EFFECTIVE SAMPLE SIZE

22

- For large T the CLT implies the variance of the MCMC estimator

$$\mathbb{V}[\hat{\pi}_T[f]] \approx \frac{\sigma^2[f]}{T} = \frac{\mathbb{V}_\pi[f]\tau_{\text{corr}}[f]}{T} = \frac{\mathbb{V}_\pi[f]}{T_{\text{ess}}[f]}$$

- Where we define the **effective sample size (ESS)** defined as

$$T_{\mathrm{ESS}}[f] = \frac{T}{\tau_{\mathrm{corr}}[f]} = \frac{T}{1 + 2\sum_{t=1}^{\infty} \rho_t[f]}$$

- The ESS estimates the number of iid samples from π a Monte Carlo estimator would require to achieve a comparable variance to the MCMC

- The goal is to design MCMC kernels to reduce the auto-corellations

- ESS is a useful tool: but use with caution!!

- Only valid **after burn-in** until chain has converged is meaningless

- Generally not fun to compute and can be unstable

- ESS of a Markov chain doesn't mean anything, it depends on the statistics of interest

EFFECTIVE SAMPLE SIZE

- For large T the CLT implies the variance of the MCMC estimator

$$\mathbb{V}[\hat{\pi}_T[f]] \approx \frac{\sigma^2[f]}{T} = \frac{\mathbb{V}_\pi[f]\tau_{\text{corr}}[f]}{T} = \frac{\mathbb{V}_\pi[f]}{T_{\text{ess}}[f]}$$

- Where we define the **effective sample size (ESS)** defined as

$$T_{\text{ESS}}[f] = \frac{T}{\tau_{\text{corr}}[f]} = \frac{T}{1 + 2\sum_{t=1}^{\infty}\rho_t[f]}$$

- The ESS estimates the number of iid samples from π a Monte Carlo estimator would require to achieve a comparable variance to the MCMC
- The goal is to design MCMC kernels to reduce the auto-corellations
- ESS is a useful tool: but use with caution!!
  - Only valid **<u>after burn-in</u>** until chain has converged is meaningless
  - Generally not fun to compute and can be unstable
  - ESS of a Markov chain doesn't mean anything, it depends on the statistics of interest

EXAMPLE RANDOM WALK ON A CIRLE