UNIVERSITÀ DEGLI STUDI DI BERGAMO

Corso di
Elettrotecnica NO

ver. 0000B

Corso di Elettrotecnica NO

Angelo Baggini

Cap. 4

Rappresentazione e analisi delle reti elettriche in regime variabile
Regime PAS

Ipotesi

Abbiamo già rimosso $\frac{d}{dt} = 0$

$$\oint_e \overline{E}\,\overline{dl} = -\frac{d\phi}{dt} \qquad \left(\oint_e \overline{E}\,\overline{dl} = 0 \rightarrow \sum V = 0\right)$$

Adesso rimuoviamo l'ipotesi di impossibilità di accumulo di carica

$$\oint_S \overline{J} \cdot \overline{S} = -\frac{dq}{dt} \qquad \left(\oint_S \overline{J}\,\overline{S} = 0 \rightarrow \sum I = 0\right)$$

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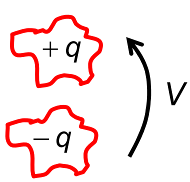
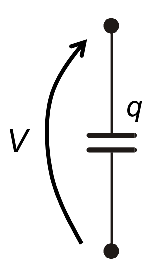
Fenomeno capacitivo



$$\frac{q}{V} = \text{costante} = C$$

U.M.

farad F



$$C = \frac{q}{V} \qquad i = \frac{dq}{dt} \qquad i = C\frac{dv}{dt}$$

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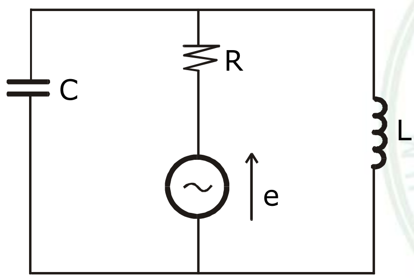
Circuito in regime variabile - Esempio



$$e(t) = 10\ sen\ t\ V$$

$$C = 1\ \mu F$$

$$R = 2\ \Omega$$

$$L = 1\ m\ H$$

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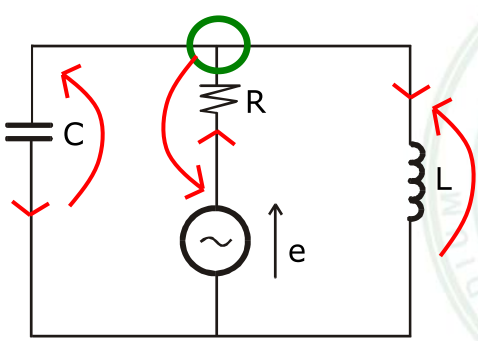
Circuito in regime variabile - Esempio



$$i_R = i_C + i_L$$

$$v_C + v_R - e = 0$$

$$e - v_R - v_L = 0$$

$$v_R = Ri$$

$$v_L = L \frac{di_L}{dt}$$

$$v_C = \int_{-\infty}^{t} \frac{i_c}{c} dt$$

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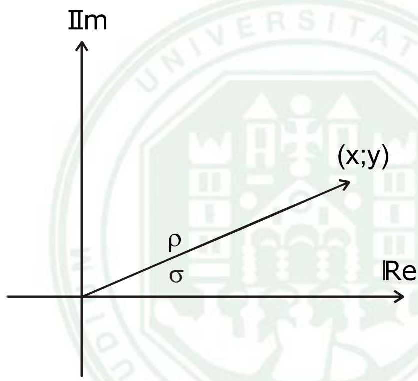
Richiami sui numeri complessi

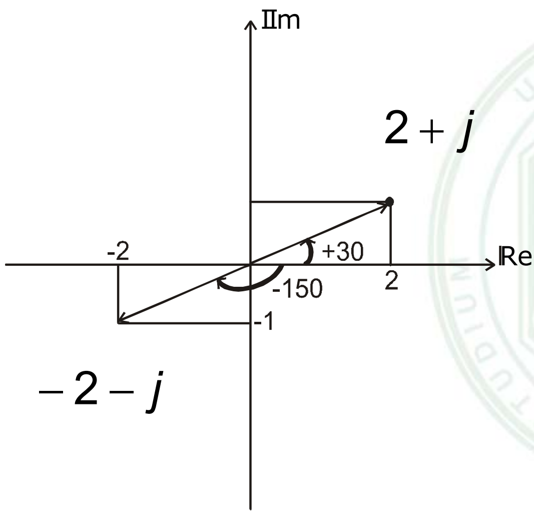


$$x + jy = \rho e^{j\vartheta} = \rho\angle\vartheta$$

$$x = \rho\cos\vartheta \rightarrow \rho = \sqrt{x^2 + y^2}$$

$$y = \rho\, sen\,\vartheta \rightarrow \vartheta = atg\,\frac{y}{x}$$

Richiami sui numeri complessi



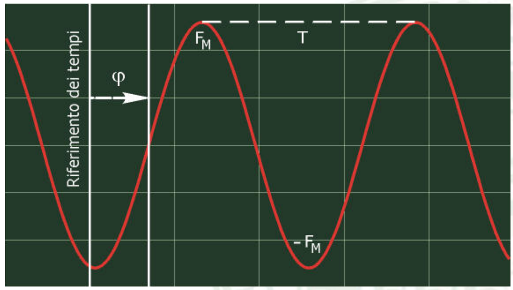
$$\rho_1 = \sqrt{4+1}$$

$$\rho_2 = \sqrt{4+1}$$

$$\vartheta_1 = atg\frac{1}{2}$$

$$\vartheta_2 = atg\frac{1}{2}$$

Funzione Periodica Alternata Sinusoidale (PAS)



$$T = \frac{1}{f}$$

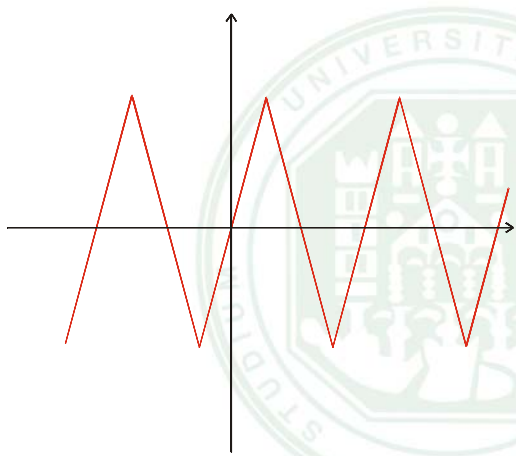
$$\omega = 2\pi f$$

$$f(t) = F_M \cos(\omega t + \varphi)$$

$$(F_M; \omega; \varphi)$$

$$f' \longrightarrow PAS - \text{stessa } \omega$$

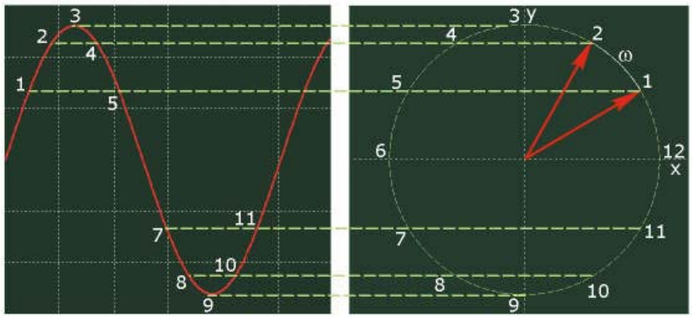
$$\int f dt \longrightarrow PAS - \text{stessa } \omega$$



Forma di Eulero di una funzione PAS

$$f(t) = F_M \cos(\omega t + \varphi) = \mathrm{Re}\,(F_M e^{\,j(\omega t+\varphi)}) =$$

$$= \mathrm{Re}\,(F_M[\cos(\omega t + \varphi) + j\,\mathrm{sen}(\omega t + \varphi)])$$



Derivazione e integrazione

$$f' = \mathrm{Re}(j\omega F_M e^{\,j(\omega t+\varphi)}) = \mathrm{Re}(\omega F_M e^{\,j(\omega t+\varphi+\frac{\pi}{2})})$$

$$\int f dt = \mathrm{Re}(\frac{1}{j\omega} F_M e^{\,j(\omega t+\varphi)}) = \mathrm{Re}(\frac{F_M}{\omega} e^{\,j(\omega t+\varphi-\frac{\pi}{2})})$$

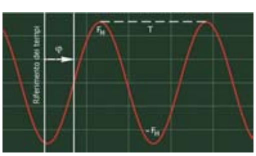
Funzione "cappello"

$$\bar{f}(t) = F_M e^{j(\omega t+\varphi)}$$

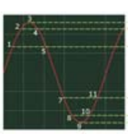
$$f' = j\omega F_M e^{j(\omega t+\varphi)} = \omega F_M e^{j(\omega t+\varphi+\frac{\pi}{2})}$$

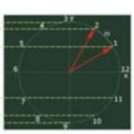
$$\int f dt = \frac{1}{j\omega} F_M e^{j(\omega t+\varphi)} = \frac{F_M}{\omega} e^{j(\omega t+\varphi-\frac{\pi}{2})}$$

Dominio del tempo



$f(t) = F_M \cos(\omega t + \varphi)$



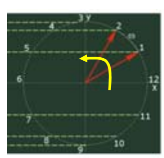


$f(t) = F_M \cos(\omega t + \varphi) = \mathrm{Re}(F_M e^{j(\omega t+\varphi)})$

$\frac{df}{dt} = -\omega F_M \mathrm{sen}(\omega t + \varphi) = \mathrm{Re}(j\omega F_M e^{j(\omega t+\varphi)})$

$\int$......

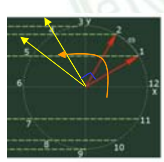
Dominio dei vettori rotanti



$\bar{f}(t) = F_M e^{j(\omega t+\varphi)}$

$\frac{d\bar{f}}{dt} = j\omega\bar{f}(t)$

$\int \bar{f} dt = \frac{\bar{f}(t)}{j\omega}$



Supponendo tutti con la stessa w

Dominio dei fasori

$\bar{F} = \frac{F_M}{\sqrt{2}} e^{j\varphi}$

$\frac{d\bar{F}}{dt} = \bar{F} j\omega$

$\int \bar{F} dt = \frac{\bar{F}}{j\omega}$

Derivate e integrali nel tempo: idem, ma non ruotano

Rappresentazione fasoriale

$$f(t) = \sqrt{2}\,10\cos(50t + \frac{\pi}{3}) \Leftrightarrow \bar{F} = 10e^{j\frac{\pi}{3}}$$

PAS
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Rappresentazione fasoriale

$$\bar{I} = 5\,e^{j\frac{\pi}{2}}\ \text{A} \quad \text{nota}\ \omega = 10\ \text{rad}\cdot s^{-1}$$

$$\rightarrow i(t) = \sqrt{2}\,5\cos(10t + \frac{\pi}{2})$$

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Rappresentazione fasoriale

$$\overline{E} = 5V \quad nota\,\omega = 10\,rad\,s^{-1} \rightarrow e(t) = \sqrt{2}\,5\cos(10t)$$

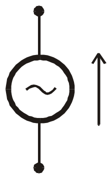
Rappresentazione fasoriale

$$\overline{G} = 5 + j5 \text{ nota } \omega = 10\,\text{rad}\,\text{s}^{-1}$$

$$\overline{G} = \sqrt{50}\;e^{j\frac{\pi}{4}}$$

$$\rightarrow g(t) = \underbrace{\sqrt{2}\cdot\sqrt{50}}_{10}\cos(10t + \frac{\pi}{4})$$

Generatore di tensione



$$e(t) = V_M \cos(\omega t + \varphi)$$

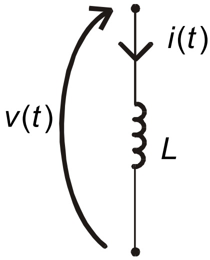
$$\bar{E} = \frac{V_M}{\sqrt{2}} e^{j\varphi}$$

Generatore di corrente

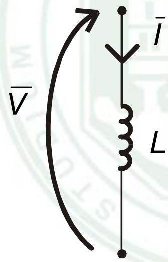
$$a(t) = A_M \cos(\omega t + \varphi)$$

$$\bar{A} = \frac{A_M}{\sqrt{2}} e^{j\varphi}$$

Induttore

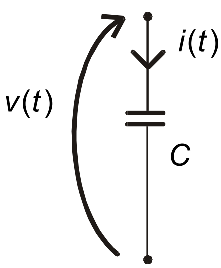


$$v(t) = L\frac{di(t)}{dt}$$

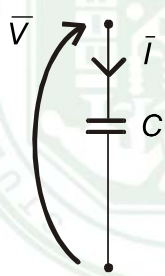


$$\bar{V} = j\omega L\,\bar{I}$$

Condensatore



$$i(t) = C\frac{dv(t)}{dt}$$

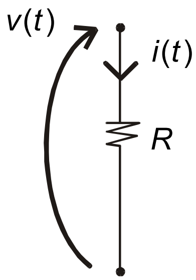


$$\bar{I} = C\,j\omega\bar{V}$$

$$\bar{V} = \frac{-j}{\omega C}\bar{I}$$

< 0

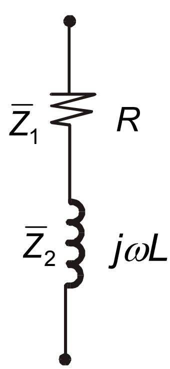
Resistore



$$v(t) = Ri(t)$$

$$\bar{V} = R\bar{I}$$

Impedenza



$$\bar{Z} = R + j\omega L = \bar{Z}_1 + \bar{Z}_2$$

$$\bar{V} = \bar{Z}\,\bar{I}$$

Impedenza

Resistenza Reattanza

$$\overline{Z} = R \pm jX$$

$$\overline{V} = \overline{Z}\,\overline{I}$$

Ammettenza

$$\overline{Y} = \frac{1}{\overline{Z}} = G \pm jB$$

Conduttanza Suscettanza

Impedenza

$$X = \omega L > 0 \ \text{ reattanza induttiva}$$

$$X = \frac{-1}{\omega C} < 0 \ \text{ reattanza capacitiva}$$

Ammettenza

$$\overline{Y} = G \pm jB = \frac{1}{\overline{Z}} = \frac{1}{R + jX} \neq \frac{1}{R} \pm j\frac{1}{X}$$

$$\overline{Y} = \frac{1}{R + jX} = \frac{R}{R^2 + X^2} - \frac{jX}{R^2 + X^2}$$

Impedenza

Impedenze e fasori sono rappresentati con numeri complessi, ma sono due cose diverse

Le impedenze non sono fasori!!

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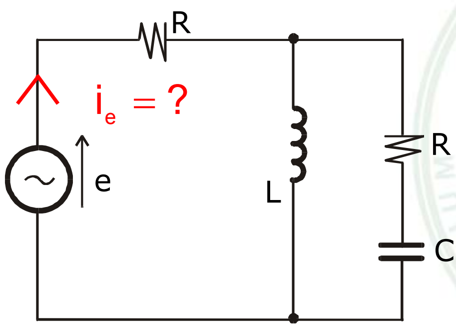
Esempio



$$e = \sqrt{2}\,10\cos(10t + \frac{\pi}{2})$$

$$L = 1H \qquad R = 5\Omega$$

$$C = \frac{1}{10}F$$

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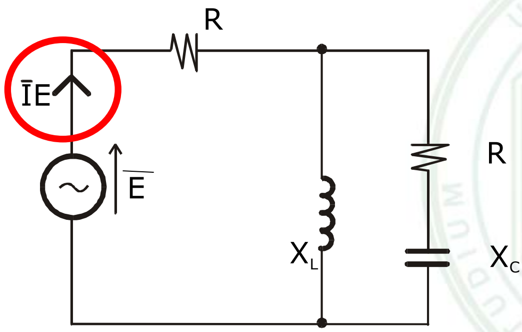
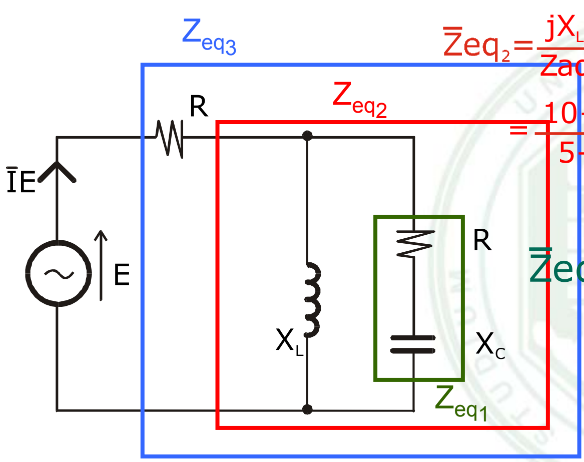
Trasformazione nel dominio dei fasori



$$\overline{E} = 10e^{j\frac{\pi}{2}}\ \mathrm{V} = 0 + j10\ \ \mathrm{V}$$

$$X_L = \omega L = 10 \cdot 1 = 10\Omega$$

$$X_C = \frac{-1}{\omega C} = \frac{-1}{\frac{1}{10}\cdot 10} = -1\Omega$$



$$\overline{Z}eq_2 = \frac{jX_L\ Zaq_1}{Zaq_1 + jX_L} = \frac{j10(5\text{-}j)}{5\text{-}j+j10}$$

$$= \frac{10+j50}{5+j9} = 4{,}72 + j1{,}51\Omega$$

$$\overline{Z}eq_1 = R + jX_c = 5\text{-}j\Omega$$

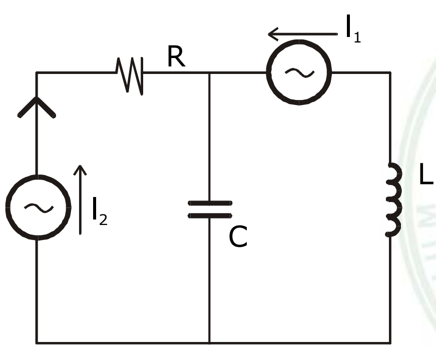
$$\overline{Z}eq_3 = R + \overline{Z}eq_2 = 5 + 4{,}72 + j1{,}51 = 9{,}72 + j1{,}51\Omega$$

$$\bar{I}_E = \frac{\bar{E}}{\bar{Z}_{eq3}} = \frac{10e^{j\frac{\pi}{2}}}{9{,}72 + j1{,}51} = \underbrace{0{,}16 + j} = \boxed{\sqrt{0{,}16^2 + 1^2}}\, e^{j\boxed{\operatorname{atg}\frac{1}{0{,}16}}}\ A$$

$$i_e = \boxed{\sqrt{2} \cdot 1{,}027} \cos(10t + \boxed{1{,}41})\ A$$

Se volessi le altre correnti potrei procedere con un partitore di corrente

Esempio 2

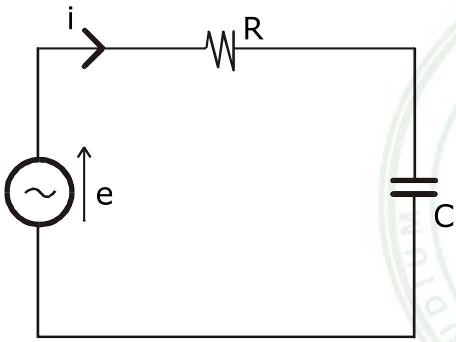


$$I_1 = \sqrt{25}\ sen(5t + \frac{\pi}{3})$$

cos

$$I_2 = \sqrt{23}\cos(5t + \frac{\pi}{6})$$

Esempio 3



$$e = \sqrt{2}10\ \text{sen}(10t)\ \text{V}$$

$$R = 10\Omega$$

$$C = 1mF$$

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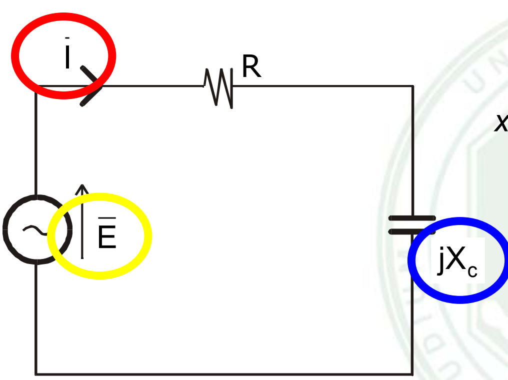
Trasformazione nel dominio dei fasori



$$X_C = \frac{-1}{\omega C} = \frac{-1}{10 \cdot 10^{-3}} = -100\Omega$$

$$e = \sqrt{2}10\ \text{sen}(10t) = \sqrt{2}10\ \cos(10t - \frac{\pi}{2})$$

$$\bar{E} = 10e^{-j\frac{\pi}{2}}$$

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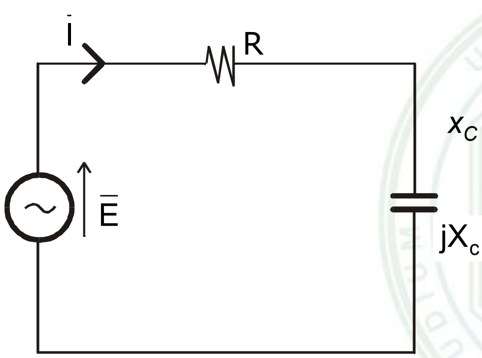
Soluzione nel dominio dei fasori

$$\bar{I} = \frac{\bar{E}}{\bar{Z}_{eq}} = \frac{10e^{-j\frac{\pi}{2}}}{10 - j\,100} = 0{,}099 - j\,0{,}0099 = 0{,}1e^{-j\,0{,}0996} \quad A$$

RI-trasformazione nel dominio del tempo

$$i = \sqrt{2}\,0{,}1\cos(10t - 0{,}099) \quad A$$

Esempio 3 bis – Trasformazione fasori



$$x_C = \frac{-1}{\omega C} = \frac{-1}{10 \cdot 10^{-3}} = -100\Omega$$

$$e = \sqrt{2}\,10\ \mathrm{sen}(10t) \qquad \bar{E} = 10V$$

Soluzione nel dominio dei fasori

$$\bar{I} = \frac{\bar{E}}{\bar{Z}} = \frac{10}{10 - j\,100} = 0{,}0099 + j\,0{,}099 = 0{,}1 e^{j\,1{,}471}$$

RI-trasformazione nel dominio del tempo

$$i = \sqrt{2} \cdot 0{,}1\,\mathrm{sen}(10t + 1{,}471)\ \ A$$

$$= \sqrt{2}\,0{,}1\cos(10t + 1{,}471 - \frac{\pi}{2}) =$$

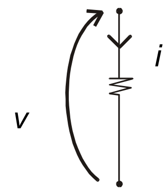
$$= \sqrt{2}\,0{,}1\cos(10t - 0{,}099)\ \ A$$

Potenza

$$P = V \cdot I$$

$$p = v \cdot i$$

Potenza - Resistore



$$v = V_M \cos(\omega t + \delta)$$

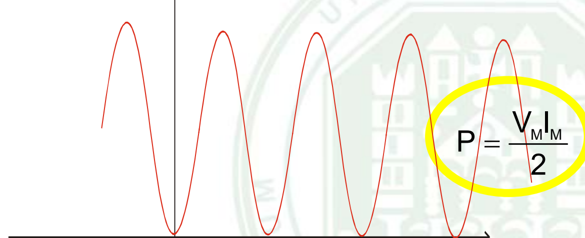
$$i = \frac{V}{R}$$

$$i = \frac{V_M}{R}\cos(\omega t + \delta) = I_M \cos(\omega t + \delta)$$

$$p = v \cdot i = V_M \cdot I_M \cdot \cos^2(\omega t + \delta) =$$

$$= V_M \cdot I_M \frac{1 + \cos\left(2(\omega t + \delta)\right)}{2} = \frac{V_M \cdot I_M}{2} + \frac{V_M \cdot I_M}{2}\cos 2(\omega t + \delta)$$

Potenza - Resistore



Definiamo

Potenza attiva = valor medio potenza istantanea

Simbolo P - Unità di misura watt (W)

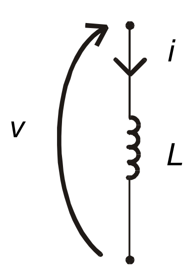
Valore efficace

$$P = \frac{V_M I_M}{2} = \frac{V_M}{\sqrt{2}} \frac{I_M}{\sqrt{2}} = V\,I$$

PAS: $$V_{eff} = \frac{V_M}{\sqrt{2}}$$

f ∀ : $$Veff = \sqrt{\frac{1}{T}\int f^2 dt}$$ RMS

Potenza - Induttore



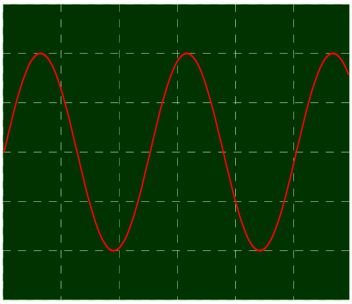
$$v = V_M \cos(\omega t + \delta) \qquad \overline{V} = \frac{V_M}{\sqrt{2}} e^{j\delta}$$

$$\bar{I} = \frac{\overline{V}}{j\omega L} = \frac{\overline{V}}{\omega L} e^{-j\frac{\pi}{2}} = \frac{V_M}{\omega L\sqrt{2}} e^{j(\delta - \frac{\pi}{2})}$$

$$i = \frac{V_M}{\omega L}\cos(\omega t + \delta - \frac{\pi}{2}) = I_M \,\mathrm{sen}(\omega t + \delta)$$

Potenza - Induttore

$$p = V_M I_M \cos(\omega t + \delta)\,\mathrm{sen}(\omega t + \delta) =$$
$$= \frac{V_M I_M}{2}\,\mathrm{sen}\,2(\omega t + \delta)$$



Definiamo

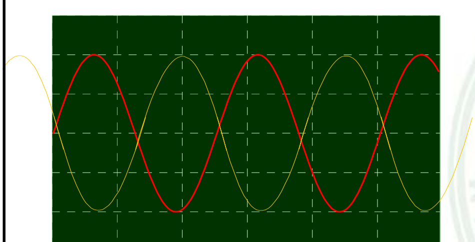
Potenza reattiva = Valore massimo della potenza PAS

Simbolo **Q**

Unità di misura voltamperereattivo (var)

$$Q_L = \frac{V_M I_M}{2} = VI$$

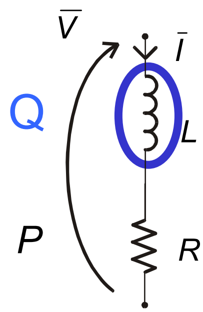
Potenza reattiva

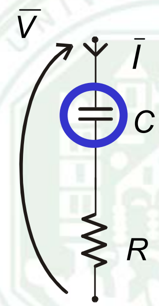


< 0 $\qquad Q_C = -VI$

> 0 $\qquad Q_L = \frac{V_M I_M}{2} = VI$

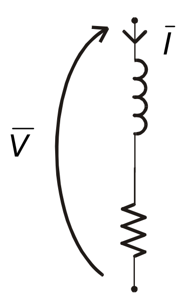
Potenza Apparente complessa

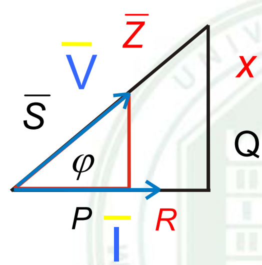




$$P \pm jQ = \bar{S} = \bar{V}\,\bar{I}^*$$

Potenza apparente





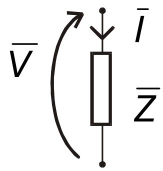
$$|\bar{S}| = S \text{ } \textit{Potenza Apparente VA}$$

$$\bar{S} = \bar{V}\,\bar{I}^* = VI\cos\varphi + jVI\mathrm{sen}\varphi$$

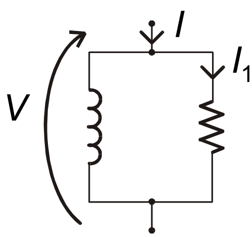
$$\cos\varphi = \text{Fattore di potenza}$$

Potenza apparente

$$\bar{S} = P + jQ = \bar{V}\,\bar{I}^*$$

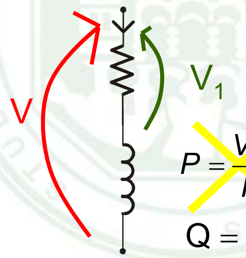


$$\bar{V} = \bar{Z}\,\bar{I} \qquad \bar{S} = \frac{V^2}{\bar{Z}^*} = \bar{Z}\,I^2$$



$$P = \frac{V^2}{R} = \cancel{RI^2}$$

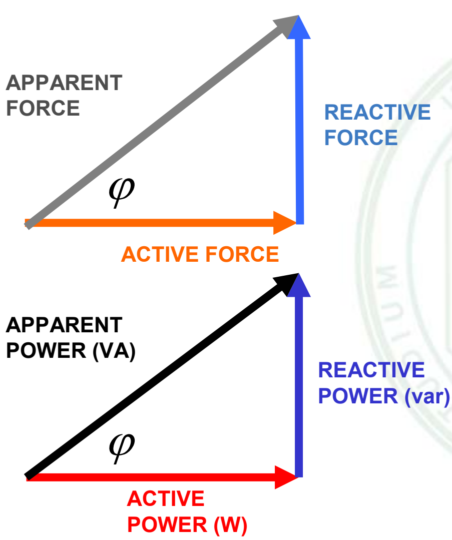
$$Q = ...$$

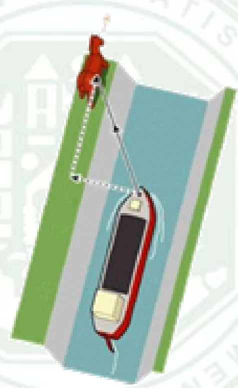


$$P = \cancel{\frac{V_1^2}{R}} = RI^2$$

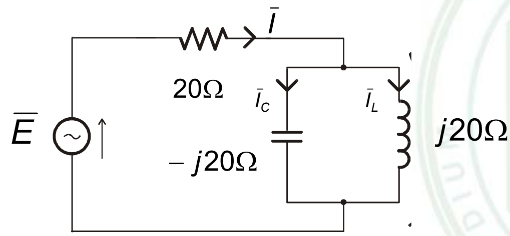
$$Q = ...$$

Potenza apparente





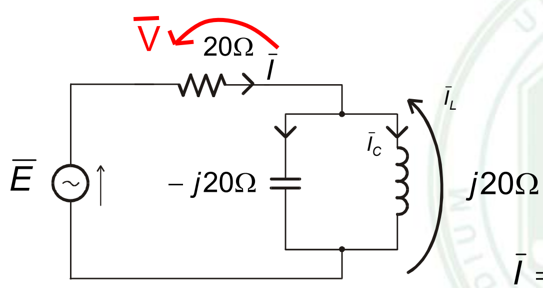
Esercizio



$$\bar{E} = 100\,V$$

$$\bar{I};\bar{I}_C;\bar{I}_L = ?$$

Esercizio

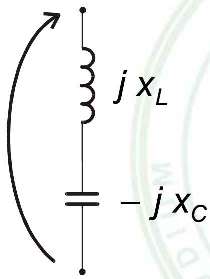


$$\bar{Z}_{//} = \frac{-j20\,j20}{j20-j20} = \infty$$

$$\bar{I} = 0 \qquad \bar{V} = 0 \qquad \bar{V}_{//} = \bar{E}$$

$$\bar{I}_C = \frac{100}{-j20} = 5jA \qquad \bar{I}_L = \frac{100}{j20} = -5jA$$

Risonanza serie



$$\rightarrow \overline{Z} = 0$$

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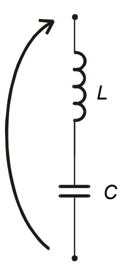
Facoltà di Ingegneria

Risonanza serie



$$\overline{Z}(\omega) \angle_{0}^{\infty}$$

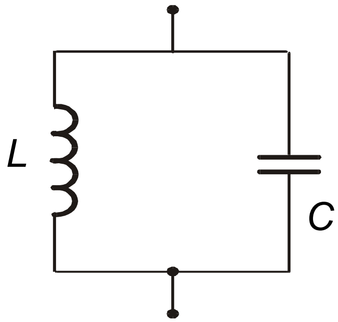
$$\overline{Z}(\omega) = +j\omega L - \frac{j}{\omega C} = \frac{j(\omega^2 LC - 1)}{\omega C} =$$

$$N = 0 \qquad \omega^2 LC - 1 = 0 \qquad \omega = \sqrt{\frac{1}{LC}}$$

$$D = 0 \qquad \omega = 0$$

Corso di Elettrotecnica NO - Capitolo 4 – **Rappresentazione e Analisi delle reti elettriche in regime variabile – regime PAS**

Risonanza parallelo



$$\overline{Z}(\omega) = ?$$

$$= \frac{j\omega L\left(\frac{-j}{\omega C}\right)}{j\omega L + \frac{-j}{\omega C}} = \frac{\frac{L}{C}}{\frac{j(\omega^2 LC - 1)}{\omega C}} = \frac{L}{C}\frac{\omega C}{j(\omega^2 LC - 1)}$$

$\longrightarrow$

$N = 0 \qquad \omega = 0$

$D = 0 \qquad \omega = \sqrt{\frac{1}{LC}}$

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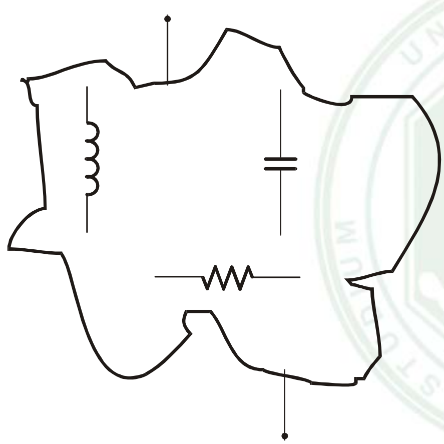
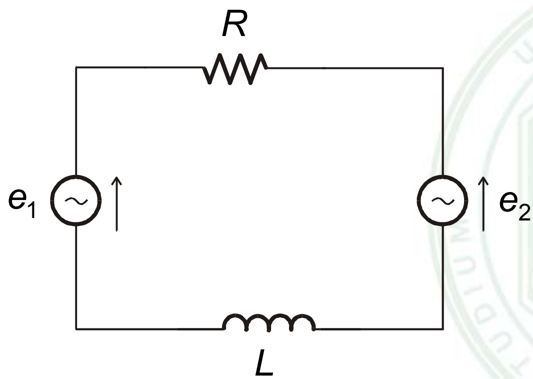
Risonanza in una rete



$$\overline{Z}(\omega) = ?$$

Corso di Elettrotecnica NO - Capitolo 4 – **Rappresentazione e Analisi delle reti elettriche in regime variabile – regime PAS**

Esercizio



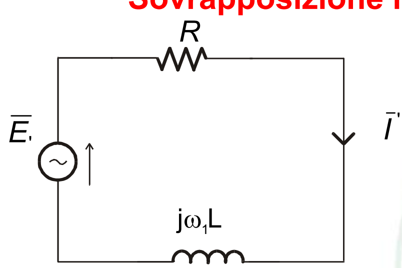
$$e_1 = 10\sqrt{2}\ sen\ 10\ t$$

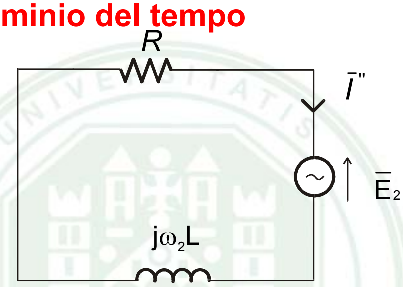
$$e_2 = 10\sqrt{2}\ sen\ 100\ t$$

$$R = 5\Omega$$

$$L = 100mH$$

Sovrapposizione nel dominio del tempo





$R = 5\Omega \quad \bar{E}_{1'} = 10V \quad x_L = \omega_1 L = 1\Omega$

$$\bar{I}' = \frac{\bar{E}_{\cdot}}{5+j} = \frac{10}{5+j} = 1{,}92 - 0{,}38j\,A = 1{,}96e^{-j0{,}197}A$$

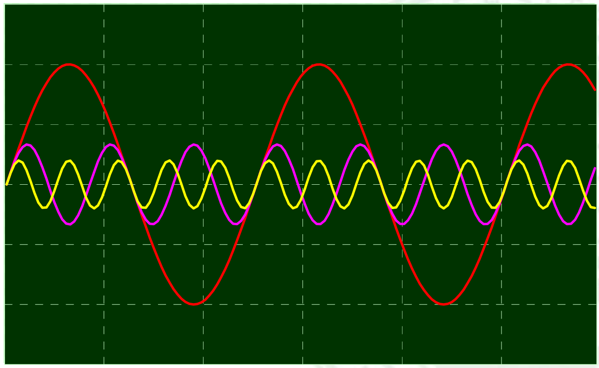
$X_L = \omega_2 L = 100\,10^{-1} = 10\Omega \quad \bar{E}_2 = 10V$

$$\bar{I}'' = -\frac{\bar{E}_2}{5+j10} = 0{,}89e^{+j2}$$

$$\cancel{\bar{I} = \bar{I}' + \bar{I}''}\ con\ 2\omega \neq !$$

$$i = i' + i'' = \sqrt{2}\cdot 1{,}96\,sen(10t - 0{,}197) + \sqrt{2}\,0{,}89\,sen(100t + 2)$$

Armoniche



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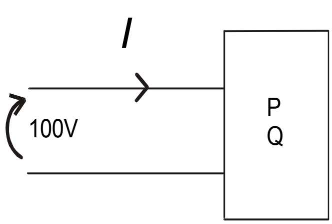
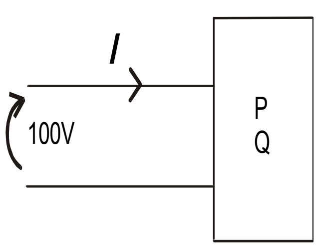
Rifasamento



1000 W
1000 var

$$I = \left| \frac{1000+j1000}{100} \right| = 14{,}14A$$

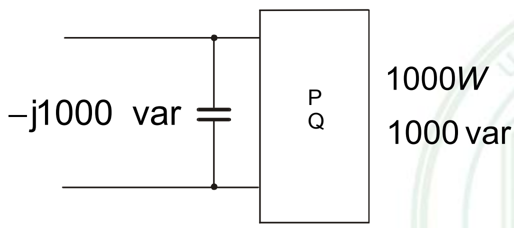


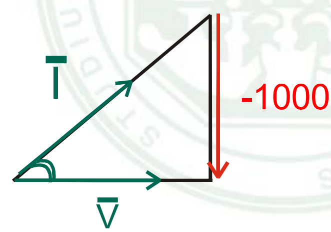
1000 W
0 var

$$\bar{I} = \frac{1000}{100} = 10A$$

RI^2
- Effetti sulle perdite
- Cavi di sezione maggiore

Rifasamento





Esercizio

$P = 1500 \ \text{kW}$

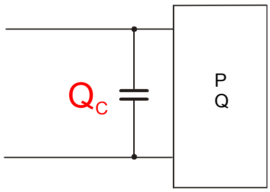
$Q = 1800 \ \text{k var} \quad \text{induttivo}$

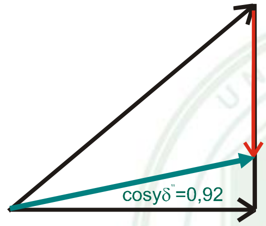
$\cos\varphi = 0{,}92$

$V = 400V$

$Q_C = ?$

Esercizio





Q_C 1800-640=1160 kvar

$$atg\frac{1800}{1500} \cong 50$$

$$\cos 50 = 0{,}64 = \cos\delta$$

$$\text{tg acos } 0{,}92 = \frac{Q_{dr}}{P} = \frac{Q_{dr}}{1500}$$

$$\cos\delta'' = \cos\text{ atg}\frac{Q_{dr}}{P}$$

Q_{dr}=1500tg acos0,92=
≅640kvar