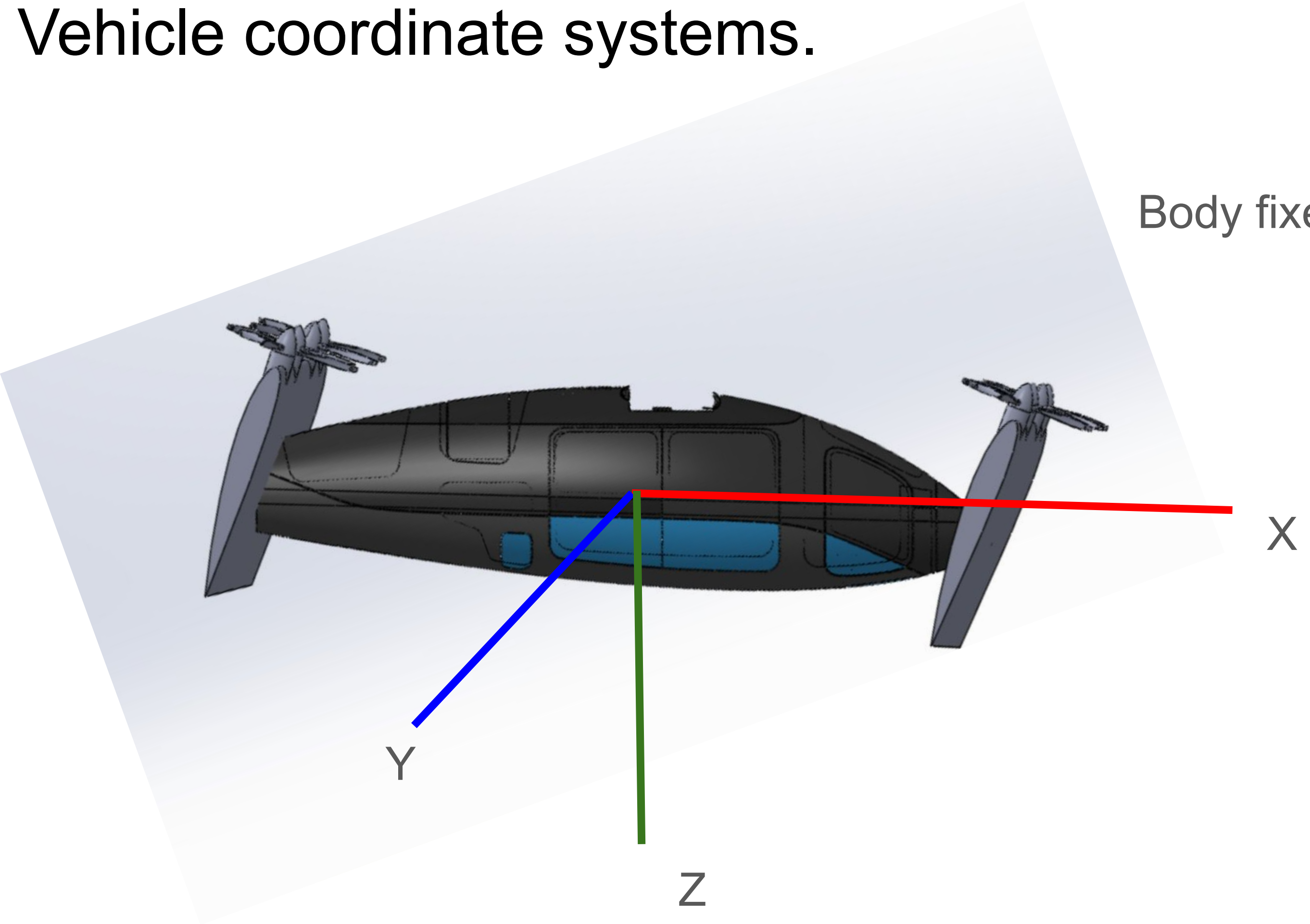
Kinematics and Dynamics

Vehicle coordinate systems.



Kinematics of the vehicle

Translational Kinematics

$$\begin{pmatrix} \dot{p}_n \\ \dot{p}_e \\ \dot{p}_d \end{pmatrix} = \begin{pmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

p_n, p_e, p_d are positions of the vehicle in inertial frame.

u, v, w are inertial velocities expressed in body frame.

Kinematics of the vehicle

Rotational Kinematics

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

Φ,Θ,Ψ are angular positions in three different coordinate sysems. p,q,r are angular rates of the vehicle in body frame.

Dynamics of the vehicle

Translational dynamics

$$\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} rv - qw \\ pw - ru \\ qu - pv \end{pmatrix} + \frac{1}{\mathrm{m}} \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix}$$

f_x, f_y, and f_z are forces acting on the vehicle expressed in body frame. We will model these forces for our vehicle.

Dynamics of the vehicle

Rotational dynamics

$$\dot{\boldsymbol{\omega}}^b_{b/i} = \mathbf{J}^{-1}\left[-\boldsymbol{\omega}^b_{b/i} \times \left(\mathbf{J}\boldsymbol{\omega}^b_{b/i}\right) + \mathbf{m}^b\right]$$

$$\begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} \frac{J_z}{\Gamma} & 0 & \frac{J_{xz}}{\Gamma} \\ 0 & \frac{1}{J_y} & 0 \\ \frac{J_{xz}}{\Gamma} & 0 & \frac{J_x}{\Gamma} \end{pmatrix} \left[\begin{pmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{pmatrix} \begin{pmatrix} J_x & 0 & -J_{xz} \\ 0 & J_y & 0 \\ -J_{xz} & 0 & J_z \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} + \begin{pmatrix} l \\ m \\ n \end{pmatrix} \right]$$

Here, J will be obtained from the CAD model of the vehicle which is the inertia matrix.

p,q,r are angular rates and l,m and n are the moments acting on the vehicle with acting about the center of mass.

Gravitational Force

Weight of the vehicle: acting along z-axis of inertial frame.

$$\mathbf{f}_g^b = \mathcal{R}_v^b \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix}$$

$$= \begin{pmatrix} -mg \sin\theta \\ mg \cos\theta \sin\phi \\ mg \cos\theta \cos\phi \end{pmatrix}$$

Since our translational dynamics equations are expressed in body frame, we have to represent this force in body frame.

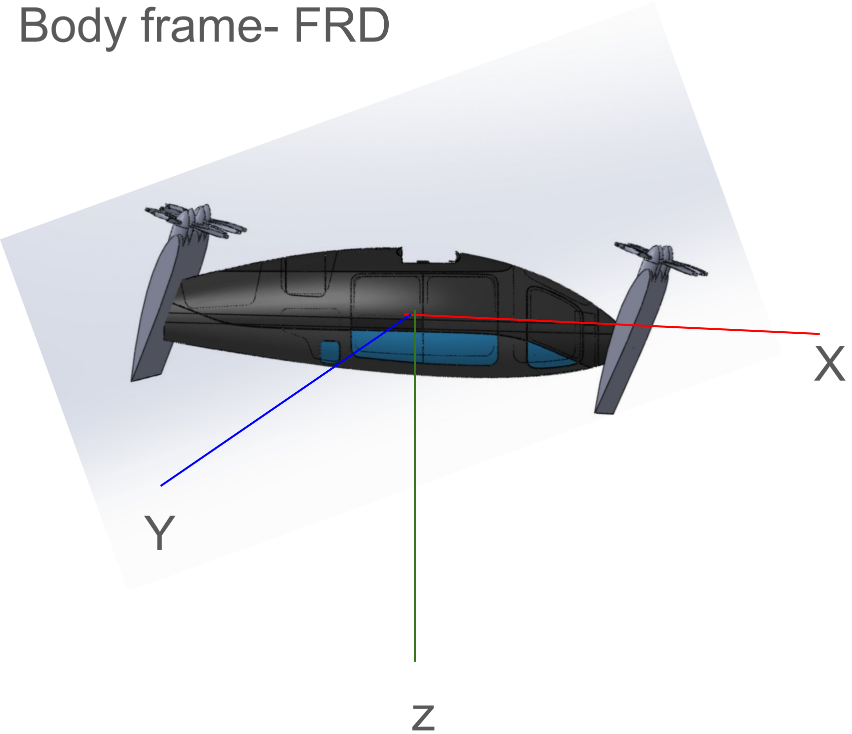
Aerodynamic forces Propulsive forces

These forces have to be modelled for our vehicle according to the non linear lift coefficient formula which takes care of higher angles of attack considering the region after the stall aoa as well.

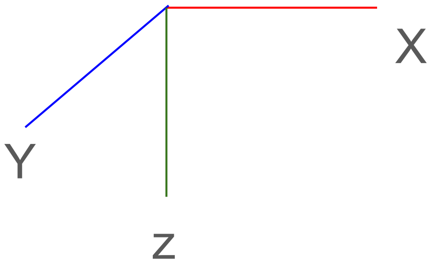
The Rotor aerodynamic analysis will provide us with the C_T and that will be utilised for the calculation of thrust and then, their components in the body frame will be in x and -ve z directions.

coordinate systems

Body frame- FRD



Inertial frame



Modelling the external forces and moments.

- Gravitational force
- Aerodynamic forces(Lift and drag)
- Propulsive forces(Thrust)

Gravitational Force

Weight of the vehicle: acting along z-axis of inertial frame.

$$\mathbf{f}_g^b = \mathcal{R}_v^b \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix}$$

$$= \begin{pmatrix} -mg \sin\theta \\ mg \cos\theta \sin\phi \\ mg \cos\theta \cos\phi \end{pmatrix}$$

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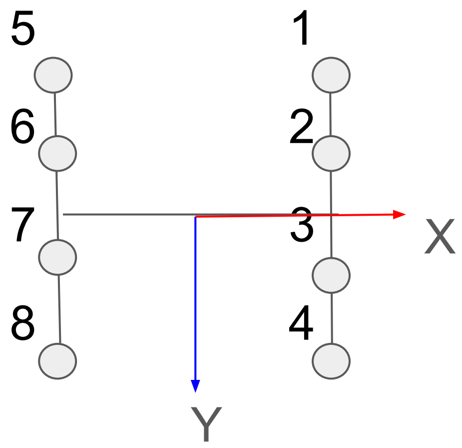
Propulsive Forces

Schematic of the vehicle's top view



1,2,5 and 6 are rotating in counter-clockwise direction
3,4,7 and 8 are rotating in clockwise direction.

The thrust produced by each rotor is represented as
T_i .

Forces from 8 rotors:

T_{bz} is the force in z direction , T_{by} is the force in y direction, T_{bx} is the force in x direction

$$T_{bz} = -\left(\sum_{i=1}^{8} T_i\right) \sin 45\, \hat{k}$$

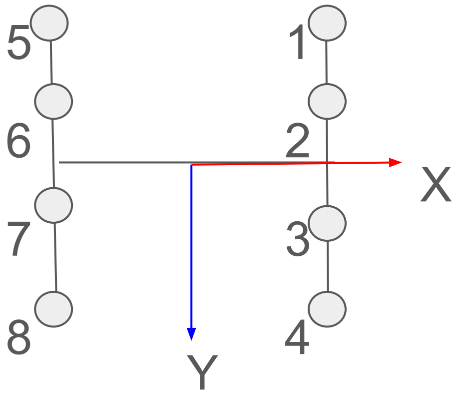
$$T_{by} = 0$$

$$T_{bx} = \left(\sum_{i=1}^{8} T_i\right) \cos 45\, \hat{i}$$

Moments due to 8 rotors

d= distance between fuselage centre to the rotor centers of 2,3,6 and 7

2d = distance between fuselage centre to the rotor centers of 1,5,4 and 8



2l = distance between the 2 wings.

M_{bx}, M_{by} and M_{bz} are moments in the respective body frame directions.

$$M_{bx} = [(T_1 + T_5 - T_4 - T_8)2d + (T_2 + T_6 - T_3 - T_7)d] \sin 45$$
$$+(M_3 + M_4 + M_7 + M_8 - M_1 - M_2 - M_5 - M_6) \cos 45$$

$$M_{by} = (T_1 + T_2 + T_3 + T_4) - (T_5 + T_6 + T_7 + T_8)l \sin 45$$

$$M_{bz} = [(T_1 + T_5 - T_4 - T_8)l + (T_2 + T_6 - T_3 - T_7)l] \cos 45$$
$$+ [-(M_3 + M_4 + M_7 + M_8) + M_1 + M_2 + M_5 + M_6] \sin 45$$

Aerodynamic forces

Points to be considered

- A relative angle of 22.5 degrees between the wing and and the rotor would lead to stall aoa.
- Control surfaces on the wings.