



High performance implementation of tomography inversion with error correction



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U.S. Department of Energy laboratory
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Summer Argonne Student
Symposium, 8/28/20

Basics

- Radon transform : Real \rightleftarrows Sinogram space.
- $Rf(\tau, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(\tau - x\cos(\theta) - y\sin(\theta)) dx dy$

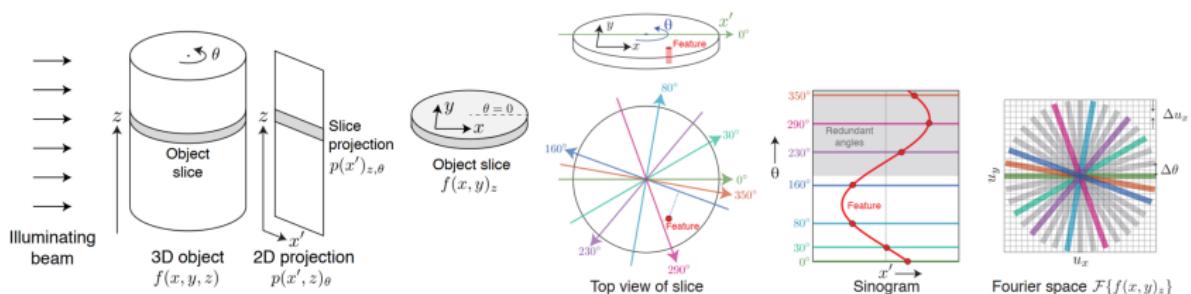


Figure: Spinning the object to obtain "sinograms", reconstruct each slice independently. Figure taken from Jacobsen [2019]

Center of rotation drifts

- $P_\theta = x_\theta^*(1 - \cos(\theta) + y_\theta^*\sin(\theta))$
- $Rf(\tau, \theta, 0, 0) = Rf(\tau - P_\theta, \theta, x_\theta^*, y_\theta^*)$
- Translation of sinogram by P_θ achieved by convolution with Gaussian.
- Recover P_θ to obtain accurate reconstruction as shown in Austin et al. [2019]

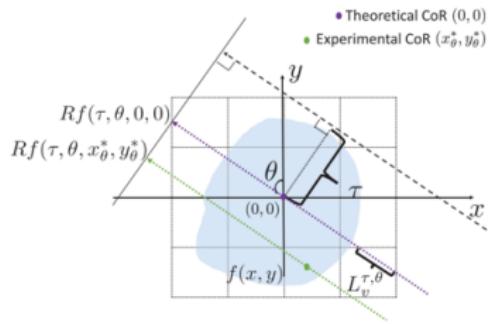


Figure: Center of rotation drift causes us to measure the shifted sinograms, figure from Austin et al. [2019]

Optimization formulation

Discretize & Vectorize

- \mathcal{W} : object vector
- \mathcal{L} : discretized Radon transform
- \mathcal{D} : measure sinogram

Least squares cost function

- To recover both shifts and object :

$$\min_{\mathcal{W} \geq 0, P_\theta} \phi(\mathcal{W}, P_\theta) = \frac{1}{2} \|\mathcal{L}\mathcal{W} - g(\mathcal{D}, P_\theta)\|$$

- First order derivatives analytically computable :

$$\nabla \phi(\mathcal{W}, P_\theta) = [\mathcal{L}^T, \nabla_{P_\theta} \phi(\mathcal{W}, P_\theta)]^T (\mathcal{L}\mathcal{W} - g(\mathcal{D}, P_\theta))$$

Implementation

- Implemented in C/C++ using :
 - PETSc (optimization routines, data management and parallel I/O)
 - Boost (geometry routines)
 - FFTW (fourier space convolution)

Joint

- Combine shifts and sample into one vector and optimize for both together.

Alternating

- Alternate between optimizing with respect to sample and with respect to shifts.

Accuracy

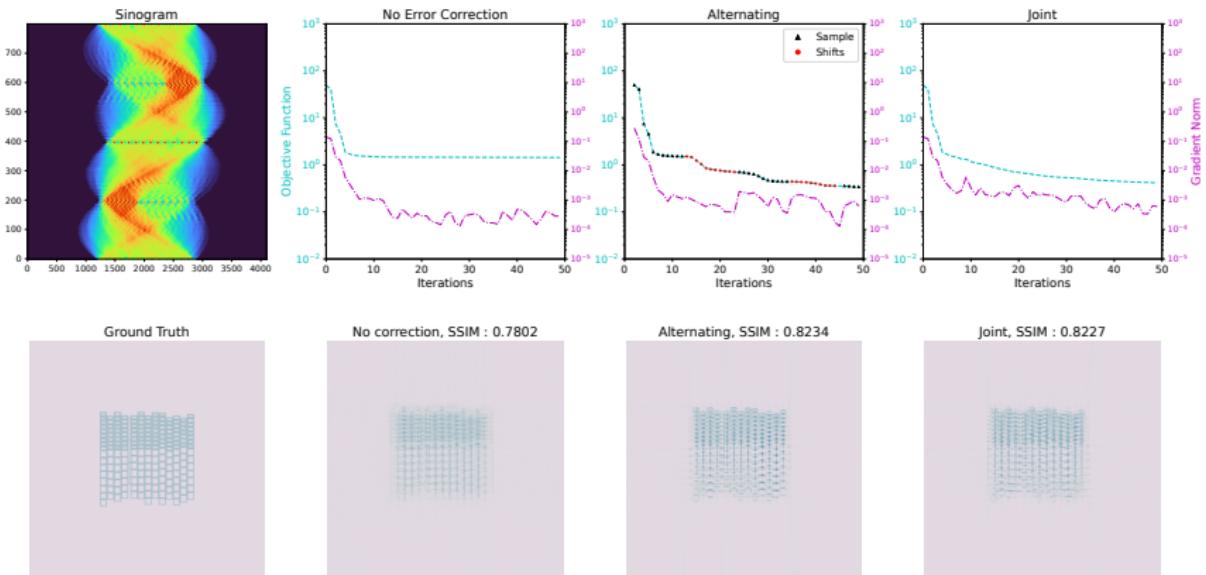


Figure: Dimensions of unknowns : $800 + 4096 \times 4096$, size of sinogram : 4096×800

Scaling

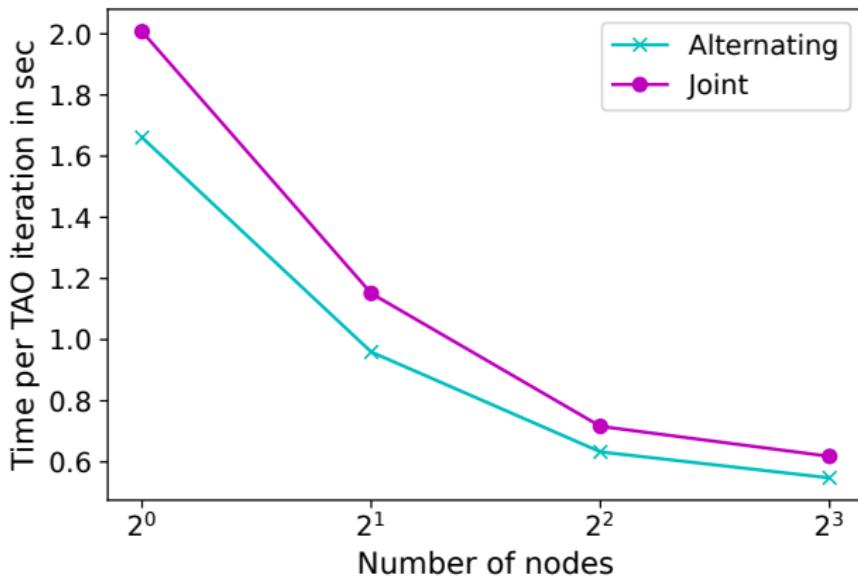


Figure: Strong scaling plots for alternating and joint reconstruction algorithms performed on bebop (dual socket broadwell), problem size : 2048x2048+400 unknowns.

Summary

Tasks completed

- Refactor & fix several bugs.
- Profiling : most of solve is spent doing MatMult & MatMultTranspose
- Minor performance enhancements

Ongoing & Planned

- Port application to GPU : preliminary results show reduction in solve time but load balance becomes challenging.
- Invert 3D tomography data by replicating the 2D solve on sub-communicators.

The background of the slide features a grayscale aerial photograph of a rural or semi-rural area. It shows a network of roads, some buildings, and large fields, possibly agricultural land. The terrain is somewhat hilly or uneven.

Thank you!

Acknowledgements:

- Matthew Kehoe
- PETSc-developers and PETSc-Users mailing list for advice, bug fixes and features.

References I

- [Austin et al. 2019] AUSTIN, Anthony P. ; WENDYDI, Zichao ; LEYFFER, Sven ; WILD, Stefan M.: Simultaneous Sensing Error Recovery and Tomographic Inversion Using an Optimization-Based Approach. In: *SIAM Journal on Scientific Computing* 41 (2019), Nr. 3, S. B497–B521
- [Jacobsen 2019] JACOBSEN, Chris: *X-Ray Microscopy*. Cambridge University Press, 2019 (Advances in Microscopy and Microanalysis)