CENTRAL LIMIT THEOREM

21

- **Theorem (CLT):** Under certain regularity assumptions, a Harris recurrent, π-invariant Markov chain satisfying enough regularity conditions (e.g. reversible geometrically ergodic), as $T \to \infty$

$$\sqrt{T}(\hat{\pi}_T[f] - \pi[f]) \quad \Longrightarrow \quad N(0, \sigma^2(f))$$

- The asymptotic variance decomposes as:

$$\sigma^2[f] = \lim_{T\to\infty} T\mathbb{V}[\hat{\pi}_T[f]] = \mathbb{V}_\pi[f]\tau_{\mathrm{corr}}[f]$$

- $\rho_t[f]$ is the t-th lag defined as the autocorrelation coefficient at stationarity

$$\rho_t[f] = \frac{\mathrm{Cov}[f(X_0), f(X_t)]}{\mathbb{V}_\pi[f]}, \qquad (X_0, X_t) \sim \pi \otimes K^t$$

- τ_{corr} is the **integrated autocorrelation time**:

$$\tau_{\mathrm{corr}}[f] = 1 + 2\sum_{t=1}^{\infty} \rho_t[f]$$

- τ_{corr} measures how long it takes to forget a stationary sample $X_0 \sim \pi$

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EFFECTIVE SAMPLE SIZE