Revisiting Event Study Designs

Borusyak and Jaravel (2017)

`https://github.com/s-saisw/readingSummary`

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1 Research questions & contribution

- This paper shows that it is impossible to identify the linear component of pre-trends in the presence of unit and time FE. Then it shows that we can overcome this under-identification issue by
  1. restricting the pre-trends in the fully dynamic specification
  2. replacing unit fixed effects with unit random effects.

  $\Rightarrow$ We can test the existence of pre-trends using F-test.
- It also shows that, when average dynamic treatment effects in the usual way, short-run effect is severely overweight and long-run effects negatively weighted. Then it proposes to weigh the dynamic treatment effect in a convex way.

2 Setup

Let $t = 1, ..., T$ be the calendar time, E_i period in which unit i receives treatment, and $K_{it} = t - E_{it}$ relative time related to the event.

Consider the following specifications:

1. Fully dynamic specification
$$Y_{it} = \tilde{\alpha}_i + \tilde{\beta}_i + \sum_{k=-\infty}^{\infty} \tilde{\gamma}_k \mathbf{1}\{K_{it} = k\} + \tilde{\epsilon}_{it} \tag{1}$$
2. Semi-dynamic specification: consider only the contemporaneous impact of event and post treatment impact.
$$Y_{it} = \tilde{\alpha}_i + \tilde{\beta}_i + \sum_{k=0}^{\infty} \tilde{\gamma}_k \mathbf{1}\{K_{it} = k\} + \tilde{\epsilon}_{it} \tag{2}$$
3. Specification commonly used
$$Y_{it} = \alpha_i + \beta_i + \sum_{k=-A}^{B-1} \gamma_k \mathbf{1}\{K_{it} = k\} + \gamma_{B+} \mathbf{1}\{K_{it} \geq B\} + \epsilon_{it} \tag{3}$$
γ_{B+} captures all the long-run effects.
4. Static specification
$$Y_{it} = \alpha_i + \beta_t + \gamma_{0+} D_{it} + \epsilon_{it} \tag{4}$$

3 Underidentification of the Fully Dynamic Specification

- For any constant h, we can show that

$$\begin{aligned}\hat{Y}_{it} =&\alpha_i + \beta_t + \sum_{k=-\infty}^{\infty} \gamma_k \mathbf{1}\{K_{it} = k\} \\ =&\alpha_i + \beta_t + \sum_{k=-\infty}^{\infty} \gamma_k \mathbf{1}\{K_{it} = k\} - hK_{it} + hK_{it} \\ =&\alpha_i + \beta_t + \sum_{k=-\infty}^{\infty} \gamma_k \mathbf{1}\{K_{it} = k\} - h(t - E_{it}) + hK_{it} \\ =&(\alpha_i + hE_{it}) + (\beta_t - ht) + \sum_{k=-\infty}^{\infty} (\gamma_k + hk)\mathbf{1}\{K_{it} = k\}\end{aligned}$$

  Dynamic causal effect $\gamma_k + hk$ fits the data just as well as the original γ_k.

- Suppose $\alpha_i = \lambda + \alpha E_i$, $\beta_t = \beta t$, and $\gamma_k = \gamma K_{it}$. Then,

$$Y_{it} = \lambda + \alpha E_i + \beta t + \gamma K_{it} + u_{it}.$$

  Since $t - E_i = K_{it}$, this estimation equation is unidentified.

- Solution 1: Restricting pre-trends
  - Restricting pre-trends is justified when event timing is unpredictable conditional on unit characteristics.
  - F-test the pre-trends of the following model
    H_0: semi-dynamic
    H_1: fully dynamic that drops two terms corresponding to $k_1, k_2 < 0$.
  - This test cannot detect linear pre-trends, but actually there is no reason for pre-trends to be linear.
  - k_1, k_2 shoud be chosen far apart from each other. e.g. $k_1 = -1$ and $k_2 =$ the most negative value of K in the sample.
  - Once the researcher is comfortable with the assumption of no pre-trends, all γ_k for $k < 0$ should be set to zero.

- Solution 2: Unit random effects
  - Random effects model is justified when the timing of the event is randomly assigned across units.
  - If event is independent of α_i, we can just drop α_i. However, this will reduce efficiency. This paper proposes using random effects model.
  - In a random effects model, the unobserved variables are assumed to be uncorrelated with (or, more strongly, statistically independent of) all the observed variables. $\Rightarrow$ We can test for independence using F-test.
  - When we can impose independence assumption, we can estimate γ_k using RE model.

4 Negative Weighting in Canonical Regression

- when average dynamic treatment effects in the usual way, short-run effect is severely overweight and long-run effects negatively weighted.
- Solution 1: if pre-trends can be assumed away, flexibly estimate semi-dynamic model.
- Solution 2: if some anticipation effects are possible, run regression with no restriction on post-treatment, i.e. set $B = \infty$ and $A < \infty$.
- Solution 3: find a control group that never experiences treatment. But if control group is small relative to control group, this would not help.

References

Borusyak, K. and Jaravel, X. (2017). Revisiting event study designs. *Available at SSRN 2826228*.