

# **Indian Institute of Technology, Kanpur**

## **Department of Aerospace Engineering**



## **Attitude Control Actuators for a Simulated Spacecraft**

*submitted by*

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## **Abstract:**

This report explores the principles behind attitude control systems in spacecraft, focusing on the use of Control Moment Gyroscopes (CMGs) and reaction wheels. The original research was centred on developing and validating a CMG array for the Air Force Institute of Technology's satellite simulator, SimSat, aimed at providing a platform for testing advanced control algorithms. Due to the hardware-specific nature of the study, this report will provide a comprehensive overview of the system's operation, including the dynamics and control mechanisms involved. Additionally, a simplified PID control model is implemented using Simulink to simulate the fundamental concepts of attitude control. This simulation aims to illustrate how PID controllers can manage spacecraft orientation, providing a practical demonstration of the core concepts presented in the original research.

## **Introduction:**

Controlling a satellite's orientation, known as attitude control, is crucial for successful space missions. Whether it's keeping communication antennas pointed towards Earth, aligning scientific instruments, or ensuring solar panels receive maximum sunlight, maintaining the right orientation is essential. This is typically achieved using a variety of devices that adjust the satellite's position. Among the most effective are internal systems like Control Moment Gyroscopes (CMGs) and reaction wheels, which allow precise orientation changes without using up propellant. The research that this report is based on involved the Air Force Institute of Technology's SimSat, a satellite simulator. The original study focused on building and testing a CMG system that could be used for developing and verifying advanced attitude control algorithms in a lab setting. Improvements to the existing reaction wheel system were also a key part of the project, aimed at increasing the momentum storage and making the system more stable. Since this research was heavily hardware-oriented, replicating the experiment is not practical. Instead, this report will delve into the mechanics of how CMGs and reaction wheels function in attitude control. Additionally, a simplified simulation will be created in Simulink, using a PID controller to illustrate the basic principles of controlling a satellite's orientation. The aim of this report is to break down the complexities of attitude control, making it easier to understand how CMGs and reaction wheels contribute to satellite stability and manoeuvrability. By doing so, it will lay a foundation for anyone interested in the development of control algorithms and the role they play in modern aerospace engineering.

## **Rigid Body Dynamics:**

Angular momentum:  $\vec{H} = I\vec{\omega}$

where  $\vec{\omega}$  – Angular Velocity of the Object

Moment of Inertia Tensor:  $I_b = \begin{matrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{matrix}$

Moment Vector:  $\vec{M} = \frac{d\vec{H}}{dt}$  (w.r.t inertial frame)

Using Chain Rule to expand:

$$\frac{d\vec{H}}{dt} = \left\{ \frac{dI}{dt} \right\} \vec{\omega} + I \left\{ \frac{d\vec{\omega}}{dt} \right\}$$

Expressing  $I_b$  in Inertial frame:

$$\frac{d\vec{H}}{dt} = I_b \frac{d\vec{\omega}_{bI}}{dt} + \vec{\omega}_{bI} \times I_b \vec{\omega}_{bI}$$

Euler's Equation for Rotational Bodies:

$$\vec{M} = I_b \frac{d\vec{\omega}_{bI}}{dt} + \vec{\omega}_{bI} \times I_b \vec{\omega}_{bI}$$

Angular Momentum Exchange:

$$\vec{H}_{net} = I_b \vec{\omega}_{bI} + \vec{h}_{act}$$

where  $I_b$  contains both the vehicle's static MOI and the MOI of each actuator combined using the parallel axis theorem. The actuator's angular momentum  $\vec{h}_{act}$  contains only the dynamic angular momentum of the actuators

Moment:

$$\vec{M} = I_b \frac{d\vec{\omega}_{bI}}{dt} + \frac{d\vec{h}_{act}}{dt} + \vec{\omega}_{bI} \times (I_b \vec{\omega}_{bI} + \vec{h}_{act})$$

If there are no external moments, the above equation can be rearranged to,

$$I_b \frac{d\vec{\omega}_{bI}}{dt} = -\frac{d\vec{h}_{act}}{dt} - \vec{\omega}_{bI} \times (I_b \vec{\omega}_{bI} + \vec{h}_{act})$$

The Last term is the non-linear part. To address this non-linear feedback controller can be used.

## Control Actuators:

There are two types of Control Actuators:

- Reaction Wheels
- Control Moment Gyroscopes

Reaction Wheels:

It consists of Flywheel, Electric Motor and Supporting Electronics. The reaction wheel motor is mounted rigidly to the spacecraft body, fixing its axis of rotation in the body frame. The angular momentum of an individual reaction wheel is

$$\vec{h}_1 = I_{rw} \vec{\psi}_1$$

where  $I_{rw}$  is the reaction wheel's scalar moment of inertia along its axis of rotation and  $\vec{\psi}_1$  is the angular rate vector of the reaction wheel defined in the body frame of the spacecraft. In order to control the vehicle in all three axes, a minimum of three reaction wheels are required. The Angular Momentum of the Reaction Wheel Array is

$$\overrightarrow{h}_{rwa} = \begin{bmatrix} I_{rw} \dot{\Psi}_1 \\ I_{rw} \dot{\Psi}_2 \\ I_{rw} \dot{\Psi}_3 \end{bmatrix}$$

The Moment Equation for the reaction wheel is

$$\vec{M} = \mathbf{I}_b \dot{\vec{\omega}}_{bi} + \begin{bmatrix} I_{rw} \dot{\Psi}_1 \\ I_{rw} \dot{\Psi}_2 \\ I_{rw} \dot{\Psi}_3 \end{bmatrix} + \vec{\omega}_{bi} \times \left( \mathbf{I}_b \vec{\omega}_{bi} + \begin{bmatrix} I_{rw} \Psi_1 \\ I_{rw} \Psi_2 \\ I_{rw} \Psi_3 \end{bmatrix} \right)$$

It should be noted that most spacecraft use four reaction wheels in a pyramidal arrangement so that if any one reaction wheel fails, torques can still be generated in all three directions. Because reaction wheels do not change orientation relative to the body, the formulation of the spacecraft dynamics as described by Eq. (22) does not change as the reaction wheels change angular velocity. To summarize Reaction Wheels

- What They Do: Reaction wheels are used to fine-tune the satellite's orientation. They work by spinning up or down to create small, precise changes in the satellite's angular momentum, helping it stabilize or maintain a desired orientation.

- Modifications in the Study: In addition to the CMGs, the researchers improved the existing reaction wheel system. They increased the wheels' momentum storage capacity, making them more effective at maintaining orientation and handling small disturbances.

**Role in the System:** Reaction wheels were used for accurate adjustments and steady-state control, complementing the rapid manoeuvring capability provided by the CMGs.



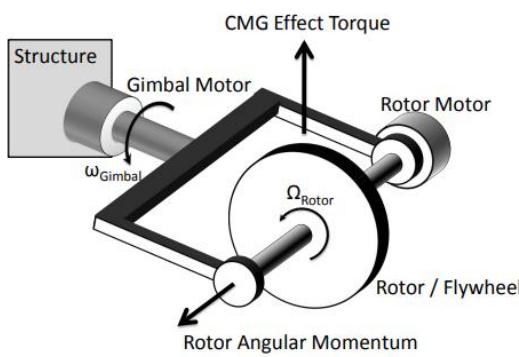
#### Control Moment Gyroscope:

The second category of momentum exchange devices is the control moment gyroscope (CMG). A CMG exchanges momentum with the spacecraft by rotating a flywheel mounted on a gimballed platform. The flywheel is typically spun at a constant rate, and therefore maintains a fixed magnitude of angular momentum. Torque is applied to the gimbals to change the direction of the angular momentum vector, which imparts a torque on the vehicle.

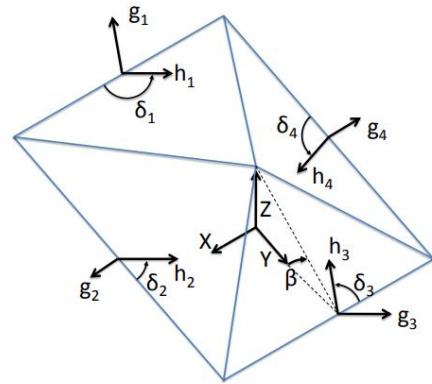
Neglecting the gimbal inertia, the torque produced is

$$\vec{\tau} = \overrightarrow{\omega_{\text{gimbal}}} \times \overrightarrow{h_{\text{rotor}}}$$

where  $\vec{\omega}$  is the gimbal rate and  $\vec{h}_i$  is the angular momentum of the CMG flywheel. Torque multiplication occurs because it requires only a small amount of torque to impart an angular velocity on the gimbal  $\vec{\omega}$  which changes the direction of the angular momentum stored in the rotor and imparts a large CMG effective torque on the spacecraft.



Single Gimbal CMG



Four CMG Pyramid Config.

The number and orientation of CMGs determines the overall performance of the array. There are several factors to consider when selecting the CMG array design: system dynamics, physical space limitations, costs, and singularity concerns. While a three unit CMG array can provide three axis control, the need for redundancy leads to four unit CMG arrays being the minimum number used on spacecraft. Because the CMG gimbals and rotors rotate relative to the body, the system dynamics are highly dependent on the CMG array configuration. Therefore, the control laws developed for one CMG array design may not be applicable to others. The four unit pyramid design was chosen for SimSat because it offers a near spherical momentum envelope.

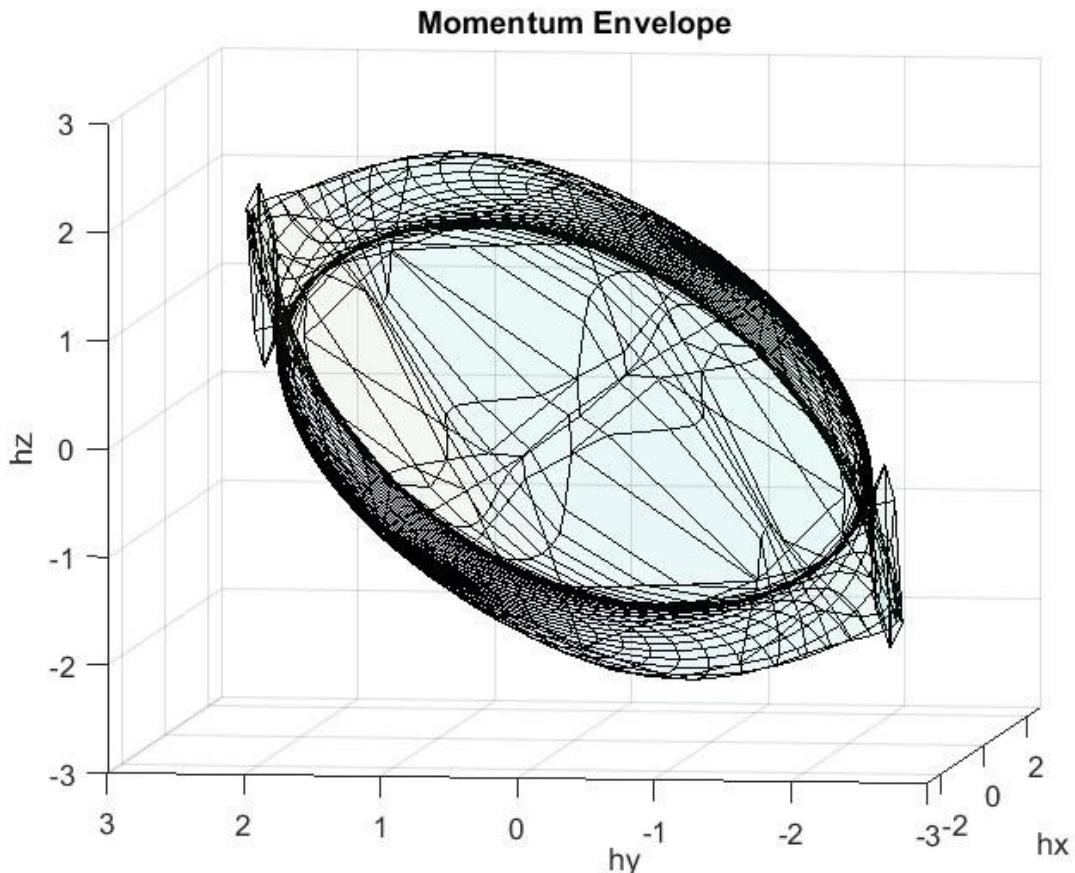
To summarize CMGs,

#### Control Moment Gyroscopes (CMGs)

- **What They Do:** CMGs are devices that help adjust a satellite's orientation quickly by generating strong control torques. Unlike reaction wheels, which rely solely on spinning to change momentum, CMGs use the gyroscopic effect, allowing them to produce high torque with less power.
- **Setup in the Research:** The project used a set of four CMGs arranged in a pyramidal configuration. This layout gave the system full control over all three axes, making it possible to handle any required adjustments in orientation.
- **How They Work:** Each CMG contains a spinning rotor that maintains momentum. When the angle of the rotor is changed (gimbaled), it generates a torque that shifts the satellite's orientation. The researchers implemented a specialized algorithm called the Moore-Penrose Pseudoinverse Steering Law to coordinate the gimbal angles effectively, ensuring precise control.

- Benefits: CMGs are particularly useful for quick and powerful manoeuvres, making them ideal for counteracting unexpected disturbances.

The following momentum envelope is done for a single envelope by me.



Momentum Envelope of a Single CMG

The change in Angular Momentum Array can be found from the following equation

$$\dot{\vec{H}}_{cmga} = \mathbf{J}_H \dot{\vec{\delta}}$$

where  $\dot{\vec{H}}_{cmga}$  is the controller solution and must be solved for  $\dot{\vec{\delta}}$ , however the Jacobian  $\mathbf{J}_H$  is not square and is therefore not directly invertible.

The CMG array was tested using the Moore-Penrose Pseudoinverse Steering Law (MPPSL) to validate its performance. The purpose of the steering law is to manipulate the angular momentum vector within this envelope to provide the desired torque, preferably without encountering a singularity

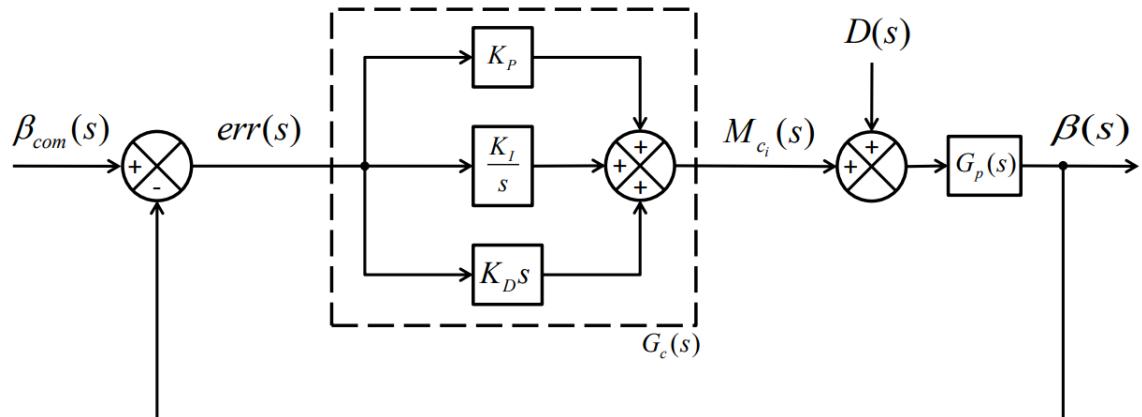
The steering law implemented in this research was the Moore-Penrose Pseudoinverse Steering Law (MPPSL), also known as the right-inverse, is a method of solving the system where the matrix has more columns than rows by creating the Moore-Penrose Pseudoinverse matrix

## Control System:

A Linearised PID Controller was used by the original research to control the parameters. In the project I have designed a simple PID model controlling the pitch angle when a disturbance is applied for a given Step input. When tuned properly, PID control can provide satisfactory, but not necessarily optimal, control for a range of inputs and disturbances. Because of these properties, PID control, as well as proportional-integral and proportional-derivative controls are used in over half of the industrial controllers used today. PID control operates on the difference between a desired state  $q_D$  and the actual state  $q_A$ , defined as the error  $q_E$ , to calculate a corrective action  $M_C$  which is fed to the plant  $G_P$ . The components of the PID control are the proportional, integral, and derivative controllers.

The PID Controller was used to control the attitude and speed of the motors in CMGs in the Original research.

# Control Algorithms used in the Research



PID Controller Block Diagram

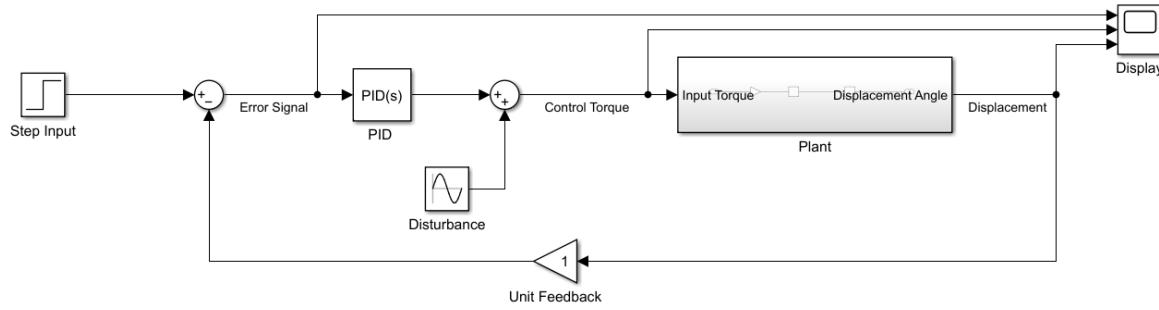
## 1. PID Controller

- The primary control mechanism in the study was a PID (Proportional-Integral-Derivative) controller, which is commonly used for stabilization and orientation adjustments.
- How It Works: The PID controller calculates how much torque to apply by looking at the difference between the desired orientation and the current orientation. It adjusts the control input to minimize this error over time.
- Fine-Tuning: The PID parameters were carefully tuned to ensure the system responded quickly without overshooting, allowing the satellite to stabilize efficiently.

## 2. CMG Steering Algorithm

- A specialized algorithm, known as the Moore-Penrose Pseudoinverse Steering Law, was used to control the gimbal angles of the CMGs. This algorithm ensures that the generated torque is distributed optimally across the four CMGs, avoiding configurations that could limit their effectiveness.
- Gimbal Control: The gimbals were moved within specific limits to prevent excessive motion, ensuring reliable torque output for quick and precise attitude adjustments.

PID control for a pitch angle of a Satellite model:



Simulink Block Diagram of Pitch angle PID Controller

The dynamic equation used is Newton's Law of rotation,

$$\tau = I \cdot \dot{\omega}$$

The moment of Inertia used is  $15 \text{ Kg.m}^2$ . The desired pitch angle is set to be  $30^\circ$ . And the desired value of Pitch angle changes with respect to different operations.

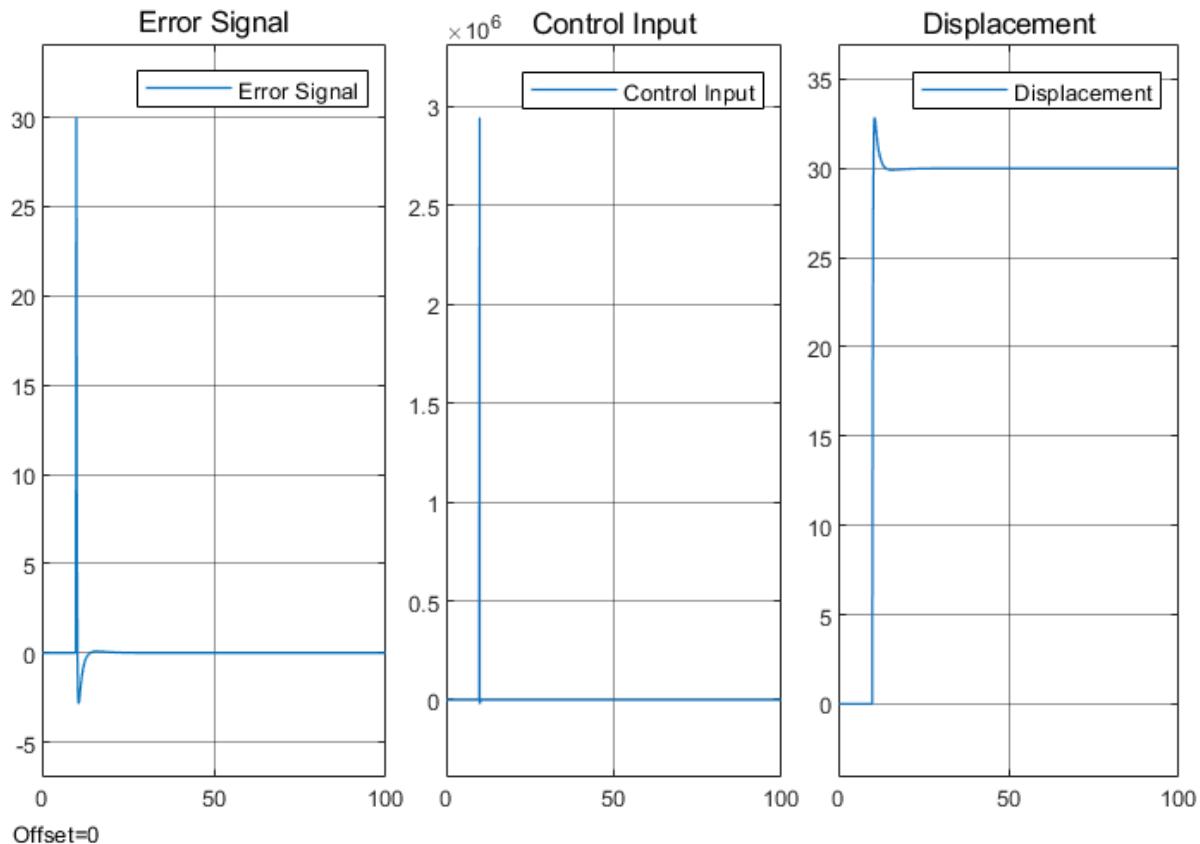
The typical values for a pitch controller are:

- Rise Time: 1–5 seconds.
- Settling Time: 5–10 seconds (with no significant oscillations).
- Overshoot: Less than 10% (ideally below 5%).

The PID is tuned for these values.

## Result and Recommendations:

The following is the results of PID controller tuned for desired output of pitch angle  $30^\circ$ .



This simple control technique is used in the original research paper. Further Feedback Linearization Block are added to nullify the effect of Non-Linear terms encountered in the Rigid Dynamics part. And sensor blocks are added in the feedback loop to provide the output signal.

This Control techniques can be extended to all other axes of rotations and for Controlling the speed of the motor in an CMG.

References:

1. Ryan E. Snider. Attitude Control of a Satellite Simulator Using Reaction Wheels and a PID Controller. MS Thesis, Air Force Institute of Technology (AU), March 2010.
2. Frederick A. Leve. Novel Steering and Control Algorithms for Single-Gimbal Control Moment Gyroscopes. PhD thesis, The University of Florida, 2010.
3. Attitude Control Actuators for a Simulated Spacecraft, Christopher McChesney ,Air Force Institute of Technology, Wright-Patterson AFB, Ohio, 45430.