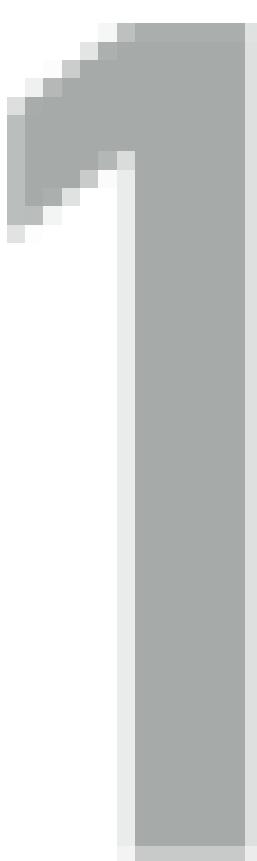


from DIVERGENCES



► Let $f: \mathbb{R}_+ \rightarrow \mathbb{R}$ be an convex function such that $f(1) = 0$, define the f -divergence:

$$D_f(\mu' || \mu) = \mu \left[f \circ \frac{d\mu'}{d\mu} \right] = \int_{\mathbb{X}} f\left(\frac{d\mu'}{d\mu}(x)\right) \mu(x) dx$$

► The likelihood ratio or Radon-Nikodym derivative $\mu(dx) = \mu(x)dx$ and $\mu'(dx) = \mu'(x)dx$

$$\frac{d\mu'}{d\mu} : X \rightarrow \mathbb{R},$$

$$\frac{d\mu'}{d\mu}(x) = \frac{\mu'(x)}{\mu(x)}$$

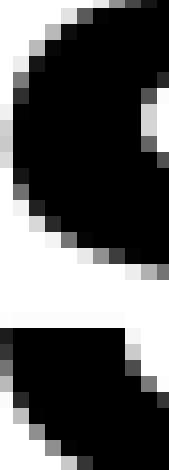
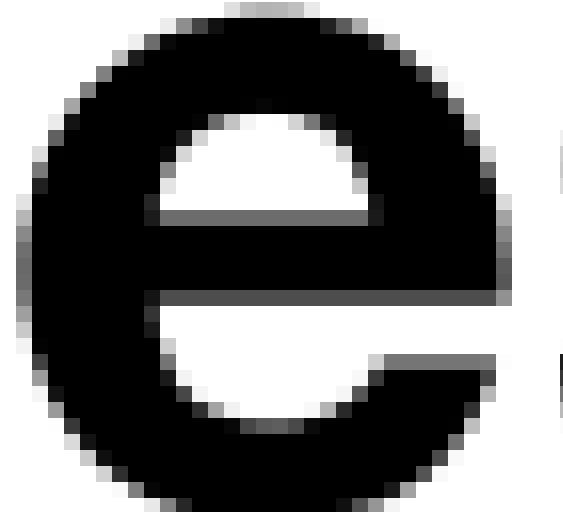
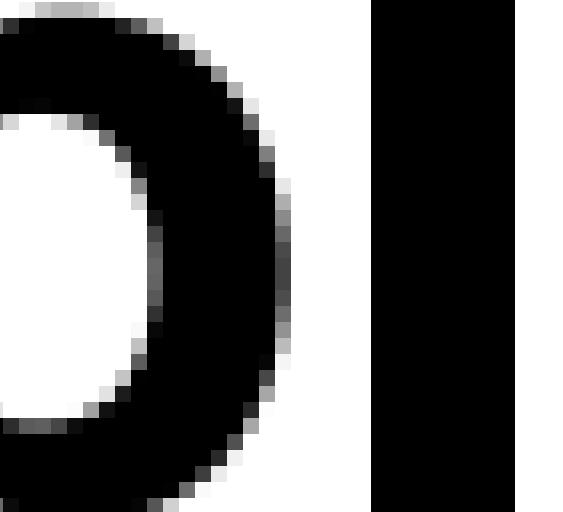
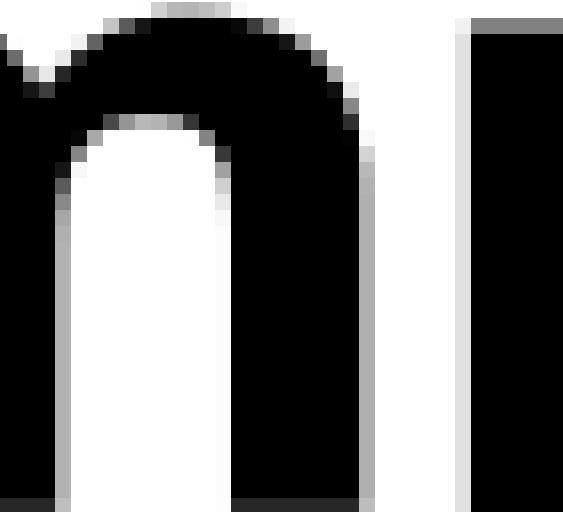
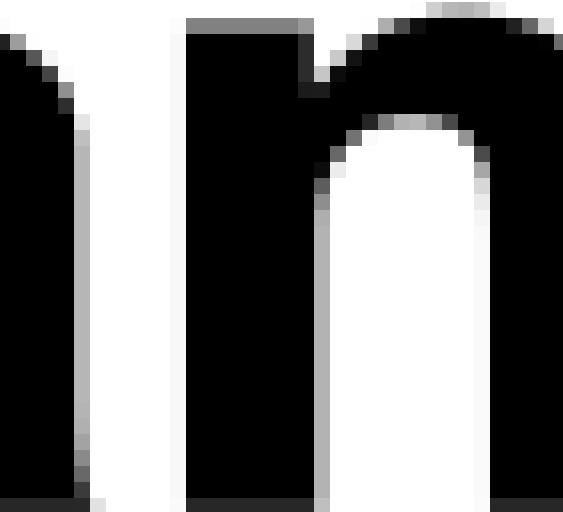
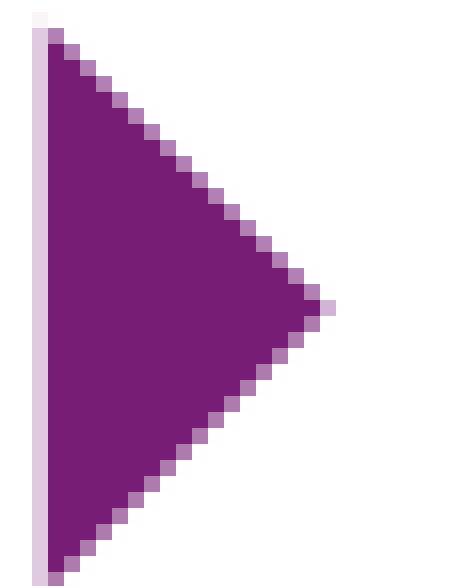
$$f(r) = \frac{1}{2} |1 - r|$$

$$\text{TV}(\mu, \mu') = \frac{1}{2} \int_{\mathbb{X}} |\mu(x) - \mu'(x)| dx$$

$$f(r) = (1 - r)^2$$
$$\chi^2(\mu' || \mu) = V_\mu \left[\frac{d\mu'}{d\mu} \right] = \int_X \left(\frac{\mu'(x)}{\mu(x)} - 1 \right)^2 \mu(x) dx$$

$$f(r) = r \log r - \int_{\mathbb{X}} \mu'(x) \log \frac{\mu'(x)}{\mu(x)} dx$$

We have $D_f(\mu' \parallel \mu) \geq 0$ and with equality if and only if $\mu = \mu'$.



f -DIVERGENCES

- ▶ The likelihood ratio or Radon-Nikodym derivative $\mu(dx) = \mu(x)dx$ and $\mu'(dx) = \mu'(x)dx$

$$\frac{d\mu'}{d\mu} : \mathbb{X} \rightarrow \mathbb{R}, \quad \frac{d\mu'}{d\mu}(x) = \frac{\mu'(x)}{\mu(x)}$$

- ▶ Let $f: \mathbb{R}_+ \rightarrow \mathbb{R}$ be a convex function such that $f(1) = 0$, define the f -divergence:

$$D_f(\mu' \parallel \mu) = \mu \left[f \circ \frac{d\mu'}{d\mu} \right] = \int_{\mathbb{X}} f\left(\frac{d\mu'}{d\mu}(x)\right) \mu(x) dx$$

- ▶ We have $D_f(\mu' \parallel \mu) \geq 0$ and with equality if and only if $\mu = \mu'$

▶ Examples:

$$f(r) = \frac{1}{2} |1 - r|$$

$$\text{TV}(\mu, \mu') = \frac{1}{2} \int_{\mathbb{X}} |\mu(x) - \mu'(x)| dx = 1 - \int_{\mathbb{X}} \mu(x) \wedge \mu'(x) dx$$

$$f(r) = (1 - r)^2$$

$$\chi^2(\mu' \parallel \mu) = \mathbb{V}_{\mu} \left[\frac{d\mu'}{d\mu} \right] = \int_{\mathbb{X}} \left(\frac{\mu'(x)}{\mu(x)} - 1 \right)^2 \mu(x) dx$$

$$f(r) = r \log r$$

$$\text{KL}(\mu' \parallel \mu) = \int_{\mathbb{X}} \mu'(x) \log \frac{\mu'(x)}{\mu(x)} dx$$

CONVERGENCE