High performance implementation of tomography inversion with error correction

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Basics

- Radon transform : Real $\rightleftarrows$ Sinogram space.
- $Rf(\tau,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(\tau - x\cos(\theta) - y\sin(\theta))dxdy$

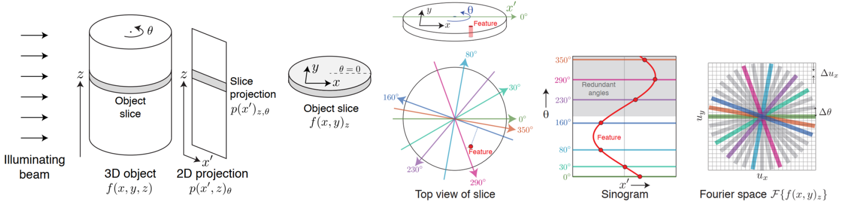

Figure: Spinning the object to obatin "sinograms", reconstruct each slice independently. Figure taken from Jacobsen [2019]

Center of rotation drifts

- $P_\theta = x^*_\theta(1 - cos(\theta) + y^*_\theta sin(\theta))$
- $Rf(\tau, \theta, 0, 0) =$
  $Rf(\tau - P_\theta, \theta, x^*_\theta, y^*_\theta)$
- Translation of sinogram by P_θ achieved by convolution with gaussian.
- Recover P_θ to obtain accurate reconstruction as shown in Austin et al. [2019]

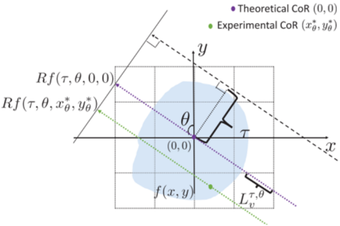

Figure: Center of rotaion drift causes us to measure the shifted sinograms, figure from Austin et al. [2019]

Optimization formulation

Discretize & Vectorize

- $\mathcal{W}$: object vector
- $\mathcal{L}$: discretized radon transform
- $\mathcal{D}$: measure sinogram

Least squares cost function

- To recover both shifts and object :
  $\underset{\mathcal{W}\geq 0, P_\theta}{min} \phi(\mathcal{W}, P_\theta) = \frac{1}{2}||\mathcal{L}\mathcal{W} - g(\mathcal{D}, P_\theta)||$
- First order derivatives analytically computable :
  $\nabla\phi(\mathcal{W}, P_\theta) = [\mathcal{L}^T, \nabla_{P_\theta}\phi(\mathcal{W}, P_\theta)]^T(\mathcal{L}\mathcal{W} - g(\mathcal{D}, P_\theta))$

Implementation

- Implemented in C/C++ using :
  - PETSc (optimization routines, data management and parallel I/O)
  - Boost (geometry routines)
  - FFTW (fourier space convolution)

Joint

- Combine shifts and sample into one vector and optimize for both together.

Alternating

- Alternate between optimizing with respect to sample and with respect to shifts.

Accuracy

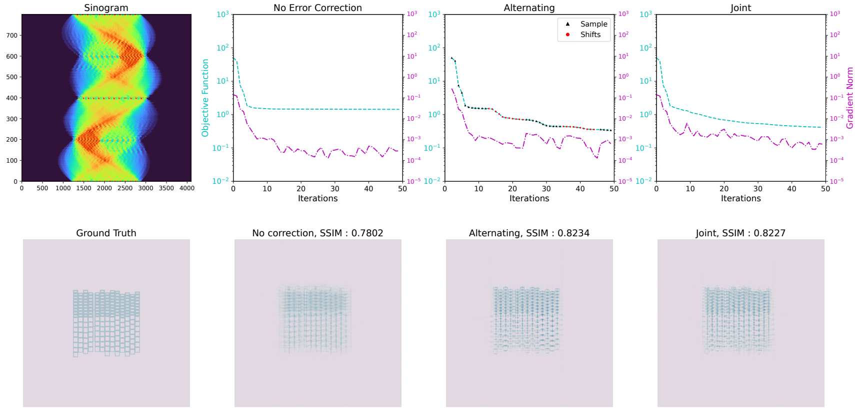

Figure: Dimensions of unknowns : $800 + 4096x4096$, size of sinogram : $4096x800$

Scaling

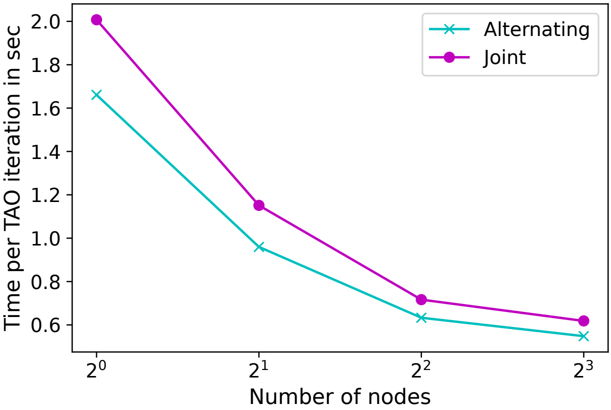

Figure: Strong scaling plots for alternating and joint reconstruction algorithms performned on bebop (dual socket broadwell), problem size : 2048x2048+400 unknowns.

Summary

Tasks completed

- Refactor & fix several bugs.
- Profiling : most of solve is spent doing MatMult & MatMultTranspose
- Minor performance enhancements

Ongoing & Planned

- Port application to GPU : premliminary results show reduction in solve time but load balance becomes challenging.
- Invert 3D tomography data by replicating the 2D solve on sub-communicators.

Thank you!

Acknowledgements:

- Matthew Kehoe
- PETSc-developers and PETSc-Users mailing list for advice, bug fixes and features.

References I

[Austin et al. 2019] Austin, Anthony P. ; WendyDi, Zichao ; Leyffer, Sven ; Wild, Stefan M.: Simultaneous Sensing Error Recovery and Tomographic Inversion Using an Optimization-Based Approach. In: *SIAM Journal on Scientific Computing* 41 (2019), Nr. 3, S. B497–B521

[Jacobsen 2019] Jacobsen, Chris: *X-Ray Microscopy*. Cambridge University Press, 2019 (Advances in Microscopy and Microanalysis)