



# SAMPLES VS EXPECTATIONS

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- Can estimate the average feature using expectations.

$$\pi[f] = \mathbb{E}_{X \sim \pi}[f(X)]$$

Expectations, probabilities fully characterize distributions:

► If  $\mu$  is some distribution such that  $\mu[f] = \pi[f]$  for all bounded  $f$  then  $\mu = \pi$

► If  $\mu$  is some distribution such that  $\mu[A] = \pi[A]$  for all  $A \subseteq X$  then  $\mu = \pi$

► We will say that we can efficiently sample from  $X$  if we can provide we can efficiently compute  $\pi[f]$  for any  $f$



► Given a sample  $X \sim \pi$ , we can code features  $f(X)$  of  $X$  using a statistic  $f: \mathcal{X} \rightarrow \mathbb{R}$

▶ This is basically a numerical computing integral

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- ▶ Given a sample  $X \sim \pi$ , we can code features  $f(X)$  of  $X$  using a statistic  $f: \mathbb{X} \rightarrow \mathbb{R}$
- ▶ Can estimate the average feature using expectations.

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- ▶ Expectations, probabilities fully characterise distributions:
  - ▶ If  $\mu$  is some distribution such that  $\mu[A] = \pi[A]$  for all  $A \subset \mathbb{X}$  then  $\mu = \pi$
  - ▶ If  $\mu$  is some distribution such that  $\mu[f] = \pi[f]$  for all bounded  $f$  then  $\mu = \pi$
- ▶ We will say that we can efficiently sample from  $X$  if we can provide we can efficiently compute  $\pi[f]$  for any  $f$
- ▶ This is basically a course on numerically computing integrals

# MONTE CARLO METHODS: