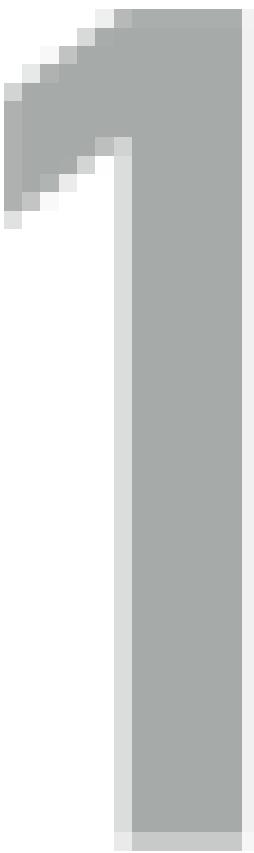
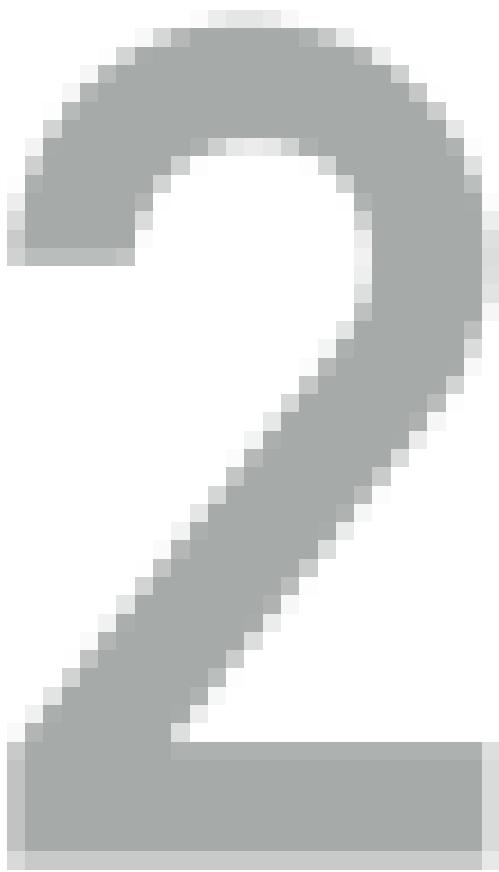


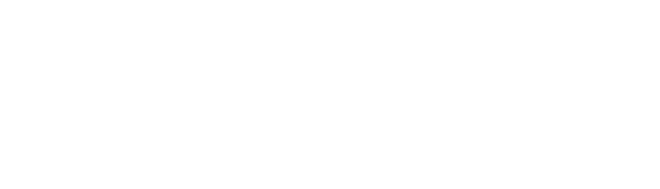


CONFIDENTIAL



- **Theorem (CLT):** Under certain regularity assumptions, a Harris recurrent,  $\pi$ -invariant Markov chain satisfying enough regularity conditions (e.g. reversible geometrically ergodic), as  $T \rightarrow \infty$

$$\sqrt{T}(\hat{\pi}_T[f] - \pi[f]) \implies N(0, \sigma^2(f))$$



The asymptotic variance decomposes as:

$$\sigma^2[f] = \lim_{T \rightarrow \infty} T \mathbb{V}[\hat{\pi}_T[f]] = \mathbb{V}_{\pi}[f] \tau_{\text{corr}}[f]$$

- $\rho_t[f]$  is the  $t$ -th lag defined as the autocorrelation coefficient at stationarity

$$\rho_t[f] = \frac{\text{Cov}[f(X_0), f(X_t)]}{\text{Var}_\pi[f]}, \quad (X_0, X_t) \sim \pi \otimes K^t$$

- ▶  $\tau_{\text{corr}}$  is the **integrated autocorrelation time**:

$$\tau_{\text{corr}}[f] = 1 + 2 \sum_{t=1}^{\infty} \rho_t[f]$$

measures how long it takes to forget a stationary sample  $x_0 \sim \pi$

# CENTRAL LIMIT THEOREM

- ▶ **Theorem (CLT):** Under certain regularity assumptions, a Harris recurrent,  $\pi$ -invariant Markov chain satisfying enough regularity conditions (e.g. reversible geometrically ergodic), as  $T \rightarrow \infty$

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$$\rho_t[f] = \frac{\text{Cov}[f(X_0), f(X_t)]}{\mathbb{V}_\pi[f]}, \quad (X_0, X_t) \sim \pi \otimes K^t$$

- ▶  $\tau_{\text{corr}}$  measures how long it takes to forget a stationary sample  $X_0 \sim \pi$

# EFFECTIVE SAMPLE SIZE