MARKOV KERNELS V4: MARKOV CHAINS

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- A Markov Chain $X_0, X_1, \ldots \in \mathbb{X}$ is a stochastic process in $\mathbb{X}$ such that

$$\mathbb{P}[X_t \in A \,|\, X_0, \ldots, X_{t-1}] = \mathbb{P}[X_t \in A \,|\, X_{t-1}]$$

- Given a Markov kernel K can construct a chain X_t by iterating over $X_0 \sim \mu$

$$X_t \sim K(X_{t-1}, \mathrm{d}x_t)$$

- The marginal law μ_t of X_t after t steps satisfies

$$\mu_t = \mathrm{Law}(X_t) = \mu_{t-1}K = \mu K^t$$

- The joint law of $X_{0:t} = (X_0, \ldots, X_t)$

$$\mu \otimes K \cdots \otimes K(\mathrm{d}x_{0:t}) = \mu(\mathrm{d}x_0) \prod_{s=1}^{t} K(x_{t-1}, \mathrm{d}x_t)$$

- Given a Markov chain defines a Markov kernel as the one step transition

$$K(x, A) = \mathbb{P}[X_1 \in A \,|\, X_0 = x] = \mathbb{P}_x[X_1 \in A]$$

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