NORMALISING CONSTANT

- **Unbiased:**

$$\mathbb{E}[\hat{Z}] = \frac{1}{N}\sum_{n=1}^{N}\mathbb{E}[w(X_n)] = Z$$

- **Consistent:** convergence is a.s.

$$\lim_{N\to\infty}\hat{Z} = \lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N} w(X_n) = Z$$

- **Variance:**

$$\mathbb{V}\left[\frac{\hat{Z}}{Z}\right] = \frac{1}{N}\mathbb{V}_{X\sim\eta}\left[\frac{\pi(X)}{\eta(X)}\right] = \frac{e^{D_2(\pi\|\eta)} - 1}{N}$$

  - The variance grows exponentially as η deviates from the target

$$D_2(\pi\|\eta) = \log\left(1 + \mathbb{V}_{X\sim\eta}\left[\frac{\pi(X)}{\eta(X)}\right]\right) \geq \mathrm{KL}(\eta\|\pi)$$

INTEGRALS ARE NORMALISING CONSTANTS