ERGODIC THEOREM

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- **Theorem**: If X_t is a π-irreducible, π-invariant, Harris recurrent Markov chain, then for any integrable function $f : \mathbb{X} \to \mathbb{R}$ the following limit a.s. holds

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} f(X_t) = \pi[f]$$

- Unlike the Monte Carlo estimator, this is biased for a finite time T

- Given a function $f : \mathbb{X} \to \mathbb{R}$ we can approximate expectation using the average

$$\hat{\pi}_T[f] = \frac{1}{T} \sum_{t=1}^{T} f(X_t)$$

- How long does it take to forget the initial distribution $X_0 \sim \mu$ and enter the stationary regime approximating the target π?

- A μ-irreducible Markov chain X_t is Harris recurrent if $\forall x \in \mathbb{X}$, and $\mu(A) > 0$,

$$\mathbb{P}_x \left[\sum_t 1_A(X_t) = \infty \right] = 1$$

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MIXING-TIME