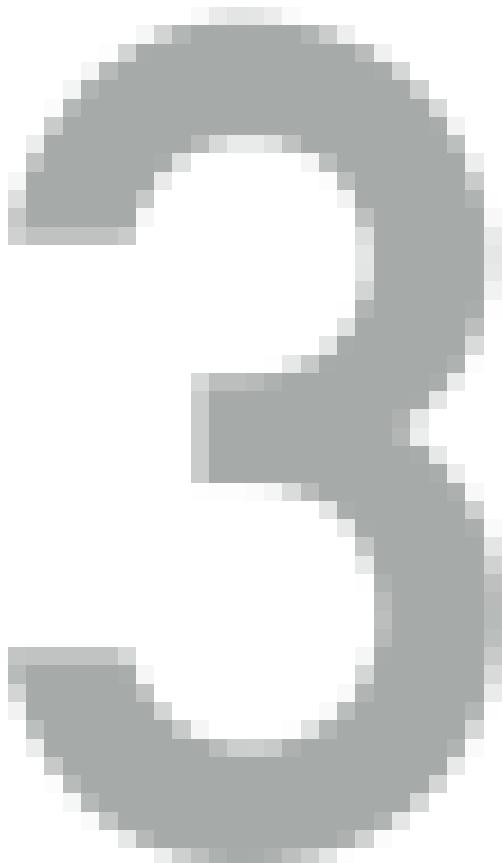
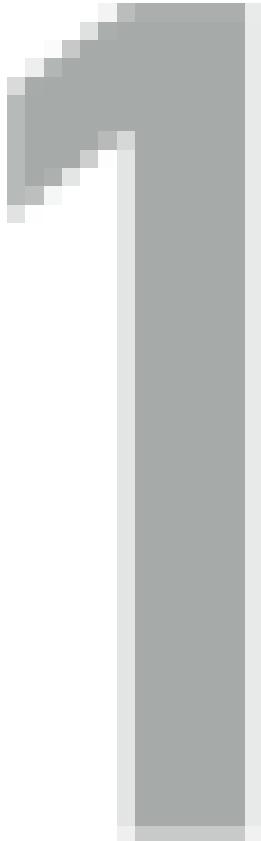
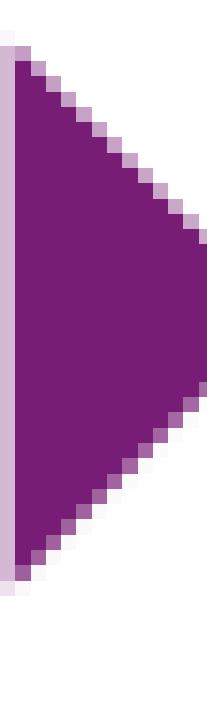


TRANSPORTATION



$$\langle Kf, g \rangle_{\pi} = \int_X Kf(x)g(x)\pi(dx)$$

Prove definition we compute both sides: 

$$\langle f, K^T g \rangle_{\pi} = \int_X f(x) K^T g(x) \pi(dx)$$

- Given an π -invariant kernel, K in $L^2(\pi)$ we define the adjoint (or transpose) as K^\top if all $f, g \in L^2(\pi)$ we have

$$\langle Kf, g \rangle_\pi = \langle f, K^\top g \rangle_\pi$$

→ **see also** **detained** **and** **valence**



Proposition: The adjoint kernel satisfies

$$\pi(dx)K(x, dx') = \pi(dx')K^\top(x', dx)$$

K is π -reversible if and only if $K = K^\top$

$$= \int_{x'} K(x, dx') f(x') g(x) \pi(dx)$$

$$= \int_x f(x) K^T(x', dx) g(x) \pi(dx')$$

TRANSPOSE OF KERNEL

- Given an π -invariant kernel, K in $L^2(\pi)$ we define the adjoint (or transpose) as K^\top if all $f, g \in L^2(\pi)$ we have

$$\langle Kf, g \rangle_\pi = \langle f, K^\top g \rangle_\pi$$

- Proposition:** The adjoint kernel satisfies

$$\pi(dx)K(x, dx') = \pi(dx')K^\top(x', dx)$$

- K is π -reversible if and only if K is self-adjoint $K = K^\top$
- Proof:** Using the definition we compute both sides:

$$\langle Kf, g \rangle_\pi = \int_{\mathbb{X}} Kf(x)g(x)\pi(dx) = \int_{\mathbb{X}} \int_{\mathbb{X}} K(x, dx')f(x')g(x)\pi(dx)$$

$$\langle f, K^\top g \rangle_\pi = \int_{\mathbb{X}} f(x')K^\top g(x')\pi(dx') = \int_{\mathbb{X}} \int_{\mathbb{X}} f(x')K^\top(x', dx)g(x)\pi(dx')$$

- We see they are equivalent if and only if detailed balance holds

ALGEBRA OF KERNELS