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Summary of Doing It Now or Later

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Introduction

- Purpose: to explore the *behavioral* and *welfare* implications of *present-biased preferences* in a model where a person must engage in an activity once during some length of time.
- Flaws of assumption of time consistency: To capture impatience, economists usually discount utility over time exponentially. This implies that people have time-consistent preference. However, this ignores human tendency to grab *immediate reward* and to avoid *immediate costs*. This tendency is called present-biased preferences, i.e. when consider two future moments, a person gives stronger relative weight to the earlier moment.
- The model presented in this paper consists of two distinctions:
    1. Whether the choices involve *immediate costs* (i.e. delayed rewards) or *immediate rewards* (i.e. delayed costs) → to unify the investigation of phenomena such as procrastination and overeating that have often been explored separately but which clearly have similar underlying aspects
    2. Whether people are *sophisticated* or *naïve.* Sophisticates foresee they will have self-control problem in the future why naifs do not

Present-Biased Preferences

- Simplified form of present-biased preferences (Phelps & Pollak 1968, Laibson 1994) : a person always gives extra weight to well-being now over any future moment, but weighs all future moments equally.
- Let u_t be a person's instantaneous utility, $U_t(u_t, u_{t+1}, \dots, u_T)$ be a person's intertemporal preferences.
    - The simple model: For all t,

$$U_t(u_t, u_{t+1}, \dots, u_T) = \sum_{\tau=t}^{T} \delta^\tau u_\tau$$

    where $\delta \in (0,1]$ is a discount factor

    - Exponential discounting only captures the fact that people are impatient
    - It implies that preferences are time consistent: a person's relative preference at an earlier date over a later date is the same no matter when she is asked.
      e.g. a person may prefer \$205 to be received 366 days from now to \$200 to be received 365 days from now. However, if the choices are to receive \$205

tomorrow, or \$200 today, she may prefer \$200 today. → present-biased

- [Def 1] Two-parameter model (β, δ)- preferences: For all t,

$$U_t(u_t, u_{t+1}, \dots, u_T) = \delta^t u_t + \beta \sum_{\tau=t+1}^{T} \delta^\tau u_\tau$$

where $\beta > 0$ represents bias for the present and $\delta \in (0,1]$ is a long-run time consistent discounting.

  - When $\beta = 1$, (β, δ)- preferences are simply exponential discounting and is the same as the previous model
  - When $\beta < 1$, β discounts future utility, implying present-biased preferences

Doing It Once

- The person must engage in activity once during some length of time. At each moment the person can choose only whether or not to do it now, and cannot choose when later she will do it.
- Settings:
  - Suppose there is an activity that a person must perform exactly once, and there are $T < \infty$ periods that she can do it.
  - For $t \in \{1, 2, \dots, T\}$, let $v \equiv (v_1, v_2, \dots, v_T)$ be the reward schedule (reward at each period) and $c \equiv (c_1, c_2, \dots, c_T)$ be the cost schedule (cost incurred at each period) However, a person does not necessarily receive reward and cost immediately upon completion of the activity. There are immediate costs (delayed reward) and immediate reward (delayed costs)
  - In any period before the last period ($t \leq T - 1$), the person must choose to do it or to wait. If she waits, she will face the same choice again in the next period. $(t + 1)$ If the person waits until period T, she must do it then.
  - Assume there is no long-term discounting ($\delta = 1$) and simplify (β, δ)- preferences. Interpret delayed rewards or costs as being experienced in T+1.
- <u>Intertemporal utility function</u> from the perspective of period t:
  - Immediate costs

$$U^t(\tau) = \begin{cases} \beta v_\tau - c_\tau \ if \ \tau = t \\ \beta v_\tau - \beta c_\tau \ if \ \tau > t \end{cases}$$

  - Immediate rewards

$$U^t(\tau) = \begin{cases} v_\tau - \beta c_\tau \ if \ \tau = t \\ \beta v_\tau - \beta c_\tau \ if \ \tau > t \end{cases}$$

- <u>Strategy for different types of people</u>: People who have time-consistent preferences(TC) have β=1, but those who have present-biased preferences have β<1. Sophisticates know exactly what future selves would do, while naifs do not. They have identical preferences.

- Strategy $s \equiv (s_1, s_2, \dots, s_T)$ where $s_t \in \{Y, N\}$
  - If a person chooses to do it in period t, then $s_t = Y$ but if she waits in that period $s_t = N$.
  - A strategy also specifies what the person would do even after she has done it.
  - As the person must do it in period T if she has not yet done it, specify $s_T = Y$.
  - Assume *perception-perfect strategy*: a person chooses the optimal action given current preferences and her perceptions of future behavior.
- [Def 2] A perception-perfect strategy for TCs is a strategy $s^{tc} \equiv ({s_1}^{tc}, {s_2}^{tc}, \dots, {s_T}^{tc})$ that satisfies for all t<T. $s_t^{tc} = Y$ if and only if $U^t(t) \geq U^t(\tau)$ for all $\tau > t$.
  - TCs will complete the activity in period t if it is the optimal period of those remaining
- [Def 3] A perception-perfect strategy for naifs is a strategy $s^n \equiv ({s_1}^n, {s_2}^n, \dots, {s_T}^n)$ that satisfies for all t<T. $s_t^n = Y$ if and only if $U^t(t) \geq U^t(\tau)$ for all $\tau > t$.
  - Naifs will complete the activity in period t if it is the optimal period of those remaining
  - Naifs and TCs decision process are the same. Naifs believe they are going to behave like TCs in the future but the problem is that they are present-biased.
- [Def 4] A perception-perfect strategy for TCs is a strategy $s^s \equiv ({s_1}^s, {s_2}^s, \dots, {s_T}^s)$ that satisfies for all t<T. $s_t^s = Y$ if and only if $U^t(t) \geq U^t(\tau')$ where $\tau' \equiv \min_{\tau > t}\{\tau \mid s_\tau^s = Y\}$
  - Sophisticates know when their future selves will complete the activity if they wait now. They will complete the activity in period t if doing it now is preferred to doing it in the future

Behavior

- To see how behavior depends on whether rewards or costs are immediate, and on whether people are sophisticated on naïve.
- Example 1: Immediate costs
  - Task: write a report; Reward: received at work in the future (delayed reward); Cost: not seeing the movie shown on that day (immediate cost)
  - Let $T = 4$, $\beta = {}^{1}\!/_{2}$, $v = (\bar{v}, \bar{v}, \bar{v}, \bar{v}), c = (3, 5, 8, 13)$

    Because the cost is immediate, assume $U^t(\tau) = \begin{cases} \beta v_\tau - c_\tau \ if \ \tau = t \\ \beta v_\tau - \beta c_\tau \ if \ \tau > t \end{cases}$
  - $s^{tc} = (Y, Y, Y, Y)$

Assume β =1 and perception-perfect strategy of TCs

In period 1, $\bar{v}-3 \geq \bar{v}-5$; $\bar{v}-3 \geq \bar{v}-8$; $\bar{v}-3 \geq \bar{v}-13$ → $s_1^{tc}=Y$

In period 2, $\bar{v}-5 \geq \bar{v}-8$; $\bar{v}-5 \geq \bar{v}-13$ → $s_2^{tc}=Y$

In period 3, $\bar{v}-8 \geq \bar{v}-13$ → $s_3^{tc}=Y$

In period 4, according to the requirement → $s_4^{tc}=Y$

➢ $s^n=(N,N,N,Y)$

Assume β =0.5 and perception-perfect strategy of naifs

In period 1,$0.5\bar{v}-3<0.5\bar{v}-2.5$; $0.5\bar{v}-3 \geq 0.5\bar{v}-4$; $0.5\bar{v}-3 \geq 0.5\bar{v}-6.5$ → $s_1^n=N$

In period 2, $0.5\bar{v}-5<0.5\bar{v}-4$; $0.5\bar{v}-5 \geq 0.5\bar{v}-6.5$ → $s_2^n=N$

In period 3, $0.5\bar{v}-8<0.5\bar{v}-6.5$ → $s_3^n=N$

In period 4, according to the requirement → $s_4^n=Y$

➢ $s^s=(N,Y,N,Y)$

Assume β =0.5 and perception-perfect strategy of naifs

In period 4, according to the requirement → $s_4^s=Y$

In period 3, $0.5\bar{v}-8<0.5\bar{v}-6.5$ → $s_3^n=N$

In period 2, $0.5\bar{v}-5>0.5\bar{v}-6.5$ → $s_2^n=Y$

In period 1, $0.5\bar{v}-3<0.5\bar{v}-2.5$ → $s_1^n=N$

➢ Intuitively, TCs do the activity in the period that maximizes $v_\tau-c_\tau$. Naifs think they would not procrastinate in the future, and give in that moment. On the other hand, although sophisticates also have self-control problems due to present-biased preference but perfect foresight help them assess the cost of doing it in the future and would do it now if the cost today exceeds the cost of doing it in the future.

➢ This example illustrates that <u>sophistication is good</u> because it helps overcome self-control problems.

➢ Task: going to a movie ; Reward: seeing a movie; Cost: none as the ticket is free

➢ Let $T=4$, $\beta={}^1\!/_2$, $v=(3,5,8,13)$, $c=(0,0,0,0)$

Because the reward is immediate, assume $U^t(\tau)=\begin{cases} v_\tau-\beta c_\tau \; if \; \tau=t \\ \beta v_\tau-\beta c_\tau \; if \; \tau>t \end{cases}$

➢ $s^{tc}=(N,N,N,Y)$

✧ TCs wait to see the last movie because it yields the highest reward

➢ $s^n=(N,N,Y,Y)$

✧ In the first two periods, naifs incorrectly skip the movies believing they would see the movie that yields the highest benefit, but they finally give in in period 3.

➢ $s^s=(Y,Y,Y,Y)$

✧ Period-1 sophisticate knows that her period-2 self knows her period-3 self will give in

- Example 2: Immediate rewards → sophistication is not always good
  - Task: going to a movie; Reward: seeing a movie; Cost: none, as the ticket is free
  - Let $T = 4$, $\beta = {}^{1}\!/_{2}$, $v = (3, 5, 8, 13)$, $c = (0, 0, 0, 0)$

    Because the reward is immediate, assume $U^t(\tau) = \begin{cases} v_\tau - \beta c_\tau \; if \; \tau = t \\ \beta v_\tau - \beta c_\tau \; if \; \tau > t \end{cases}$
  - $s^{tc} = (N, N, N, Y)$; $\tau_{tc} = 4$
    - TCs wait to see the last movie because it yields the highest reward
  - $s^n = (N, N, Y, Y)$; τ_n=3
    - In the first two periods, naifs incorrectly skip the movies believing they would see the movie that yields the highest benefit, but they finally give in in period 3.
  - $s^s = (Y, Y, Y, Y)$; $\tau_s = 1$
    - Period-2 sophisticate correctly predicts that her period-3 self will give in. Period-1 sophisticate foresees this reasoning and behavior by her period-2 self so she simply sees the movie in the first period because she realizes she will give in to them in the next period. (?)
- Proposition 1: 1) If costs are immediate, then $\tau_n \geq \tau_{tc}$ 2) If rewards are immediate, then $\tau_n \leq \tau_{tc}$.
  - Interpretation:
    - Naifs believe they will behave like TCs in the future but are more impatient.
    - Naifs are influenced solely by the *present-biased effect* (How present-biased preferences affect behavior).
  - Proof

    1) Show that if naifs do it then TCs do it.

    Consider period t, and let $t' \equiv \max_{\tau > t}(v_\tau - c_\tau)$ be period after t that yields the most payoff when not consider present-biased effect

    Naifs do it in period t when $\beta v_t - c_t \geq \beta(v_{t'} - c_{t'}) \leftrightarrow v_t - \frac{1}{\beta}c_t \geq v_{t'} - c_{t'}$

    TCs do it in period t when $v_t - c_t \geq v_{t'} - c_{t'}$

    Because $\beta \leq 1$, we get $v_t - c_t \geq v_t - \frac{1}{\beta}c_t \geq v_{t'} - c_{t'}$. Therefore, we can conclude that if naifs do it ($v_t - \frac{1}{\beta}c_t \geq v_{t'} - c_{t'}$), TCs also do it as $v_t - c_t \geq v_t - \frac{1}{\beta}c_t$ QED

    2) Show that if TCs do it then naifs do it.

    Consider period t, and let $t' \equiv \max_{\tau > t}(v_\tau - c_\tau)$

TCs do it in period t when $v_t - c_t \geq v_{t'} - c_{t'}$

Naifs do it in period t when $v_t - \beta c_t \geq \beta(v_{t'} - c_{t'}) \leftrightarrow \frac{1}{\beta} v_t - c_t \geq v_{t'} - c_{t'}$

Because $\beta \leq 1$, we get $\frac{1}{\beta} v_t - c_t \geq v_t - c_t \geq v_{t'} - c_{t'}$. Therefore, we can conclude that if TCs do it $(v_t - c_t \geq v_{t'} - c_{t'})$, naifs also do it as $\frac{1}{\beta} v_t - c_t \geq v_t - c_t$ QED

- Proposition 2: For all cases, $\tau_s \leq \tau_n$
  - Interpretation: sophisticates do it before naifs
    - For immediate costs, sophistication helps mitigate the tendency to procrastinate. For immediate rewards, sophistication can exacerbate the tendency to preproperate
    - Sophisticates are influenced by both *present-biased effect* and *sophistication effect* (How awareness of self-control problems ones might have in the future affect behavior)
  - Proof: show that if naifs do it sophisticates do it.
    Naifs do it in period t only when $U^t(t) \geq U^\tau(t)$ for all $\tau > t$

    Sophisticates do it in period t if $U^t(t) \geq U^t(\tau')$ where $\tau' \equiv \min_{\tau > t}\{\tau \mid s_\tau^s = Y\}$
- Example 3: immediate costs
  - Task: write a paper; Reward: receive at work in the future (If wait until period 2 and 3, a paper can be better written); Cost: miss a sports program (pro basketball, college football, pro football)
  - Let $T = 3$, $\beta = 1/2$, $v = (12, 18, 18), c = (3, 8, 13)$
  - $s^{tc} = (N, Y, Y)$; $\tau_{tc} = 2$
    - TCs write the paper in second period because the marginal benefit of a better paper outweighs the cost of giving up college football to pro basketball (9<10)
  - $s^s = (Y, N, Y)$; $\tau_s = 1$
    - Sophisticates know that the period-2 self will procrastinate and they will end up writing the paper on Sunday if they do not do it in period 1.
    - In this example, sophisticates behave exactly opposite from what present-biased preferences would suggest
  - $s^n = (N, N, Y)$; τ_n=3

Welfare

- Purpose of this section: to show that a small bias for the present can cause severe welfare loss

- Method used in savings literature(?) is Pareto efficiency criterion: asking when all period selves weakly prefer one strategy to another. If that strategy is Pareto superior to another then it is better. However, when applied to intertemporal choice, it is hard to compare strategies e.g. one strategy is preferred by all period selves while the other is preferred only by one period self. → Better use long-run perspective to capture welfare
- Long-run perspective: Suppose there is a fictitious period 0 where the person has no decision to make and weights all future periods equally. A person's long-run utility from doing it in period τ is $U^0(\tau) \equiv v_\tau - c_\tau$
- Qualitative comparisons between sophisticates and naifs:
  - When costs are immediate, sophisticates do at least as well as naifs.
    - According to proposition 2 or $\tau_s \leq \tau_n$, sophisticates and naifs' welfare differs when sophisticates avoid costly procrastination despite incurring immediate costs. → That action is preferred by long-run self → sophisticates are better off in the long run
  - When rewards are immediate, we cannot say whether sophisticates or naifs are better off.
    - E.g. there is a future period with large immediate reward but has even larger delayed cost → sophisticates preoperate but naifs may not → sophisticates might be able to avoid the temptation trap
- Rather than compare the welfare difference between sophisticates and naifs, the focus of welfare analysis is to see how small bias for the present can cause severe welfare losses. Welfare loss: deviation from TC's long run utility
  - For sophisticates $U^0(\tau_{tc}) - U^0(\tau_s)$
  - For naifs $U^0(\tau_{tc}) - U^0(\tau_n)$
- Suppose there is an upper-bound on how large rewards and costs can be. Let the upper bound on rewards and costs be $\bar{X}$.
- Proposition 3: Suppose costs are immediate, and consider all v and c such that $v_t \leq \bar{X}$ and $c_t \leq \bar{X}$ for all t:

  1) $\lim_{\beta \to 1}(sup_{(v,c)}[U^0(\tau_{tc}) - U^0(\tau_s)]) = 0$
     - Because sophisticates know exactly when they will do it if they wait now, delaying from τ_{tc} to τ_s is a single decision to procrastinate → not very severe welfare losses
     - When bias for present is small (β → 1), the upper bound of welfare loss is zero.
  2) For any $\beta < 1, sup_{(v,c)}[U^0(\tau_{tc}) - U^0(\tau_n)]) = 2\bar{X}$

  - ✧ Because naifs make repeated decisions to procrastinate, believing they will do it next period→welfare loss is the sum of these increments→severe welfare losses
  - ✧ Small bias for the present can cause severe welfare loss for naifs
- Proposition 4: Suppose rewards are immediate, and consider all v and c such that $v_t \leq \bar{X}$ and $c_t \leq \bar{X}$ for all t:

  1) $\lim_{\beta \to 1}(sup_{(v,c)}[U^0(\tau_{tc}) - U^0(\tau_n)]) = 0$
     - ✧ Because naifs made only a single decision from the planned τ_{tc} to do it in τ_n → not very severe welfare losses
     - ✧ When bias for present is small, the welfare loss is bounded at zero.
  2) For any $\beta < 1, sup_{(v,c)}[U^0(\tau_{tc}) - U^0(\tau_s)]) = 2\bar{X}$
     - ✧ Because sophisticates know they will preoperate in the end → preoperate near the end → preoperate sooner → repeated decisions → large welfare losses
     - ✧ Small bias for the present can cause severe welfare loss for sophisticates
- If costs are immediate, sophisticates always choose a Pareto-optimal strategy (preferred by long-run self → cause small welfare loss), while naifs may not. On the other hand, if rewards are immediate, naifs always choose a Pareto-optimal strategy, while sophisticates may not

Smoking Guns

- Researchers have searched for empirical proof that people have time-inconsistent preferences. There are efforts to do so by focusing on external commitment devices, because commitment devices can limit future choices of individuals and prove time-inconsistency wrong.
- This section will show that evidence for time-inconsistency, i.e. smoking guns, also exists in one-activity model where there is no external commitment device.
- There are two properties that a person with time-consistent preferences will never violate
  1. Dominance:
     - ✧ Generally, for intertemporal choice, one strategy dominates another if it yields in every period an instantaneous utility at least as large as the instantaneous utility from the other strategy.
       - E.g.1 For v=(a, b, c) and c=(x, y, z), consider strategy1 (Y, Y, Y) and strategy2 (N, Y, Y). Strategy1 yields instantaneous utility (a-x, 0, 0) and strategy2 yields instantaneous utility (0, b-y, 0). Strategy1 dominates strategy 2 when $a - x \geq 0, 0 \geq b - y, and\ 0 \geq 0$

- E.g.2 For v=(a, b, c) and c=(x, y, z), consider strategy1 (Y, Y, Y) and strategy3 (N, N, Y). Strategy1 yields instantaneous utility (a-x, 0, 0) and strategy3 yields instantaneous utility (0, 0, c-z). Strategy1 dominates strategy 2 when $a - x \geq 0, 0 \geq 0, and\ 0 \geq c - z$

✧ In this model, strategy that provides a reward with no cost dominates strategy that provides no reward but incurs cost

- E.g.3 For v=(a, b, 0) and c=(0, y, z), strategy1 (Y, ?, Y) and strategy2 (N, N, Y), strategy1 yields instantaneous utility (a, 0, 0) and strategy2 yields instantaneous utility (0, 0, -z). Strategy1 dominates strategy 2 because $a \geq 0, 0 \geq 0, and\ 0 \geq -z$
- Consider e.g.2. When $x = c = 0$, strategy1 dominates strategy2 for all v=(a, b, c) and c=(x, y, z)
- "consider a three-period example where v = (1, x, 0) and c = (0, y, 1). Then if costs are immediate, doing it in period 1 yields the stream of instantaneous utilities (0, 0, 0, 1) while doing it in period 3 yields the stream of instantaneous utilities (-1, 0, 0, 0). Clearly the former dominates the latter."

✧ Definition 5: A person obeys dominance if whenever there exists some period τ with $v_\tau > 0\ and\ c_\tau = 0$, the person does not do it in period any period τ' with $v_{\tau'} = 0\ and\ c_{\tau'} > 0$

2. <u>Independence of irrelevant alternatives</u>:

✧ Eliminating an option that is not chosen from the choice set should not change the person's choice from the remaining options

✧ Definition 6: For any $v \equiv (v_1, v_2, \dots, v_T)$ and $c \equiv (c_1, c_2, \dots, c_T)$, define $v^{-t} \equiv (v_1, \dots, v_{t-1}, v_{t+1}, \dots,\ v_T)$ and $c^{-t} \equiv (c_1, \dots, c_{t-1}, c_{t+1}, \dots, c_T)$. A person's behavior is independent of irrelevant alternatives if whenever she chooses period $\tau' \neq t$ when facing v and c she also chooses τ' when facing v^{-t} and c^{-t}.

- Proposition 5: For any β and δ such that $0 < \delta \leq 1$ and $0 < \beta < 1$, and for both sophistication and naivete
  1. There exists (v,c) and assumptions about immediacy such that a person with (β , δ)-preferences will violate dominance.
  2. There exists (v,c) and assumptions about immediacy such that a person with (β , δ)-preferences will violate independence of irrelevant alternatives.

➢ Example where sophisticates violate dominance and independence of irrelevant alternatives. Suppose rewards are immediate, $v = (0,5,1), c = (1,8,0), \beta = 0.5$

- ✧ $s_{tc} = (N,N,Y), s_n = (N,Y,Y), s_s = (Y,Y,Y)$

  Sophisticates

  In period 3, according to the requirement → $s_3^s = Y$

  In period 2, $5 - 0.5(8) > 0.5(1) - 0.5(0)$ → $s_2^s = Y$

  In period 1, $0 - 0.5(1) > 0.5(5) - 0.5(8)$ → $s_1^s = Y$

  *Note that in period 1, $0 - 0.5(1) < 0.5(1) - 0.5(0)$

  Naifs

  In period 1, $0 - 0.5(1) > 0.5(5) - 0.5(8), 0 - 0.5(1) < 0.5(1) - 0.5(0)$ → $s_1^n = N$

  In period 2, $5 - 0.5(8) > 0.5(1) - 0.5(0)$ → $s_2^n = Y$

  In period 3, according to the requirement → $s_3^n = Y$

- ✧ According to definition 5, since period 3 has with $v_3 = 1 > 0$ *and* $c_3 = 0$, and there is period 1 with $v_1 = 0$ and $c_1 > 0$. The person should have been doing it in period 3 rather than period 1. However, sophisticates violate dominance by choosing period 1 over period 3. In this case, TCs and naifs do not violate dominance.
- ✧ In period 1, a sophisticate prefers period 3 to period 1 and period 1 to period 2. ($period\ 3 > period\ 1 > period\ 2$) However, in period 2 when the period 1 is eliminated, she chooses period 2, the choice she did not choose in the previous period, over period 3. ($period\ 2 > period\ 3$) → independence of irrelevant alternatives is violated. On the contrary, for naif, in period 1, she prefers period 1 to period 2 and period 3 to 1. ($period\ 3 > period\ 1 > period\ 2$) In period 2, when period 1 is eliminated, she still chooses period 3 to period 2 ($period\ 3 > period\ 2$) → independence of irrelevant alternatives is not violated.

➢ Example where naifs violate dominance. Suppose costs are immediate, $v = (1,8,0), c = (0,5,1)$, $\beta = 0.5$.

- ✧ $s_{tc} = (N,Y,Y), s_n = (N,N,Y), s_s = (Y,N,Y)$

  Naifs

  In period 1, $0.5(1) - 0 < 0.5(8) - 0.5(5), 0.5(1) - 0 > 0.5(0) - 0.5(1)$ → $s_1^n = N$

  In period 2, $0.5(8) - 5 < 0.5(0) - 0.5(1)$ → $s_2^n = N$

  In period 3, according to the requirement → $s_3^n = Y$

  Sophisticates

  In period 3, according to the requirement → $s_3^s = Y$

  In period 2, $0.5(8) - 5 < 0.5(0) - 0.5(1)$ → $s_2^s = N$

  In period 1, $0.5(1) - 0 > 0.5(0) - 0.5(1)$ → $s_1^s = Y$

  *Note that in period 1, $0.5(1) - 0 < 0.5(8) - 0.5(5)$

- ✧ Since period 1 has with $v_3 = 1 > 0$ *and* $c_3 = 0$, and there is period 3 with $v_1 = 0$ and $c_1 > 0$. The person should have been doing it in period 3 rather than period 1. However, naifs violate dominance by choosing period 3 over period 1. In this case, TCs and sophisticates do not violate dominance.
- ✧ In period 1, a naif prefers period 2 to period 1 and period 1 to period 3. ($period\ 2 > period\ 1 > period\ 3$) However, in period 2 when the period 1 is eliminated, she chooses period 3 over period 2. ($period\ 3 > period\ 2$) → independence of irrelevant alternatives is violated. Similarly, period-1 sophisticate compares period 1 and 3 and chooses period 1 over period 3. period 2 over period 1. ($period\ 2 > period\ 1 > period\ 3$) When period 1 is eliminated in period 2, she chooses period 3 over period 2 ($period\ 3 > period\ 2$) but she does not get to choose in period 2 → Is independence of irrelevant alternatives violated?

➢ Conclusion:
- ✧ Evidence for time inconsistency (smoking guns) does not have to involve the use of external commitment devices.
- ✧ Smoking guns exist for both naifs and sophisticates. In the literature on external commitment device, smoking guns exist only for sophisticates as naifs would not pay to limit future choice sets.

Multi-Tasking

- This section is about a task that must be performed more than once.
- Settings:
  - ➢ The person must do the activity $M \geq 1$ times, and she can do it at most once in any given period.
  - ➢ Denote the period in which a person completes the activity for the i[th] time as $\tau^i(M)$ and define $\Theta(\mathrm{M}) \equiv \{\tau^1(M), \tau^2(M), \ldots, \tau^M(M)\}$
  - ➢ For each period the person does it, she receives reward v_τ and incurs cost c_τ, which can be experienced immediately or with some delay.
  - ➢ Intertemporal utility in period t is defined as follows

<u>Immediate costs</u>

$$U^t(\Theta(M)) \equiv \begin{cases} -(1-\beta)c_t + \beta(\sum_{\tau\in\Theta(M)} v_\tau - \sum_{\tau\in\Theta(M)} c_\tau)\ if\ t \in \Theta(M) \\ \beta(\sum_{\tau\in\Theta(M)} v_\tau - \sum_{\tau\in\Theta(M)} c_\tau)\ if\ t \notin \Theta(M) \end{cases}$$

<u>Immediate rewards</u>

$$U^t(\Theta(M)) \equiv \begin{cases} -(1-\beta)v_t + \beta(\sum_{\tau\in\Theta(M)} v_\tau - \sum_{\tau\in\Theta(M)} c_\tau)\ if\ t \in \Theta(M) \\ \beta(\sum_{\tau\in\Theta(M)} v_\tau - \sum_{\tau\in\Theta(M)} c_\tau)\ if\ t \notin \Theta(M) \end{cases}$$

For example, when costs are immediate, M=2, t=1, $\Theta(M) = \{1,2\}$, the intertemporal utility is

$$U^1(\{1,2\}) \equiv -(1-\beta)c_1 + \beta(v_1 + v_2 - c_1 + c_2) = (\beta v_1 - c_1) + \beta(v_2 + c_2)$$

This is similar to the utility function defined in the previous section, but this one is intertemporal across two periods, and equal to the sum of utility in this period and the future period.

- Given these preferences, assume perception-perfect strategy of naifs, sophisticates, and TCs analogously to Definitions 2, 3 and 4.

- Example 4: Suppose rewards are immediate, T=3 and $\beta = 1/2$ for naifs and sophisticates. Let $v = (6, 11, 21)$ and $c = (0, 0, 0)$.
  - If M=1, then it is the same as the 'doing it once' case and $\tau_s = 1,\ \tau_n = 2,\ \tau_{tc} = 3$
  - When M=2,

- TCs

<u>Period 1</u>: the perceived utility for each element of $\Theta(2) = \{1,2\}, \{1,3\}, \{2,3\}$ is as follows

$$U^1(\{1,2\}) = 6 + 11 = 17$$
$$U^1(\{1,3\}) = 6 + 21 = 27$$
$$U^1(\{2,3\}) = 11 + 21 = 32$$

According to perception-perfect strategy for TCs (Def 2), because 17<32 and 27<32, the person will not do it in period 1. Therefore, she must do it in period 2 and 3.

$$\therefore \Theta_{tc}(2) = \{2,3\}$$

- Naifs

<u>Period 1</u> For $\Theta(2) = \{1,2\}, \{1,3\}, \{2,3\}$,

$$U^1(\{1,2\}) = 6 + 5.5 = 11.5$$
$$U^1(\{1,3\}) = 6 + 10.5 = 16.5$$
$$U^1(\{2,3\}) = 0.5(11 + 21) = 16$$

According to perception-perfect strategy for naifs (Def 3), because there is at least one case where utility perceived of doing it in period 1 exceeds that of starting it in period 2 (16.5>16), naifs starts the activity in period 1 thinking that she can wait until period 3 to complete the second round.

<u>Period 2</u>

Now there is only one round left, we can simplify as 'doing it once' case: $v = (11, 21)$ and $c = (0,0)$. Because 11>10.5, naifs will do the second round in the second period instead of the planned third period.

$$\therefore \Theta_n(2) = \{2,3\}$$

- Sophisticates

<u>Period 1</u> For $\Theta(2) = \{1,2\}, \{1,3\}, \{2,3\}$,

$$U^1(\{1,2\}) = 6 + 5.5 = 11.5$$
$$U^1(\{1,3\}) = 6 + 10.5 = 16.5$$
$$U^1(\{2,3\}) = 0.5(11 + 21) = 16$$

Originally, period-1 sophisticate considers doing it in period 1 and 3, because it yields higher payoff than that of waiting until period 2. However, if she does it in period 1 and proceeds to period 2, she may lose to self-control because 11>10.5 and would not be able to do it in period 3 as planned. Having perfect foresight, sophisticate decides not to do it in period 1 and waits until period 2.

$$\therefore \Theta_s(2) = \{2,3\}$$

Remarks:

- Changing M changes the behavior of sophisticates: While sophisticates always preproperate when there is one activity, they do not preproperate with two activities. When there is a second activity, a commitment device becomes available: Waiting now prevents you from doing it for the second time tomorrow; you can only do it for the first time tomorrow.
- Proposition 2: 'sophisticates do it before naifs' does not hold for the multi-activity case.

- Example 5: Suppose rewards are immediate, T=4 and $\beta = 1/2$ for naifs and sophisticates. Let $v = (12, 6, 11, 21)$ and $c = (0, 0, 0, 0)$. If M=2, $\Theta_{tc}(2) = \{1,4\}, \Theta_n(2) = \{1,3\}, \Theta_s(2) = \{3,4\}$
  - Sophistication can lead a person to behave in ways that are seemingly contrary to having present-biased preferences.
- Proposition 6: (1) For all cases and for any v and c, for each $M \in \{1, 2, \ldots, T-1\}$: $\Theta_{tc}(M) \subset \Theta_{tc}(M+1)$ and $\Theta_n(M) \subset \Theta_n(M+1)$; and (2)If costs are immediate, then for all $i \in \{1, 2, \ldots, M\}, \tau_n^i(M) \geq \tau_{tc}^i(M)$, and if rewards are immediate, then for all $i \in \{1, 2, \ldots, M\}, \tau_n^i(M) \leq \tau_{tc}^i(M)$
  - Interpretation:

    (1) If TCs or naifs must do the activity an extra time, they do it in all periods they used to do it, and some additional period.

    → As in example 4, when M=1, $\Theta_n(1) = \{\tau_n\} = 2$, $\Theta_{tc}(1) = \{\tau_{tc}\} = 3$ and when M=2, $\Theta_n(2) = \{2,3\}$ and $\Theta_{tc}(2) = \{2,3\}$, $\Theta_n(1) \subset \Theta_n(2)$ and $\Theta_{tc}(1) \subset \Theta_{tc}(2)$

    (2) If costs are immediate, naifs procrastinate. If rewards are immediate, naifs preproperate.

    → As in example 5, when rewards are immediate, $\Theta_n(2) = \{2,3\}$ and $\Theta_{tc}(2) = \{2,3\}$, $2 \leq 2$ and $3 \leq 3$

Discussion and Conclusion

- Analysis in this paper can be applied to savings or addiction models.
  - Naifs will undersave in savings model and overindulge in addiction model.
  - Sophisticates can sometimes save more than TCs and may not consume at all in the addiction model.
- Economists should be careful to clarify which results are driven by present-biased preferences per se, and which results arise from present-biased preferences in conjunction with sophistication effects.

- Two motivations for incorporating present-biased preferences into economic analysis
  - Present-biased preferences may be useful in predicting some behavior, such as procrastination, where cannot be explained by assuming time-consistency alone.
  - Present-biased preferences and time-consistent preferences can have vastly different welfare implications.
    - Suppose a person becomes fat from eating large quantity of potato chips. She may do so because of 1) self-control problem or, because 2) the pleasure from eating potato chips outweighs the costs of being fat.
    - Normative implications for the two hypotheses: 1) people buy too many potato chips at prevailing price 2) people buy it at the right amount.