INVARIANCE

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- Corresponds to the left Eigenvector with Eigenvalue of 1

- We will say a K is π-invariant or π-stationary if $\pi K = \pi$

$$\pi K(\mathrm{d}x') = \int_{\mathbb{X}} \pi(\mathrm{d}x) K(x, \mathrm{d}x') = \pi(\mathrm{d}x')$$

- Products and mixtures of π-invariant kernels are π-invariant

- Invariant distributions may not be unique

- E.g. identity kernel is invariant to every distribution

$$K(x, \mathrm{d}x') = \delta_x(\mathrm{d}x')$$

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- Algorithmically, the distribution of $X \sim \pi$ is unchanged $X' \sim K(X, \mathrm{d}x') = \pi$

- Dicrete case:

$$\sum_x \pi(x) K(x, x') = \pi(x)$$

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REVERSIBILITY