





EXAMPLE RANDOM MAN CIRCLE

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► Consider the random walk on a circle  $Z_n$  where  $n$  is even.

► Suppose  $X_0 = 0$  and  $\mathbb{P}[X_t = m \mid X_{t-1} = n] = K(n, m)$  where

$$K(n, m) = \frac{1}{4}\delta_{n-1}(m) + \frac{1}{2}\delta_n(m) + \frac{1}{4}\delta_{n+1}(m)$$

►  $X_t$  is stationary with respect to  $\pi \equiv \text{Uniform}[Z_n]$

► This is a lazy random walk on a circle, mixes after  $O(n^2)$  iterations

►  $f(n) \equiv 1$  if  $n$  is even and  $f(n) \equiv 0$  if  $n$  is odd

► For all  $t > 0$  we have  $f(X_t)$  are iid and ESS is  $T_{\text{ESS}}[f] = T$

$$\mathbb{P}[f(X_t) = 1] = \frac{1}{2} = \pi[f]$$

# EXAMPLE RANDOM WALK ON A CIRCLE

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- ▶ Consider the random walk on a circle  $\mathbb{Z}_n$  where  $n$  is even.
- ▶ Suppose  $X_0 = 0$  and  $\mathbb{P}[X_t = m \mid X_{t-1} = n] = K(n, m)$  where

$$K(n, m) = \frac{1}{4}\delta_{n-1}(m) + \frac{1}{2}\delta_n(m) + \frac{1}{4}\delta_{n+1}(m)$$

- ▶  $X_t$  is stationary with respect to  $\pi = \text{Uniform}[\mathbb{Z}_n]$
- ▶ This is a lazy random walk on a circle, mixes after  $O(n^2)$  iterations
- ▶  $f(n) = 1$  if  $n$  is even and  $f(n) = 0$  if  $n$  is odd
- ▶ For all  $t > 0$  we have  $f(X_t)$  are iid and ESS is  $T_{\text{ESS}}[f] = T$

$$\mathbb{P}[f(X_t) = 1] = \frac{1}{2} = \pi[f]$$

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