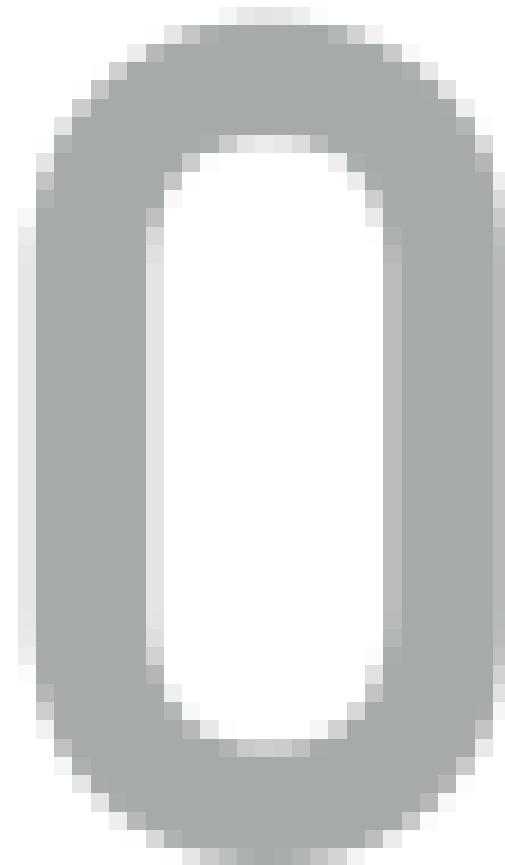
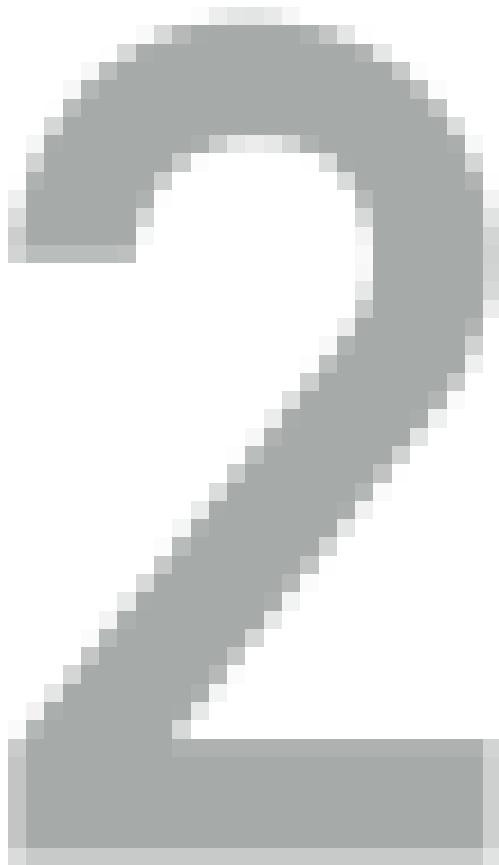




AND NORMA HANSON SINGING



► Problem: McCan't measure the normalising constant

► Solution: Introduce a reference distribution  $\pi$  (e.g. Gaussian) that dominates target

- ▶ By the Radon-Nikodym theorem

$$Z = \int_{\mathbb{X}} \gamma(x) dx = \int_{\mathbb{X}} w(x) \eta(x) dx = \eta[w], \quad w = \frac{\gamma(x)}{\eta(x)}$$

- ▶ Can estimate  $Z$  using the Monte Carlo estimator

$$X_1, \dots, X_N \sim \eta \quad \hat{Z} = \frac{1}{N} \sum_{n=1}^N w(X_n)$$

▶ **density** **and** **size** **are** **connected** **and** **in** **turn** **are** **connected** **to** **the** **number** **of** **interactions** **in** **the** **network**

# NORMALISING CONSTANT

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- ▶ **Problem:** MCMC can't measure the normalising constant  $Z$ 
  - ▶ Un-normalised density can only compare between two states
- ▶ **Solution:** Introduce a reference distribution  $\eta$  (e.g gaussian) that dominates target
  - ▶ By the Radon-Nikodym theorem

$$Z = \int_{\mathbb{X}} \gamma(x) dx = \int_{\mathbb{X}} w(x)\eta(x) dx = \eta[w], \quad w = \frac{\gamma(x)}{\eta(x)}$$

- ▶ Can estimate  $Z$  using the Monte Carlo estimator

$$X_1, \dots, X_N \sim \eta \quad \hat{Z} = \frac{1}{N} \sum_{n=1}^N w(X_n)$$

# NORMALISING CONSTANT

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- ▶ **Unbiased:**

$$\mathbb{E}[\hat{Z}] = \frac{1}{N} \sum_{n=1}^N \mathbb{E}[w(X_n)] = Z$$

- ▶ **Consistent:** convergence is a.s.

$$\lim_{N \rightarrow \infty} \hat{Z} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N w(X_n) = Z$$

- ▶ **Variance:**

$$\mathbb{V}\left[\frac{\hat{Z}}{Z}\right] = \frac{1}{N} \mathbb{V}_{X \sim \eta} \left[ \frac{\pi(X)}{\eta(X)} \right] = \frac{e^{D_2(\pi\|\eta)} - 1}{N}$$

- ▶ The variance grows exponentially as  $\eta$  deviates from the target

$$D_2(\pi\|\eta) = \log \left( 1 + \mathbb{V}_{X \sim \eta} \left[ \frac{\pi(X)}{\eta(X)} \right] \right) \geq \text{KL}(\eta\|\pi)$$