







► **Data processing inequality:** for any kernel  $K$  and  $\mu, \mu' \in \mathcal{P}(\mathbb{X})$

$$D_f(\mu' K \| \mu K) \leq D_f(\mu' \| \mu)$$

► Therefore, if  $K$  is  $\pi$ -invariant, then  $\pi = \pi K$  and,

$$D_f(\pi || K\mu) \leq D_f(\pi || \mu)$$

► A  $\mu$ -invariant kernel can not push you away from target!

If  $X$  is  $\pi$ -invariant, is  $K(X, d\alpha)$  closed?

But invariance does not guarantee that it will be robust to changes in the environment. For example, consider a robot that is trained to navigate a simple 2D environment with obstacles. The robot uses a sensor to detect the obstacles and a controller to move the robot. If the robot is trained on a specific set of obstacles, it may not be able to navigate correctly if the obstacles are moved or removed. This is because the robot's internal representation of the environment is based on the training data, and it may not be able to generalize to new situations. This is a classic example of a model that is not robust to changes in the environment.

► Example: if  $K(x, dx) = \delta_x(dx')$  is the identity kernel, then  $\mu K = \mu$  and hence,

$$D_f(\pi || K\mu) = D_f(\pi || \mu)$$

- ▶ If  $X \sim \mu$  and  $K$  is  $\pi$ -invariant, is  $X' \sim K(X, dx')$  closer to  $\pi$  than  $X$ ?
- ▶ **Data processing inequality:** for any kernel  $K$  and  $\mu, \mu' \in \mathcal{P}(\mathbb{X})$

$$D_f(\mu'K\|\mu K) \leq D_f(\mu'\|\mu)$$

- ▶ Therefore, if  $K$  is  $\pi$ -invariant, then  $\pi = \pi K$  and,

$$D_f(\pi\|K\mu) \leq D_f(\pi\|\mu)$$

- ▶ A  $\pi$ -Invariant kernel can not push you away from target!
- ▶ **BUT** invariance alone does not guarantee that it will bring you closer either.
- ▶ Example: if  $K(x, dx') = \delta_x(dx')$  is the identity kernel, then  $\mu K = \mu$  and hence,

$$D_f(\pi\|K\mu) = D_f(\pi\|\mu)$$

# CONVERGENCE OF MARKOV CHAINS