REVERSIBILITY

12

- **Proposition:** If K is π-reversible, then it is π-invariant

- In descrete setting equivalent to

$$\pi(x)K(x, x') = \pi(x')K(x', x)$$

- We will say a K is π-reversible if the **detailed balance condition** hold:

$$\pi(\mathrm{d}x)K(x, \mathrm{d}x') = \pi(\mathrm{d}x')K(x', \mathrm{d}x)$$

- **Proof:** For any bounded $f : \mathbb{X} \to \mathbb{R}$

$$\int_{\mathbb{X}} f(x')\pi K(\mathrm{dx}') = \int_{\mathbb{X}^2} f(x')\pi(\mathrm{d}x)K(x, \mathrm{d}x')$$

$$= \int_{\mathbb{X}^2} f(x')\pi(\mathrm{d}x')K(x', \mathrm{d}x)$$

$$= \int_{\mathbb{X}^2} f(x')\pi(\mathrm{d}x')$$

- **Intuition:** the mass flowing in is equivalent to the mass flowing out

REVERSIBILITY

- We will say a K is π-reversible if the **detailed balance condition** hold:

$$\pi(\mathrm{d}x)K(x, \mathrm{d}x') = \pi(\mathrm{d}x')K(x', \mathrm{d}x)$$

  - In descrete setting equivalent to

$$\pi(x)K(x, x') = \pi(x')K(x', x)$$

  - **Intuition:** the mass flowing in is equivalent to the mass flowing out

- **Proposition:** If K is π-reversible, then it is π-invariant

- **Proof:** For any bounded $f : \mathbb{X} \to \mathbb{R}$

$$\begin{aligned}
\int_{\mathbb{X}} f(x')\pi K(\mathrm{d}x') &= \int_{\mathbb{X}^2} f(x')\pi(\mathrm{d}x)K(x, \mathrm{d}x') \\
&= \int_{\mathbb{X}^2} f(x')\pi(\mathrm{d}x')K(x', \mathrm{d}x) \\
&= \int_{\mathbb{X}^2} f(x')\pi(\mathrm{d}x')
\end{aligned}$$

TRANSPOSE OF KERNEL