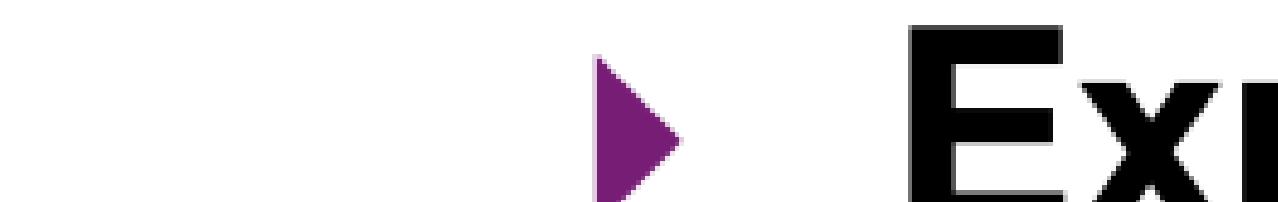






Given a probability distribution $\mu \in \mathcal{P}(\mathbb{X})$, and a function $f: \mathbb{X} \rightarrow \mathbb{R}$ we define

Let $b^2(\mu)$ denote the set of functions with finite variance or equivalent finite variance. Then $b^2(\mu)$ is a Banach space with respect to the norm $\| \cdot \|_2$.



Expectation:

$$\mu[f] = \int_X f(x) \mu(dx) = \mathbb{E}_\mu[f] = \mathbb{E}_{X \sim \mu}[f(X)]$$



Variance:

$$V_\mu[f] = \mu[f^2] - \mu[f]^2$$

► **Inner-product:**

$$\langle f, f' \rangle_{\mu} = \int_{\mathbb{X}} f(x) f'(x) \mu(dx),$$

► Norm:

$$\|f\|_{\mu}^2 = \langle f, f \rangle_{\mu} = \mathbb{V}_{\mu}[f] + \mathbb{E}_{\mu}[f]^2$$

- E.g. when $X = \{x_1, \dots, x_n\}$ is discrete we can represent f as a n -dimensional column vectors with i -th entry $f(x_i)$

$$\mu[f] = \sum_x \mu(x) f(x)$$

FUNCTIONS

- Given a probability distribution $\mu \in \mathcal{P}(\mathbb{X})$, and function $f: \mathbb{X} \rightarrow \mathbb{R}$ we define:

- ▶ **Expectation:**

$$\mu[f] = \int_{\mathbb{X}} f(x)\mu(dx) = \mathbb{E}_{\mu}[f] = \mathbb{E}_{X \sim \mu}[f(X)]$$

- ▶ **Variance:**

$$\mathbb{V}_{\mu}[f] = \mu[f^2] - \mu[f]^2$$

- ▶ **Inner-product:**

$$\langle f, f' \rangle_{\mu} = \int_{\mathbb{X}} f(x)f'(x)\mu(dx),$$

- ▶ **Norm:**

$$\|f\|_{\mu}^2 = \langle f, f \rangle_{\mu} = \mathbb{V}_{\mu}[f] + \mathbb{E}_{\mu}[f]^2$$

- Let $L^2(\mu)$ denote the set of functions with finite norm or equivalent finite variance
- E.g. when $\mathbb{X} = \{x_1, \dots, x_n\}$ is discrete we can represent f as a n -dimensional column vectors with i -th entry $f(x_i)$

$$\mu[f] = \sum_x \mu(x)f(x)$$

MARKOV KERNELS V1: PROBABILITY