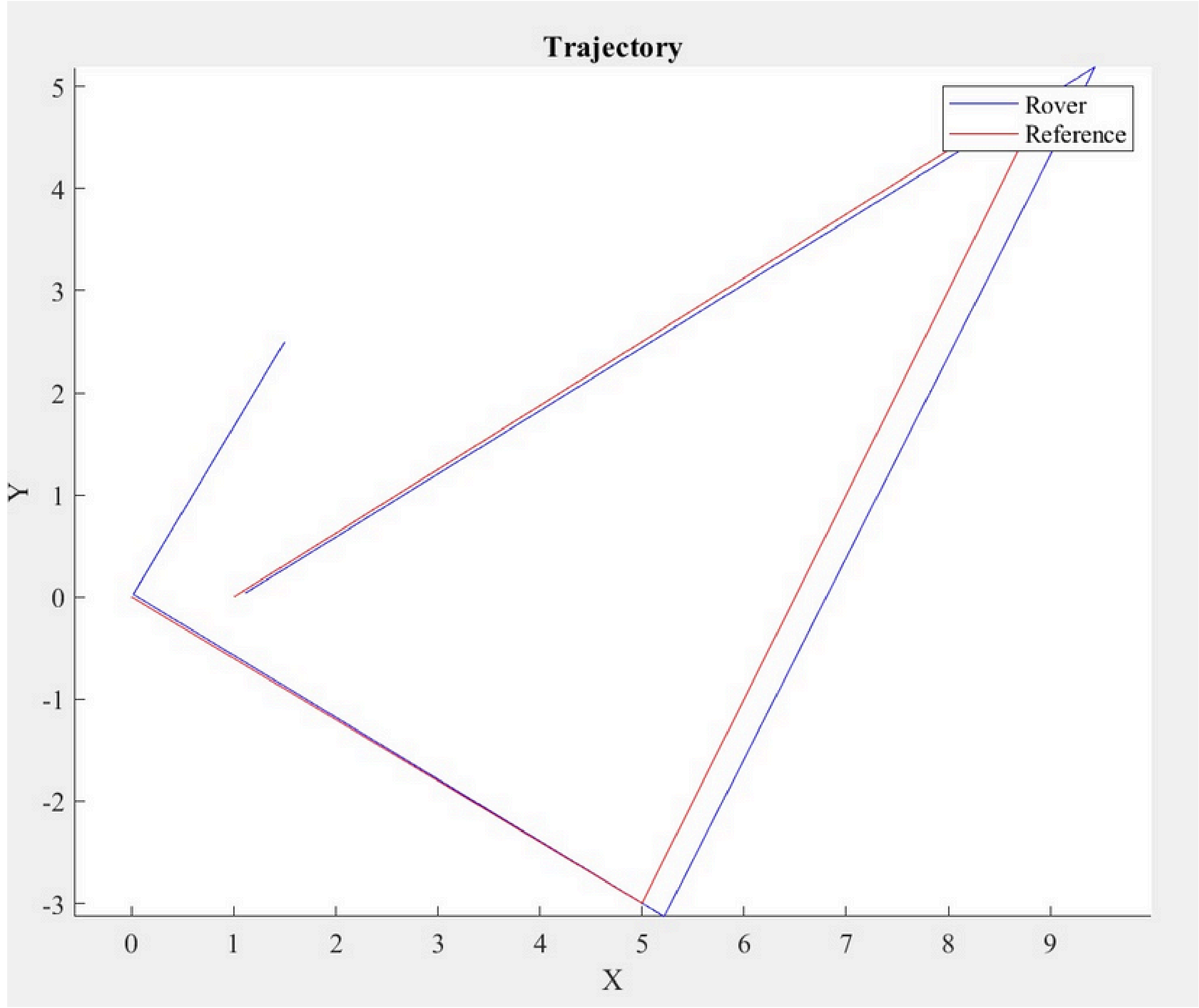
CIRCLE



SINE WAVE

RESULTS OBTAINED





SET OF DESIRED WAYPOINTS

JUSTIFICATION FOR CONTROLLER PERFORMANCE

1. **CONTROLLER GAINS**:
- THE CONTROLLER GAINS PLACE THE POLES AT SPECIFIC LOCATIONS, INFLUENCING THE SYSTEM'S STABILITY AND RESPONSE SPEED.
- WE HAVE ARBITRARILY CHOSEN OUR POLES TO BE [-20, -20].

2. **VELOCITY DEPENDENCE**:
- THE MAXIMUM VELOCITY OF THE ROVER IS DEPENDENT ON THE POLE PLACEMENT OF THE CONTROLLER AS WELL AS THE INITIAL CONDITIONS.

3. **PERFORMANCE METRICS**:
- WE GET A SETTLING TIME OF ABOUT 0.2S FOR THE SELECTED POLES.

4. **ADJUSTMENTS**:
- DECREASING THE CONTROLLER GAINS LEADS TO FASTER SETTLING TIME BUT INCREASES STEADY-STATE ERROR.

JUSTIFICATION FOR OBSERVER PERFORMANCE

1. **OBSERVER GAINS:**
- WE PLACED THE POLES OF L_TRACKING AT [-100, -100] TO ACHIEVE FAST CONVERGENCE, WHILE ALSO MAINTAINING THE ROBUSTNESS OF THE OBSERVER.

2. **POLE PLACEMENT**:
- FASTER CONVERGENCE OF THE OBSERVER REDUCES THE TIME IT TAKES TO ESTIMATE THE ACTUAL STATE.

3. **STATE ESTIMATION ACCURACY**:
- THE OBSERVER IS DESIGNED TO PROVIDE ACCURATE ESTIMATES OF THE UNMEASURED STATES OF THE SYSTEM.

4. **ROBUSTNESS TO DISTURBANCES**:
- A WELL-PERFORMING OBSERVER SHOULD EXHIBIT FAST CONVERGENCE, LOW STEADY-STATE ERROR, AND ROBUSTNESS TO DISTURBANCES.

5. **TUNING ADJUSTMENTS**:
- PLACING POLES FURTHER FROM THE ORIGIN (NEGATIVE) ENHANCES THE CONVERGENCE, OBSERVER GAINS CAN BE ADJUSTED TO BALANCE THE TRADE-OFF BETWEEN CONVERGENCE SPEED AND ROBUSTNESS.

THANK YOU