

**Proposition:** If  $K$  is  $\pi$ -reversible, then it is  $\pi$ -invariant.

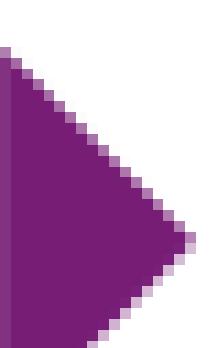


In discrete setting equivalent to

$$\pi(x)K(x, x') = \pi(x')K(x', x)$$

► We will say a  $K$  is  $\pi$ -reversible if the **detailed balance condition** hold:

$$\pi(dx)K(x, dx') = \pi(dx')K(x', dx)$$

 Proof: For any bounded  $\mathbb{R} \rightarrow \mathbb{R}$

$$\int_{x_1} f(x') \pi K(dx') = \int_{x_2} f(x) \pi(dx, dx')$$

$$= \int_{x_2} f(x') \pi(dx') K(x', dx)$$

=

$$= \int_{x_2} f(x') \pi(dx')$$

→ wing mass flow is equivalent to the mass flow in the wing.

# REVERSIBILITY

12

- ▶ We will say a  $K$  is  $\pi$ -reversible if the **detailed balance condition** hold:

$$\pi(dx)K(x, dx') = \pi(dx')K(x', dx)$$

- ▶ In discrete setting equivalent to

$$\pi(x)K(x, x') = \pi(x')K(x', x)$$

- ▶ **Intuition:** the mass flowing in is equivalent to the mass flowing out

- ▶ **Proposition:** If  $K$  is  $\pi$ -reversible, then it is  $\pi$ -invariant

- ▶ **Proof:** For any bounded  $f: \mathbb{X} \rightarrow \mathbb{R}$

$$\begin{aligned} \int_{\mathbb{X}} f(x')\pi K(dx') &= \int_{\mathbb{X}^2} f(x')\pi(dx)K(x, dx') \\ &= \int_{\mathbb{X}^2} f(x')\pi(dx')K(x', dx) \\ &= \int_{\mathbb{X}^2} f(x')\pi(dx') \end{aligned}$$

# TRANSPOSE OF KERNEL