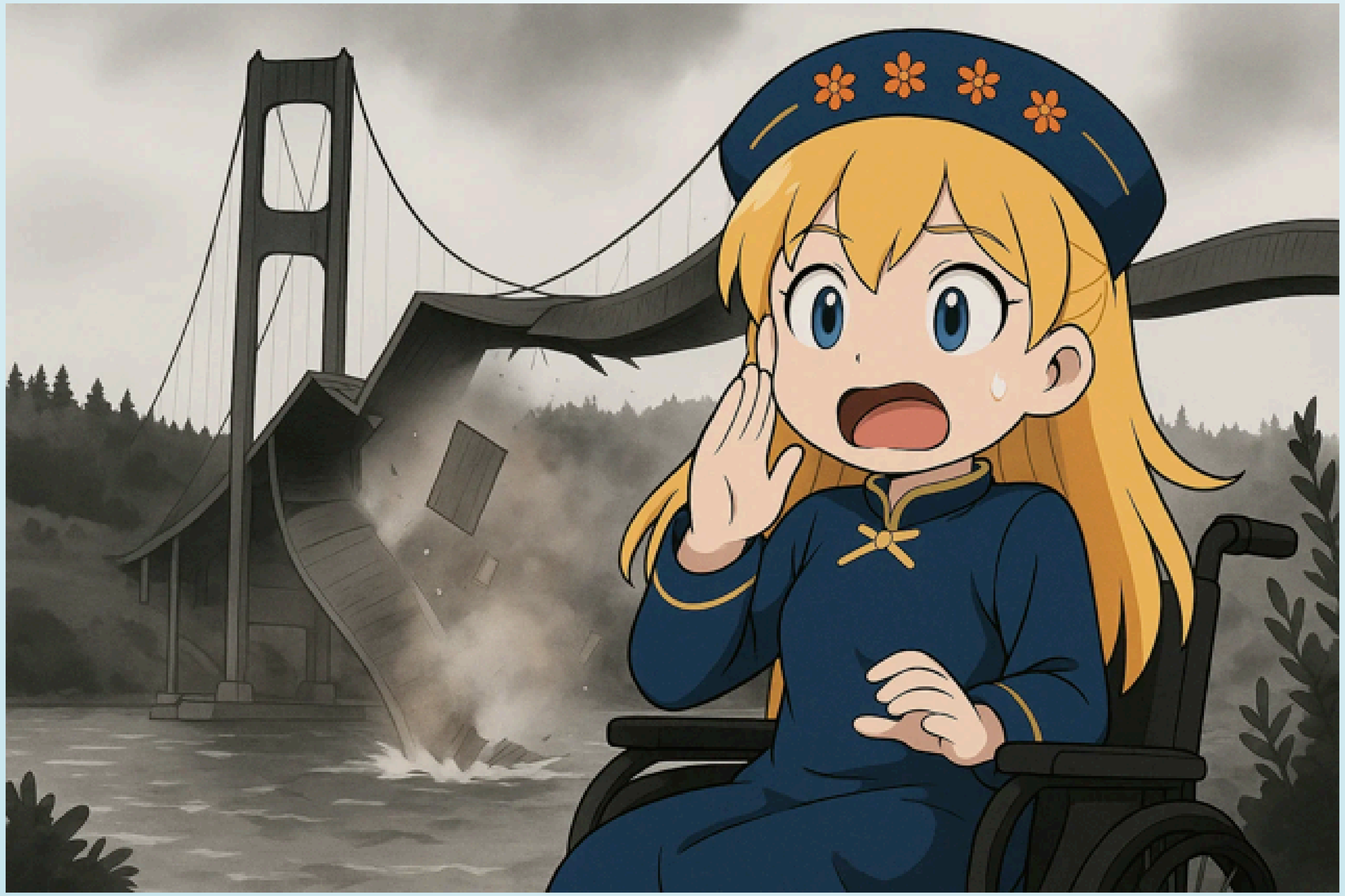
A PENDULUM WHOSE AMPLITUDE INCREASES WITH WIND!?

WHY DID THE TACOMA NARROWS BRIDGE COLLAPSE?



TODAY'S TOPICS

- Why did the Tacoma Narrows Bridge collapse?
  - The bridge collapsed on November 7, 1940
- Equations of motion for harmonic and damped oscillations
  - A simple pendulum model
- Bernoulli's theorem and lift, negative lift
- Vibration equation with negative damping due to lift
- Mechanism of **self-excited vibration**
  - A small difference in the equation of motion can drastically change the result!

THE COLLAPSE OF THE TACOMA NARROWS BRIDGE

- Collapsed on November 7, 1940
- The bridge undulated significantly due to wind!
- Let's watch the actual video!

THE CURRENT TACOMA NARROWS BRIDGE

- Rebuilt in 1950
- A second bridge was constructed in 2007

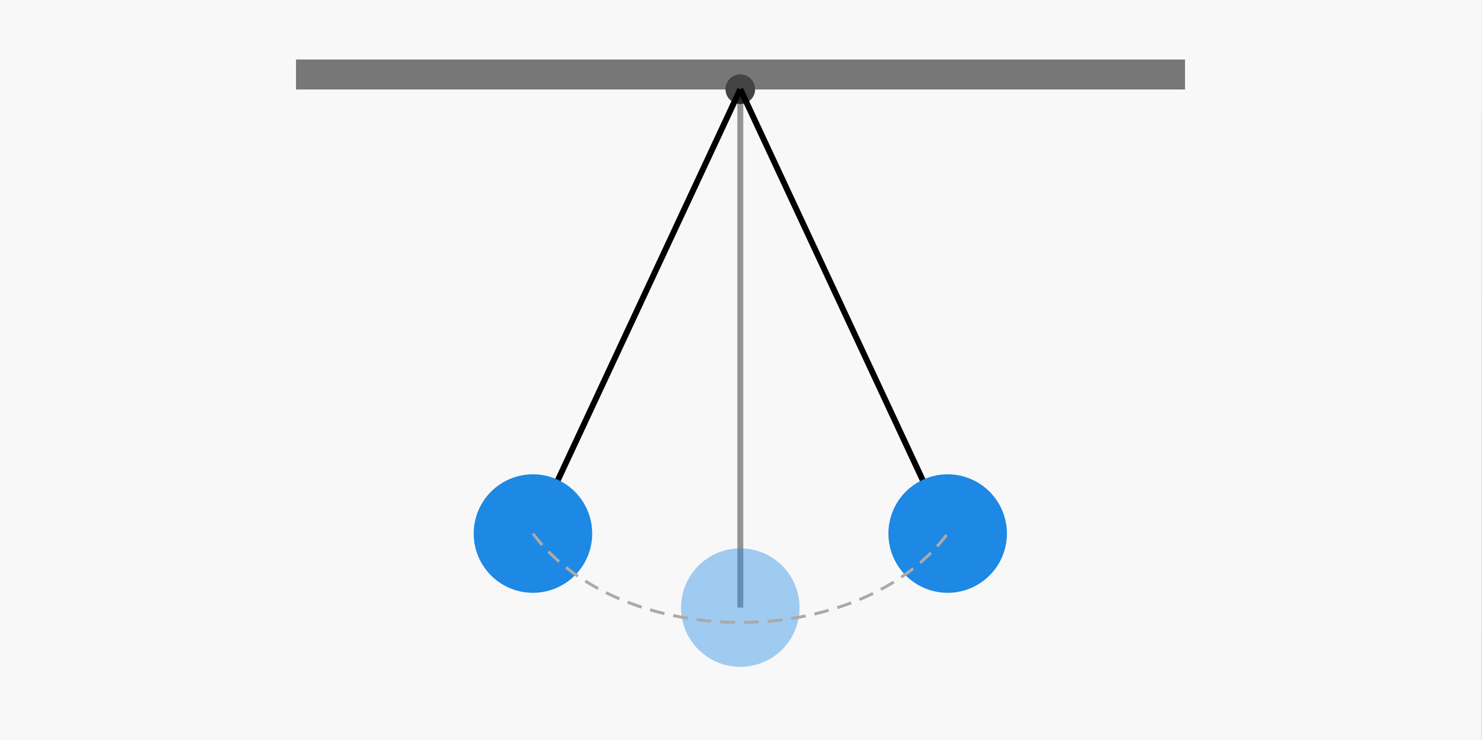
IMPORTANT NOTES

- The actual cause of the bridge collapse is more complex
  - Wind tunnel tests and numerical simulations are used in bridge design
- But today we'll consider the simplest model
- We'll focus on the mechanism of **self-excited vibration**!

EQUATIONS OF MOTION FOR VIBRATION

PENDULUM MOTION

- Lift the pendulum and release it
- A restoring force acts to move it toward the center due to gravity
- Swings to the opposite end
- This repetition causes the pendulum to oscillate



EQUATION OF MOTION FOR HARMONIC OSCILLATION

$$m\frac{d^2x}{dt^2} + \underbrace{kx}_{\text{restoring force}} = 0$$

$$x(t) = A\cos\left(\sqrt{\frac{k}{m}}t + \varphi\right)$$

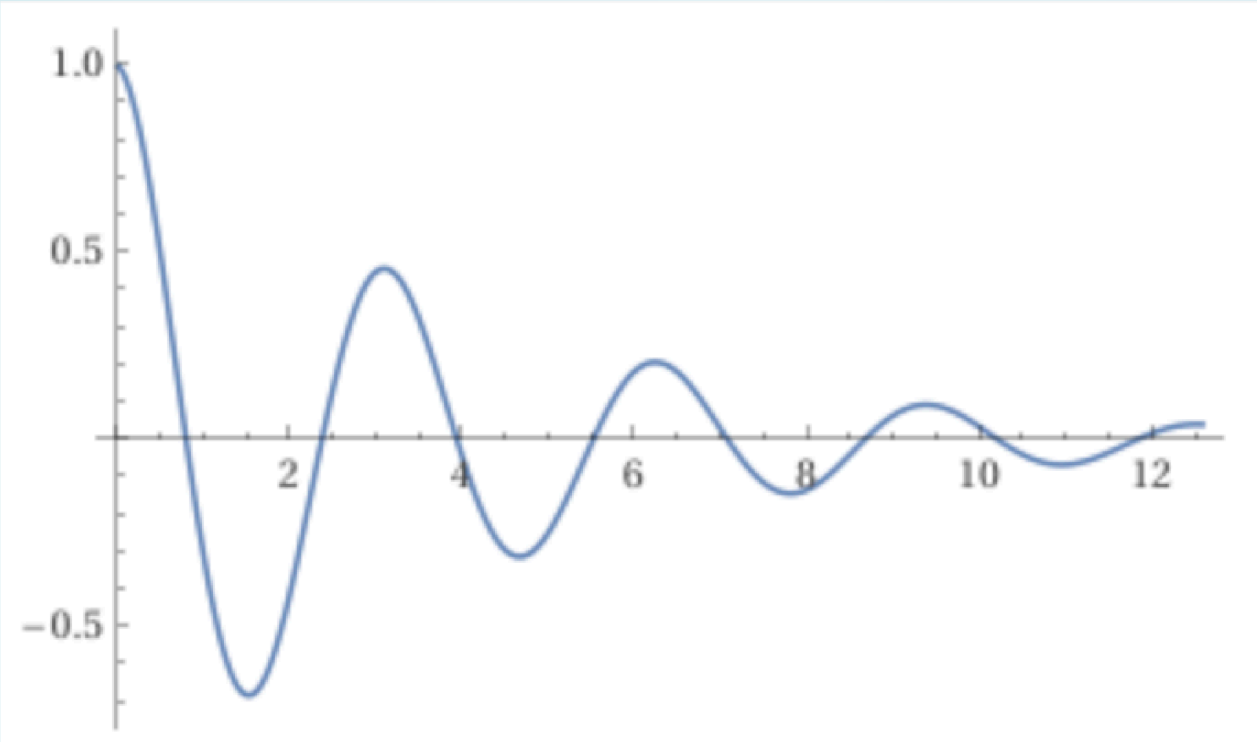
- This motion is called **harmonic oscillation**

EQUATION OF MOTION FOR DAMPED OSCILLATION

- A real pendulum doesn't continue oscillating forever
- The oscillation gradually decreases due to damping forces like air resistance and friction

$$m\frac{d^2x}{dt^2} + \underbrace{\beta\frac{dx}{dt}}_{\text{damping force}} + kx = 0$$

DAMPED OSCILLATION



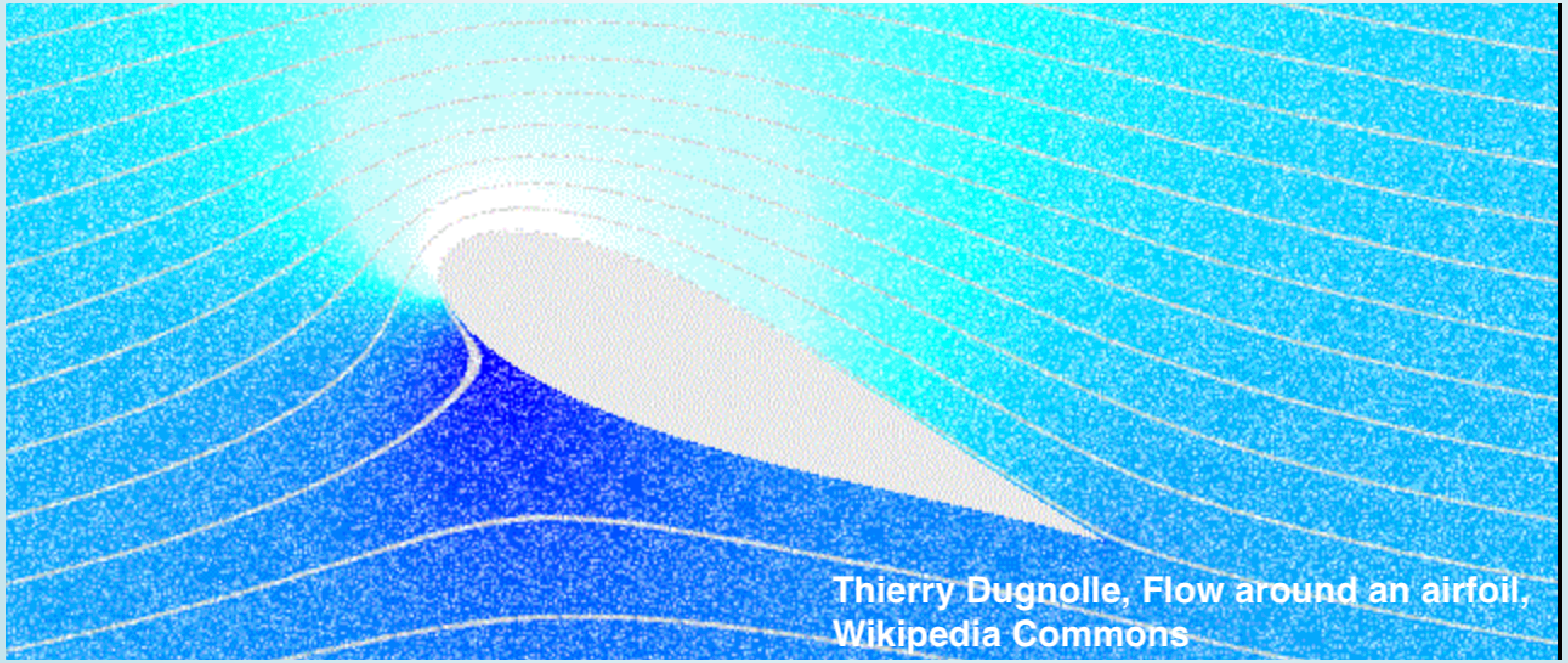
$$x(t) = A\exp(-\beta t)\cos\left(\sqrt{\frac{k}{m}}t + \varphi\right)$$

- The oscillation gradually decreases!

LIFT AND NEGATIVE LIFT

FLUID MOTION

- Air and water are examples of "flowing substances" called fluids
- Forces exerted on objects by wind or water flow can be understood through fluid motion



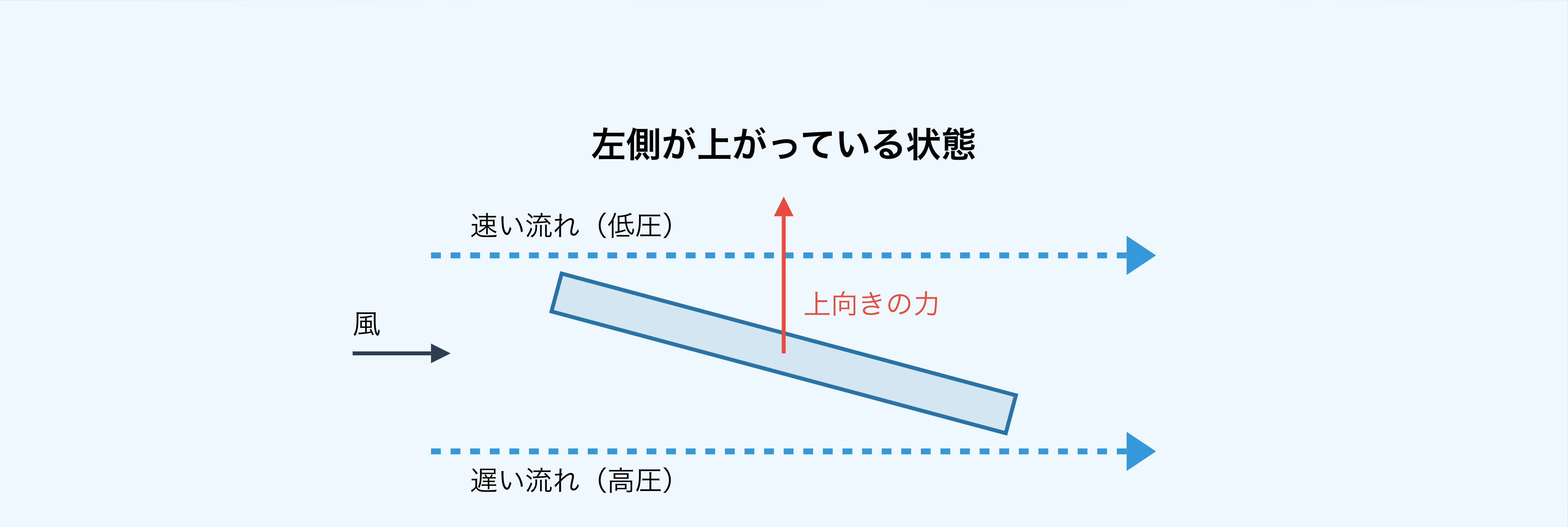
BERNOULLI'S THEOREM

- A theorem that describes the relationship between fluid velocity and pressure
  - When fluid velocity increases, pressure decreases
  - When fluid velocity decreases, pressure increases
- Since air is light, we'll ignore potential energy today

$$p + \frac{1}{2}\rho v^2 + \cancel{\rho g h} = \text{const}$$

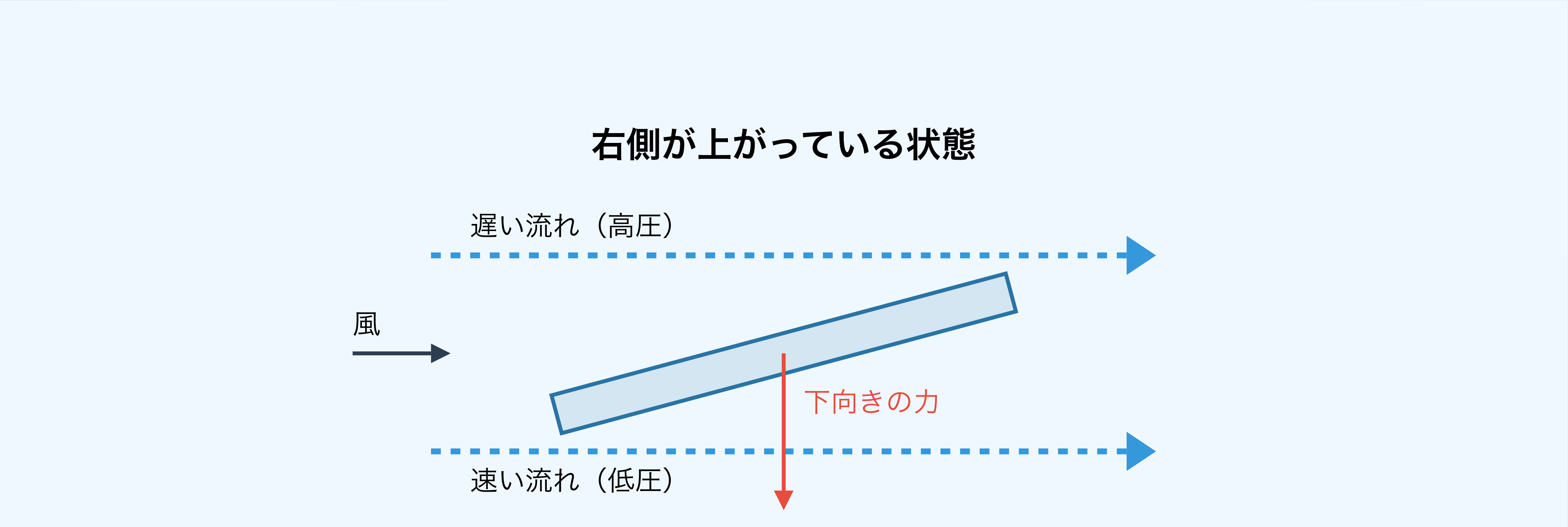
LIFT

- Fluid velocity increases on the upper side of the plate, pressure decreases
- Fluid velocity decreases on the lower side of the plate, pressure increases
- The pressure difference creates an upward force on the plate (**lift**)



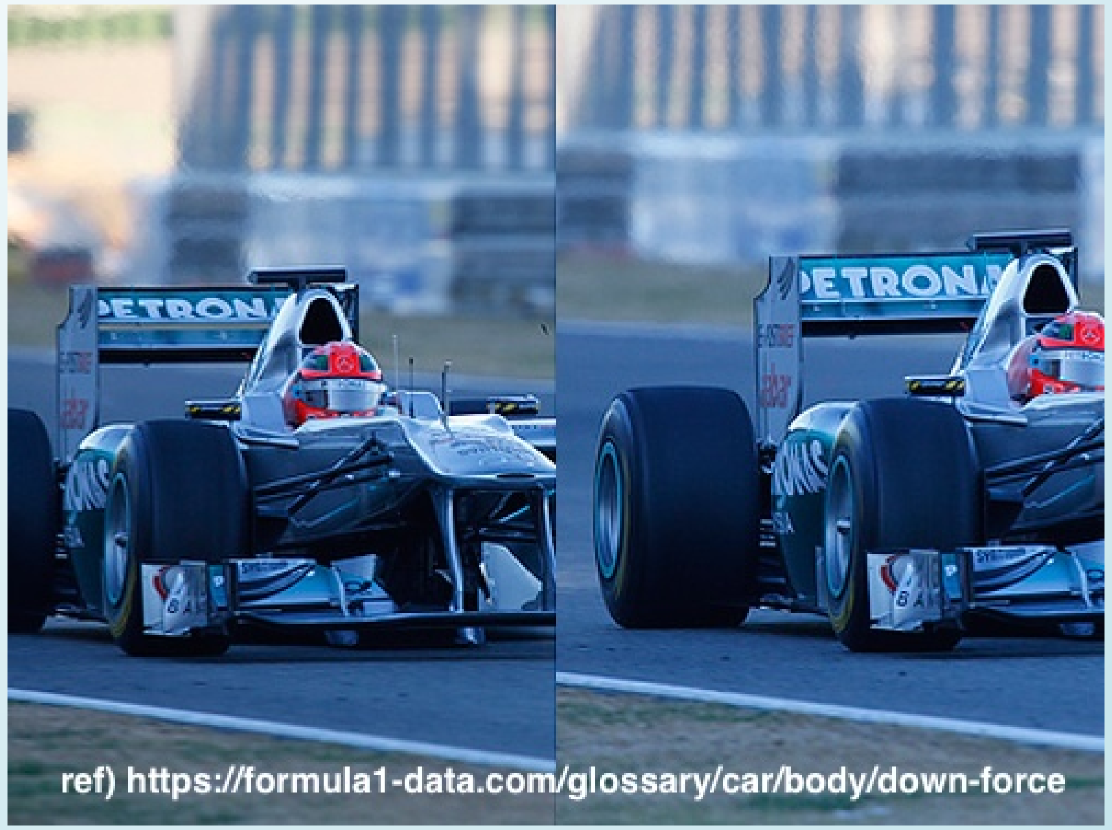
NEGATIVE LIFT

- Fluid velocity decreases on the upper side of the plate, pressure increases
- Fluid velocity increases on the lower side of the plate, pressure decreases
- The pressure difference creates a downward force on the plate (**negative lift**)



APPLICATIONS OF LIFT AND NEGATIVE LIFT

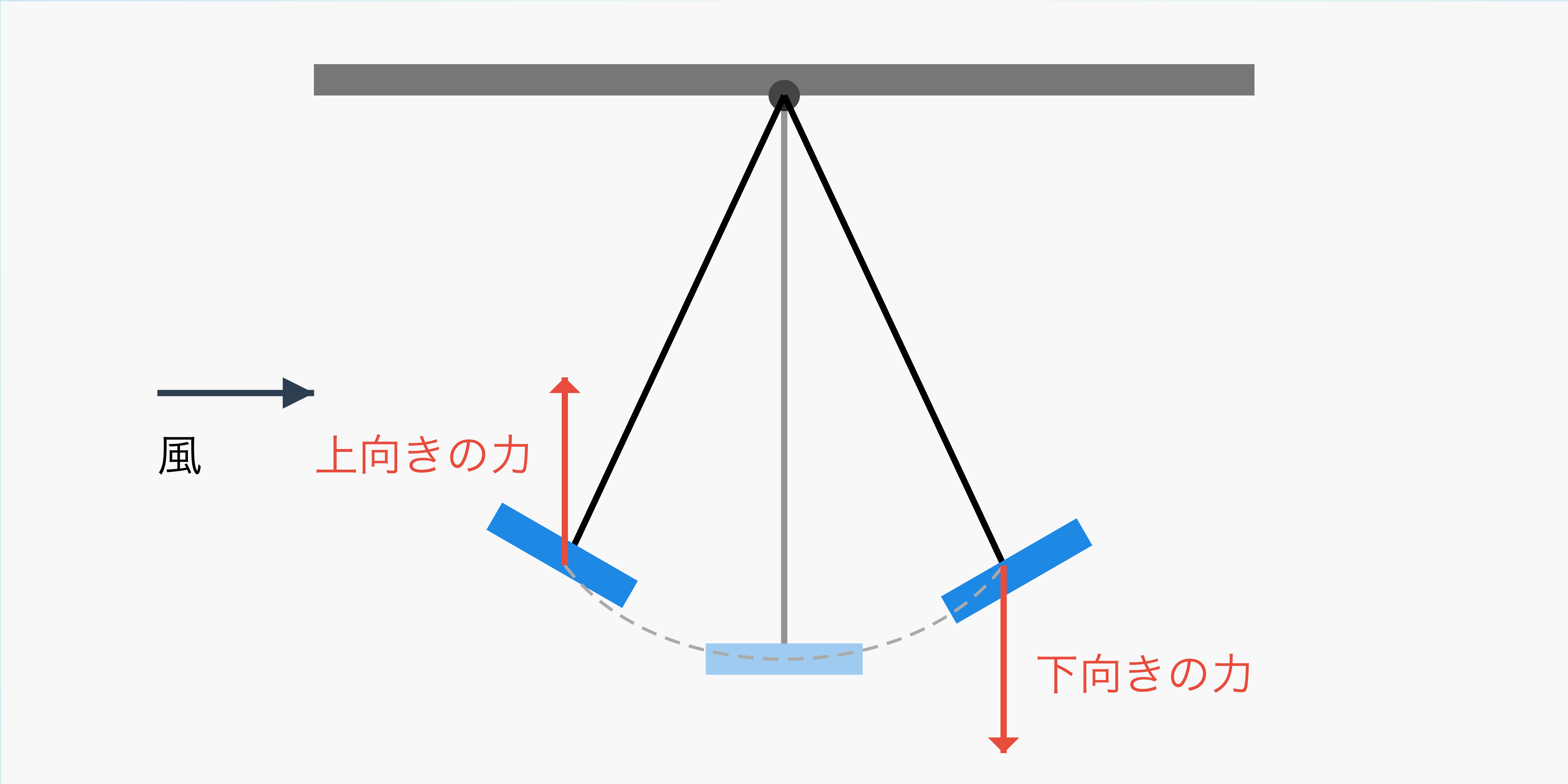
- Lift creates an upward force that allows an airplane to fly
- Negative lift creates a downward force that keeps an F1 car pressed against the track



SELF-EXCITED VIBRATION

SELF-EXCITED VIBRATION

- The pendulum is subjected to lift due to wind
- The lift creates a vibration that amplifies the motion! (**self-excited vibration**)



FAMILIAR EXAMPLE

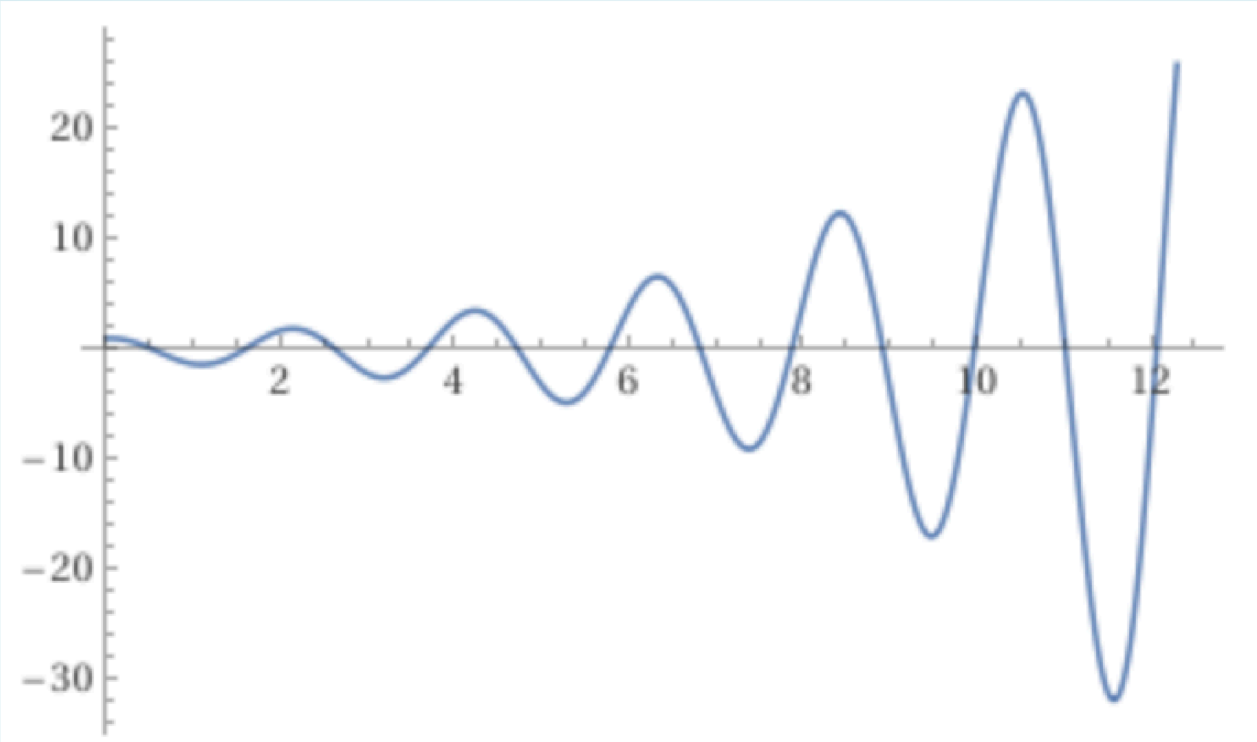
- If you swing the swing with the vibration, the vibration becomes larger!
- The same thing happens with lift!

EQUATION OF MOTION FOR SELF-EXCITED VIBRATION

$$m\frac{d^2x}{dt^2}\underbrace{-\beta\frac{dx}{dt}}_{\text{opposite sign!}}+kx=0$$

- Vibration with negative damping
- Just a change in sign…
  - What happens?

GENERAL SOLUTION



$$x(t) = A\exp(\beta t)\cos\left(\sqrt{\frac{k}{m}}t + \varphi\right)$$

- Vibration becomes larger!

WHY DID THE TACOMA NARROWS BRIDGE COLLAPSE?

- Self-excited vibration mechanism worked!
- The bridge undulated significantly due to wind!

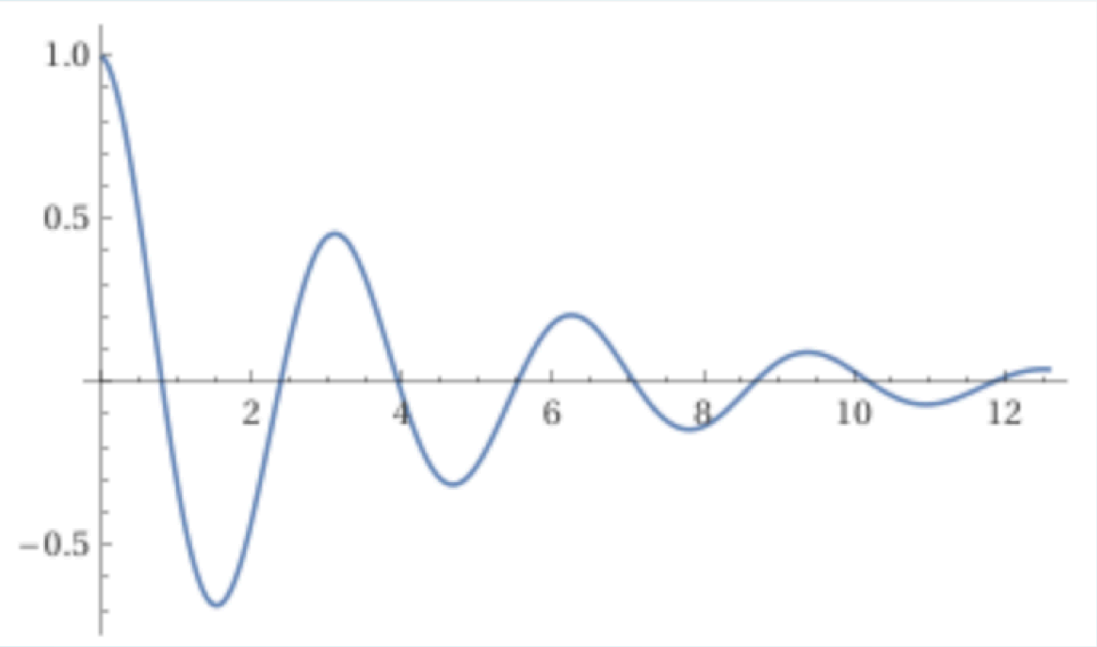
CONCLUSION

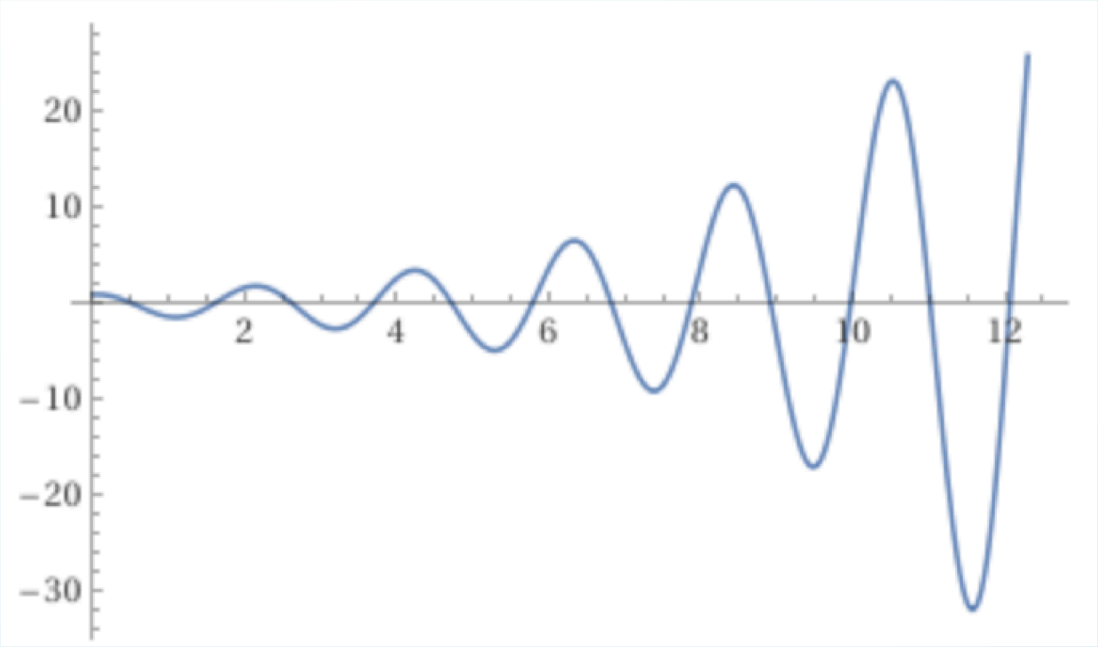
- Equations of motion for harmonic oscillation can represent the motion of a pendulum!
- Equations of motion for damped oscillation can represent the motion of a pendulum that gradually decreases in amplitude!
- Forces due to fluid flow and plate tilt create upward lift or downward negative lift!
- Equations of motion for vibration with negative damping can represent the mechanism of self-excited vibration!

ONE SIGN CHANGES THE WORLD!

$$m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = 0$$

$$m\frac{d^2x}{dt^2} - \beta\frac{dx}{dt} + kx = 0$$





ADDITIONAL NOTES

- The actual fluid dynamics of bridge structures is more complex
  - It is believed that the Tacoma Narrows Bridge collapsed due to aeroelastic flutter
- Torsional vibration is also a significant cause
- **Self-excited vibration alone is not the cause!**
- But today we focused on the simplest self-excited vibration!

LT SPEAKER RECRUITMENT

- We are recruiting LT speakers at the Physics Meeting!
  - Any genre is OK!
  - If there is no application, the host will open a Jaiyan Ressaitaru under the guise of LT...
- If you are interested, please join the Physics Meeting Discord server!

ANNOUNCEMENT

- Next meeting is scheduled for May 17
- We want everyone to watch the physics video together
- We welcome any proposals to watch the video together!