





- For large T the CLT implies the variance of the MCMC estimator

$$\mathbb{V}[\hat{\pi}_T[f]] \approx \frac{\sigma^2[f]}{T} = \frac{\mathbb{V}_\pi[f]\tau_{\text{corr}}[f]}{T} = \frac{\mathbb{V}_\pi[f]}{T_{\text{ess}}[f]}$$

- Where we define the **effective sample size (ESS)** defined as

$$T_{\text{ESS}}[f] = \frac{T}{\tau_{\text{corr}}[f]} = \frac{T}{1 + 2 \sum_{t=1}^{\infty} \rho_t[f]}$$

► The ESS estimates the number of iid samples from π a Monte Carlo estimator would require to achieve a comparable variance to the MCMC

Thermal insulation of auto-cooled tanks



Only valid after being confirmed in the chain has the **block** been **mined**.

generally not found to be computable and

→ States of a Markov chain doesn't mean the states of a process

EFFECTIVE SAMPLE SIZE

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- ▶ The ESS estimates the number of iid samples from π a Monte Carlo estimator would require to achieve a comparable variance to the MCMC
- ▶ The goal is to design MCMC kernels to reduce the auto-correlations
- ▶ ESS is a useful tool: but use with caution!!
 - ▶ Only valid after burn-in until chain has converged is meaningless
 - ▶ Generally not fun to compute and can be unstable
 - ▶ ESS of a Markov chain doesn't mean anything, it depends on the statistics of interest

EXAMPLE RANDOM WALK ON A CIRCLE