

MARKOV KERNELS V3: OPERATORS



Given $\mu \in \mathcal{P}(X)$ we can view a kernel K as operators in $L^2(\mu)$

► **Left multiplication:** given $\mu \in \mathcal{P}(\mathbb{X})$ define $\mu K \in \mathcal{P}(\mathbb{X})$

$$\mu K(\mathrm{d}x') = \int_{\mathbb{X}} \mu(\mathrm{d}x) K(x, \mathrm{d}x')$$

► **Right multiplication:** given $f : \mathbb{X} \rightarrow \mathbb{R}$ define $Kf : \mathbb{X} \rightarrow \mathbb{R}$

$$Kf(x) = \int_{\mathbb{X}} K(x, dx') f(x')$$

- When discrete equivalent to left multiplication by row vector

$$\mu K(x') = \sum_x \mu(x) K(x, x')$$

- When discrete equivalent to right multiplication by column vector

$$Kf(x) = \sum_{x'} K(x, x')f(x')$$

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KERNELS AS BUILDING BLOCKS