MONTE CARLO ESTIMATOR

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- **Consistent:** convergence is a.s.

$$\lim_{N \to \infty} \hat{\pi}[f] = \pi[f]$$

- **Variance:**

$$\mathbb{V}\left[\hat{\pi}[f]\right] = \frac{\mathbb{V}_\pi[f]}{N}, \quad \mathbb{V}_\pi[f] = \pi[f^2] - \pi[f]^2$$

- Given iid samples from $X_1, \ldots, X_N \sim \pi$, define the Monte Carlo (MC) estimator:

$$\hat{\pi}[f] = \frac{1}{N} \sum_{n=1}^{N} f(X_n)$$

- **Central limit theorem:** convergence is weak

$$\sqrt{N}(\hat{\pi}[f] - \pi[f]) \quad \Longrightarrow \quad N(0, \mathbb{V}_{\pi}[f])$$

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MARKOV CHAIN MONTE CARLO