RESEARCH STATEMENT

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My research interests span the areas of algebraic combinatorics and representation theory. A theme in my research is in developing combinatorial models to facilitate computations, particularly related to the representation theory of Lie algebras and quantum groups. My main approach is to encode complex algebraic or geometric objects as combinatorial objects, then use combinatorial methods to prove statements about the objects or do explicit computations. Computer software such as SAGE plays a role in this process; from helping test the models to executing the algorithms/computations.

Background

Crystal bases

Crystal bases have origins in the representation theory of quantum groups as well as in the theory of exactly solvable lattice models in statistical physics. They were introduced by Kashiwara [3], and provide a way to view the representation theory of quantum groups $U_q(\mathfrak{g})$ (q-deformations of the universal enveloping algebra) at $q = 0$. Quantum groups were first introduced independently by Drinfeld [1] and Jimbo [2] in the context of two-dimensional statistical mechanics. In this setting the parameter q represents temperature, so the limit $q \to 0$ corresponds to behavior near "absolute zero". In this limit, crystal bases come with a simple combinatorial structure in the form of a colored directed graph called a *crystal*. This graph contains the essential information for solving classical problems such as computing characters, decomposing tensor products, and branching rules. In order to reap the above benefits, effective realizations of crystals are needed.

There is currently a limited body of work in developing combinatorical models for crystals. In the case of highest weight crystals, Kashiwara and Nakashima [4] constructed type-dependent tableau models in classical types $A - D$. For symmetrizable Kac-Moody algebras, a type-independent model is due to Littleman [7] and is based on paths in the weight lattice. Lenart and Postnikov [6] developed another type-independent model for highest weight crystals called the *alcove model*, which is based on the affine Weyl group and enumerating saturated chains in Bruhat order. The alcove model has advantages over the Littleman path model due to its simplicity, ease of computations, and widespread applications (see below).

More recently, a generalization of the alcove model, called the *quantum alcove model*, was defined in [5]. It is based on enumerating chains in the so-called quantum Bruhat graph of the corresponding finite Weyl group, and is obtained from the Hasse diagram of the Bruhat order by adding extra down edges.

Macdonald polynomials

The symmetric Macdonald polynomials $P_\lambda(x; q, t)$, indexed by dominant weights λ, are Weyl group invariant polynomials with rational functions coefficients in q, t [8]. They generalize the irreducible characters of the corresponding simple Lie algebras, which are recovered for $q = t = 0$. A uniform combinatorial formula for $P_\lambda(x; q, t)$ was given by Ram and Yip [9] in terms of *alcove walks*. There are also non-symmetric versions of Macdonald polynomials $E_\mu(x; q, t)$, where μ is an arbitrary weight [8], as well as a Ram-Yip formula for them.

Current work

References

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