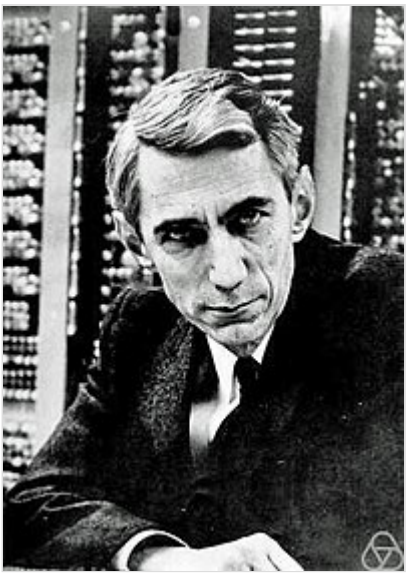
Shannon number

The **Shannon number**, named after the American mathematician Claude Shannon, is a conservative lower bound of the game-tree complexity of chess of 10^{120}, based on an average of about 10^3 possibilities for a pair of moves consisting of a move for White followed by a move for Black, and a typical game lasting about 40 such pairs of moves.



Claude Shannon

Shannon's calculation

Shannon showed a calculation for the lower bound of the game-tree complexity of chess, resulting in about 10^{120} possible games, to demonstrate the impracticality of solving chess by brute force, in his 1950 paper "Programming a Computer for Playing Chess".[1] (This influential paper introduced the field of computer chess.)

Shannon also estimated the number of possible positions, of the general order of $\frac{63!}{32!8!^2}$, or roughly $3.7 * 10^{43}$. This includes some illegal positions (e.g., pawns on the first rank, both kings in check) and excludes legal positions following captures and promotions.

Number of plies (half-moves)	Number of possible positions[2]	Number of checkmates[3]
1	20	0
2	400	0
3	8,902	0
4	197,281	8
5	4,865,609	347
6	119,060,324	10,828
7	3,195,901,860	435,767
8	84,998,978,956	9,852,036
9	2,439,530,234,167	400,191,963
10	69,352,859,712,417	8,790,619,155
11	2,097,651,003,696,806	362,290,010,907
12	62,854,969,236,701,747	8,361,091,858,959
13	1,981,066,775,000,396,239	346,742,245,764,219
14	61,885,021,521,585,529,237	
15	2,015,099,950,053,364,471,960	

After each player has moved a piece 5 times each (10 ply) there are 69,352,859,712,417 possible games that could have been played.

Tighter bounds

Upper

Taking Shannon's numbers into account, Victor Allis calculated an upper bound of 5×10^{52} for the number of positions, and estimated the true number to be about 10^{50}.[4] Later work proved an upper bound of 8.7×10^{45},[5] and showed an upper bound 4×10^{37} in the absence of promotions.[6][7]

Lower

Allis also estimated the game-tree complexity to be at least 10^{123}, "based on an average branching factor of 35 and an average game length of 80". As a comparison, the number of atoms in the observable universe, to which it is often compared, is roughly estimated to be 10^{80}.

Accurate estimates

John Tromp and Peter Österlund estimated the number of legal chess positions with a 95% confidence level at $(4.822 \pm 0.028) \times 10^{44}$, based on an efficiently computable bijection between integers and chess positions.[5]

Number of sensible chess games

As a comparison to the Shannon number, if chess is analyzed for the number of "sensible" games that can be played (not counting ridiculous or obvious game-losing moves such as moving a queen to be immediately captured by a pawn without compensation), then the result is closer to around 10^{40} games. This is based on having a choice of about three sensible moves at each ply (half-move), and a game length of 80 plies (or, equivalently, 40 moves).[8]

See also

Chess portal

- Solving chess
- Go and mathematics
- Game complexity
- Combinatorial explosion

Notes and references

1. Shannon, Claude E. (March 1950). Levy, David (ed.). "XXII. Programming a computer for playing chess" (https://web.archive.org/web/20200523062243/http://archive.computerhistory.org/projects/chess/related_materials/text/2-0%20and%202-1.Programming_a_computer_for_playing_chess.shannon/2-0%20and%202-1.Programming_a_computer_for_playing_chess.shannon.062303002.pdf) (PDF). *Philosophical Magazine*. 7. **41** (314). New York, NY: Springer: 256–275. doi:10.1080/14786445008521796 (https://doi.org/10.1080%2F14786445008521796). ISBN 978-1-4757-1970-3. ISSN 1941-5982 (https://search.worldcat.org/issn/1941-5982). Archived from the original (http://archive.computerhistory.org/projects/chess/related_materials/text/2-0%20and%202-1.Programming_a_computer_for_playing_chess.shannon/2-0%20and%202-1.Programming_a_computer_for_playing_chess.shannon.062303002.pdf) (PDF) on 2020-05-23. `{{cite journal}}`: ISBN / Date incompatibility (help)
2. "A048987" (https://oeis.org/A048987). *OEIS*.
3. "A079485" (https://oeis.org/A079485). *OEIS*.
4. Allis, Victor (1994). *Searching for Solutions in Games and Artificial Intelligence* (http://fragrieu.free.fr/SearchingForSolutions.pdf) (PDF). Maastricht, The Netherlands: Ph.D. Thesis, University of Limburg. ISBN 978-90-900748-8-7.
5. Tromp, John (2022). "Chess Position Ranking" (https://github.com/tromp/ChessPositionRanking). *GitHub*.
6. Steinerberger, Stefan (August 2015). "On the number of positions in chess without promotion". *International Journal of Game Theory*. **44** (3): 761–767. doi:10.1007/s00182-014-0453-7 (https://doi.org/10.1007%2Fs00182-014-0453-7). ISSN 0020-7276 (https://search.worldcat.org/issn/0020-7276). S2CID 31972497 (https://api.semanticscholar.org/CorpusID:31972497).
7. Gourion, Daniel (12 October 2022). "An upper bound for the number of chess diagrams without promotion" (https://univ-avignon.hal.science/hal-03483904). *ICGA Journal*. **44** (2) (version 2 ed.): 44–55. arXiv:2112.09386v2 (https://arxiv.org/abs/2112.09386v2). doi:10.3233/ICG-220210 (https://doi.org/10.3233%2FICG-220210). Retrieved 2021-12-18.
8. Grime, James (24 July 2015). *How many chess games are possible?* (https://www.youtube.com/watch?v=Km024eldY1A) (Video). Numberphile – via YouTube. Dr. James Grime talking about the Shannon Number and other chess stuff (films by Brady Haran). MSRI, Mathematical Sciences.

External links

- Mathematics and chess (http://mathworld.wolfram.com/Chess.html)