



FUNCTIONS



Given a probability distribution $\mu \in \mathcal{P}(X)$, and function $f: X \rightarrow \mathbb{R}$ we define:

► Let $L^2(\mu)$ denote the set of functions with finite norm or equivalent finite variance

► **Expectation:**

$$\mu[f] = \int_{\mathbb{X}} f(x) \mu(\mathrm{d}x) = \mathbb{E}_{\mu}[f] = \mathbb{E}_{X \sim \mu}[f(X)]$$

► **Variance:**

$$V_{\mu}[f] = \mu[f^2] - \mu[f]^2$$

► **Inner-product:**

$$\langle f, f' \rangle_{\mu} = \int_{\mathbb{X}} f(x) f'(x) \mu(\mathrm{d}x),$$

► **Norm:**

$$\|f\|_{\mu}^2 = \langle f, f \rangle_{\mu} = \mathbb{V}_{\mu}[f] + \mathbb{E}_{\mu}[f]^2$$

- E.g. when $\mathbb{X} = \{x_1, \dots, x_n\}$ is discrete we can represent f as a n -dimensional column vectors with i -th entry $f(x_i)$

$$\mu[f] = \sum_x \mu(x)f(x)$$

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MARKOV KERNELS V1: PROBABILITY