TRANSPOSE OF KERNEL

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$$\langle Kf, g \rangle_\pi = \int_{\mathbb{X}} Kf(x) g(x) \pi(\mathrm{d}x)$$

- **Proof:** Using the definition we compute both sides:

$$\langle f, K^\top g \rangle_\pi = \int_{\mathbb{X}} f(x') K^\top g(x') \pi(\mathrm{d}x')$$

- Given an π-invariant kernel, K in $L^2(\pi)$ we define the adjoint (or transpose) as $K^\top$ if all $f, g \in L^2(\pi)$ we have

$$\langle Kf, g\rangle_\pi = \langle f, K^\top g\rangle_\pi$$

- We see they are equivalent if and only if detailed balance holds

- **Proposition:** The adjoint kernel satisfies

$$\pi(\mathrm{d}x)K(x, \mathrm{d}x') = \pi(\mathrm{d}x')K^{\top}(x', \mathrm{d}x)$$

- K is π-reversible if and only if K is self-adjoint $K = K^{\mathsf{T}}$

$$= \int_{\mathbb{X}} \int_{\mathbb{X}} K(x, \mathrm{d}x') f(x') g(x) \pi(\mathrm{d}x)$$

$$= \int_{\mathbb{X}} \int_{\mathbb{X}} f(x')K^{\top}(x', \mathrm{d}x)g(x)\pi(\mathrm{d}x')$$

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$$\langle f, K^\top g \rangle_\pi = \int_{\mathbb{X}} f(x')K^\top g(x')\pi(\mathrm{d}x') = \int_{\mathbb{X}} \int_{\mathbb{X}} f(x')K^\top(x', \mathrm{d}x)g(x)\pi(\mathrm{d}x')$$

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ALGEBRA OF KERNELS