







- For  $\epsilon > 0$ , we define the **mixing time** for a  $\pi$ -invariant kernel  $K$  equals

$$\tau_{\text{mix}}(\epsilon) = \inf \left\{ t : \sup_{\mu \in \mathcal{P}(\mathbb{X})} \text{TV}(\mu K^t, \pi) < \epsilon \right\}$$

- The mixing time for a uniformly ergodic chain satisfies:

$$\tau_{\text{mix}}(\epsilon) < \frac{\log \frac{\epsilon}{M}}{\log \rho}$$

►  $\tau_{\text{mix}}$  measures how long it takes to achieve stationarity and forget  $\mu$

► A Markov chain is **geometrically ergodic** if there exists a  $M(x) > 0$  and  $\rho \in [0, 1]$

$$\forall x \in \mathcal{X}, \quad \text{TV}(K^t(x, \cdot), \pi) \leq M(x)\rho^t$$

A Markov chain is uniformly ergodic if  $M(x) \leq M$  for some  $M < \infty$

# MIXING-TIME

20

- ▶ For  $\epsilon > 0$ , we define the **mixing time** for a  $\pi$ -invariant kernel  $K$  equals

$$\tau_{\text{mix}}(\epsilon) = \inf \left\{ t : \sup_{\mu \in \mathcal{P}(\mathbb{X})} \text{TV}(\mu K^t, \pi) < \epsilon \right\}$$

- ▶  $\tau_{\text{mix}}$  measures how long it takes to achieve stationarity and forget  $\mu$
- ▶ A Markov chain is **geometrically ergodic** if there exists a  $M(x) > 0$  and  $\rho \in [0,1]$

$$\forall x \in \mathbb{X}, \quad \text{TV}(K^t(x, \cdot), \pi) \leq M(x)\rho^t$$

- ▶ A Markov chain is **uniformly ergodic** if  $M(x) \leq M$  for some  $M < \infty$
- ▶ The mixing time for a uniformly ergodic chain satisfies:

$$\tau_{\text{mix}}(\epsilon) < \frac{\log \frac{\epsilon}{M}}{\log \rho}$$

# CENTRAL LIMIT THEOREM