Assignment #1

Deliverable # 1 is due by 11:00 PM Thursday, September 17th, 2020 on Gradescope.
Deliverable # 2 is due by 11:00 PM Thursday, October 1st, 2020 on Gradescope.
Deliverable # 3 is due by 11:00 PM Friday, October 16th, 2020 on Gradescope.

Deliverable #1.

Problem 1 Use Monte Carlo simulation to compute the probability that a randomly dealt 5-hand from a deck of poker cards is a straight. How many copies of simulation do you think are roughly needed to achieve an accuracy up to level 0.0001?

Problem 2 Suppose that you run a food truck on each Wednesday between 11AM and 1PM. There is no customer waiting when the food truck opens at 11AM. Customers arrive at the food truck according to a Poisson process with rate λ persons per minute. There is a single line of queue with a first-come-first-serve rule. Customers that have arrived before 1PM will eventually get served. In other words, customers are not able to join the queue if they arrive later than 1PM. The food truck resumes its operation until all customers who have arrived before 1PM get served. You are the owner of the food truck and there is only one server (yourself). The service time for each customer is independent and identically distributed with some distribution F. The service time includes the time on taking orders and food preparation. The distribution F has a mean of $1/\mu$ and the probability distribution is to be specified. Suppose $\lambda = 2$. Simulate 100 independent days (Wednesdays) for each of the following parts.

a.) Denote E_i as the number of customers that arrive at the food truck between 11:55 AM and 12:05 PM on the i-th day. Do the following:

   i.) Compute $\frac{1}{100}\sum_{i=1}^{100} E_i$ and compute the sample variance for $\mathbb{E}_i$'s.

   ii.) Plot the histogram for $\{E_1, E_2, \ldots, E_{100}\}$.

   iii.) Group all the inter-arrival times over the 100 days, and compute the percentage of inter-arrival times that are longer than 1 minute. How does this value compare to e^{-2}?

   iv.) Suppose that the distribution F takes value on $[0, 1]$ minute. The CDF is given by $F(x) = x^2$. Simulate E_1 independent copies of the service times according to distribution F. Compute the expectation a and the standard deviation σ using the simulated copies of service times. Compute the percentage of the simulated service times that are shorter than $a+2\sigma/\sqrt{E_1}$. How does this percentage compare to the $\Phi(2)$, where $\Phi(\cdot)$ is the standard normal CDF?

Deliverable #2. Continuing on the main statement of Problem 2, do the following parts using simulation. There is no requirement to provide confidence intervals for this Deliverable, unless you insist.

Problem 2

b.) Simulate $n = 50$ independent days of operation from 11AM and 1PM and generate the sequence of customer waiting times. For this task, the service time distribution F is given by an exponential distribution with expectation as 35 seconds. Recall that the condition $\lambda = 2$ persons per minute in the problem statement implies that the exponentially distributed inter-arrival times of customers have expectation also as 30 seconds.

   i.) Suppose that you are interested in knowing that on a typical Wednesday, what is the expectation of the averaged waiting time for customers who arrive at the food truck between 11:15 AM to 11:30 AM. Compute an estimate of this quantity using simulation.

   ii.) Suppose that you are interested in knowing that on a typical Wednesday, what is the expectation of the averaged waiting time for customers who arrive at the food truck between 12:45 PM to 1:00 PM. Compute an estimate of this quantity using simulation.

   iii.) Compare the two quantities you computed in the previous two questions. Describe intuition that you may get from this comparison.

   iv.) Compute the percentage of customers who arrive at the food truck between 11:15 AM to 11:30 AM and wait for more than 3 minutes. Compute the percentage of customers who arrive at the food truck between 12:45 PM to 1:00 PM and wait for more than 3 minutes.

   v.) Suppose that the customers will immediately abandon the system and leave for other dining options, conditional on that they see more than 5 people in the system (including the one being served). Now, what is the percentage of customers who abandon the system (=food truck) upon arrival between 12:45 PM and 1:00 PM? What is the expectation of the averaged waiting time for customers who arrive at the food truck between 12:45 PM to 1:00 PM and did not abandon the system?

   vi.) Re-do the previous five parts with F being an exponential distribution with expectation as 30 seconds and everything else equal.

Deliverable #3. Continuing on the main statement of Problem 2, do the following parts using simulation. There is no requirement to provide confidence intervals for this Deliverable, unless you insist. Deliverable # 3 is due by 11:00 PM Friday, October 16th, 2020 on Gradescope.

Problem 2

c.) Suppose there are two independent streams of customers arriving at your food truck. The first stream contains regular customers who arrive according to a Poisson process with constant rate 2 per minute from 11AM to 1PM. The second stream contains customers with special membership who arrive according to a Poisson process with time-varying rate. This time-varying rate, as a function of time, is 0.5 per minute at 11AM and quadratically increasing over 11AM to 1PM. At 1PM, the arrival rate is 1 per minute. The service time requirement distribution for any customer in either stream is exponentially distributed with expectation as 35 seconds. These two streams of customers join and wait in separate lines. In lieu of the two streams of customers, you hire a second server, in additional to yourself. Each server at one time can only serve one customer, with the service length equal to the service time requirement for that customer. The second server can serve customers from both streams but always priorities customers with special membership. Specifically, whenever the second server becomes idle, there are two scenarios as follows. If the line of the second stream customers is not empty, the second server priorities serving customers from the second stream (those with special membership). If the line of the second stream customers is empty, the second server will serve the first in line from the first stream. The first server (yourself), only serves customers from the first stream.

Simulate $n = 50$ independent days of operation from 11AM and 1PM. One requirement is to use the Thinning approach to simulate the arrival process for the second stream of customers.

i.) What is the expectation of the averaged waiting time for the second stream customers who arrive at the food truck between 12:45 PM to 1:00 PM?

ii.) What is the expectation of the averaged waiting time for the first stream customers who arrive at the food truck between 12:45 PM to 1:00 PM? How does this compare to the answer you got in Deliverable 2 part b.) ii.)?