

# STAT 547E: LECTURE 1

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## INTRODUCTION

**Saifuddin Syed**

# COURSE OUTLINE

- ▶ **Logistics:** 1.5 credit topics Graduate course.
  - ▶ 12 Lectures over 6 weeks
  - ▶ MW 16:00–17:30 in ESB 4192
- ▶ **Website:**
  - ▶ Content posted on the course website:  
**[www.saifsyed.com/teaching/stat547e.html](http://www.saifsyed.com/teaching/stat547e.html)**
  - ▶ Canvas for announcements, discussion and submit assignments
- ▶ **Evaluation:** 3 Assignments 50% + Project 50%
  - ▶ 3 Assignments (~1 week each) due Week 3, 5, 7
  - ▶ Course project (~5 page report) + presentation due week 8
- ▶ **AI policy:** This is a grad course, and we are all adults.
- ▶ **Auditors:** Please formally audit!
- ▶ We are going to figure this out together, feedback is always welcome!

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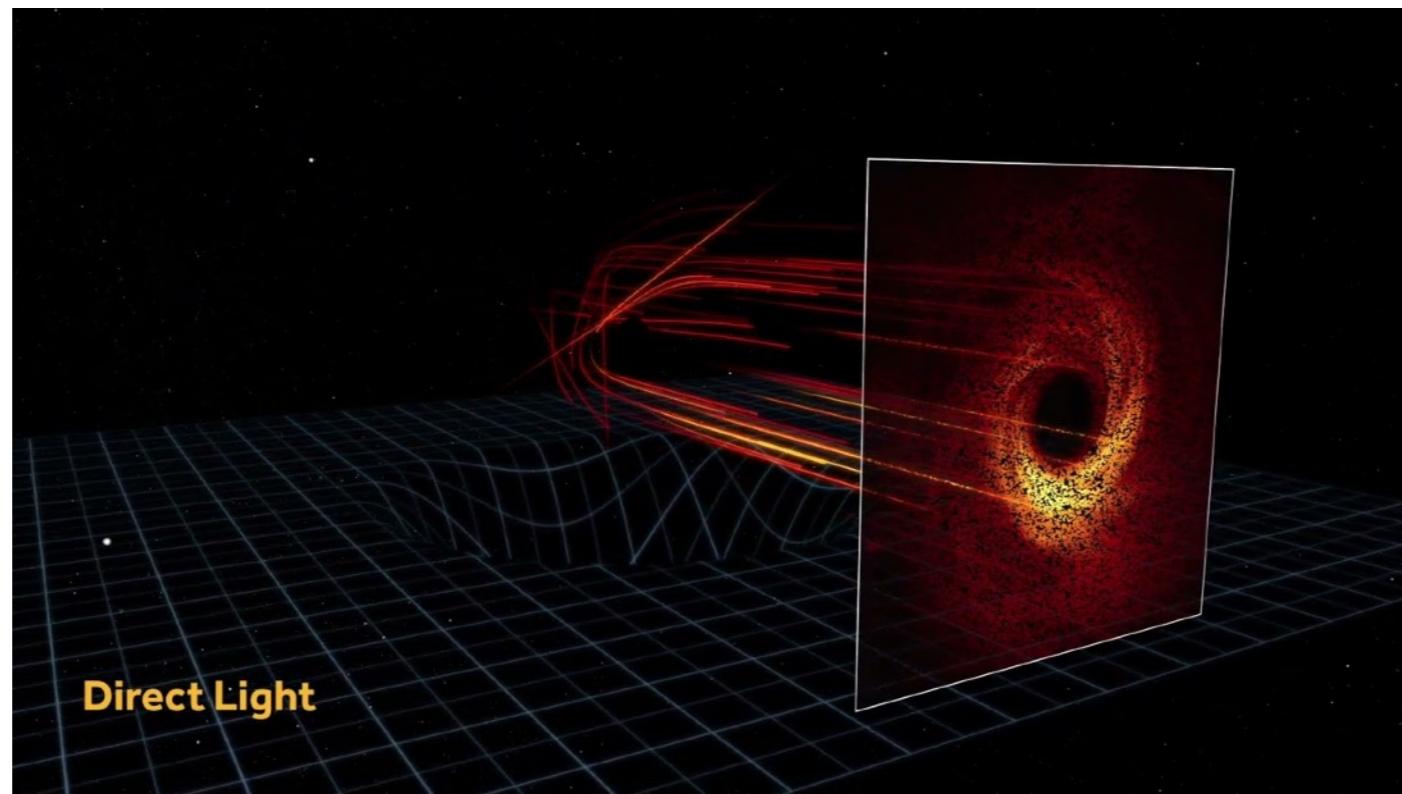
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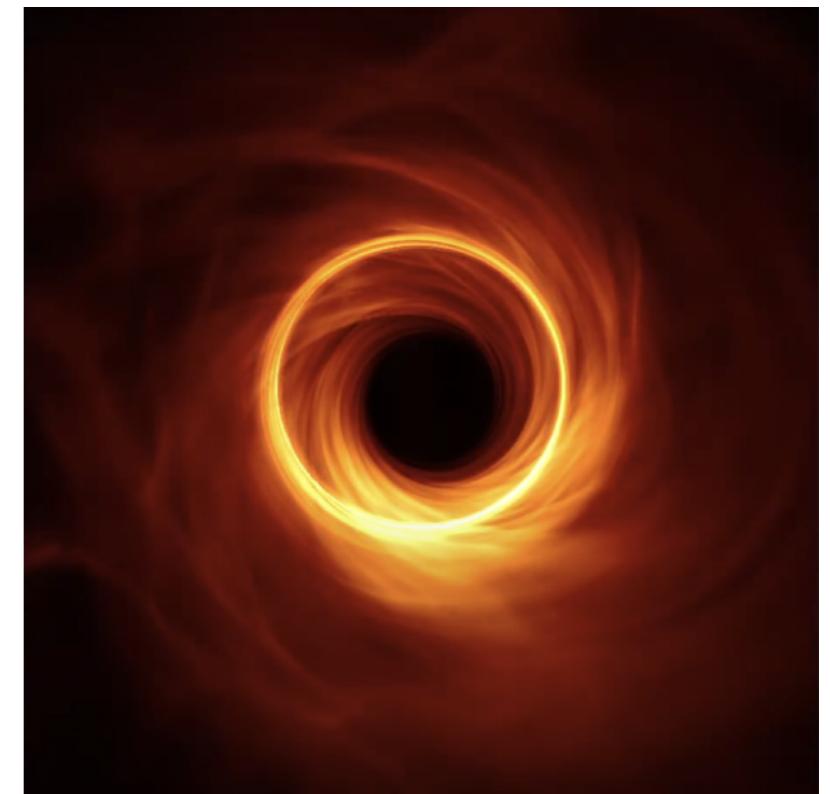
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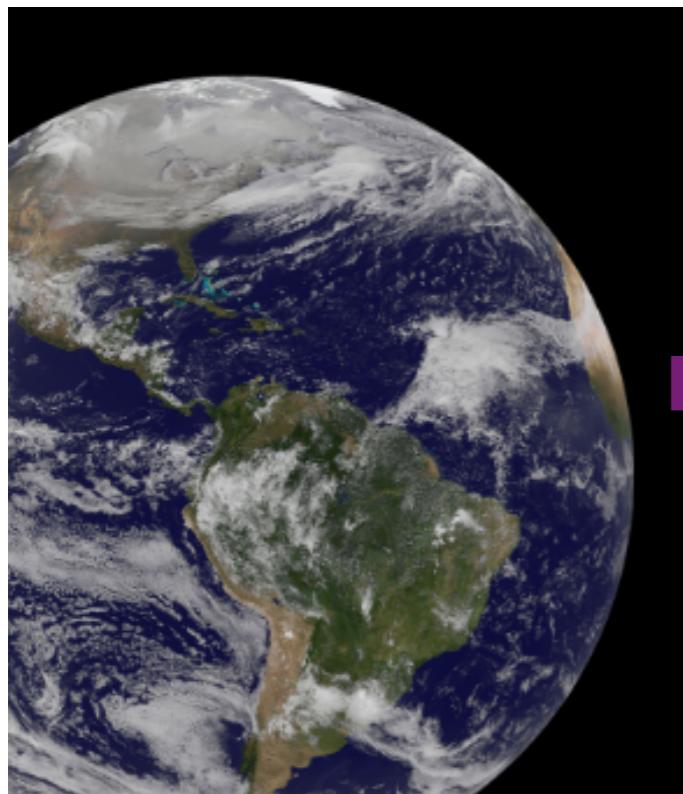
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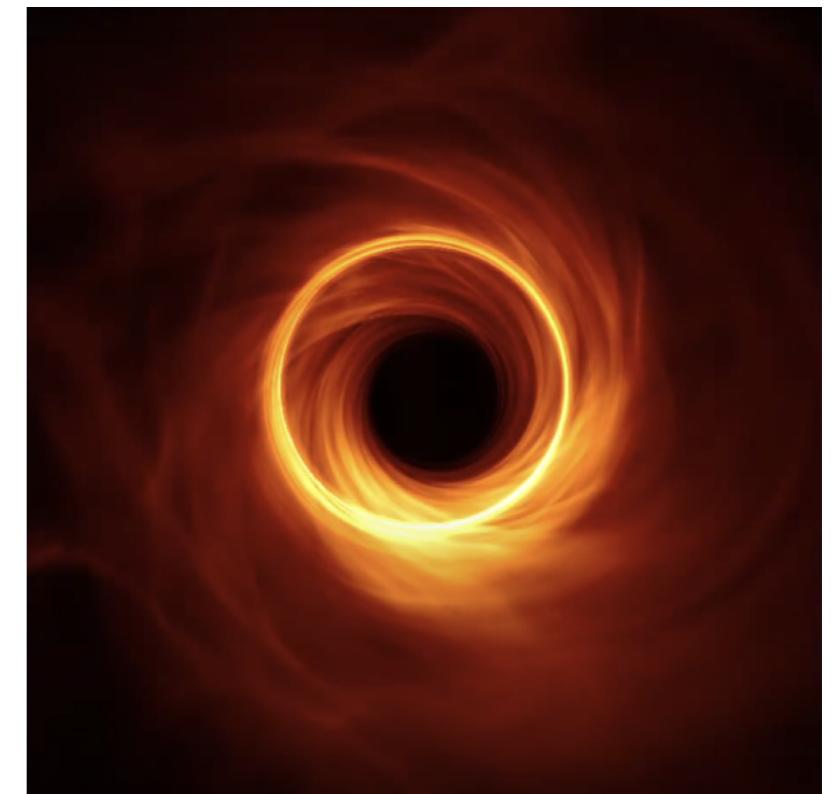
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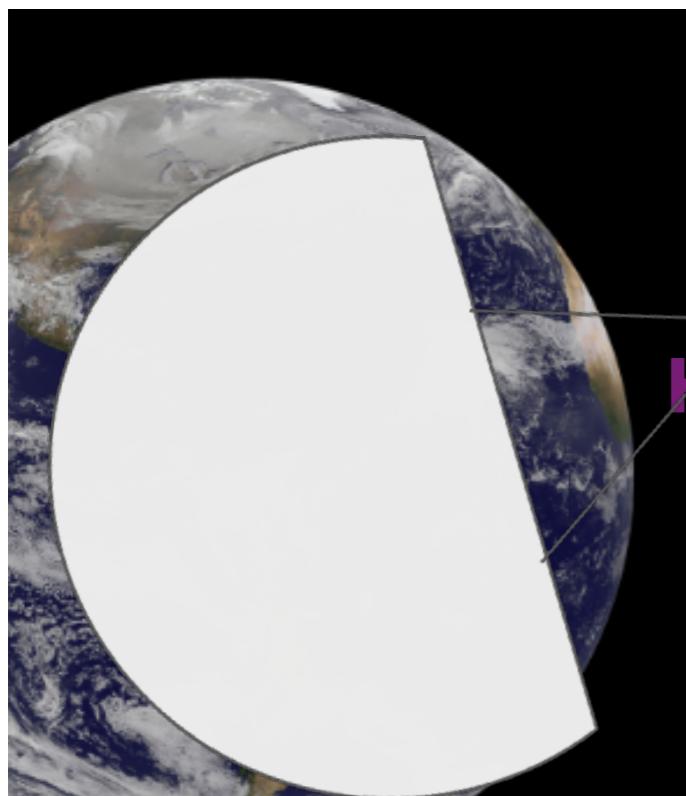
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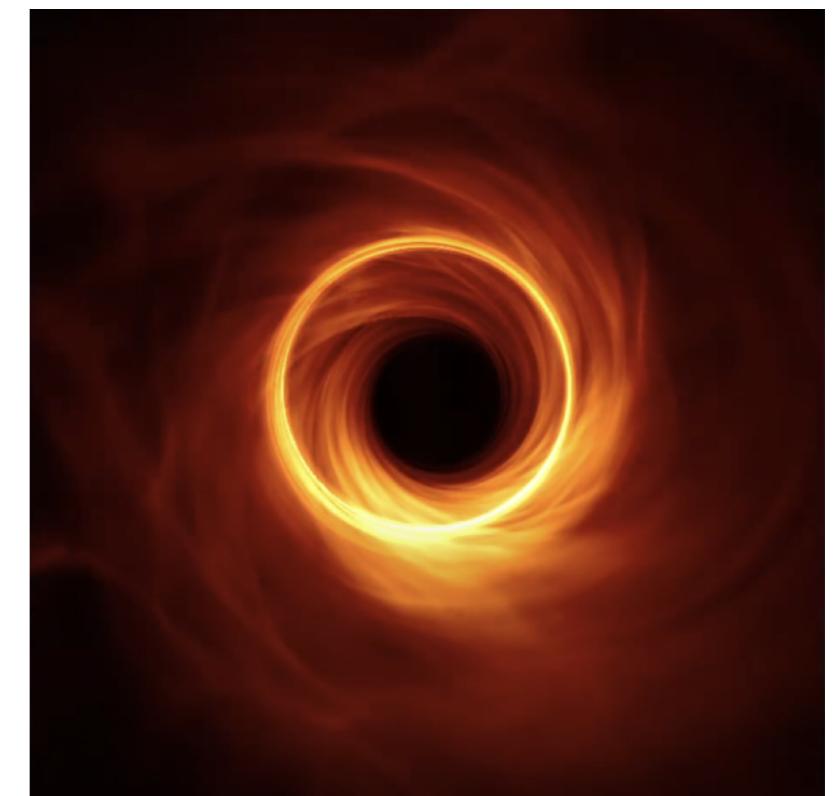
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  - ▶ Solution: turn the earth into a telescope



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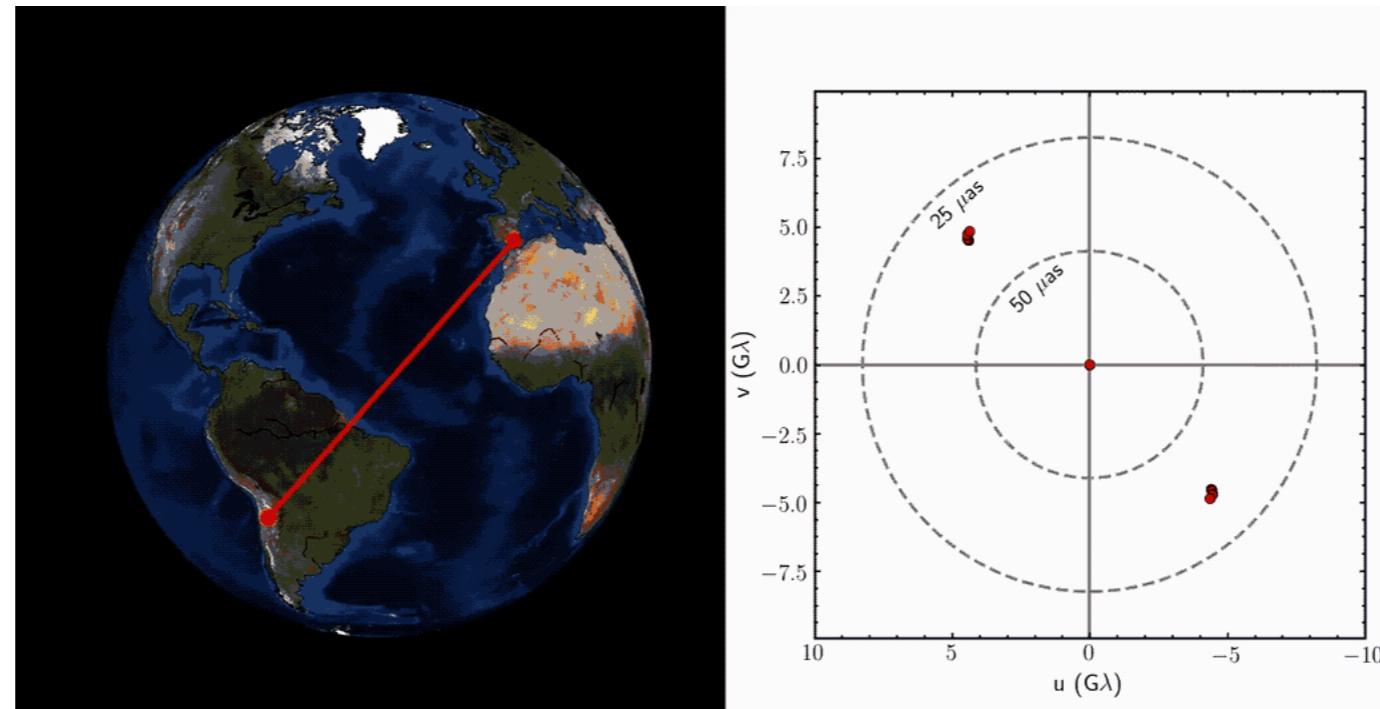
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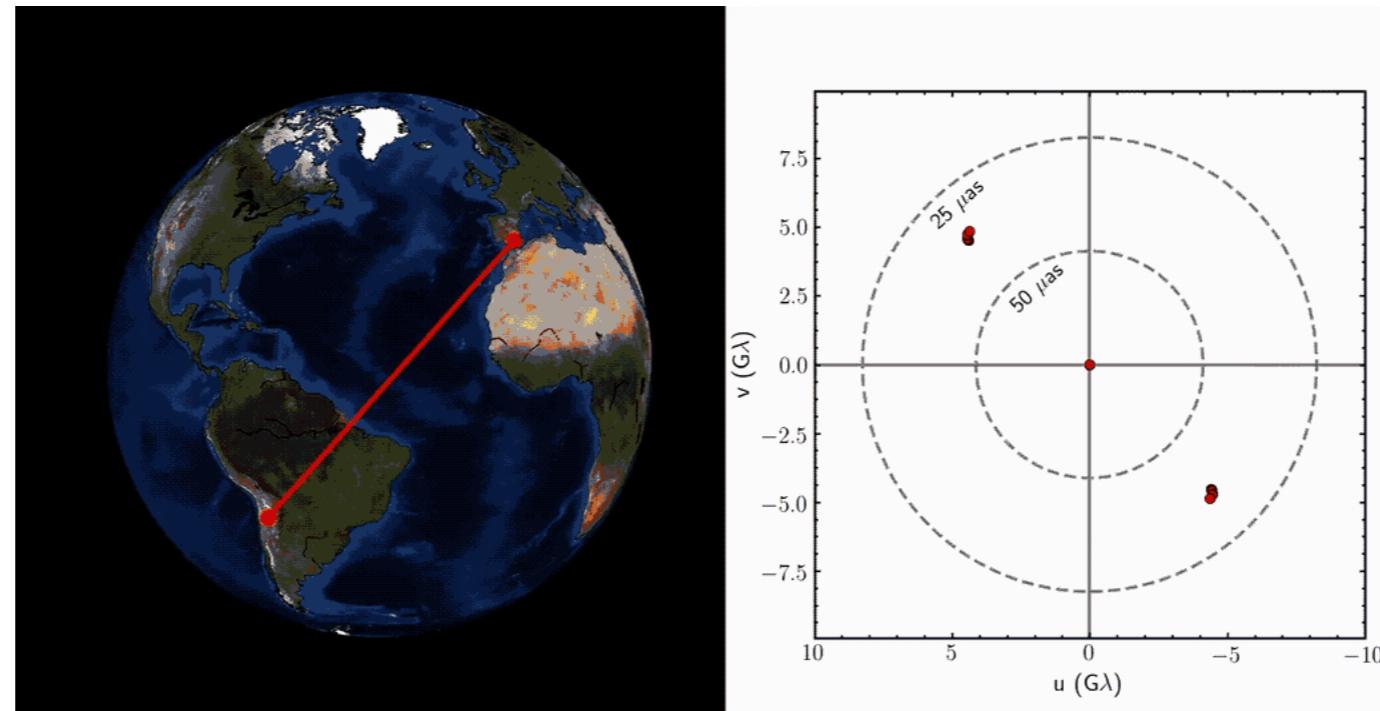
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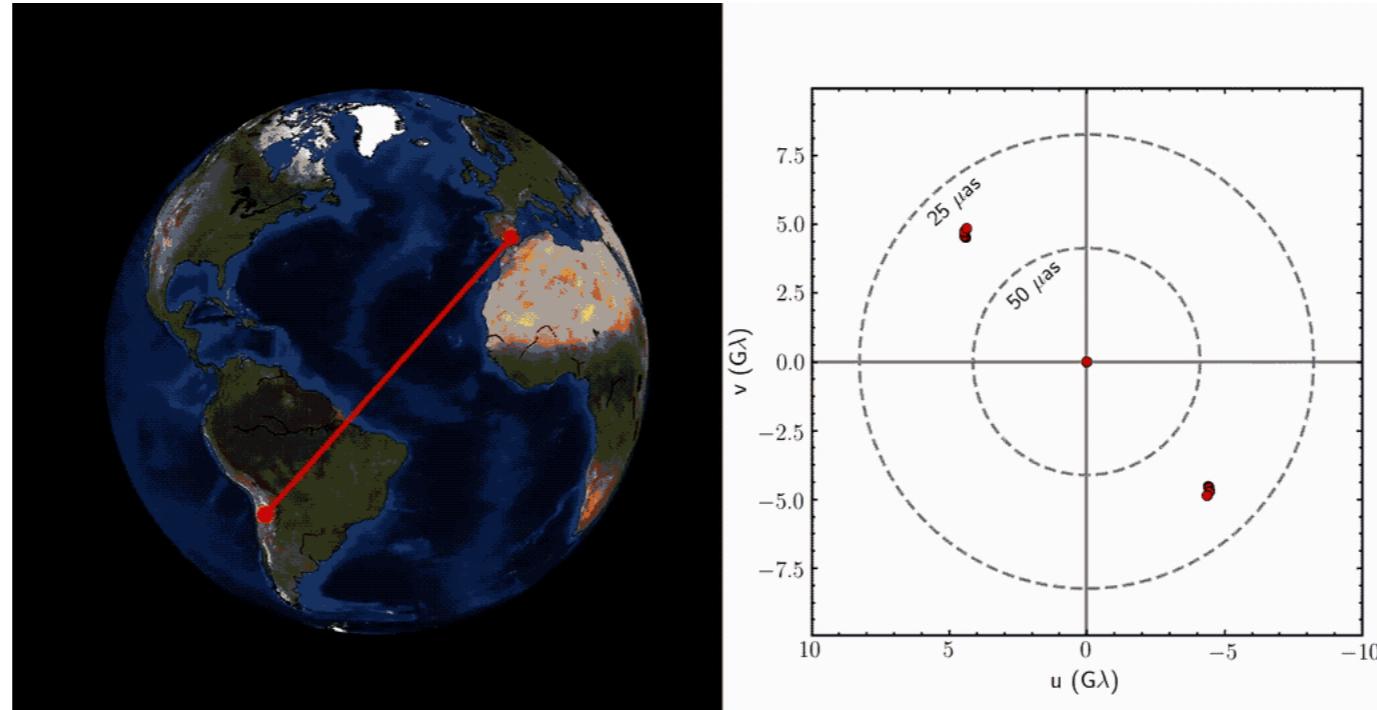
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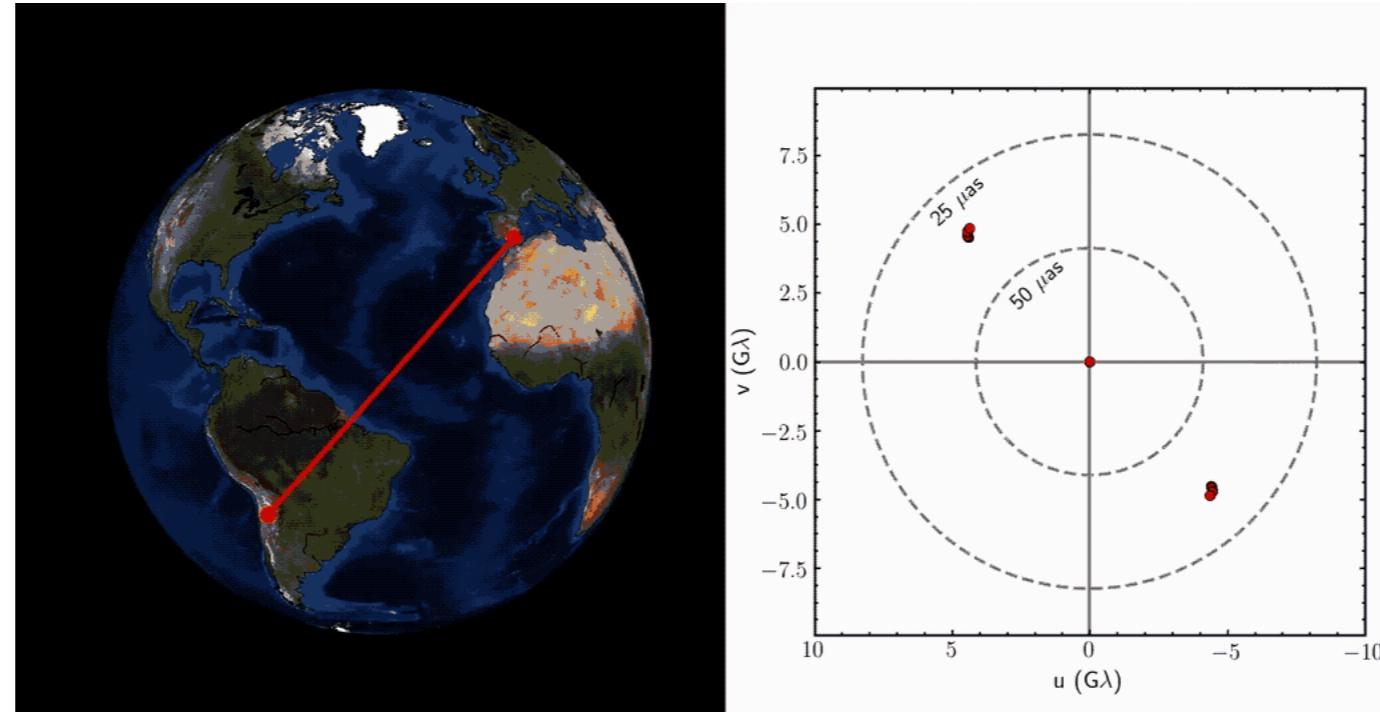


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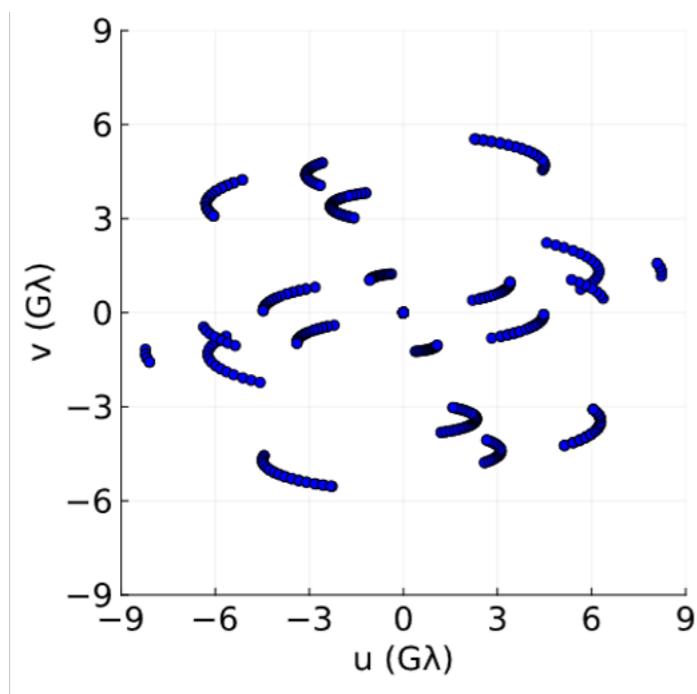
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$$y = A(x) + \epsilon, \quad \epsilon \sim N(0, \Sigma)$$
- Given the data, we want to predict what image  $x \in \mathbb{X}$  was generated.

$$x = A^{-1}(y - \epsilon)$$

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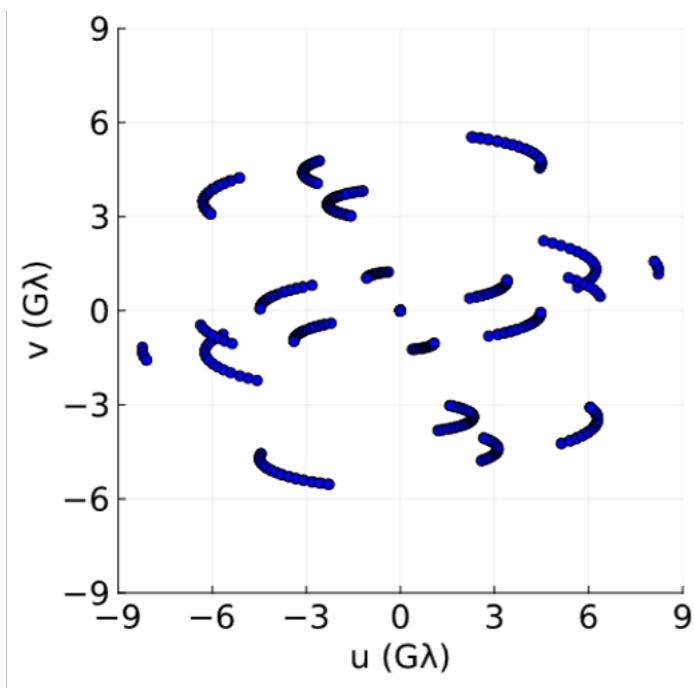
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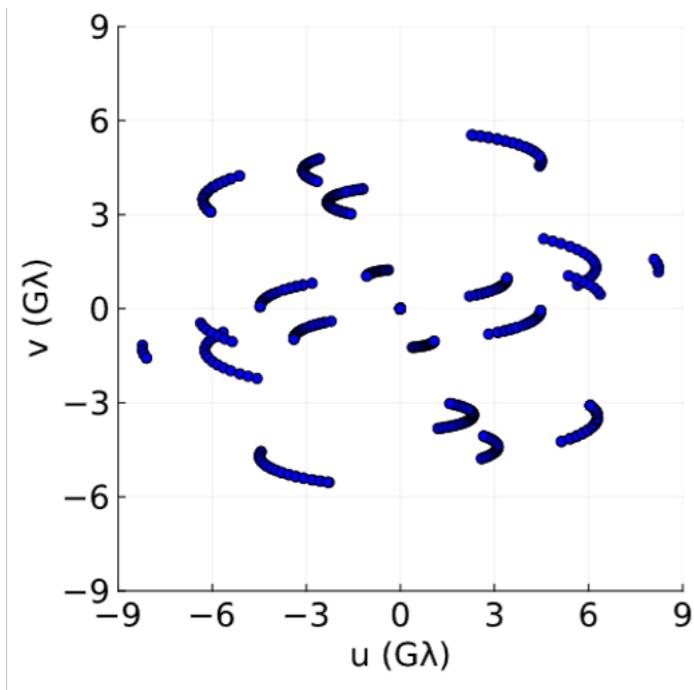
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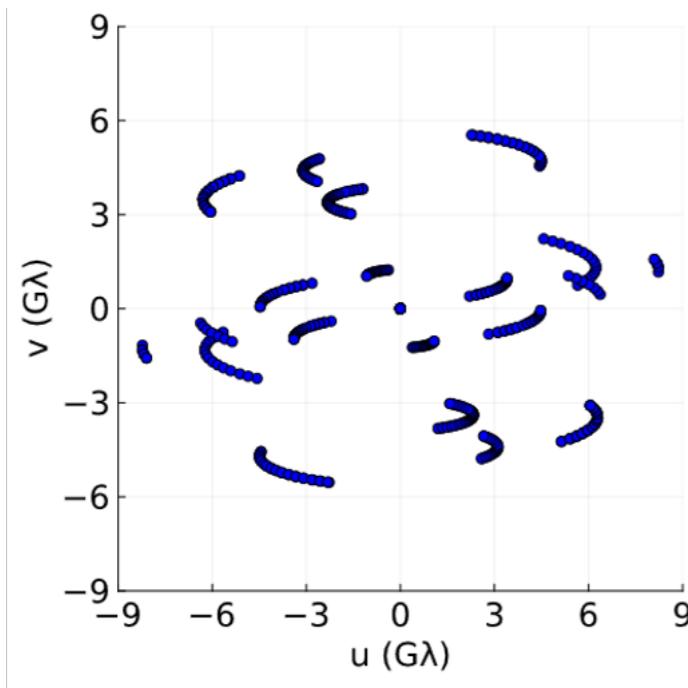
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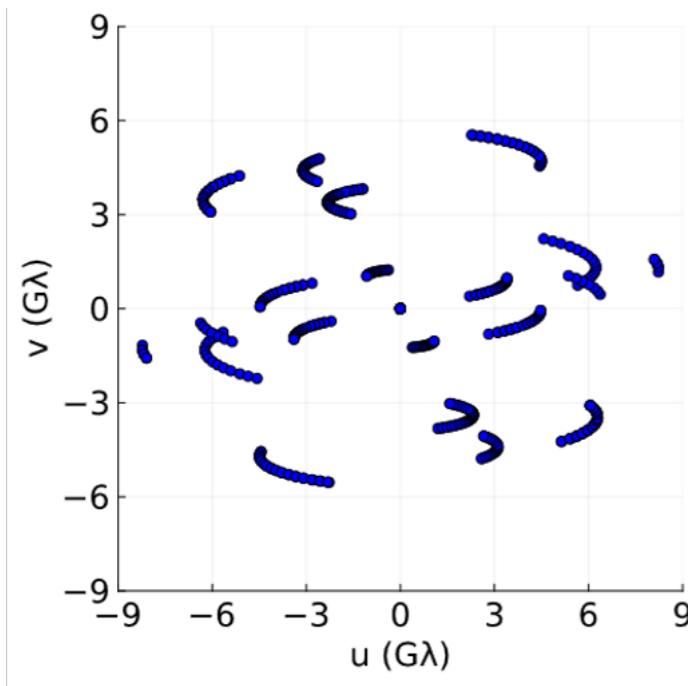
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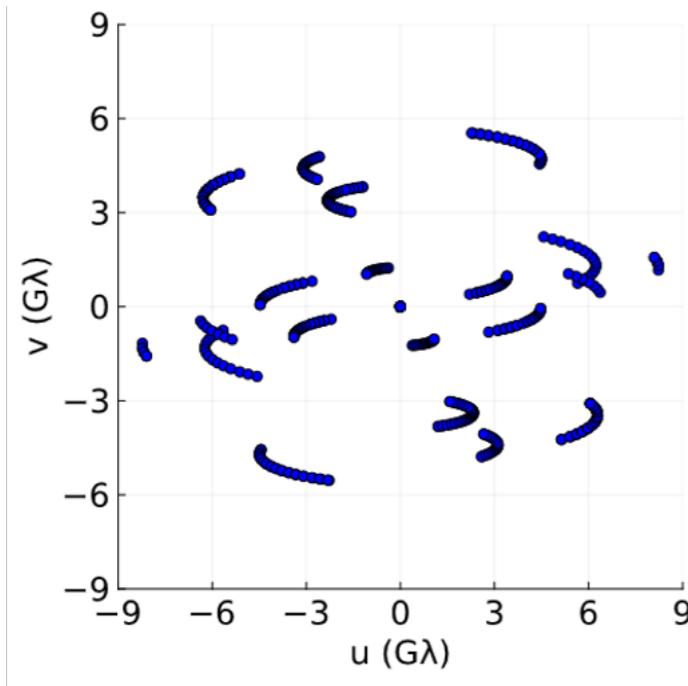
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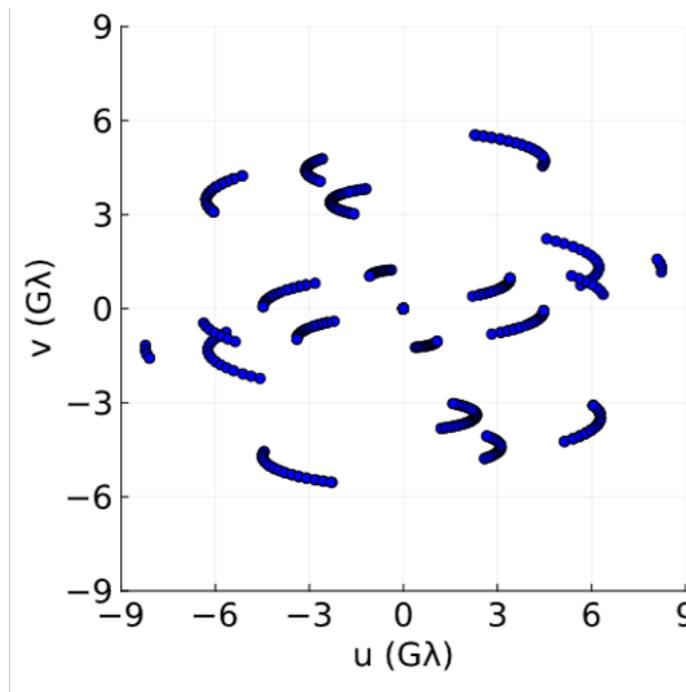
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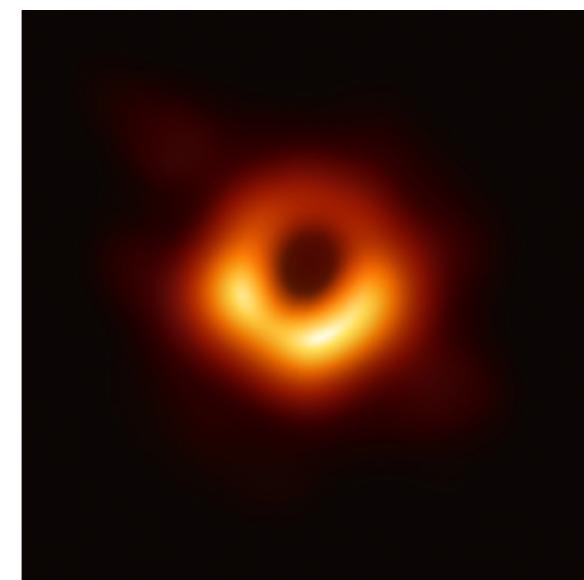


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**Inference**  
→



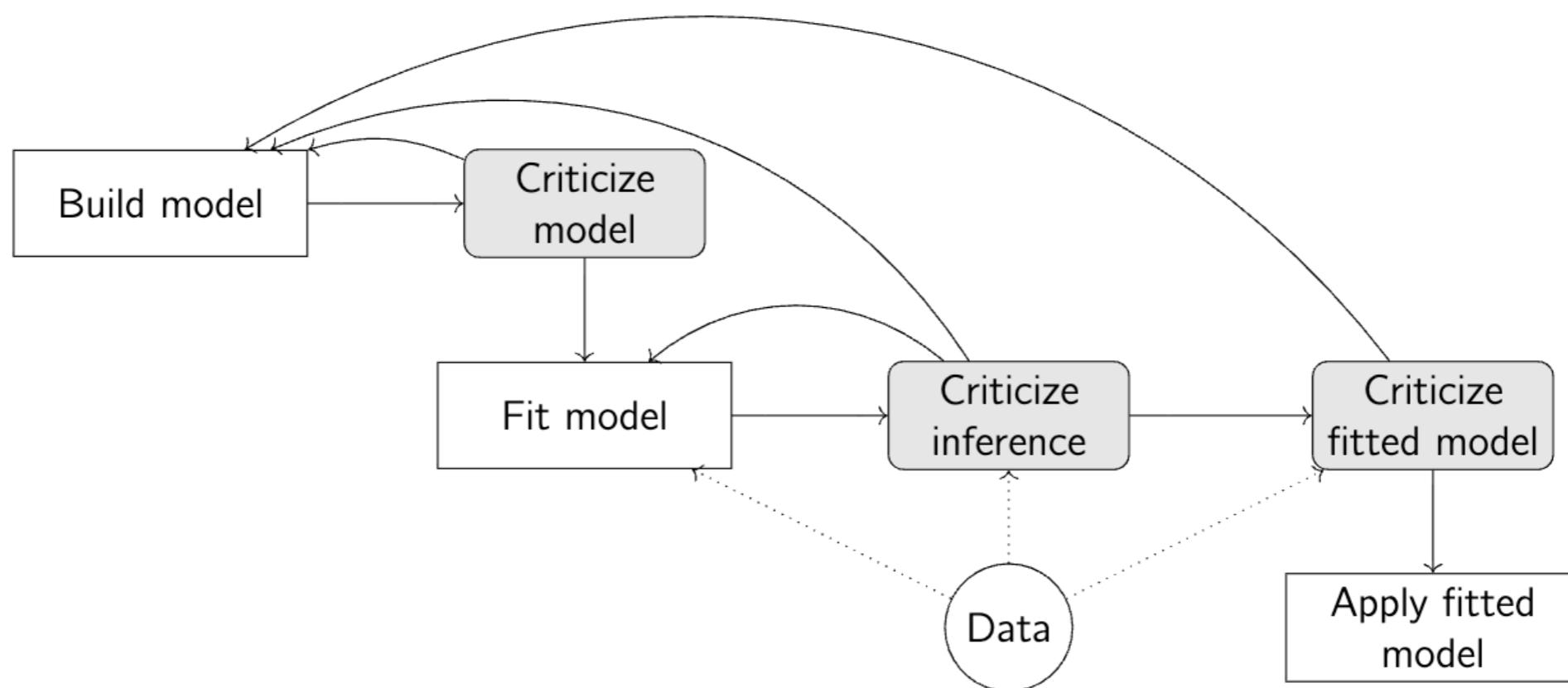
M87, 2019

$$\sim p(x | y)$$

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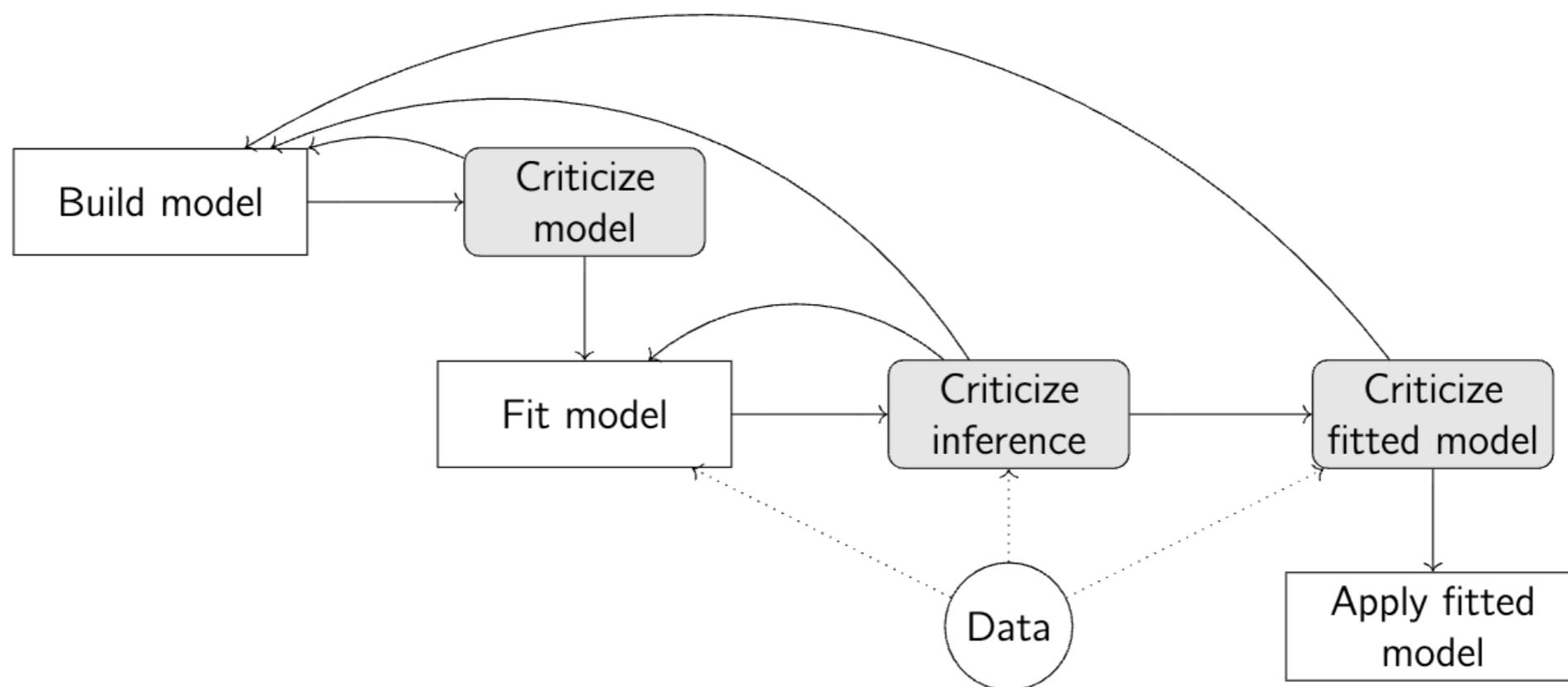
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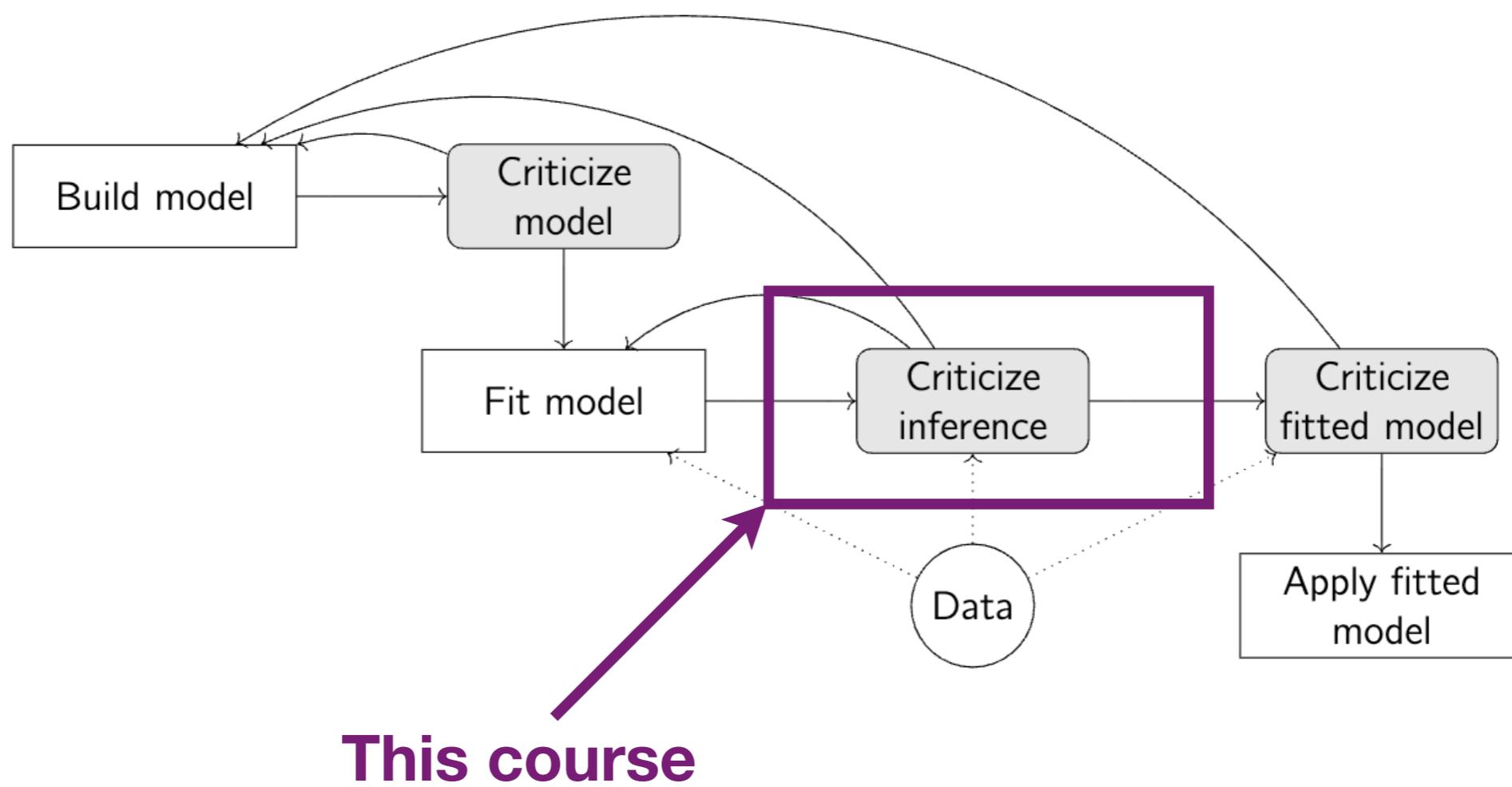
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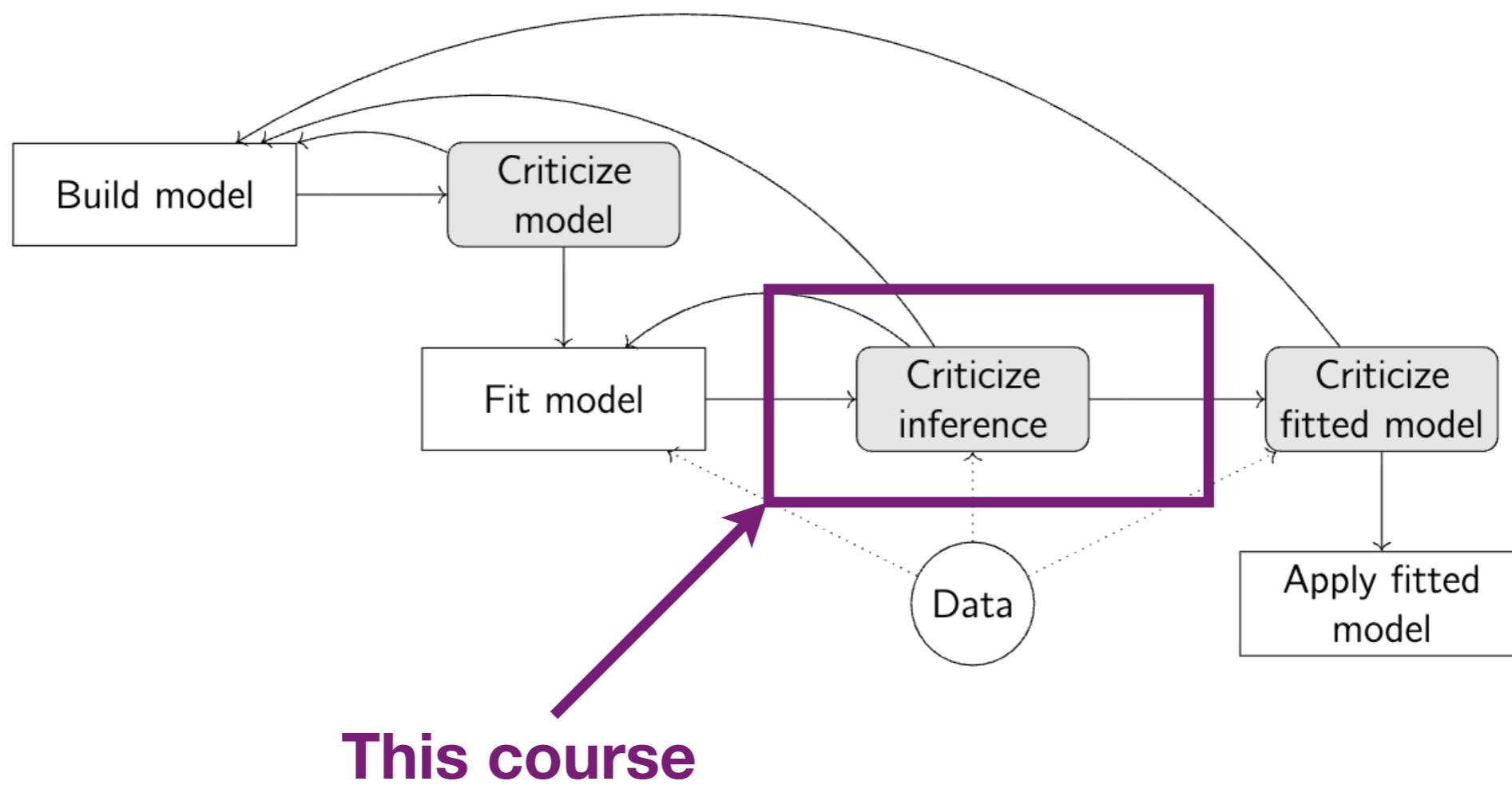
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  - ▶ We don't judge where the distribution comes from or its scientific importance



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- ▶ **Evidence:** e.g. for model comparison

$$p(y) = \int_{\mathbb{X}} L(y|x)p(x)dx$$

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- ▶ We want to be able to say something about the properties of  $\pi$

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- ▶ We will often interpolate between density and log-space

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- ▶ For us, the data is an abstraction

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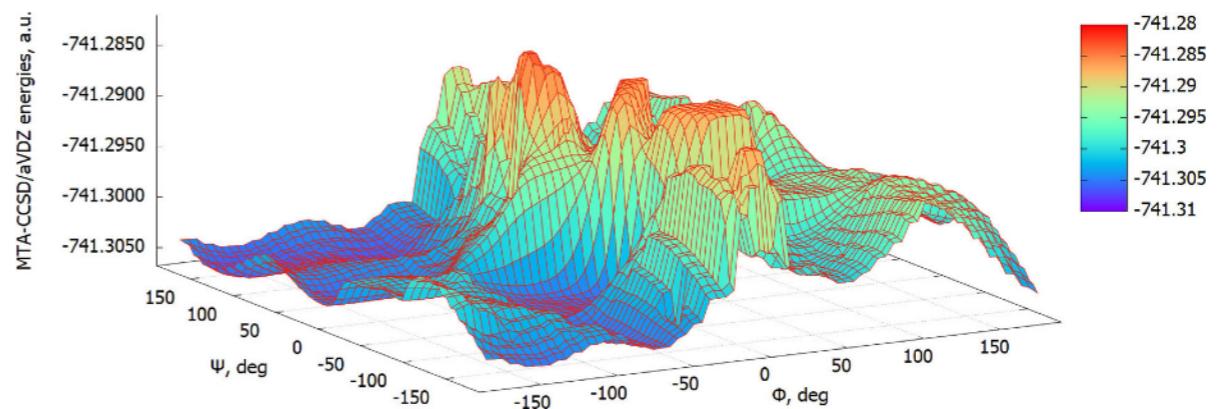
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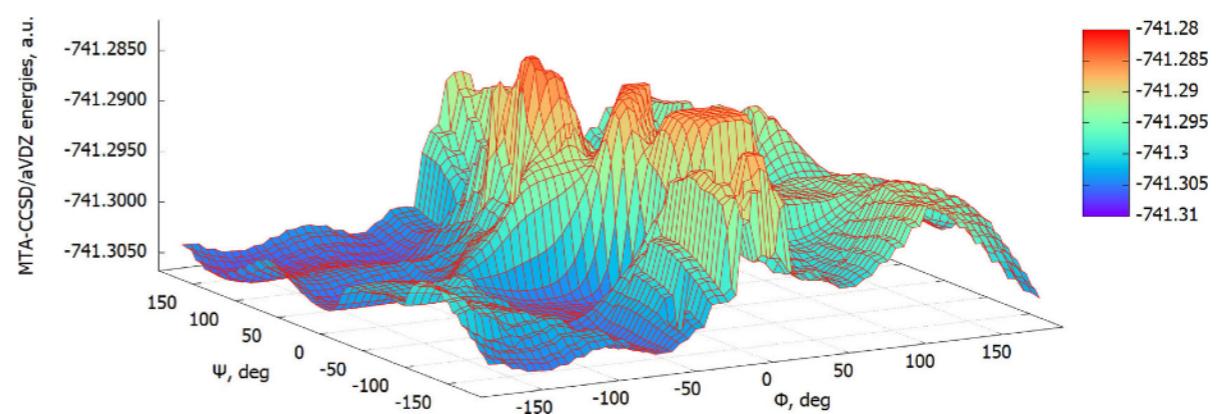
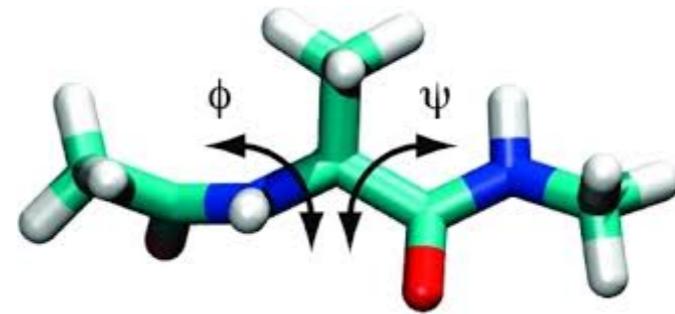
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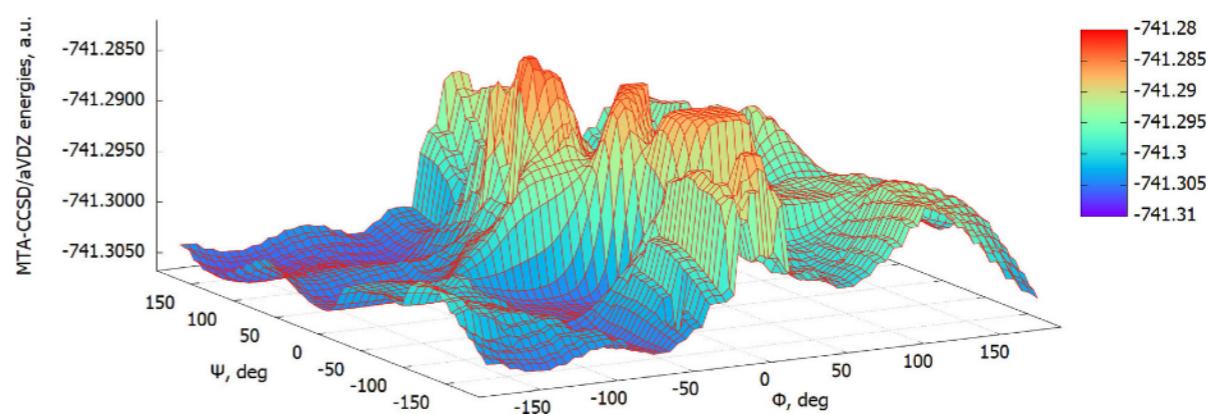
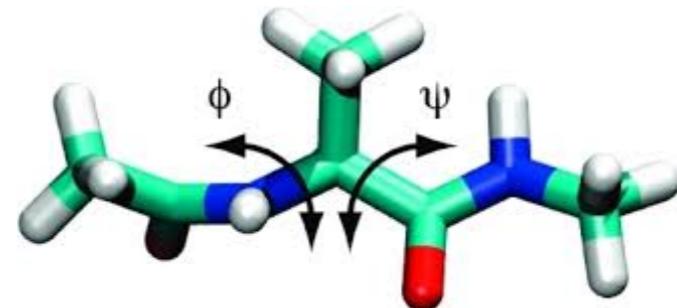
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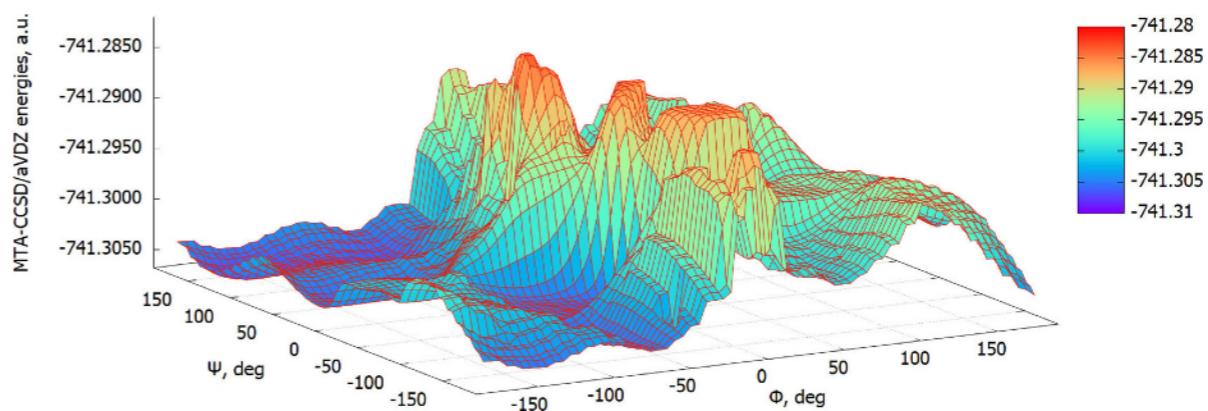
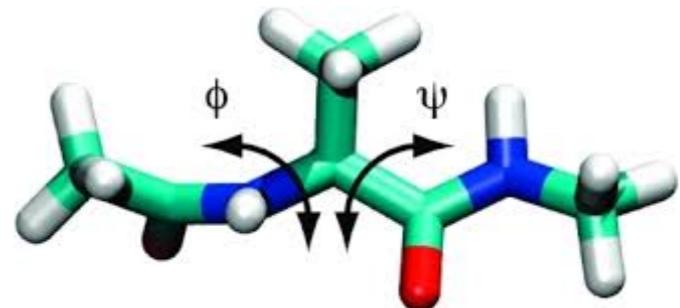


- ▶ Molecular dynamics simulates these systems to infer physical properties

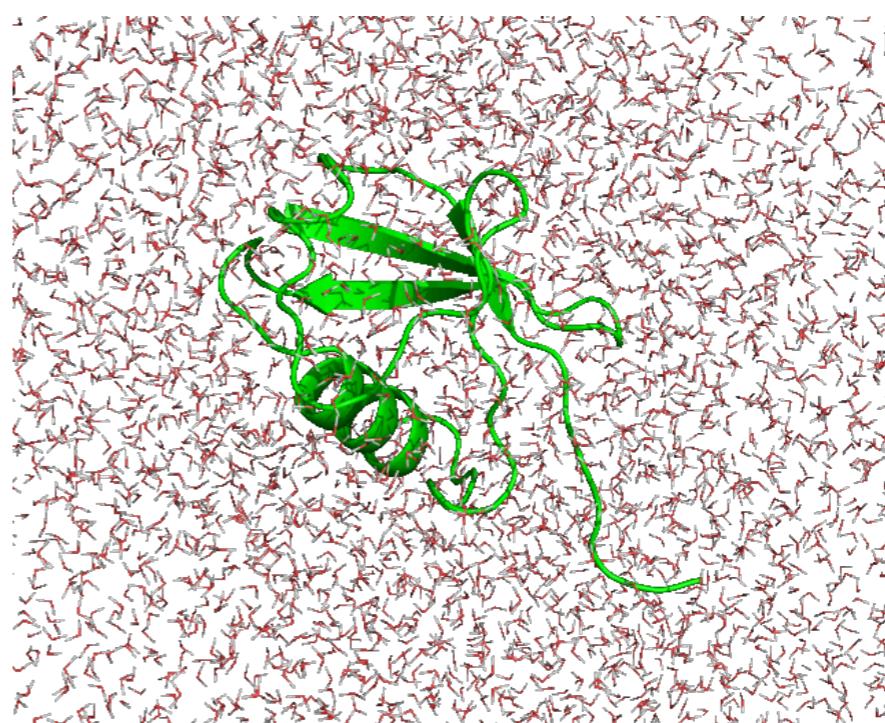
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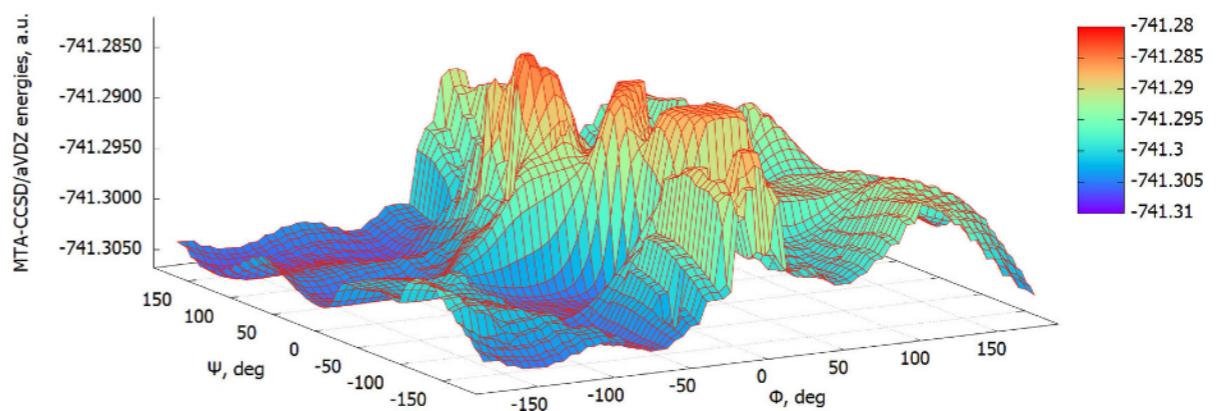
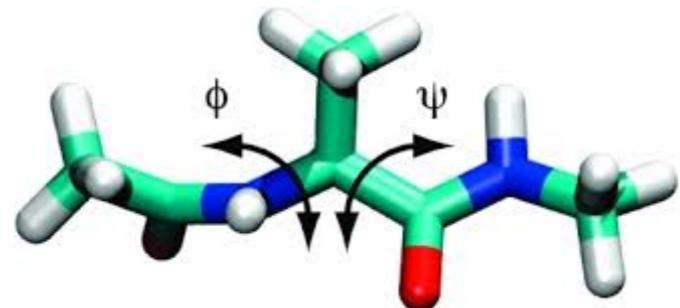


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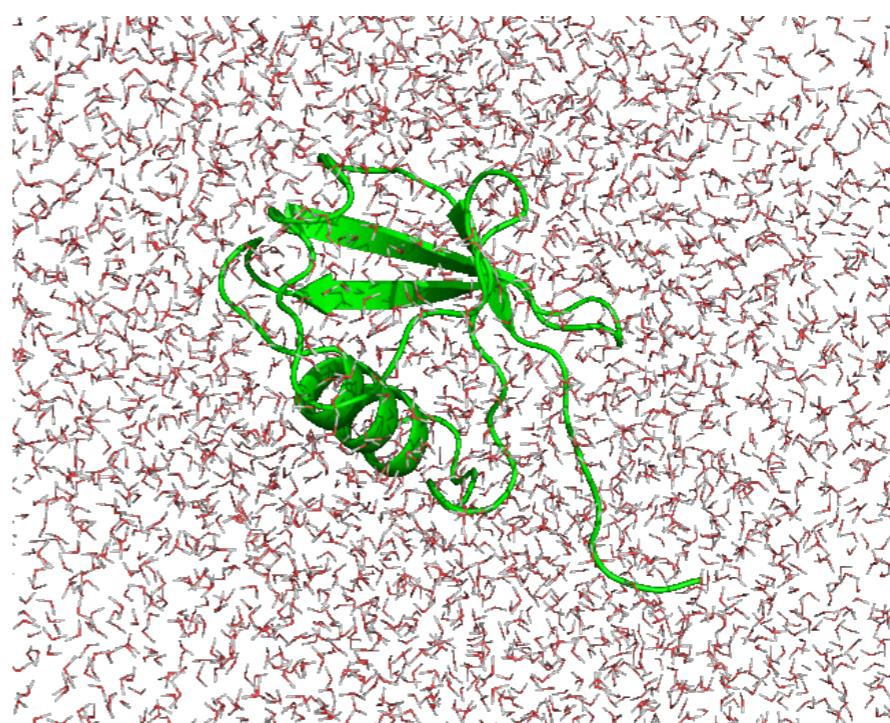


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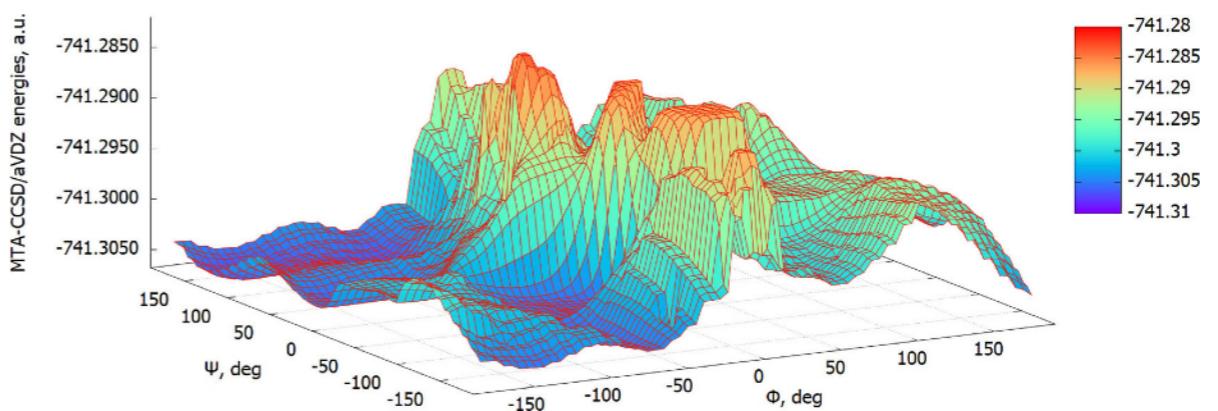
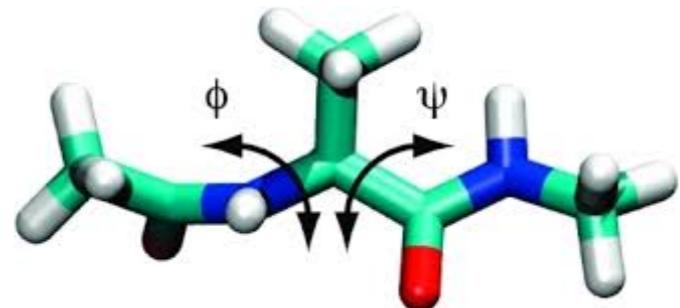


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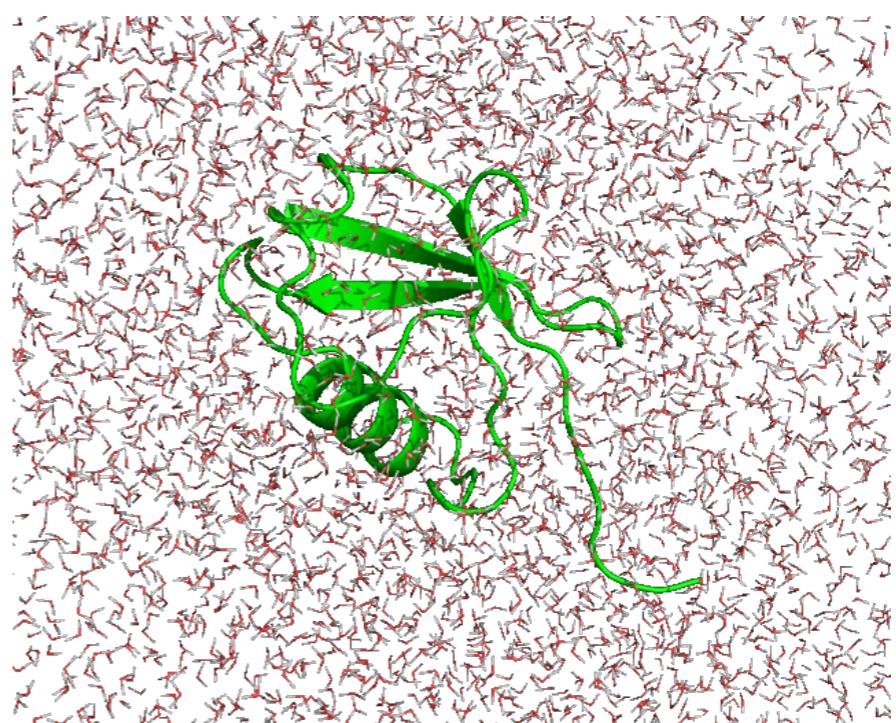


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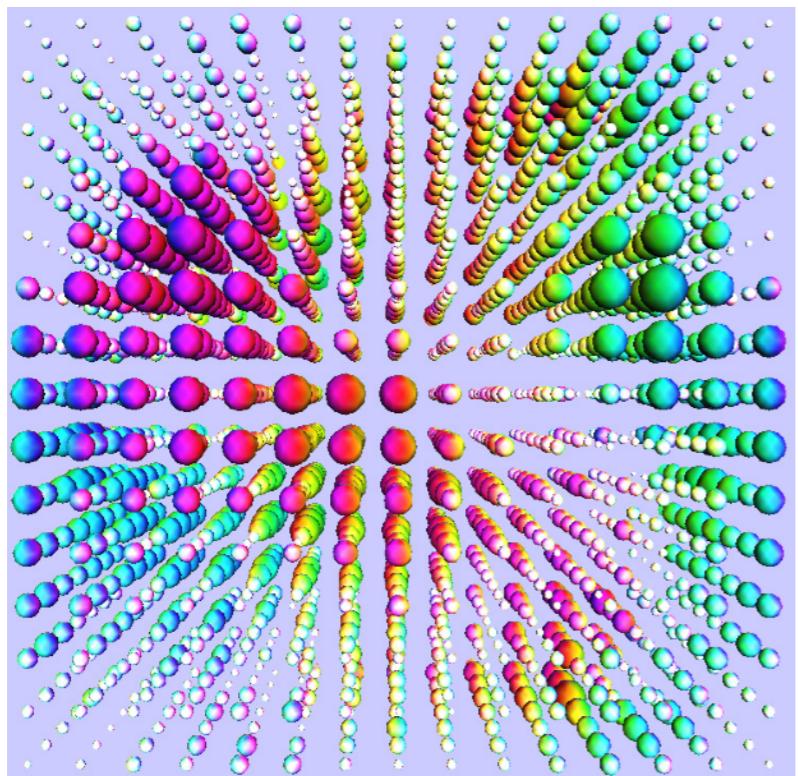


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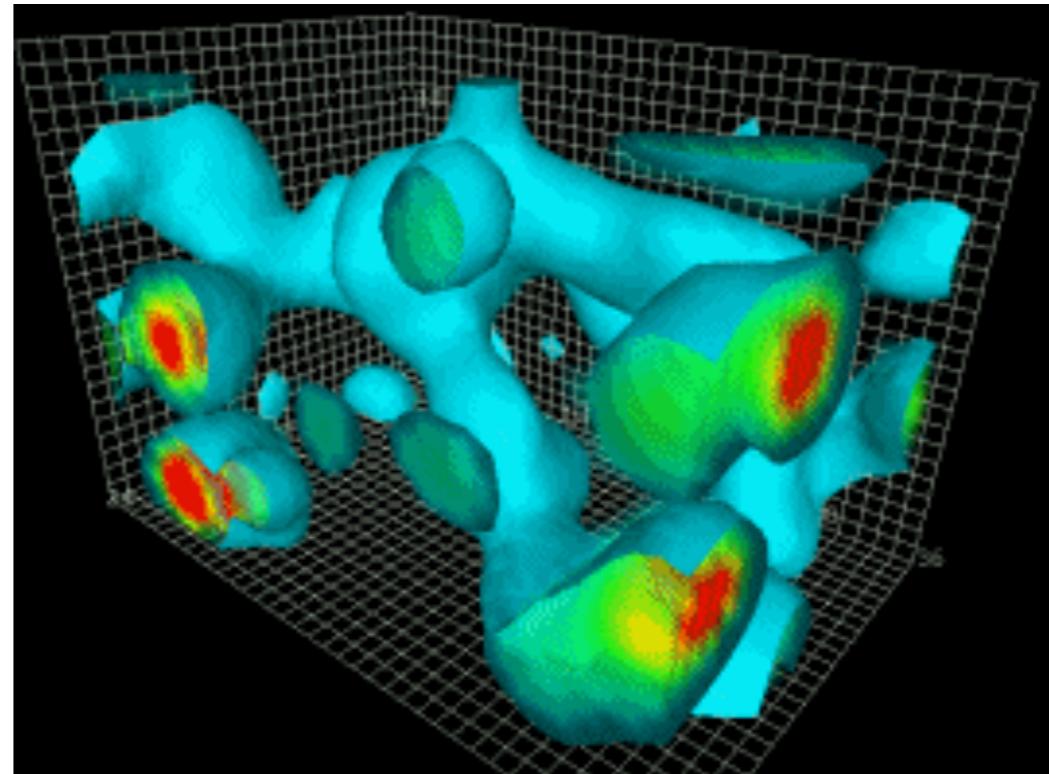
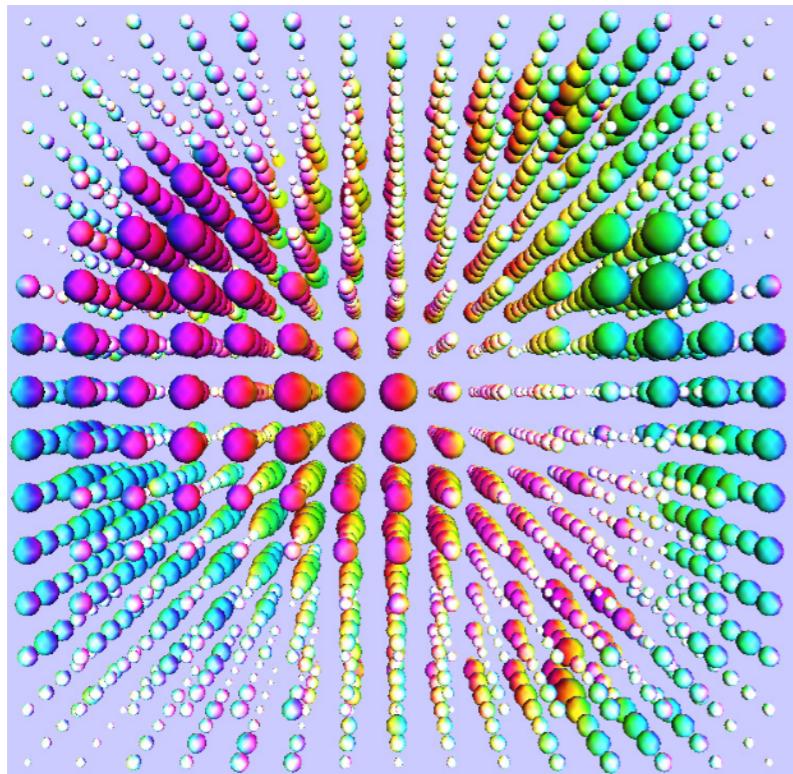
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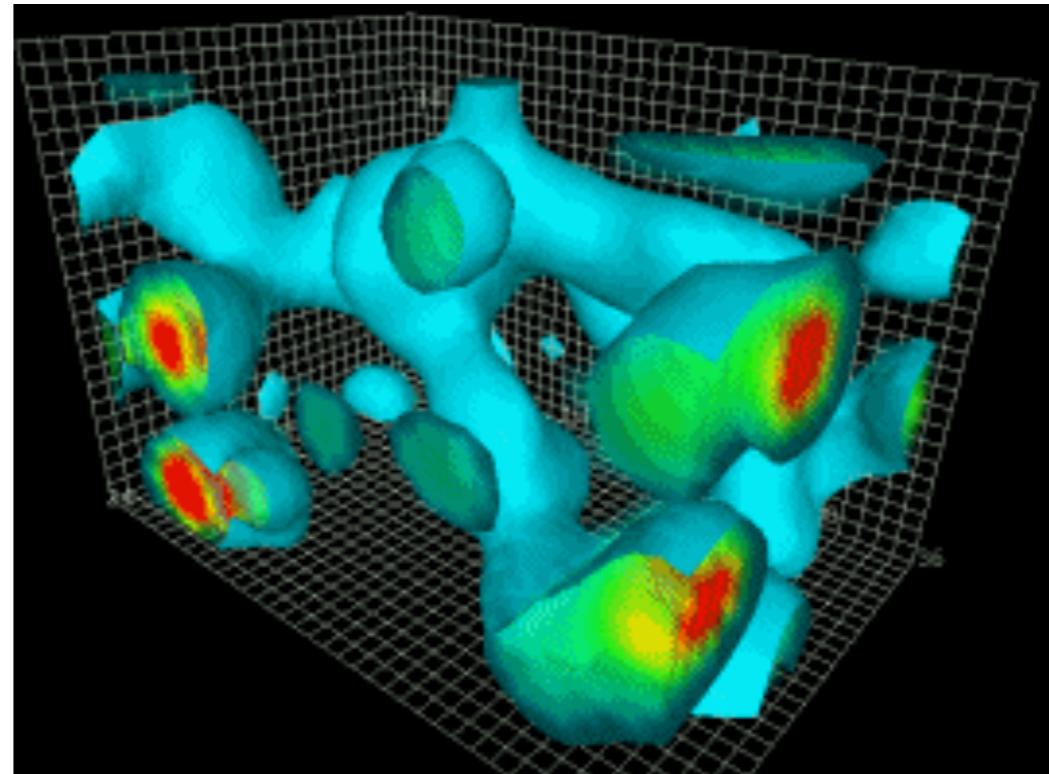
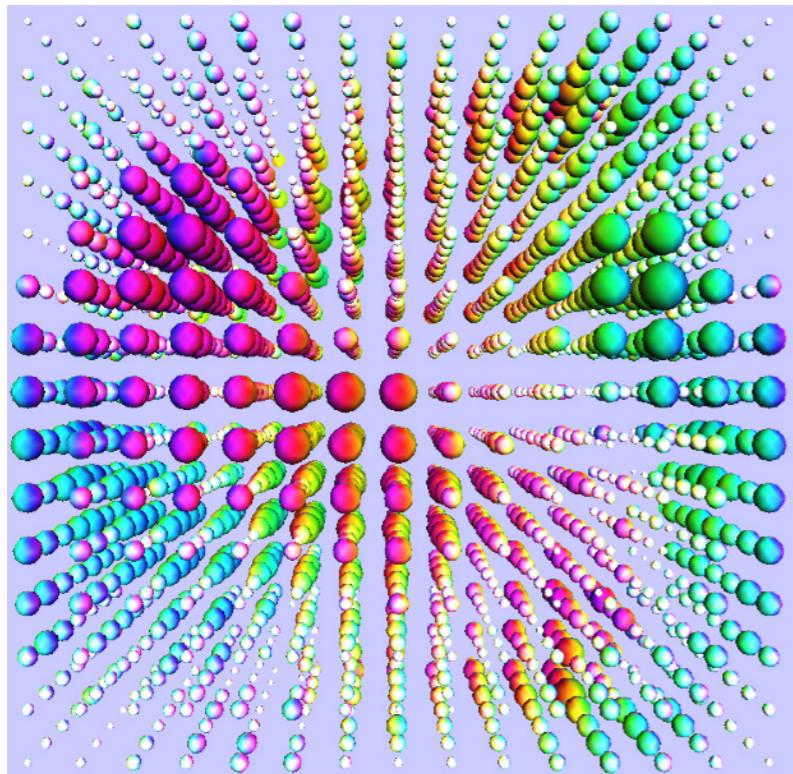
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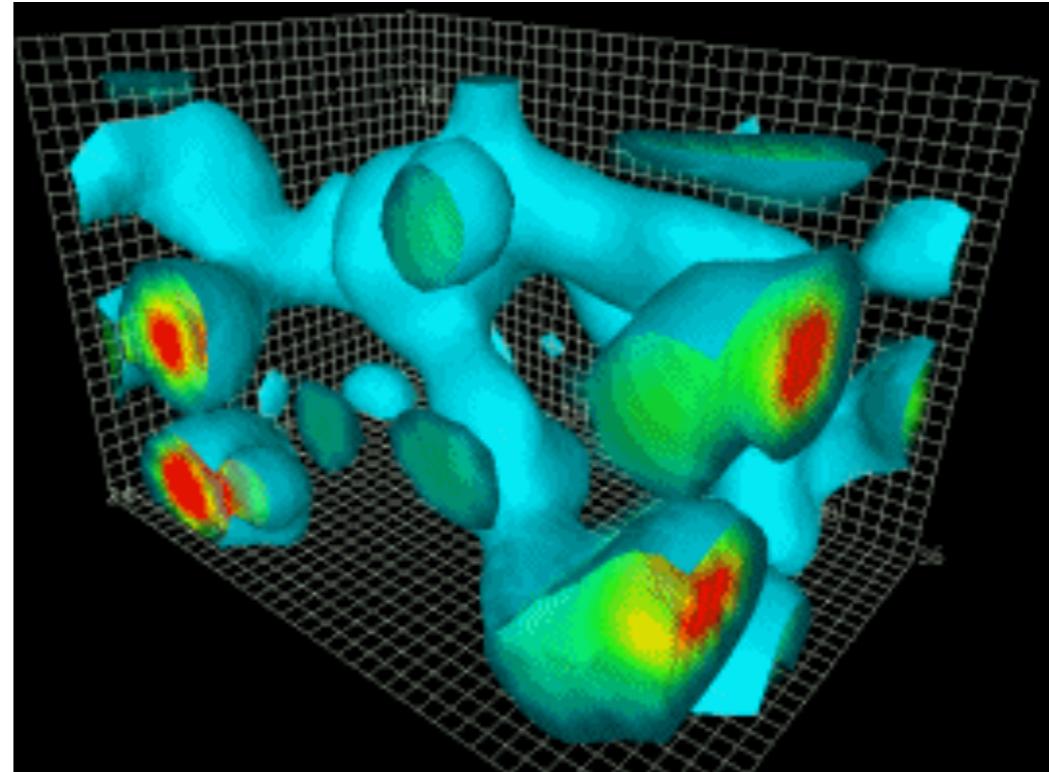
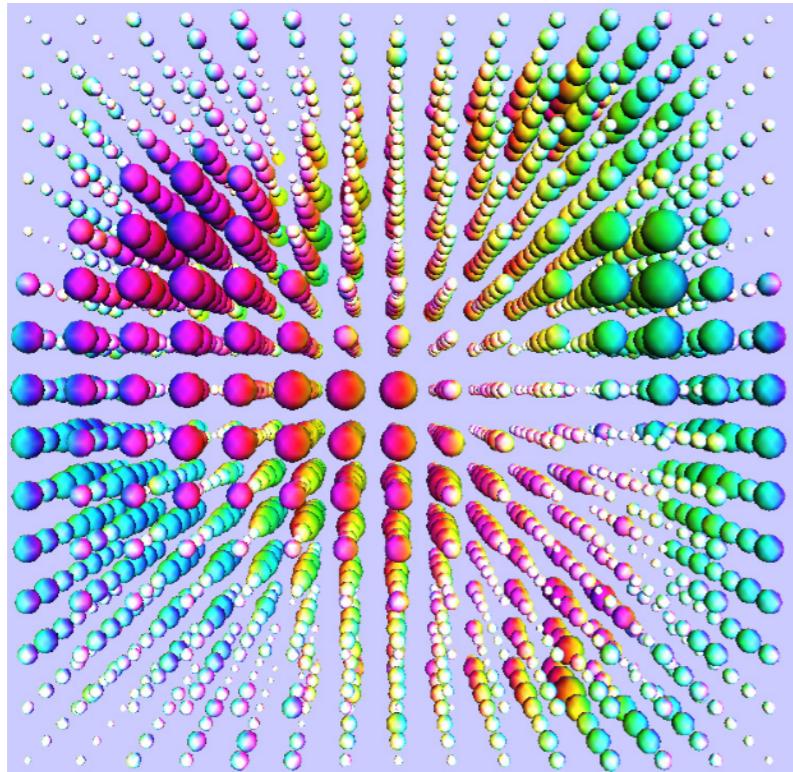
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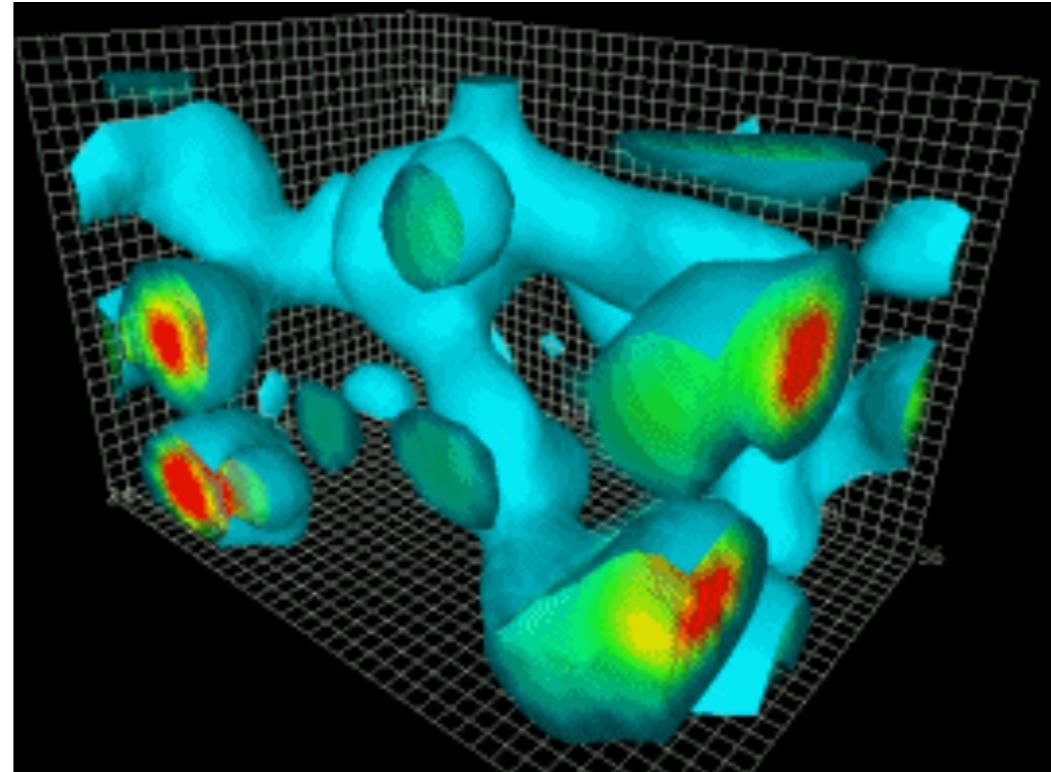
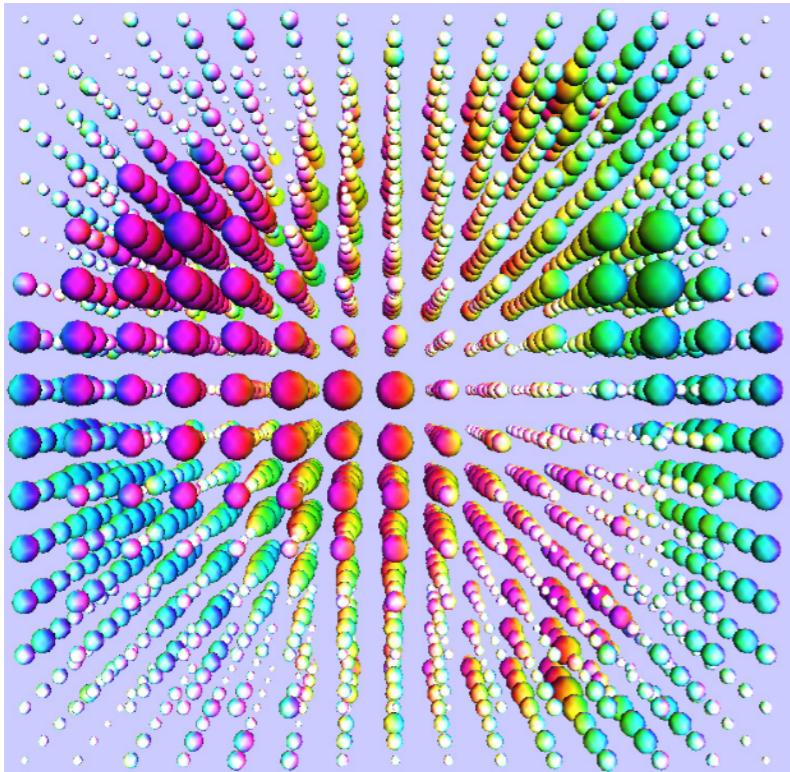
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- ▶ Run simulations for months at a time, use ~15% of global compute resources



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- ▶ This is basically a course on numerically computing integrals

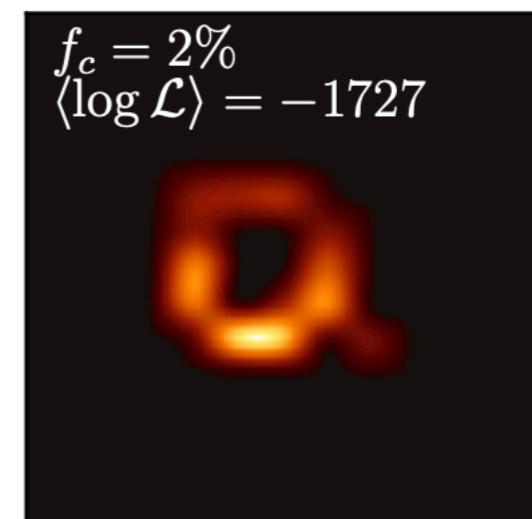
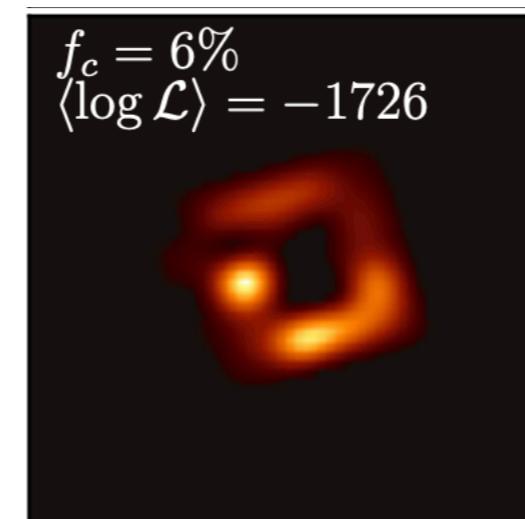
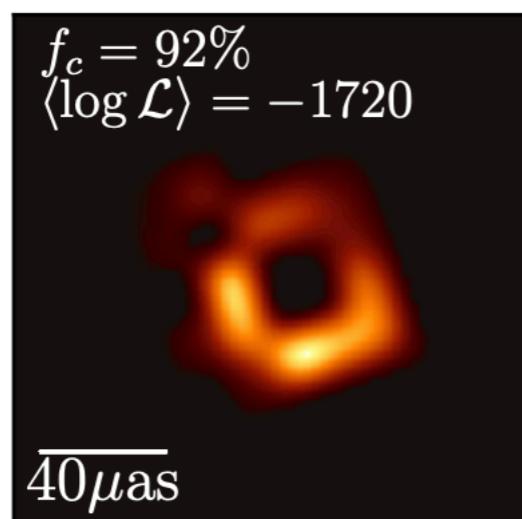
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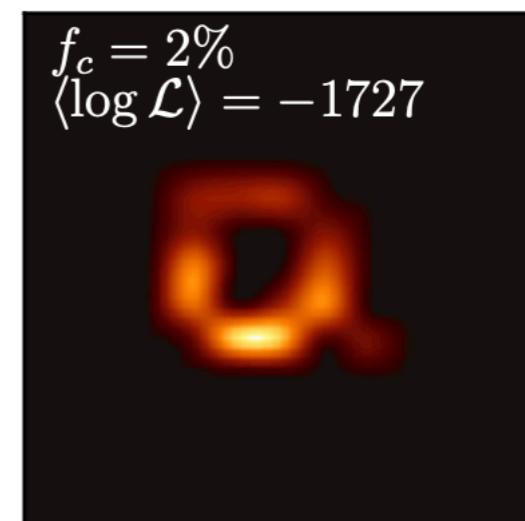
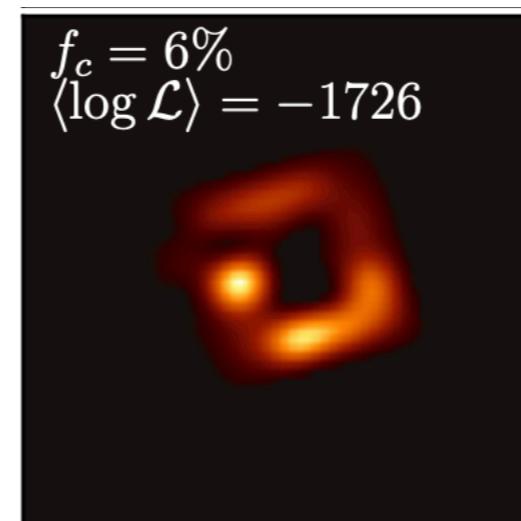
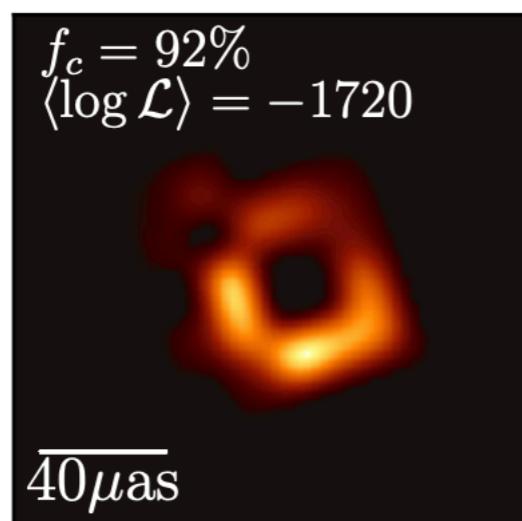
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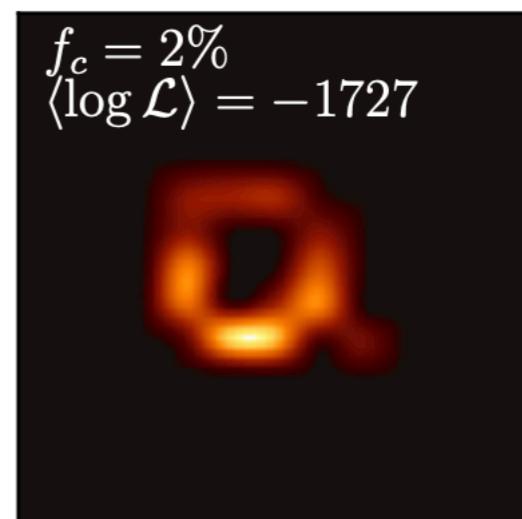
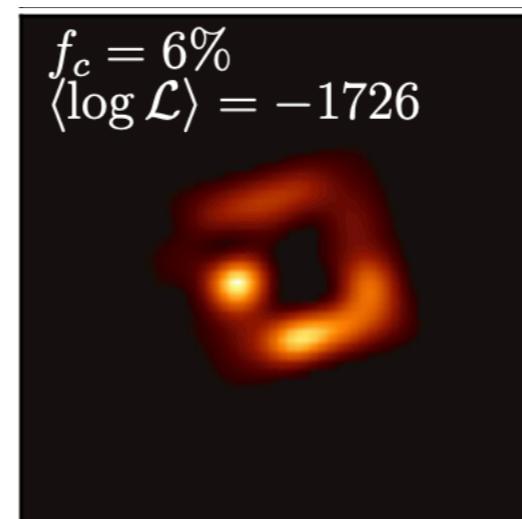
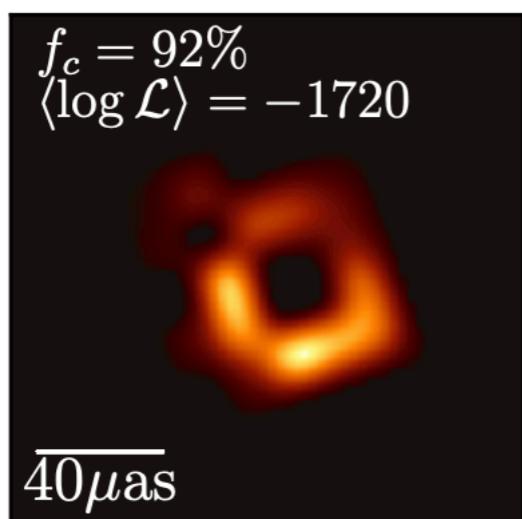
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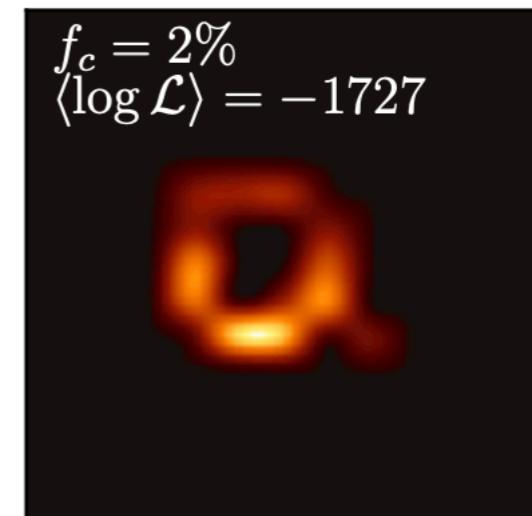
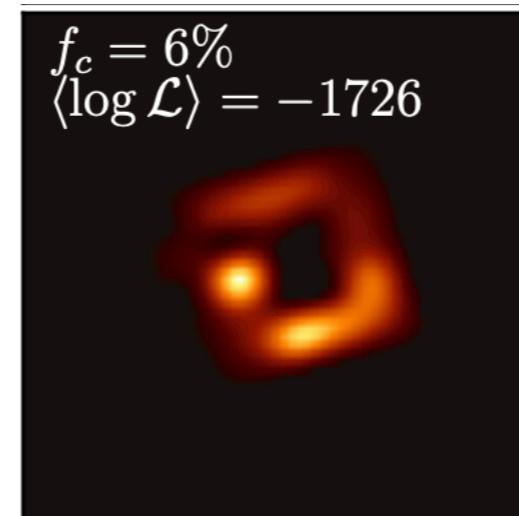
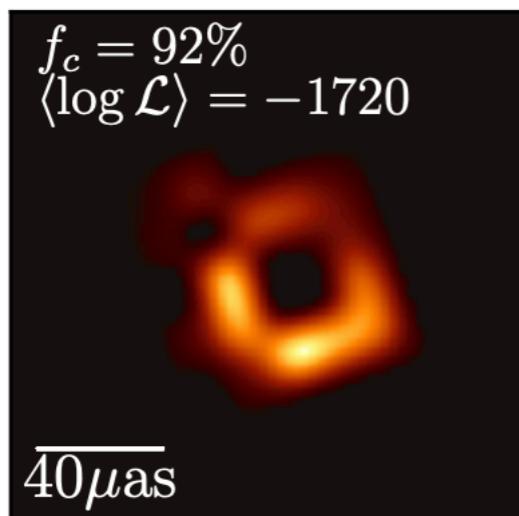
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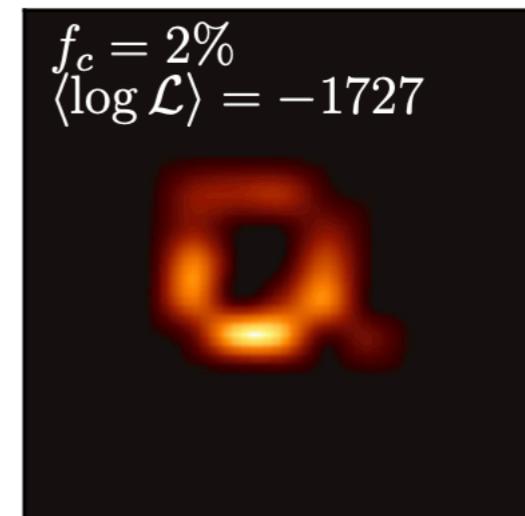
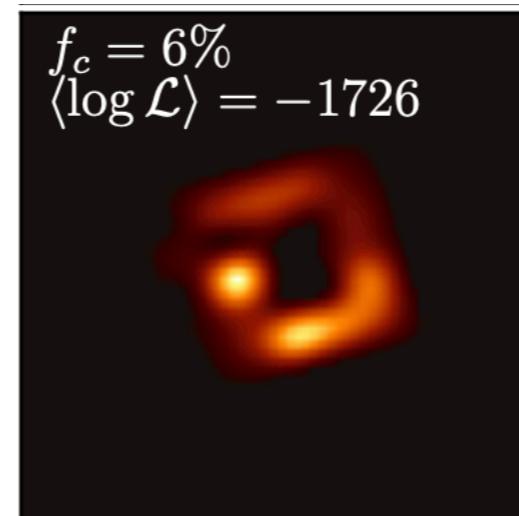
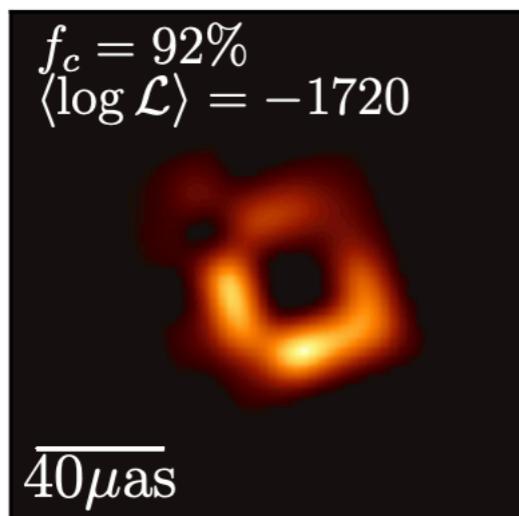
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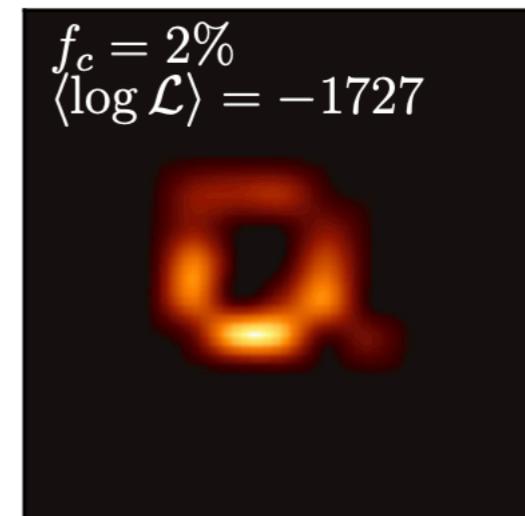
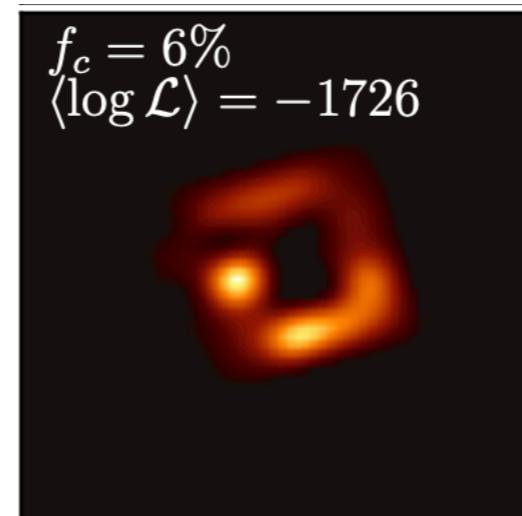
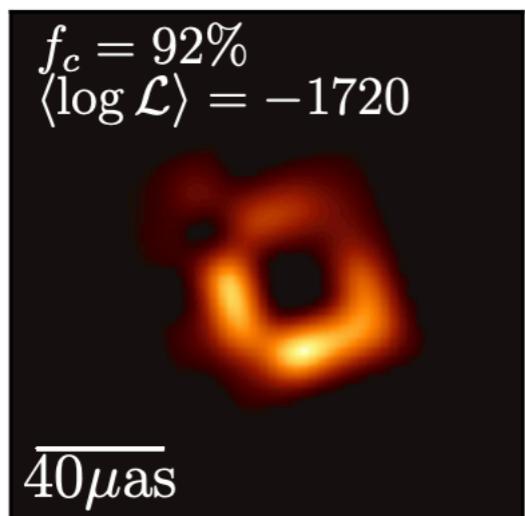
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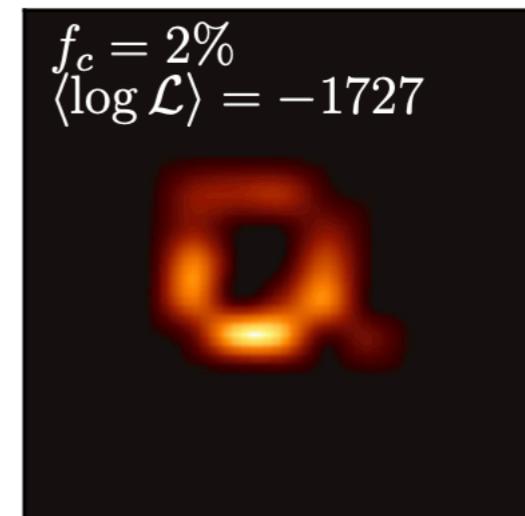
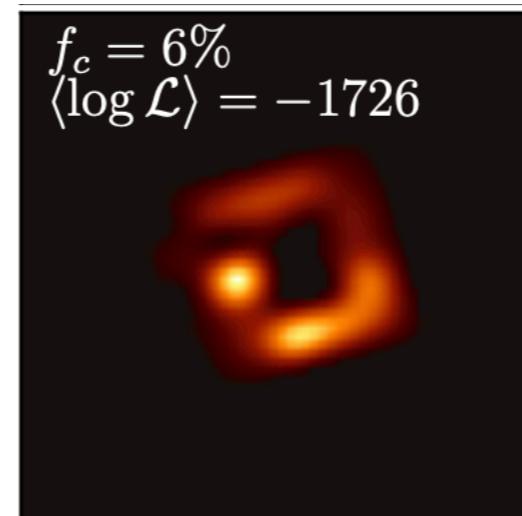
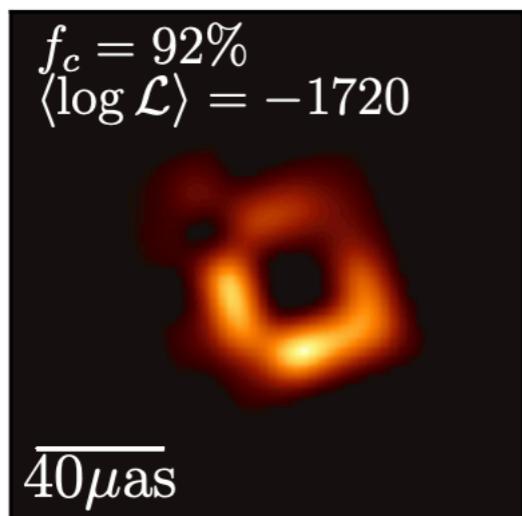
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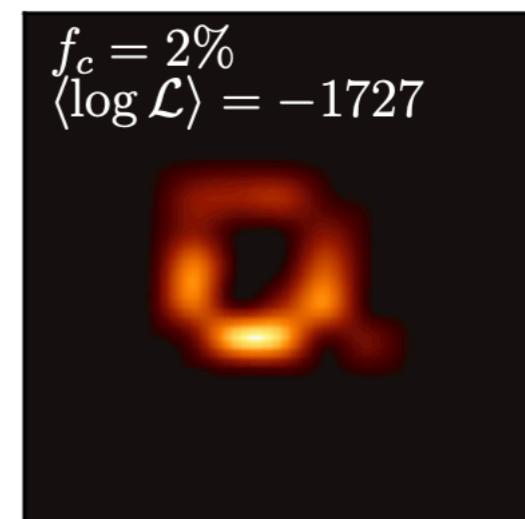
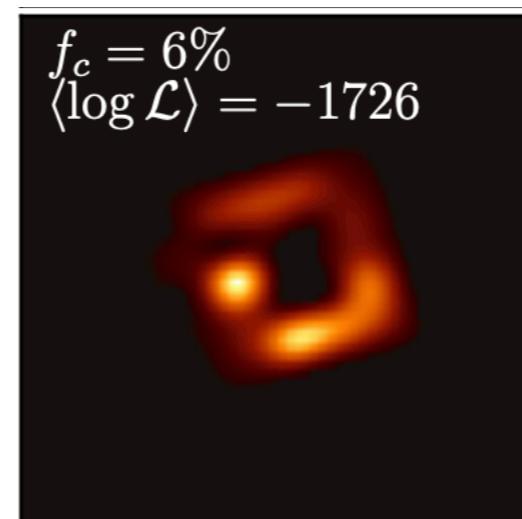
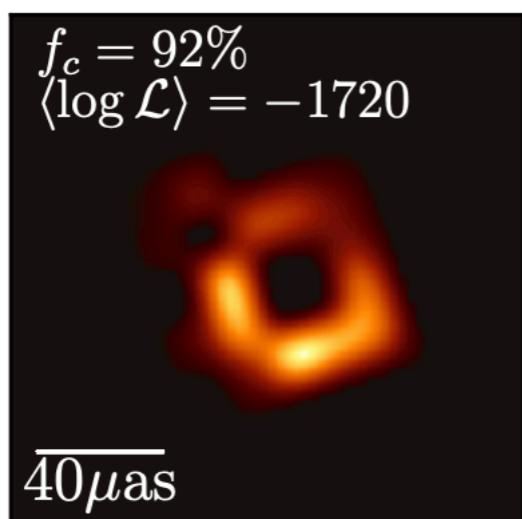
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$$\sqrt{N}(\hat{\pi}[f] - \pi[f]) \implies N(0, \mathbb{V}_\pi[f])$$

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  - ▶ By the law of large numbers, the average a.s. converges to  $\pi[f]$  as  $T \rightarrow \infty$

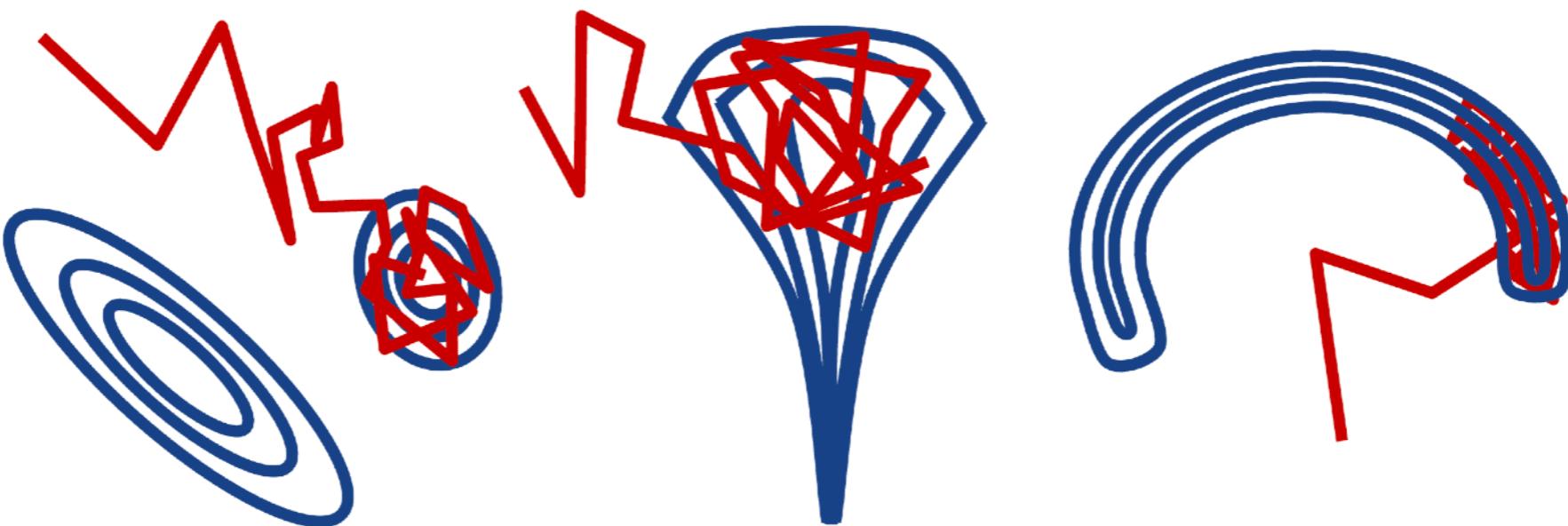
$$\pi[f] = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T f(X_t)$$



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19

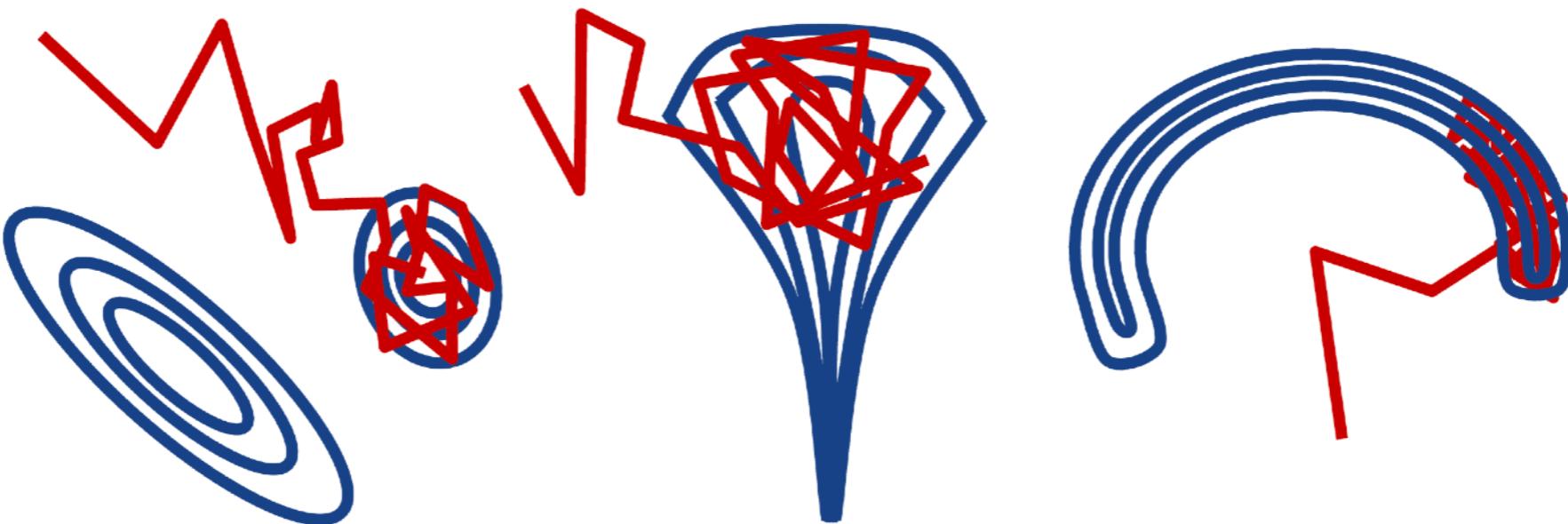
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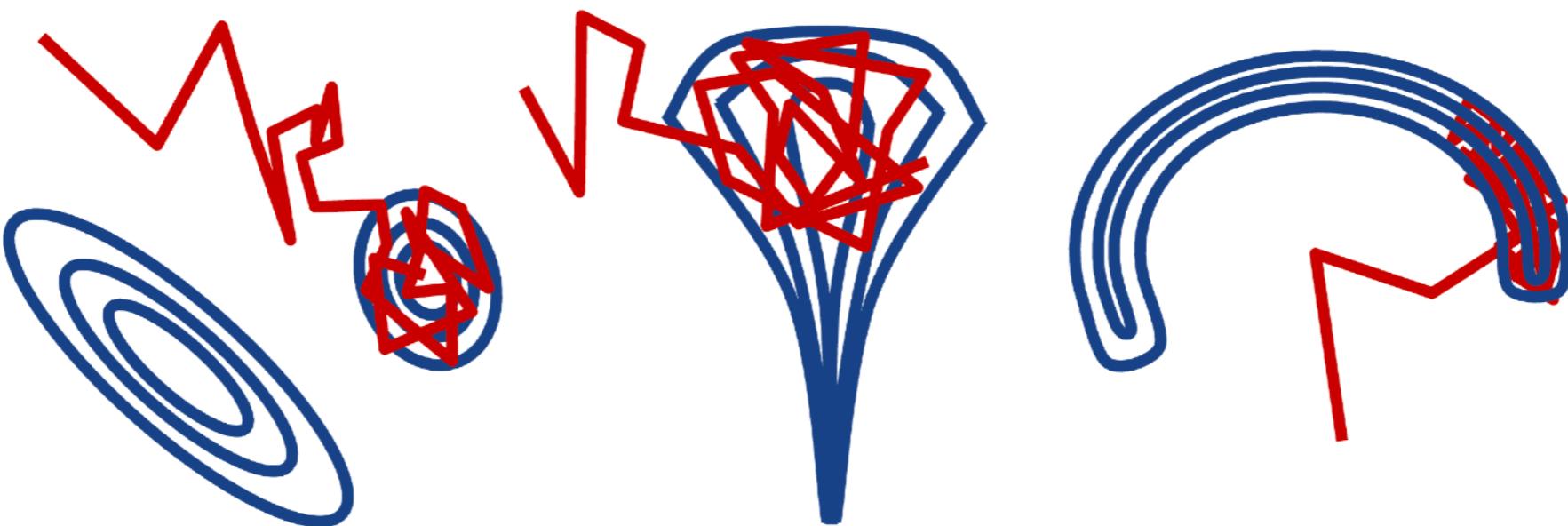
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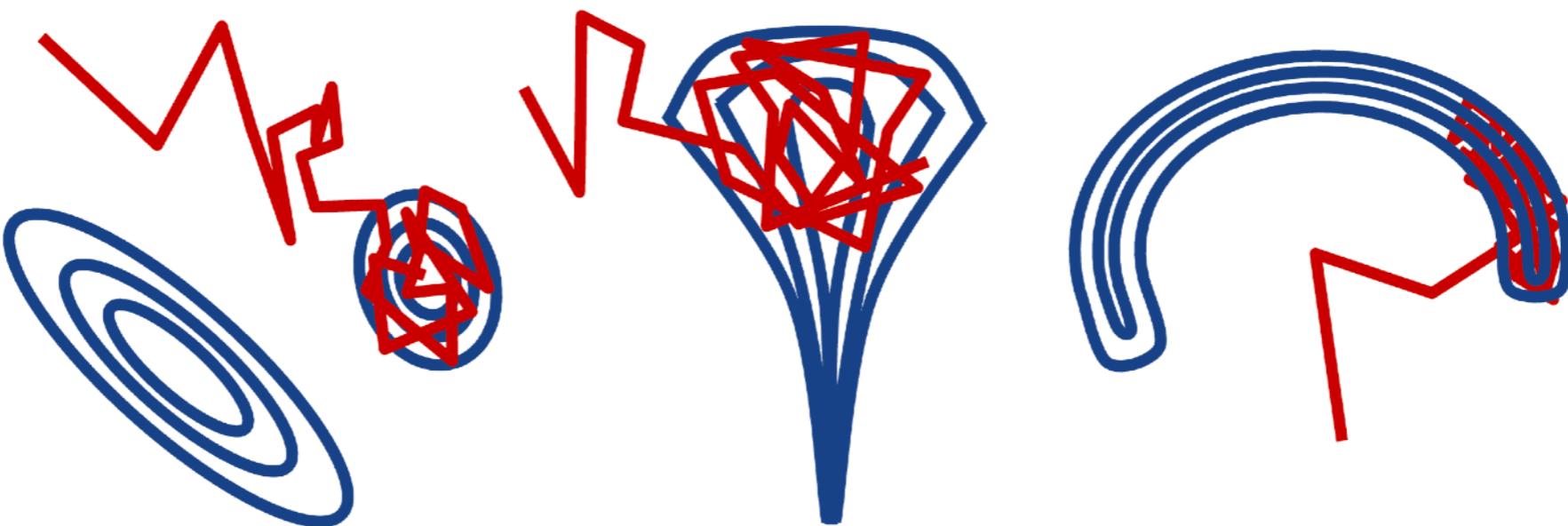


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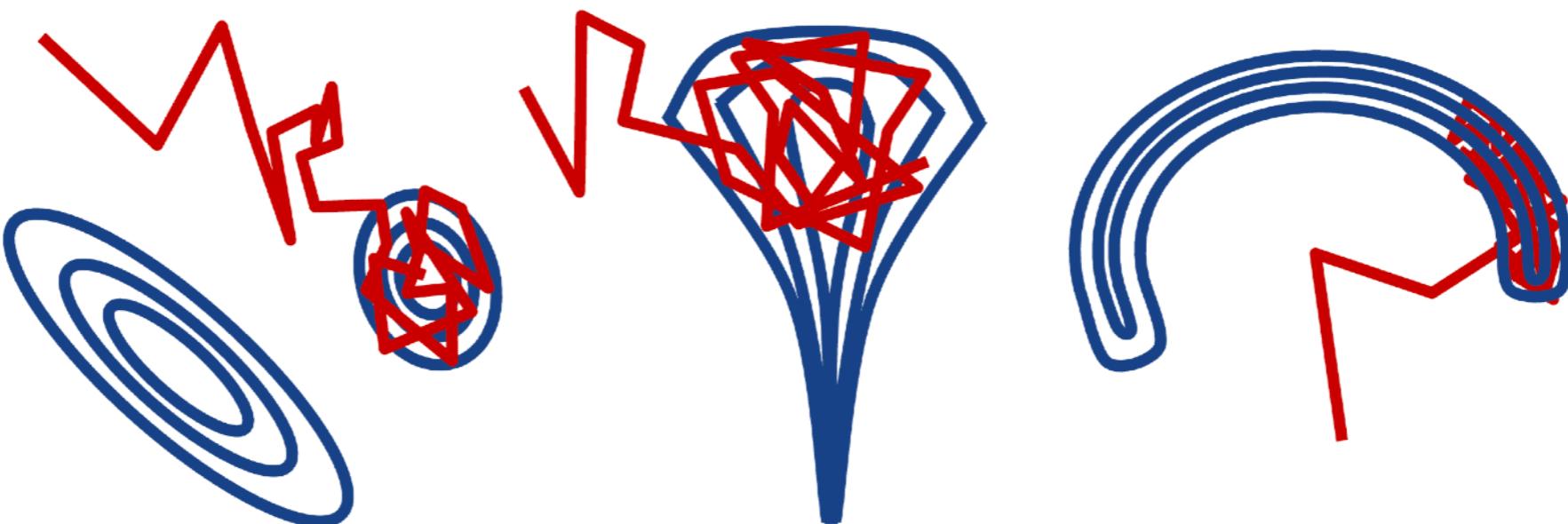


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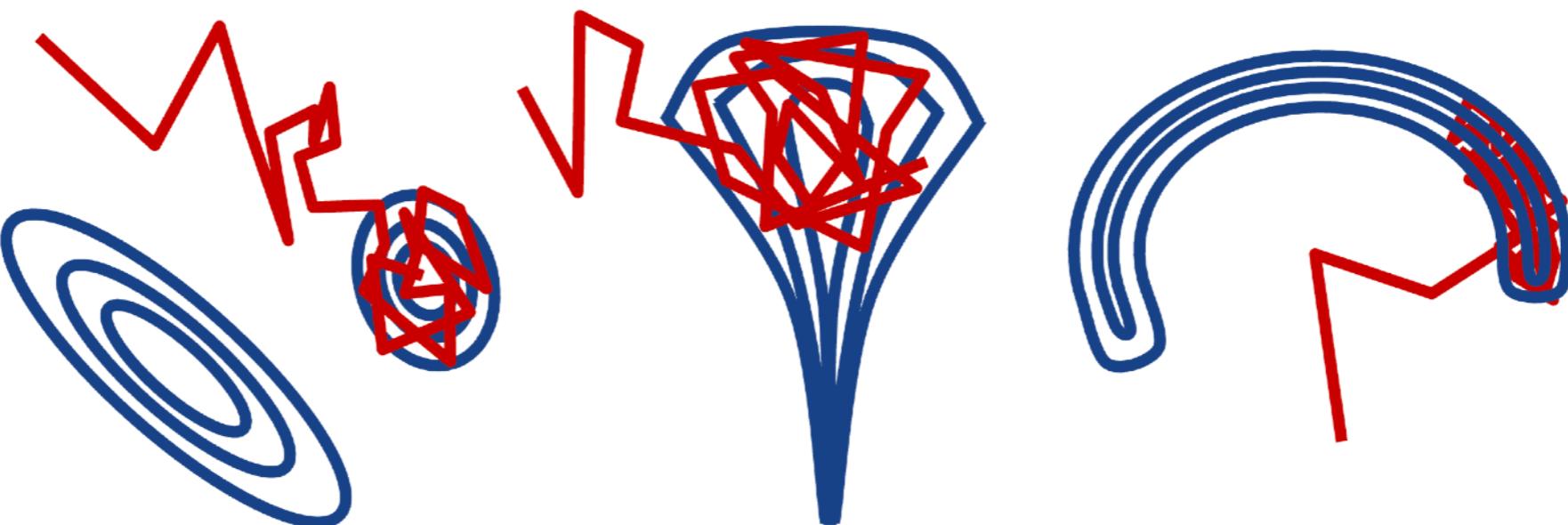


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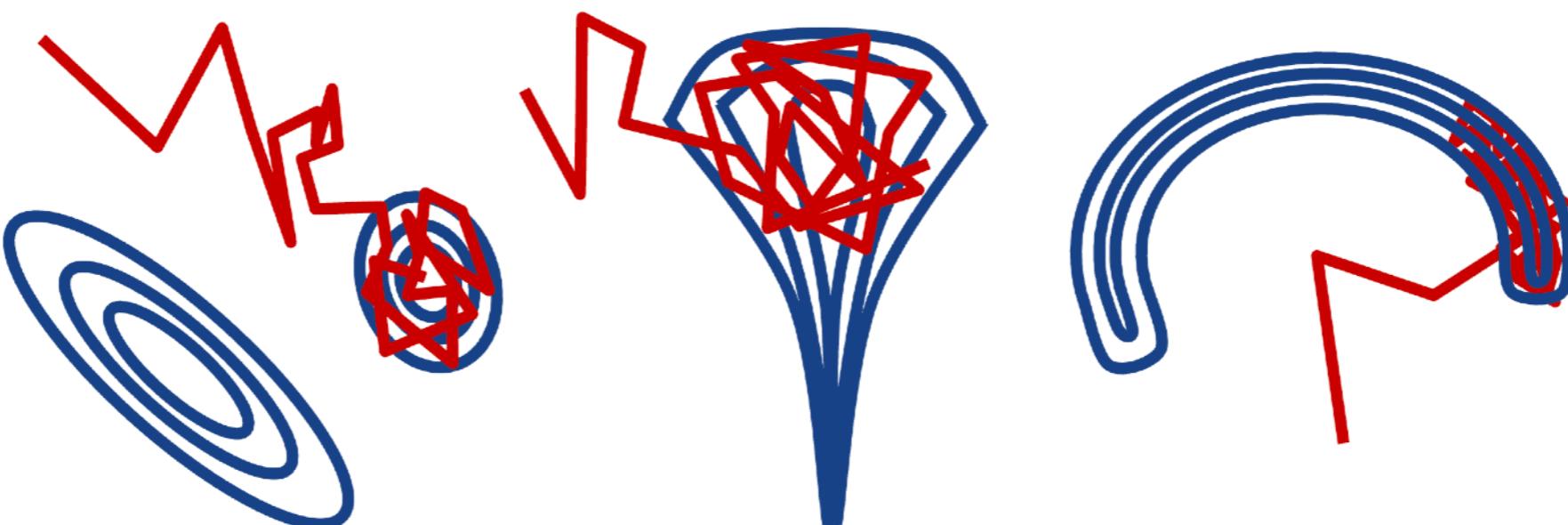


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- ▶ Converge exponentially fast within a mode but exponentially slow between modes



# NORMALISING CONSTANT

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- ▶ Can estimate  $Z$  using the Monte Carlo estimator

$$X_1, \dots, X_N \sim \eta \quad \hat{Z} = \frac{1}{N} \sum_{n=1}^N w(X_n)$$

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- ▶ **Unbiased:**

$$\mathbb{E}[\hat{Z}] = \frac{1}{N} \sum_{n=1}^N \mathbb{E}[w(X_n)] = Z$$

- ▶ **Consistent:** convergence is a.s.

$$\lim_{N \rightarrow \infty} \hat{Z} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N w(X_n) = Z$$

- ▶ **Variance:**

$$\mathbb{V}\left[\frac{\hat{Z}}{Z}\right] = \frac{1}{N} \mathbb{V}_{X \sim \eta} \left[ \frac{\pi(X)}{\eta(X)} \right] = \frac{e^{D_2(\pi\|\eta)} - 1}{N}$$

- ▶ The variance grows exponentially as  $\eta$  deviates from the target

$$D_2(\pi\|\eta) = \log \left( 1 + \mathbb{V}_{X \sim \eta} \left[ \frac{\pi(X)}{\eta(X)} \right] \right) \geq \text{KL}(\eta\|\pi)$$

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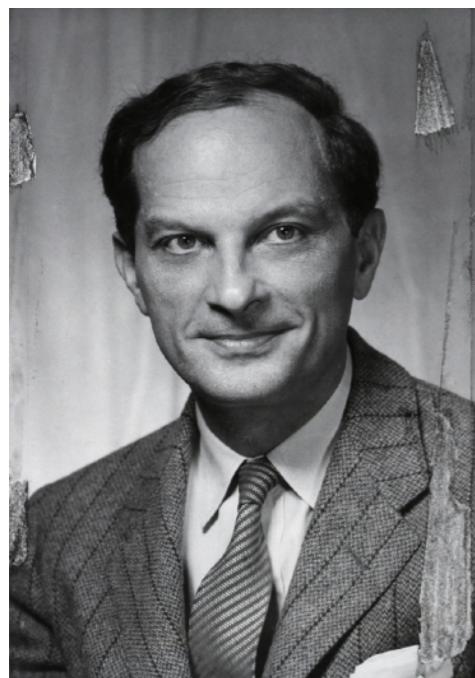
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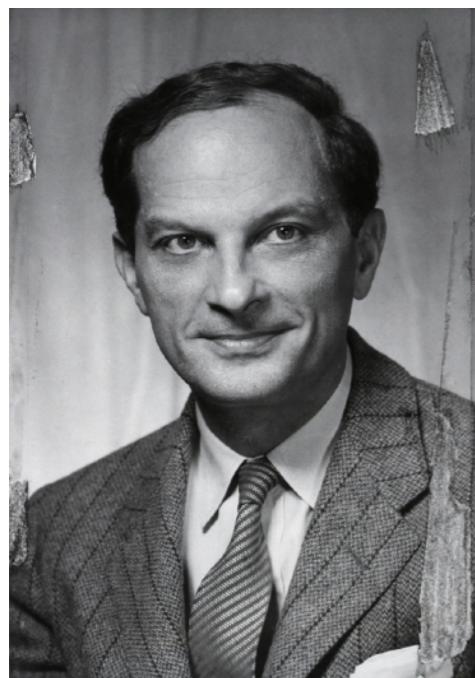
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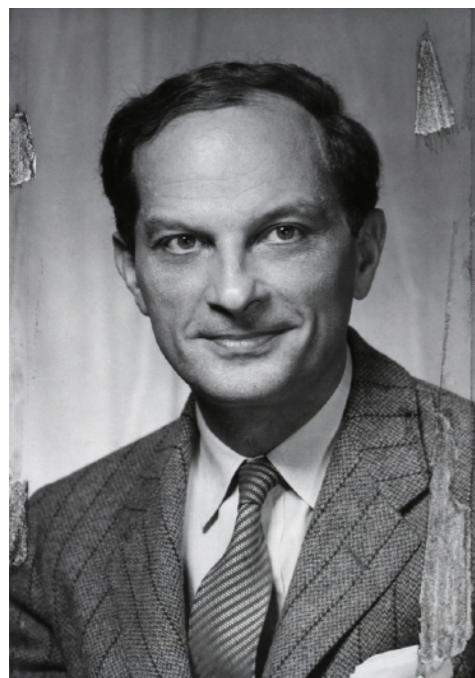
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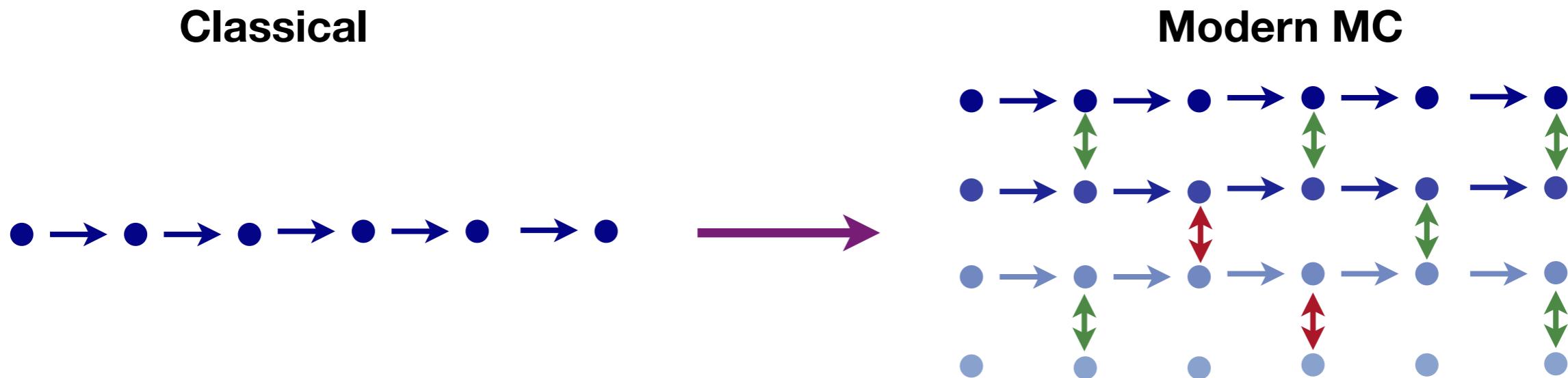
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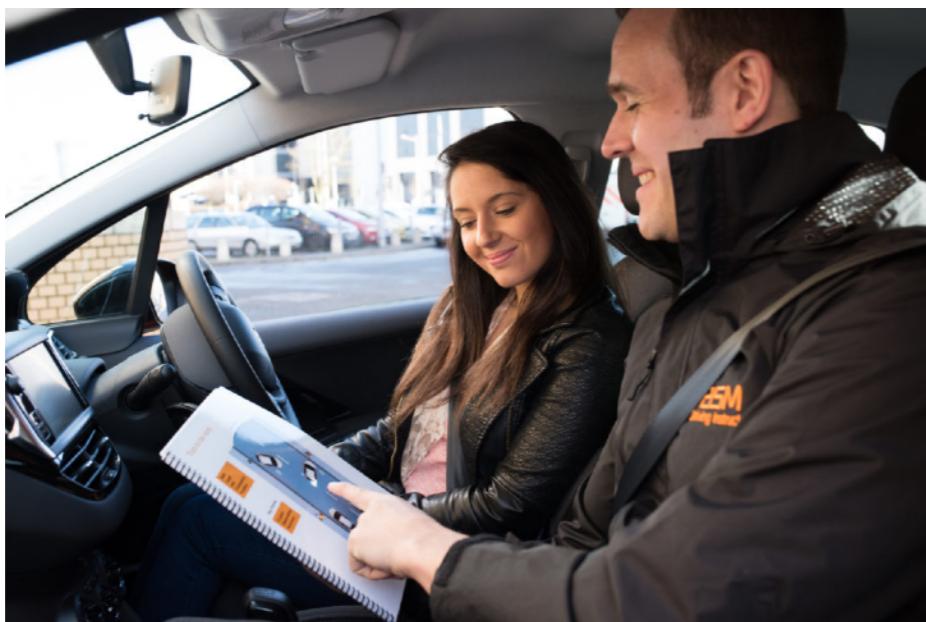
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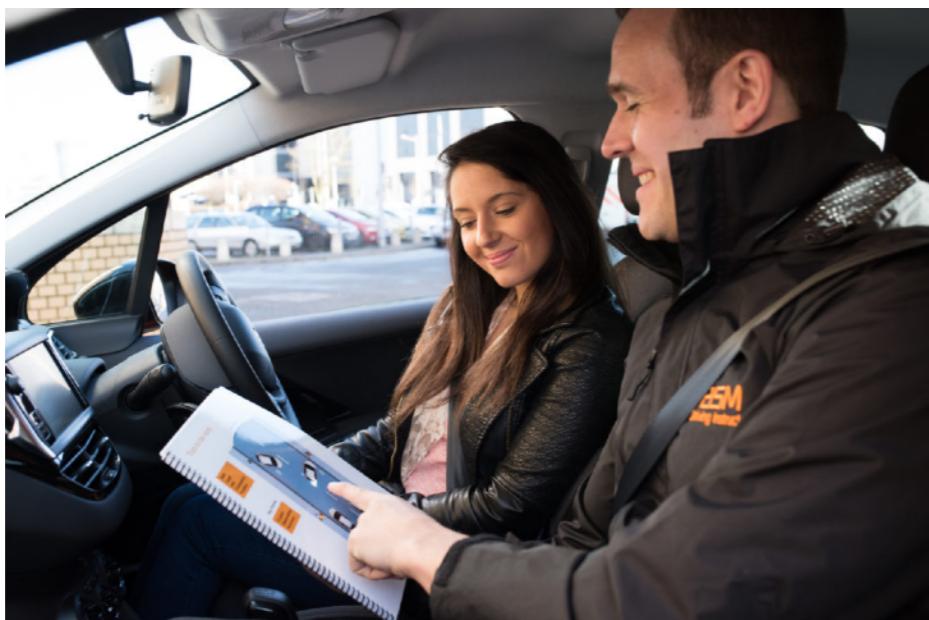
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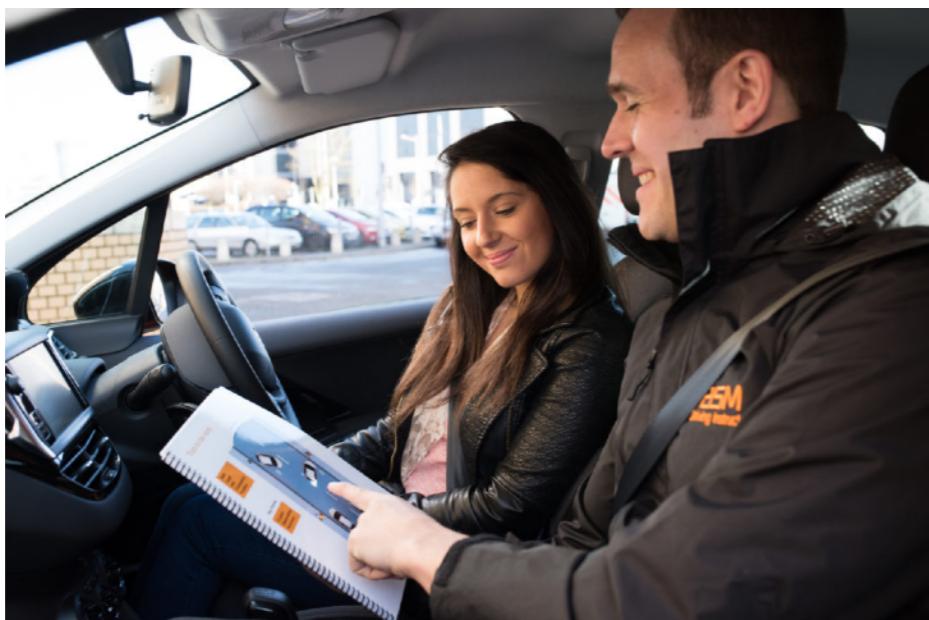
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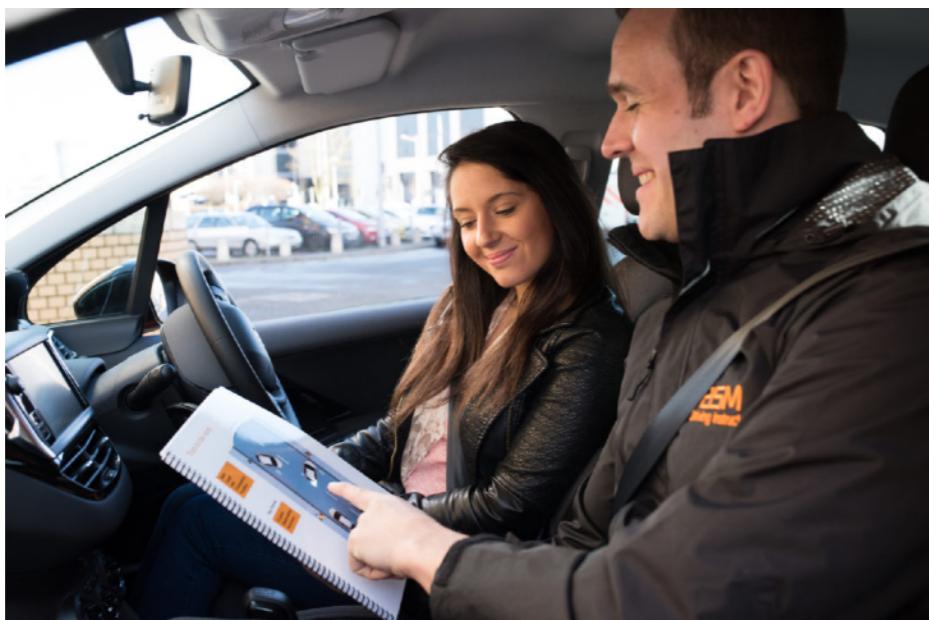
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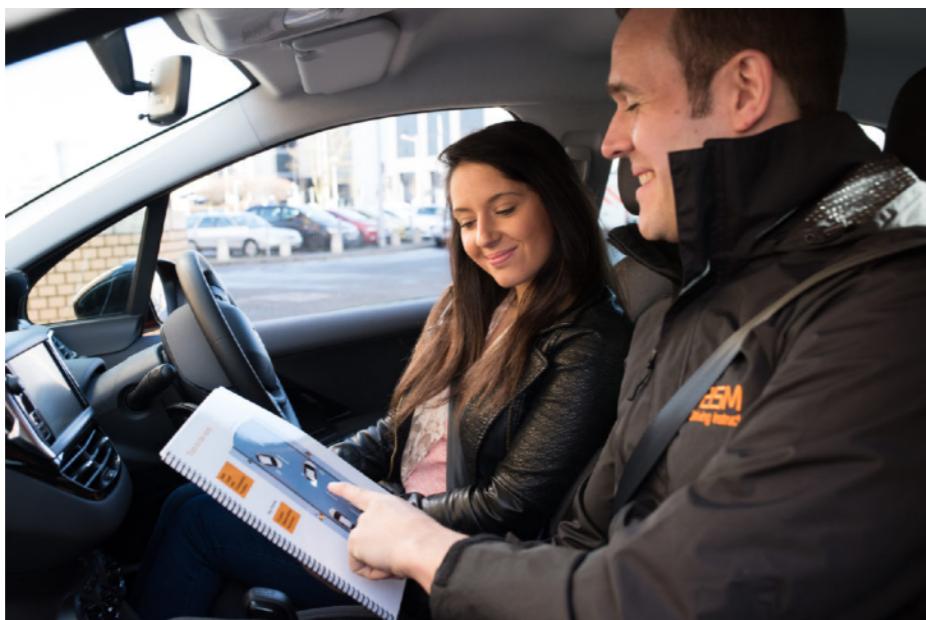
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Double sided printing, flexible plastic kid safe



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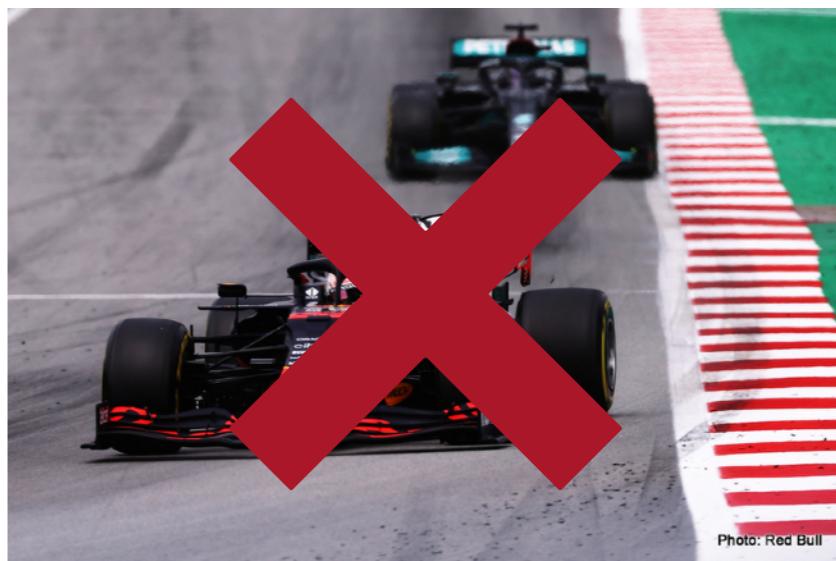
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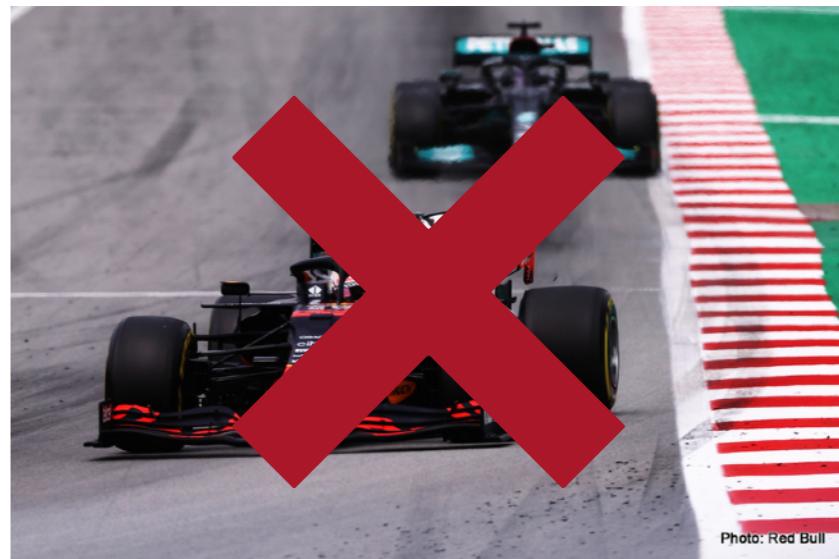
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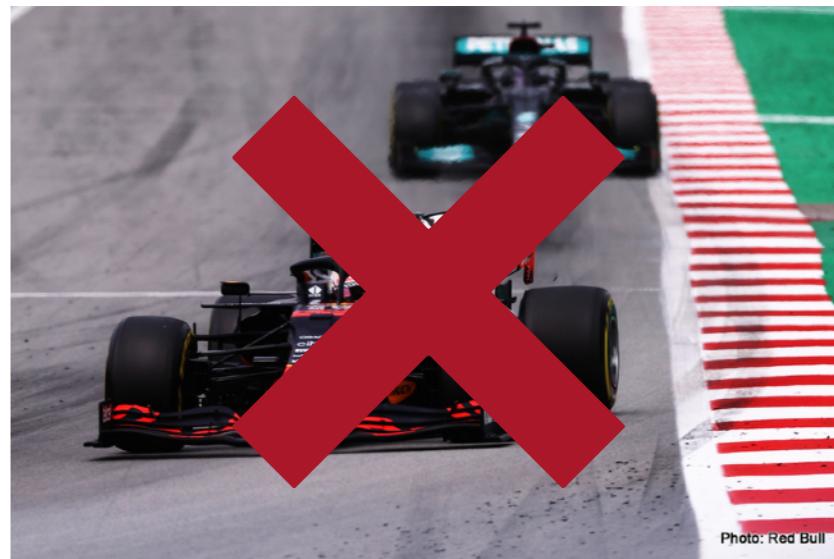
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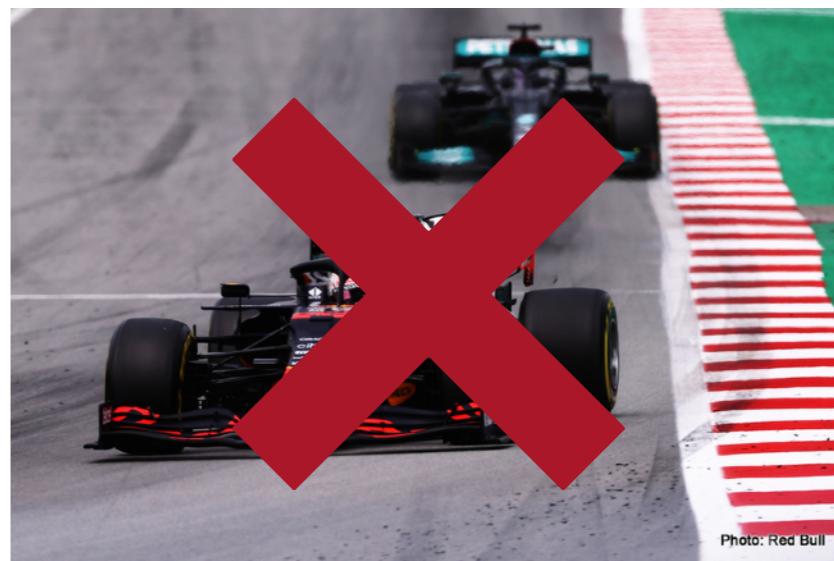
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  - ▶ Implementations in practice have their own challenges that are not our focus



# COURSE OUTLINE

## ▶ **Part 1: Foundations**

- ▶ MCMC theory
- ▶ Local inference algorithms
- ▶ Annealing

## ▶ **Part 2: Annealing algorithms**

- ▶ Parallel annealing
- ▶ Sequential annealing

## ▶ **Part 3: Free energy methods**

- ▶ Acceleration methods
- ▶ Enhanced sampling
- ▶ Optional topics