INTEGRALS ARE NORMALISING CONSTANTS

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- What if I don't care about *every* statistic? I just want to compute $\pi[f]$ for some f

- Define a the f-tilted target π_f distribution:

$$\pi_f(\mathrm{d}x) = \frac{\gamma_f(x)}{Z_f}\mathrm{d}x \qquad\qquad \gamma_f(x) = f(x)\gamma(x)$$

- The normalising constant of the tilted target satisfies:

$$Z_f = \int_{\mathbb{X}} \gamma_f(x)\mathrm{d}x = \int_{\mathbb{X}} f(x)w(x)\eta(x)\mathrm{d}x = \eta[fw],$$

$$\pi[f] = \int_{\mathbb{X}} f(x)\pi(x)\mathrm{d}x$$

$$= \frac{\int_{\mathbb{X}} f(x)\gamma(x)\mathrm{d}x}{Z}$$

$$= \frac{Z_f}{Z}$$

$$= \frac{\eta[fw]}{\eta[w]}$$

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MONTE CARLO AND COMPUTATION