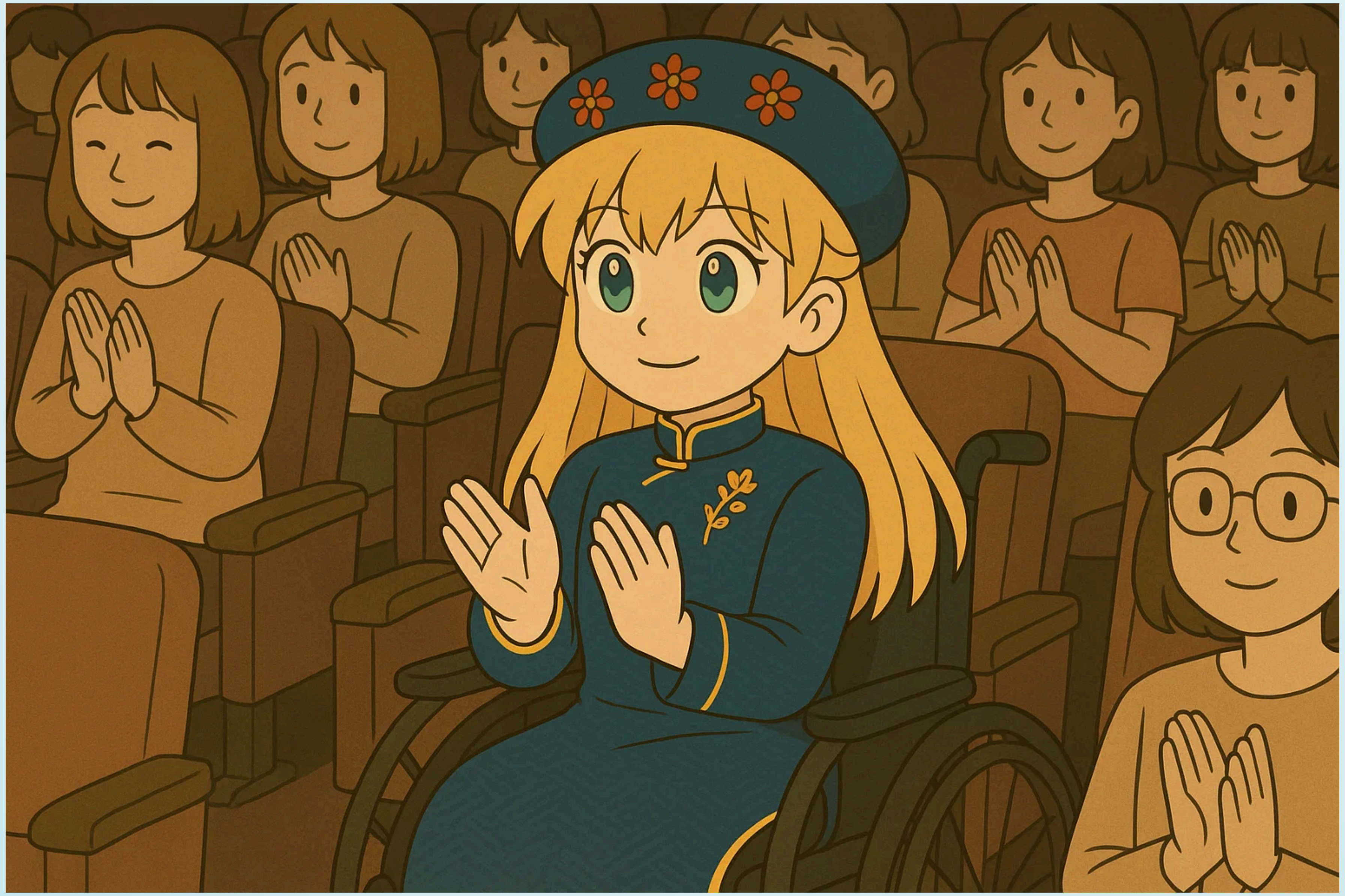
SECRETS OF RHYTHM UNVEILED BY THE KURAMOTO MODEL

WHY DOES APPLAUSE SYNCHRONIZE?



ABOUT ME

- Same (мег-сск)
  - 🧑‍💻 Freelance Software Engineer
  - 🧑‍🎓 Working professional studying at an online university
- Areas of expertise:
  - 📸 Computer Vision (Image Recognition/Point Cloud Processing)
  - 🌍 Spatial Information Processing (GIS/Remote Sensing)
  - ☁️ Cloud

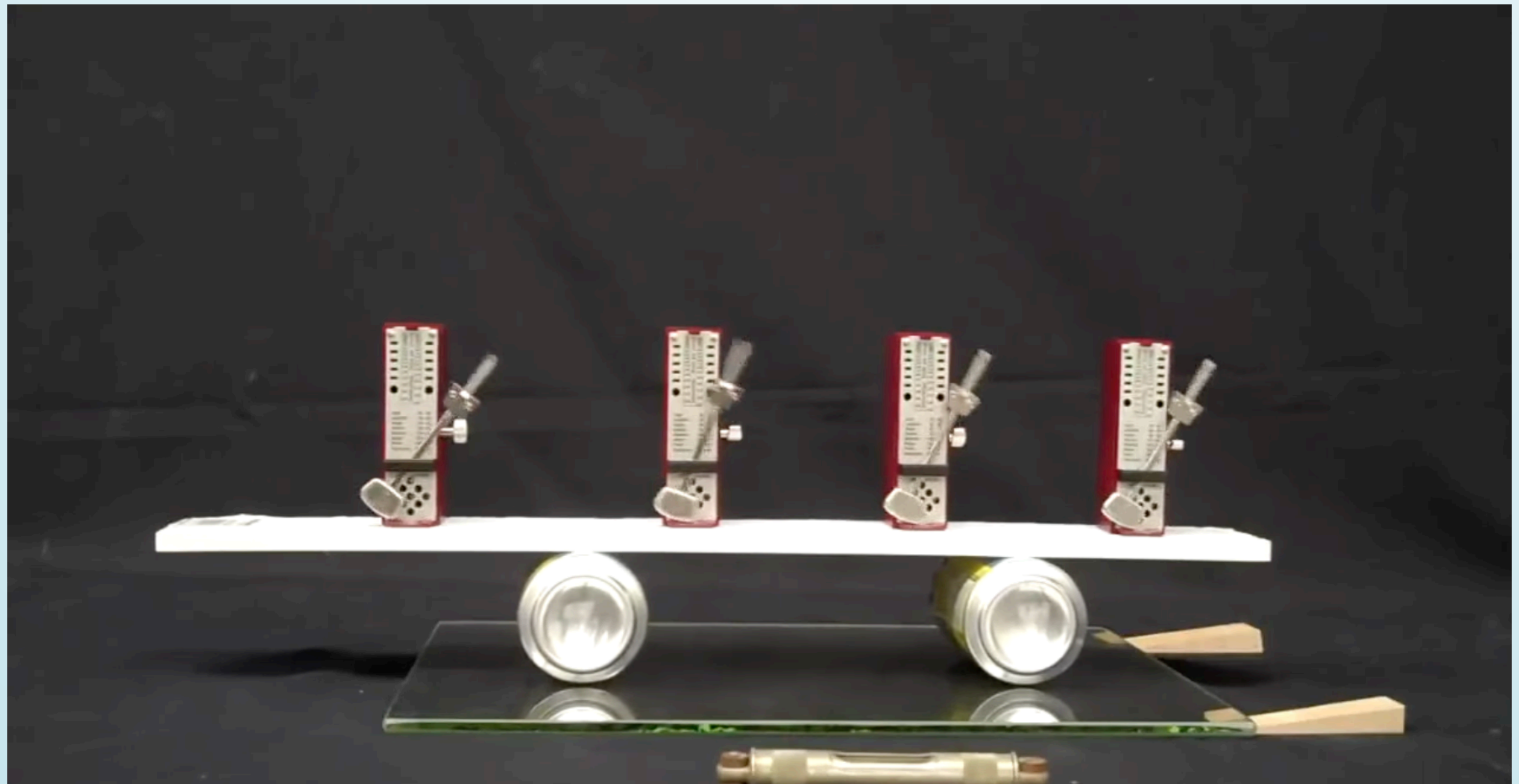
TODAY'S TOPICS

- We'll discuss the Kuramoto model, which can model "synchronization phenomena" (when random rhythms align)!
- We'll break down the meaning of the Kuramoto model's equation
- Let's observe a simulation of the Kuramoto model recreating "firefly flashing"
- The Kuramoto conjecture and the critical coupling constant
- Introduction to representative "phase transitions"

OVERVIEW OF THE KURAMOTO MODEL

THE KURAMOTO MODEL

- The Kuramoto model is a mathematical model for describing phenomena where "rhythms synchronize"
- It was proposed by Dr. Yoshiki Kuramoto in 1975
- Let's watch a video of metronomes synchronizing!



APPLICATIONS OF THE KURAMOTO MODEL

- It can model "rhythm synchronization phenomena"!
    - 👏 Applause rhythms
    - Firefly illumination rhythms
    - Power grids (when generator frequencies get out of sync, blackouts occur!)
    - Heartbeat rhythms (disruption leads to atrial fibrillation)

THE KURAMOTO MODEL EQUATION

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i)$$

- θ_i is the phase of oscillator i
- ω_i is the angular velocity of oscillator i
- K is the strength of interaction (coupling constant)
- N is the number of oscillators

THE SECOND TERM

$$\frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j-\theta_i)$$

- Think of θ_i as your applause rhythm and θ_j as other people's applause rhythms
- This term represents the average deviation between your rhythm and everyone else's
- The larger the coupling constant K, the easier it is for rhythms to synchronize

MEANING OF THE COUPLING CONSTANT K

- The larger the coupling constant K, the easier it is for rhythms to synchronize
  - When K is small, rhythms don't synchronize
- In the applause example, K represents how much you're consciously trying to match your applause to others
  - When $K = 0$, you're not paying attention to others' applause at all and maintaining your own rhythm

REVISITING THE KURAMOTO MODEL EQUATION

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i)$$

- The "rate of change of your rhythm" is determined by the "average deviation from others' rhythms" and coupling constant K

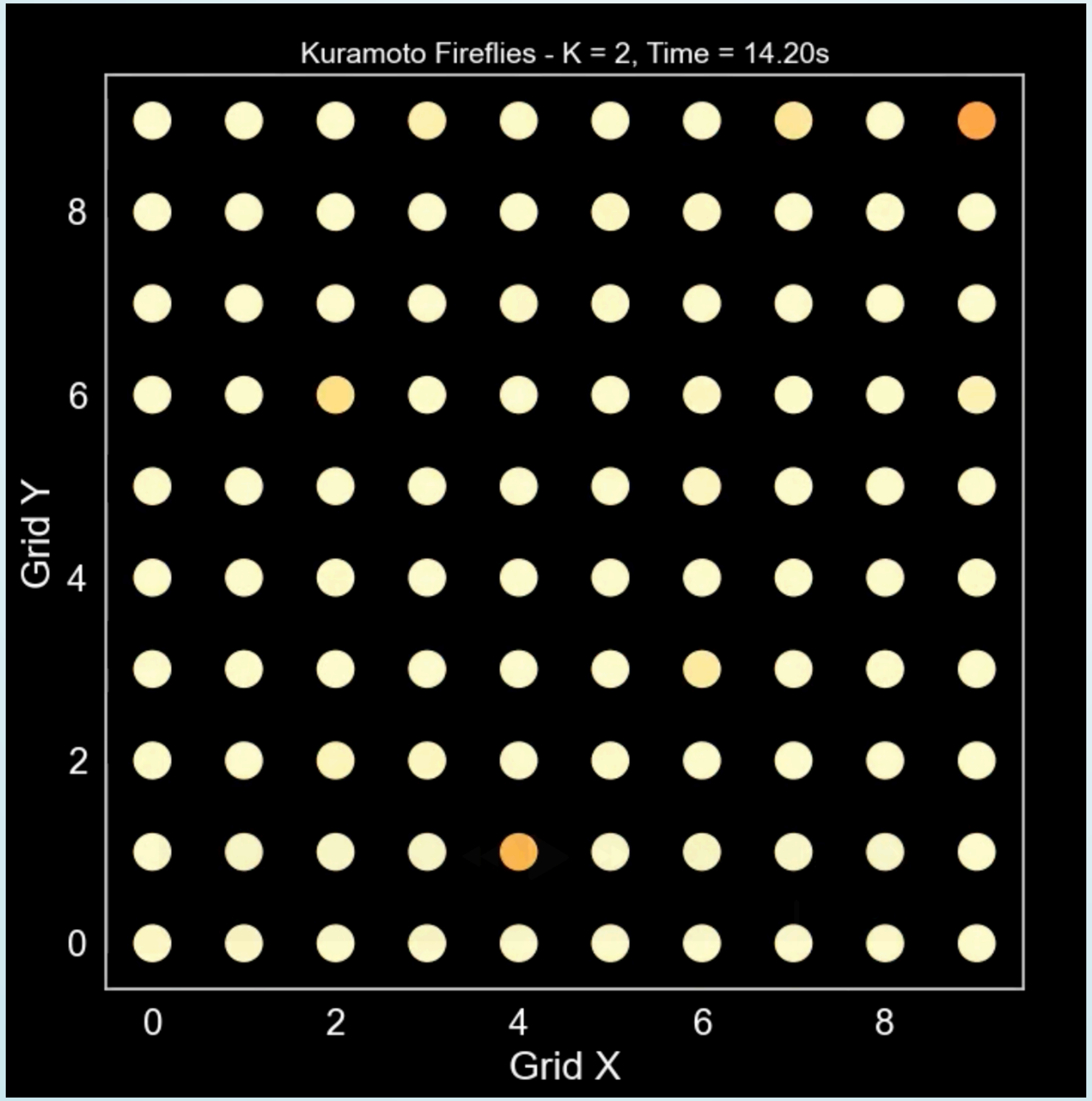
SIMULATION OF THE KURAMOTO MODEL

SIMULATING THE KURAMOTO MODEL

- Someone shared Kuramoto model simulation code on GitHub, so I modified their code to play around with it!
  - https://github.com/fabridamicelli/kuramoto
- Let's use this code to observe how firefly flashing rhythms synchronize!

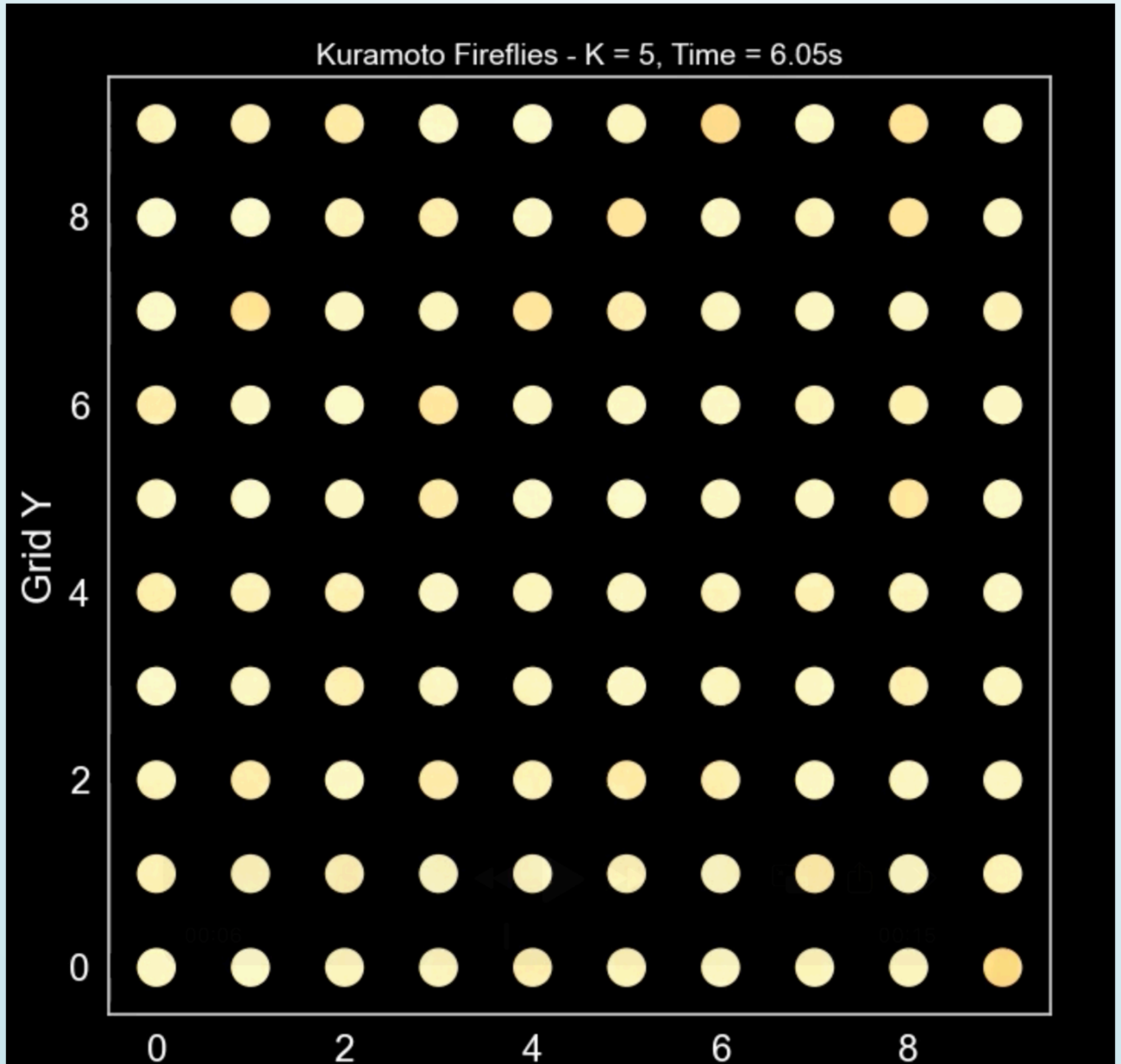
FIREFLY FLASHING: $K = 2$

- Rhythms gradually synchronize over time!



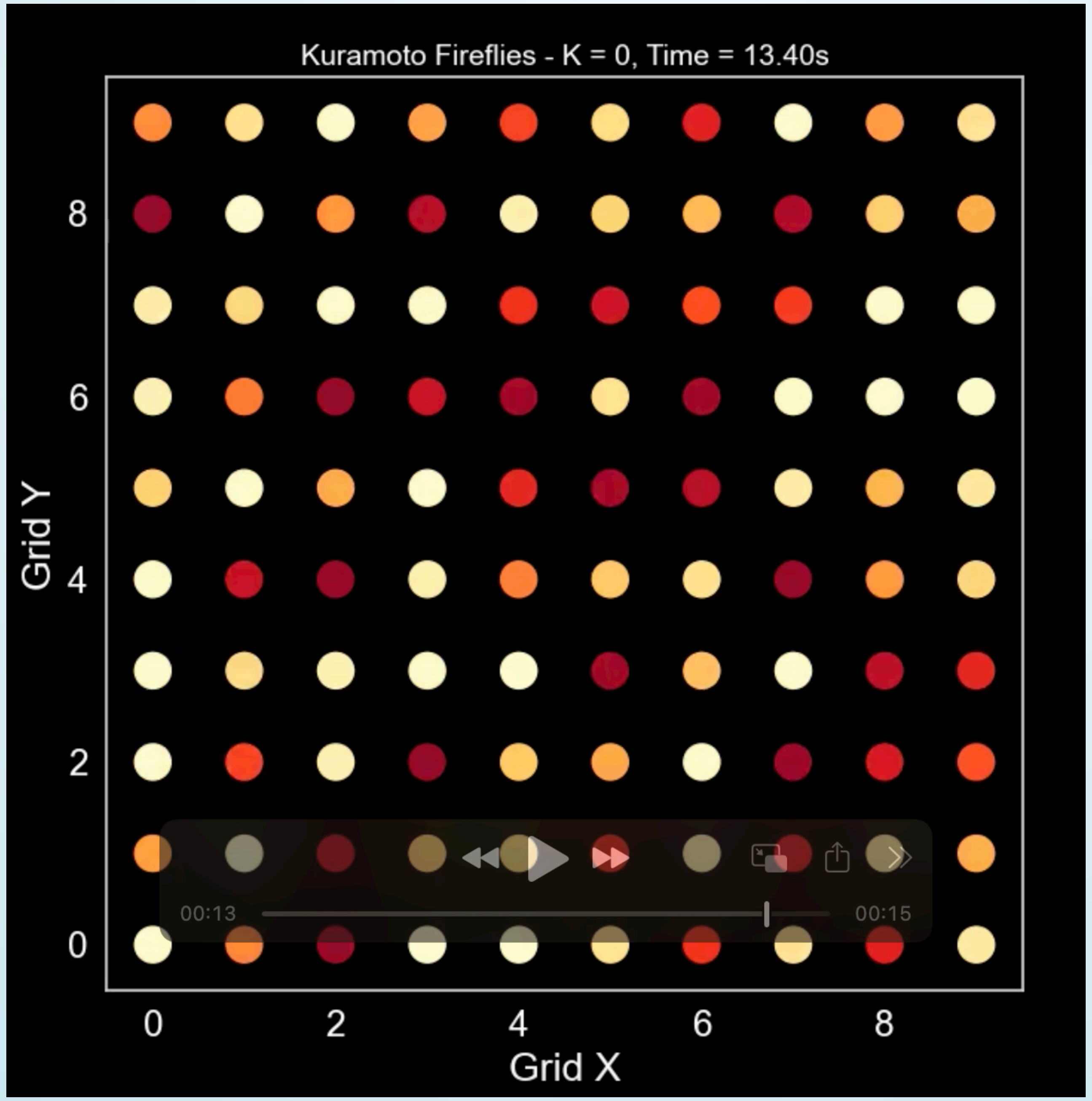
FIREFLY FLASHING: $K = 5$

- With a larger coupling constant, rhythms synchronize quickly!



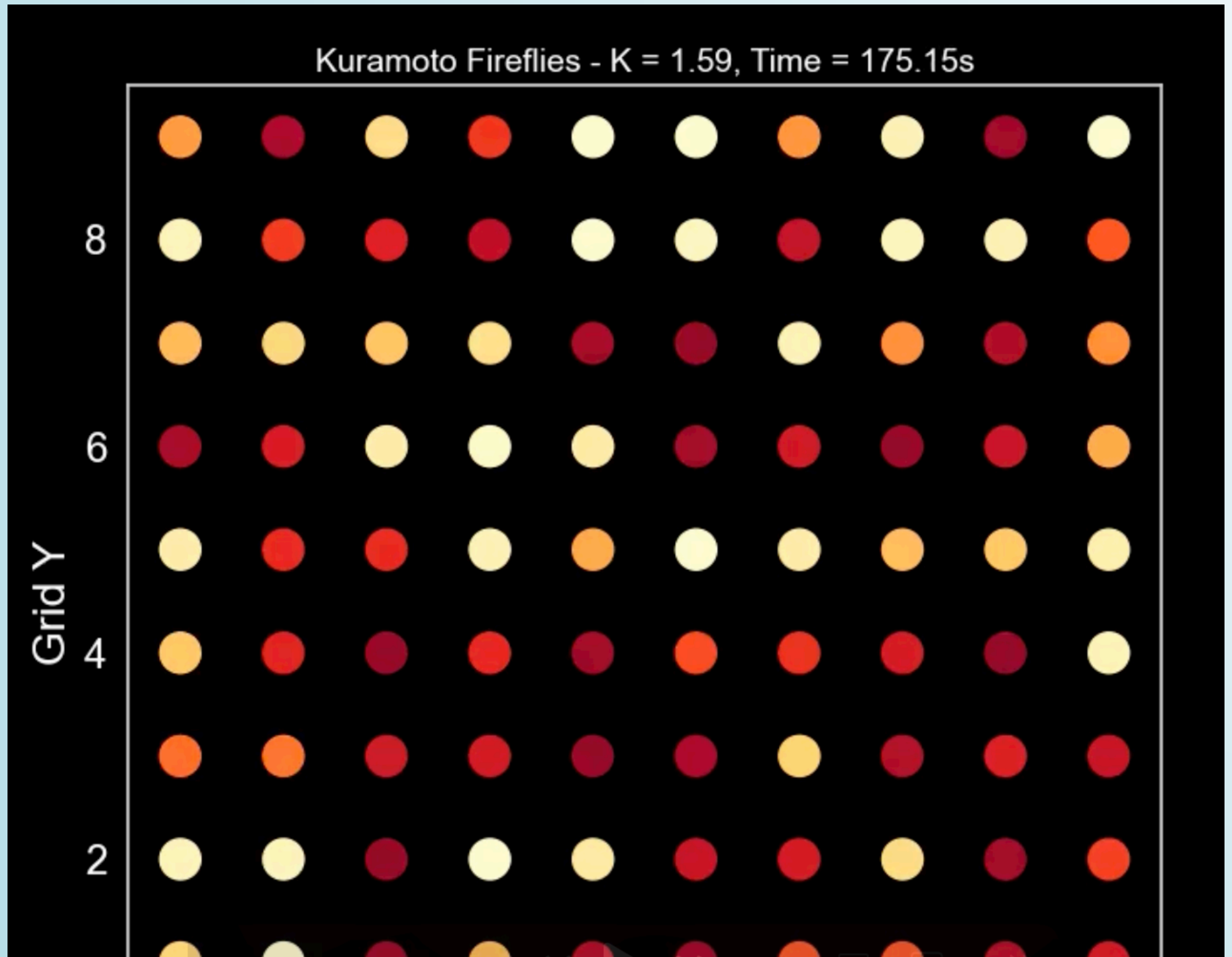
FIREFLY FLASHING: $K = 0$

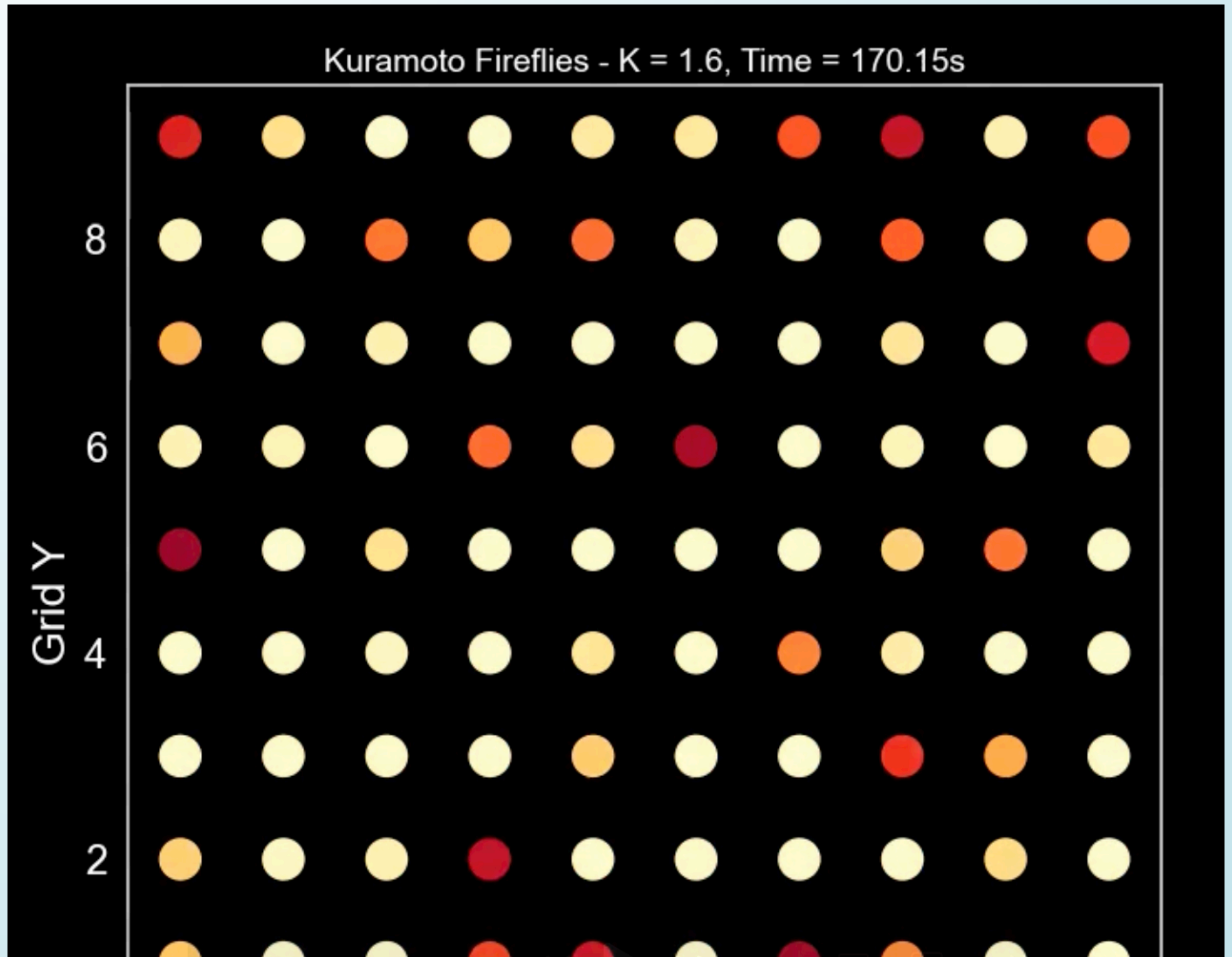
- When the coupling constant is 0, rhythms never synchronize!



THE MYSTERIOUS BEHAVIOR OF THE COUPLING CONSTANT

- When $K = 1.59$, rhythms don't synchronize
- When $K = 1.6$, rhythms synchronize (though it takes time)





THE KURAMOTO CONJECTURE AND PHASE TRANSITIONS

THE KURAMOTO CONJECTURE

- When the coupling constant exceeds the critical point K_c, rhythms suddenly begin to synchronize
- It's like water suddenly turning to ice at 0°C

The critical point K_c is:

$$K_c = \frac{2}{\pi g(0)}$$

- $g(\omega)$ is the distribution function of angular velocities

CALCULATING THE CRITICAL POINT

- In our simulation, the angular velocity distribution function $g(\omega)$ is the standard normal distribution $N(0,1)$

$$g(\omega) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\omega^2}{2}}$$

MEAN FIELD FREQUENCY

- The average angular velocity of all oscillators becomes 0
- This average angular velocity is called the "mean field frequency"
- The entire system's distribution can be represented by $g(0)$

$$g(0) = \frac{1}{\sqrt{2\pi}}$$

CALCULATING THE CRITICAL POINT

$$K_c = \frac{2}{\pi g(0)} = \frac{2}{\pi \frac{1}{\sqrt{2\pi}}} = \sqrt{\frac{8}{\pi}} \simeq 1.596$$

- $K = 1.6$ barely exceeds the critical point, so rhythms synchronize!
- This phenomenon, where properties change dramatically beyond a critical point, is called a "**phase transition**"

EXAMPLES OF PHASE TRANSITIONS

- Phase transitions appear in various physical phenomena
  - Water turning to ice at 0°C
  - Water evaporating at 100°C
  - Magnets losing magnetism at high temperatures
  - Vacuum phase transitions (the beginning of the universe)

PROOF OF THE KURAMOTO CONJECTURE

- The Kuramoto model was proposed in 1975, and the Kuramoto conjecture was formulated in 1984
- The Kuramoto conjecture was proven in 2012 by Dr. Hayato Chiba, who was at Kyushu University at the time!
  - https://arxiv.org/abs/1008.0249
- Therefore, it should now be called the "Kuramoto-Chiba Theorem"

SUMMARY

- The Kuramoto model can explain phenomena where "rhythms synchronize"
- The coupling constant represents the tendency to adjust to deviations from surrounding rhythms
- When the coupling constant exceeds the critical point, rhythms suddenly synchronize (Kuramoto conjecture, proven in 2012)
- The Kuramoto model is a representative model of phase transition phenomena

CALL FOR LIGHTNING TALK PRESENTERS

- The Physics Study Group is looking for lightning talk presenters!
  - Any genre is welcome!
  - If no one volunteers, the organizer will have to do another "lightning talk" that's actually a giant recital...
- If you're interested, please join the Physics Study Group Discord server!

ANNOUNCEMENTS

- The next meeting is scheduled for June 28th
- In addition to lightning talks, we also welcome suggestions for physics YouTube videos that you'd like to watch together with everyone!