

ALGEBRAIC
COMBINATORICS



▶ Can use ratio analysis and a lineal approach to analyze kernelels and algorithms from a theoretical perspective

 Example if and else if statements

The order of parallel steps is not fixed (can be interleaved).
The parallel steps are independent of each other.

▶ Example and purpose if *is reversible* and *is irreversible*

What are eigenvectors, eigenvalues, orthogonality, etc? See exercises.



The operators commute:

$$K_1 K_2 = K_2 K_1$$

 Operators are self-adjoint

$$K^T = K$$

- Product of reversible kernels is not always reversible:

$$(K_1 K_2)^T = K_2^T K_1^T$$



Mixture of reversible kernels is:

$$(\alpha K_1 + (1 - \alpha)K_2)^\top = \alpha K_1^\top + (1 - \alpha)K_2^\top$$

► Adjoint corresponds to chain running backward in time

ALGEBRA OF KERNELS

- ▶ Can use intuition from linear algebra to analyse kernels and algorithms
- ▶ **Example:** Kernels commute if and only if:

- ▶ The operators commute:

$$K_1 K_2 = K_2 K_1$$

- ▶ The order of algorithms doesn't matter (can parallelise)
- ▶ **Example:** Kernels is reversible if and only if:

- ▶ Operators are self-adjoint

$$K^\top = K$$

- ▶ Product of reversible kernels is not always reversible:

$$(K_1 K_2)^\top = K_2^\top K_1^\top$$

- ▶ Mixutre of reversible kernels is:

$$(\alpha K_1 + (1 - \alpha) K_2)^\top = \alpha K_1^\top + (1 - \alpha) K_2^\top$$

- ▶ Adjoint corresponds to chain running backward in time
- ▶ Exercise: what the interpretations of orthogonality, eigenvalues, eigenvectors, normality, etc

MARKOV KERNELS V4: MARKOV CHAINS