CONVERGENCE OF MARKOV CHAINS

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- A Markov chain is μ-irreducible if $\forall x \in \mathbb{X}$, and $\mu(A) > 0$, there exists $t \geq 0$ such that $K^t(x, A) > 0$

- μ-irreducible Markov chain is **aperiodic** if for all $\mu(A) > 0$,

$$\gcd(t : \mathbb{P}[X_t \in A \,|\, X_0 \in A] > 0) = 1$$

- **Theorem**: If K is a π-irreducible, π-invariant and aperiodic, then for any μ

$$\lim_{t \to \infty} \|\mu K^t - \pi\|_{\mathrm{TV}} = 0$$

- Implies X_t is an approximate sample from π when t is large enough

- Irreducible chains always bring your closer to the target, but not always quickly

- E.g. multi-modal distribution…

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ERGODIC THEOREM