



# PROBABILITY DISTRIBUTIONS



► Suppose  $(X, \mathcal{F})$  is a measurable space

►  $\mathcal{P}(X)$  be the space of probability distributions over  $X$

► Given a probability distribution  $\mu \in \mathcal{P}(X)$ , we will assume there is a density over  $\mathrm{d}x$

$$\mu(\mathrm{d}x) = \mu(x)\mathrm{d}x$$

- Define the product measure  $\mu_1 \otimes \mu_2 \in \mathcal{P}(X \times X)$

$$\mu \otimes \mu'(\mathrm{d}x, \mathrm{d}x') = \mu(\mathrm{d}x)\mu'(\mathrm{d}x') = \mu(x)\mu'(x')\mathrm{d}x\mathrm{d}x'$$

- E.g. when  $X = \{x_1, \dots, x_n\}$  is discrete we can represent  $\mu$  as a  $n$ -dimensional row vectors with  $i$ -th entry  $\mu(x_i)$



Given  $\mu, \mu' \in \mathcal{P}(X)$  with  $\mu(dx) = \mu(x)dx$  and  $\mu'(dx) = \mu'(x)dx$ :

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- ▶ Given  $\mu, \mu' \in \mathcal{P}(\mathbb{X})$  with  $\mu(\mathrm{d}x) = \mu(x)\mathrm{d}x$  and  $\mu'(\mathrm{d}x) = \mu'(x)\mathrm{d}x$ :
  - ▶ Define the product measure  $\mu_1 \otimes \mu_2 \in \mathcal{P}(\mathbb{X} \times \mathbb{X})$

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# FUNCTIONS