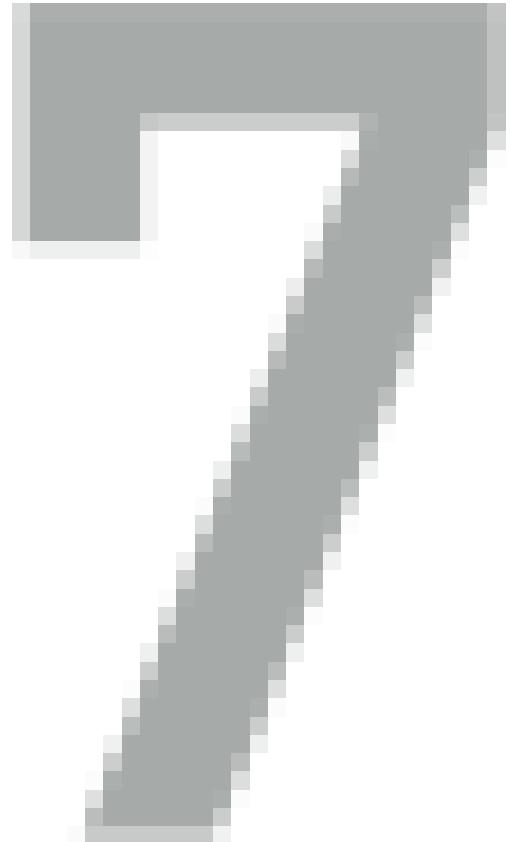
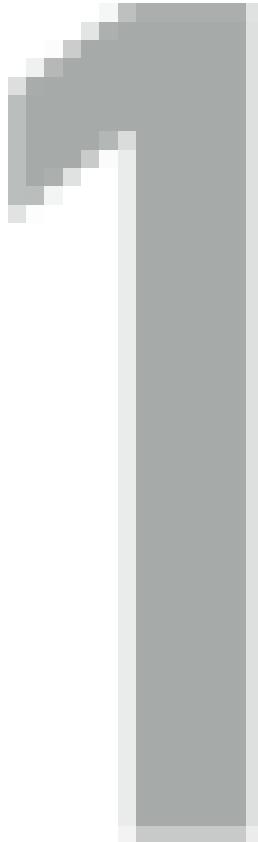


INTERLOCUTORIAL



► Unbiased:

$$\mathbb{E}[\hat{\pi}[f]] = \pi[f]$$



Consistent: convergence is a.s.

$$\lim_{N \rightarrow \infty} \hat{\pi}[f] = \pi[f]$$



Variance:

$$\mathbb{V}[\hat{\pi}[f]] = \frac{\mathbb{V}_{\pi}[f]}{N}, \quad \mathbb{V}_{\pi}[f] = \pi[f^2] - \pi[f]^2$$

- Given iid samples from $X_1, \dots, X_N \sim \pi$, define the Monte Carlo (MC) estimator:

$$\hat{\pi}[f] = \frac{1}{N} \sum_{n=1}^N f(X_n)$$

► Central limit theorem: convergence is weak

$$\sqrt{N}(\hat{\pi}[f] - \pi[f]) \implies N(0, \text{V}_{\pi}[f])$$

MONTE CARLO ESTIMATOR

- Given iid samples from $X_1, \dots, X_N \sim \pi$, define the Monte Carlo (MC) estimator:

$$\hat{\pi}[f] = \frac{1}{N} \sum_{n=1}^N f(X_n)$$

- Unbiased:** $\mathbb{E}[\hat{\pi}[f]] = \pi[f]$
- Consistent:** convergence is a.s.

$$\lim_{N \rightarrow \infty} \hat{\pi}[f] = \pi[f]$$

- Variance:** $\mathbb{V}[\hat{\pi}[f]] = \frac{\mathbb{V}_\pi[f]}{N}, \quad \mathbb{V}_\pi[f] = \pi[f^2] - \pi[f]^2$
- Central limit theorem:** convergence is weak

$$\sqrt{N}(\hat{\pi}[f] - \pi[f]) \implies N(0, \mathbb{V}_\pi[f])$$

MARKOV CHAIN MONTE CARLO