微分の公式と性質

2024.06.26

復習 (微分係数•導関数)

定義 (質問)

ullet 微分係数 f'(a) は定点 a における接線の傾き

$$f'(a) = \lim_{z o a}$$

問 0626-1 f'(a) の定義式をかけ

 \bullet 導関数 f'(x) は a を変数と考え,x とおいたもの

$$f'(x) = \lim_{z o x}$$

問 0626-2 f'(x) の定義式をかけ

導関数の書き方

- ullet 導関数 f'(x) を求めることを「微分する」
- 関数 y=f(x) を変数 x で微分する $y',\ f'(x)$ (ラグランジュ) $\frac{dy}{dx}$ (ライプニッツ)

[例]
$$y=f(x)=x^2$$

$$y'=f'(x)=f'=\left(x^2\right)'=2x$$

$$rac{dy}{dx} = rac{df}{dx} = rac{d}{dx}f(x) = rac{d}{dx}(x^2) = 2x$$

いろいろな関数の微分

$$\bullet \ (c)' = \lim_{z \to x} \frac{c - c}{z - x} = 0$$

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$$\bullet \ (ax)' = \lim_{z \to x} \frac{az - ax}{z - x} = \lim_{z \to x} \frac{a(z - x)}{z - x} = a$$

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$$=\lim_{z o x}rac{a(z-x)(z+x)}{z-x}$$

$$=\lim_{z o x}a(z+x)=2ax$$



•
$$(x^3)' = \lim_{z \to x} \frac{z^3 - x^3}{z - x}$$
 (1)

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$$z^3 - x^3 = (z - x)(z^2 + zx + x^2)$$

$$\bullet \ (x^3)' = \lim_{z \to x} \frac{z^3 - x^3}{z - x} \tag{1}$$

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$$ullet (1) = \lim_{z o x} rac{(z-x)(z^2+zx+x^2)}{z-x} = \lim_{z o x} (z^2+zx+x^2) = 3x^2$$

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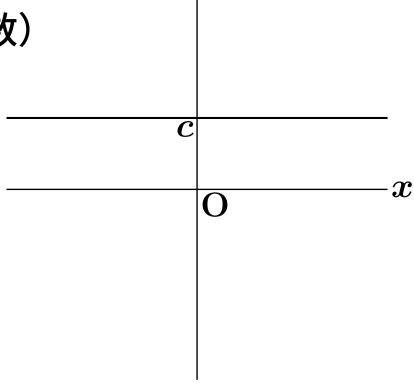
• 次の因数分解公式を用いる

$$z^3 - x^3 = (z - x)(z^2 + zx + x^2)$$

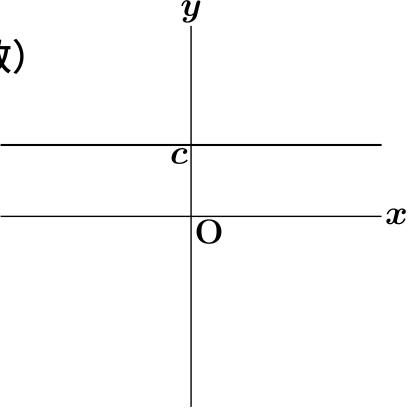
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問 0626-3 $(x^4)'$ を求めよ

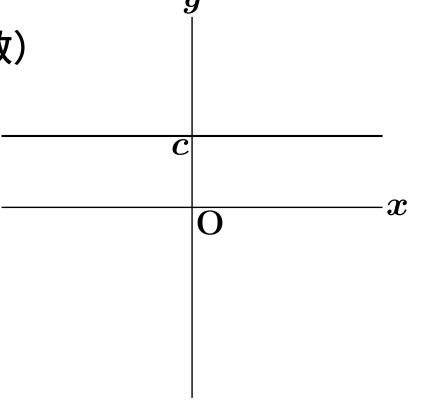
ullet 定数関数 f(x)=c(c は定数) (c)'=0



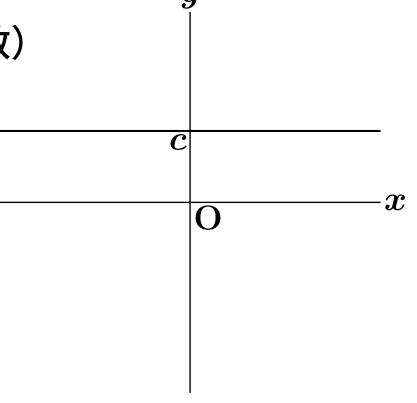
- 定数関数 f(x) = c (cは定数) (c)' = 0
- ullet f(x) = x $(x)' = \lim_{z o x} rac{z-x}{z-x} = 1$



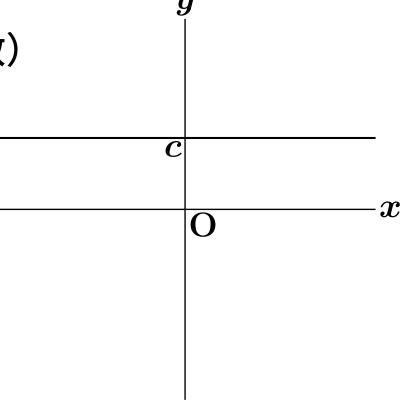
- 定数関数 f(x) = c (cは定数) (c)' = 0
- $ullet f(x) = x \ (x)' = \lim_{z o x} rac{z-x}{z-x} = 1$
- $\bullet \ (x^2)' = 2x$



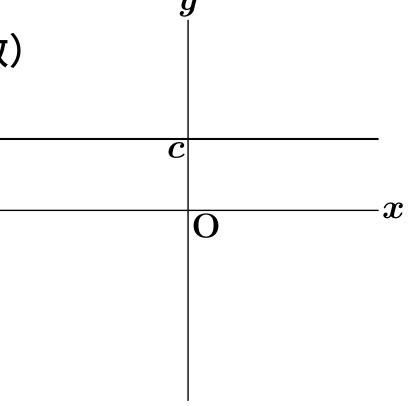
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- ullet 一般に $(x^n)'= \boxed{nx^{n-1}}$



微分の性質 (和と定数倍)

 $f(x),\ g(x)$ と定数 c について

$$\bullet (f+g)' = f'+g', (f-g)' = f'-g'$$

$$\bullet \ (cf)' = cf'$$

微分の性質 (和と定数倍)

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例)
$$(x^2+3x+4)'=(x^2)'+(3x)'+(4)'=2x+3$$

微分の性質 (和と定数倍)

f(x), g(x) と定数 c について

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例)
$$(x^2+3x+4)'=(x^2)'+(3x)'+(4)'=2x+3$$

問 0626-4 微分せよ

$$[1] \ y = 2x^2 - 3x + 2$$

[2]
$$y = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x + 1$$

積と商の微分・記法

$$ullet$$
 $|(fg)'=f'g+fg'|$ 積の微分公式

(fg)' = f'g + fg' 積の微分公式

$$\left(f(x)g(x)
ight)' = \lim_{z o x}rac{f(z)g(z)-f(x)g(x)}{z-x}$$

$$ullet$$
 $(fg)'=f'g+fg'$ 積の微分公式

$$egin{aligned} ig(f(x)g(x)ig)' &= \lim_{z o x} rac{f(z)g(z)-f(x)g(x)}{z-x} \ &= \lim_{z o x} rac{ig(f(z)-f(x)ig)g(z)+f(x)ig(g(z)-g(x)ig)}{z-x} \ &= \lim_{z o x} igg(rac{f(z)-f(x)}{z-x}g(z)+f(x)rac{g(z)-g(x)}{z-x}igg) \end{aligned}$$

$$egin{aligned} ig(f(x)g(x)ig)' &= \lim_{z o x} rac{f(z)g(z) - f(x)g(x)}{z - x} \ &= \lim_{z o x} rac{ig(f(z) - f(x)ig)g(z) + f(x)ig(g(z) - g(x)ig)}{z - x} \ &= \lim_{z o x} igg(rac{f(z) - f(x)}{z - x}g(z) + f(x)rac{g(z) - g(x)}{z - x}igg) \ &= f'(x)g(x) + f(x)g'(x) \end{aligned}$$

例
$$y' = \left((x+1)(x^2+2x+3)\right)'$$

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= $(x+1)'(x^2+2x+3)+(x+1)(x^2+2x+3)'$

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 $= (x+1)'(x^2+2x+3)+(x+1)(x^2+2x+3)'$
 $= (x^2+2x+3)+(x+1)(2x+2)$
 $= 3x^2+6x+5$

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 $= 3x^2+6x+5$

問 0626-5 積の微分公式で微分せよ.

[1]
$$y = (x+1)(x+3)$$
 [2] $y = x^2(x+2)$

商の微分

$$ullet \left(rac{f}{g}
ight)' = rac{f'\,g - f\,g'}{g^2}$$
 商の微分公式

$$ullet$$
 $\left(rac{f}{g}
ight)' = rac{f'\,g - f\,g'}{g^2}$ 商の微分公式

[例
$$(1)$$
] $\left(\frac{2x+1}{3x+1}\right)'$

$$ullet$$
 $\left(rac{f}{g}
ight)' = rac{f'\,g - f\,g'}{g^2}$ 商の微分公式

[例
$$(1)$$
] $\left(\frac{2x+1}{3x+1}\right)'$
$$= \frac{(2x+1)'(3x+1)-(2x+1)(3x+1)'}{(3x+1)^2}$$

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 $\left(rac{f}{g}
ight)' = rac{f'\,g - f\,g'}{g^2}$ 商の微分公式

[例 (1)]
$$\left(\frac{2x+1}{3x+1}\right)'$$

$$= \frac{(2x+1)'(3x+1) - (2x+1)(3x+1)'}{(3x+1)^2}$$

$$= \frac{2(3x+1) - 3(2x+1)}{(3x+1)^2}$$

$$ullet \left(rac{f}{g}
ight)' = rac{f'\,g - f\,g'}{g^2}$$
 商の微分公式

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$$ullet$$
 $\left|\left(rac{f}{g}
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$$\left[oldsymbol{\emptyset}\left(2
ight)
ight]\,\left(rac{1}{x}
ight)'$$

$$ullet \left(rac{f}{g}
ight)' = rac{f'\,g - f\,g'}{g^2}$$
 商の微分公式

[例 (1)]
$$\left(\frac{2x+1}{3x+1}\right)'$$

$$= \frac{(2x+1)'(3x+1) - (2x+1)(3x+1)'}{(3x+1)^2}$$

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[例(2)]
$$\left(\frac{1}{x}\right)' = \frac{(1)'(x) - 1(x)'}{x^2}$$

$$ullet \left(rac{f}{g}
ight)' = rac{f'\,g - f\,g'}{g^2}$$
 商の微分公式

[例 (1)]
$$\left(\frac{2x+1}{3x+1}\right)'$$

$$= \frac{(2x+1)'(3x+1) - (2x+1)(3x+1)'}{(3x+1)^2}$$

$$= \frac{2(3x+1) - 3(2x+1)}{(3x+1)^2} = \frac{-1}{(3x+1)^2}$$
[例 (2)] $\left(\frac{1}{x}\right)' = \frac{(1)'(x) - 1(x)'}{x^2} = \frac{0-1}{x^2}$

$$ullet \left(rac{f}{g}
ight)' = rac{f'\,g - f\,g'}{g^2}$$
 商の微分公式

$$\left[\cancel{ \bigcirc } (1) \right] \left(\frac{2x+1}{3x+1} \right)'$$

$$= \frac{(2x+1)'(3x+1) - (2x+1)(3x+1)'}{(3x+1)^2}$$

$$= \frac{2(3x+1) - 3(2x+1)}{(3x+1)^2} = \frac{-1}{(3x+1)^2}$$

[例(2)]
$$\left(\frac{1}{x}\right)' = \frac{(1)'(x) - 1(x)'}{x^2} = \frac{0 - 1}{x^2} = -\frac{1}{x^2}$$

$$ullet$$
 $\left(rac{f}{g}
ight)' = rac{f'\,g - f\,g'}{g^2}$ 商の微分公式

[例 (1)]
$$\left(\frac{2x+1}{3x+1}\right)'$$

$$= \frac{(2x+1)'(3x+1)-(2x+1)(3x+1)'}{(3x+1)^2}$$

$$= \frac{2(3x+1)-3(2x+1)}{(3x+1)^2} = \frac{-1}{(3x+1)^2}$$
[例 (2)] $\left(\frac{1}{x}\right)' = \frac{(1)'(x)-1(x)'}{x^2} = \frac{0-1}{x^2} = -\frac{1}{x^2}$
問 0626-6 $y = \frac{x}{x+1}$ を微分せよ

ullet n が正の整数のとき $|(x^n)'=nx^{n-1}|$

$$(x^n)' = nx^{n-1}$$

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• 分数乗

$$(x^{\frac{1}{2}})'=(\sqrt{x})'$$

ullet n が正の整数のとき $|(x^n)'=nx^{n-1}|$

$$(x^n)' = nx^{n-1}$$

• 分数乗

$$(x^{rac{1}{2}})'=(\sqrt{x})'=\lim_{z o x}rac{\sqrt{z}-\sqrt{x}}{z-x}$$

ullet n が正の整数のとき $|(x^n)'=nx^{n-1}|$

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• 分数乗

$$(x^{rac{1}{2}})'=(\sqrt{x})'=\lim_{z o x}rac{\sqrt{z}-\sqrt{x}}{z-x}$$
 $\sqrt{z}=w,\sqrt{x}=u$ とおくと $z=w^2,x=u^2$

ullet n が正の整数のとき $|(x^n)' = nx^{n-1}|$

$$(x^n)' = nx^{n-1}$$

$$(x^{rac{1}{2}})' = (\sqrt{x})' = \lim_{z o x} rac{\sqrt{z} - \sqrt{x}}{z - x} = \lim_{w o u} rac{w - u}{w^2 - u^2}$$
 $\sqrt{z} = w, \sqrt{x} = u$ とおくと $z = w^2, x = u^2$

x^p の微分 |

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 $\sqrt{z} = w, \sqrt{x} = u$ とおくと $z = w^2, x = u^2$
 $= \lim_{w o u} rac{1}{w + u}$

x^p の微分 |

ullet n が正の整数のとき $|(x^n)' = nx^{n-1}|$

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 $\sqrt{z} = w, \sqrt{x} = u$ とおくと $z = w^2, x = u^2$ $= \lim_{w o u} rac{1}{w + u} = rac{1}{2u}$

ullet n が正の整数のとき $|(x^n)' = nx^{n-1}|$

$$(x^n)' = nx^{n-1}$$

$$(x^{rac{1}{2}})' = (\sqrt{x})' = \lim_{z o x} rac{\sqrt{z} - \sqrt{x}}{z - x} = \lim_{w o u} rac{w - u}{w^2 - u^2}$$
 $\sqrt{z} = w, \sqrt{x} = u$ とおくと $z = w^2, x = u^2$
 $= \lim_{w o u} rac{1}{w + u} = rac{1}{2u} = rac{1}{2\sqrt{x}}$

$oldsymbol{x}^p$ の微分

ullet n が正の整数のとき $|(x^n)' = nx^{n-1}|$

$$(x^n)' = nx^{n-1}$$

$$(x^{rac{1}{2}})' = (\sqrt{x})' = \lim_{z o x} rac{\sqrt{z} - \sqrt{x}}{z - x} = \lim_{w o u} rac{w - u}{w^2 - u^2} \ orall_{z = w, \sqrt{x} = u}
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分数乗

$$(x^{rac{1}{2}})' = (\sqrt{x})' = \lim_{z o x} rac{\sqrt{z} - \sqrt{x}}{z - x} = \lim_{w o u} rac{w - u}{w^2 - u^2} \ orall_{z = w, \sqrt{x} = u} ag{55} < c \quad z = w^2, x = u^2 \ = \lim_{w o u} rac{1}{w + u} = rac{1}{2u} = rac{1}{2\sqrt{x}} = rac{1}{2}x^{-rac{1}{2}}$$

問 0626-7 $y=x^{rac{3}{2}}=x\sqrt{x}$ を微分せよ.

$$ullet (x^p)' =$$

$$ullet (x^p)' = ig| px^{p-1}$$

- $ullet (x^p)' = px^{p-1}$
- ulletマイナス乗も同じ $(rac{1}{x})'$

- $ullet (x^p)' = \boxed{px^{p-1}}$
- マイナス乗も同じ $(rac{1}{x})'=(x^{-1})'$

$$ullet (x^p)' = \boxed{px^{p-1}}$$

• マイナス乗も同じ
$$(\frac{1}{x})' = (x^{-1})' = -x^{-2}$$

$$ullet (x^p)' = igg| px^{p-1}$$

• マイナス乗も同じ
$$(\frac{1}{x})'=(x^{-1})'=-x^{-2}=-\frac{1}{x^2}$$

$$ullet (x^p)' = igg| px^{p-1}$$

• マイナス乗も同じ

$$(\frac{1}{x})' = (x^{-1})' = -x^{-2} = -\frac{1}{x^2}$$

問 0626-8 次の関数を微分せよ.

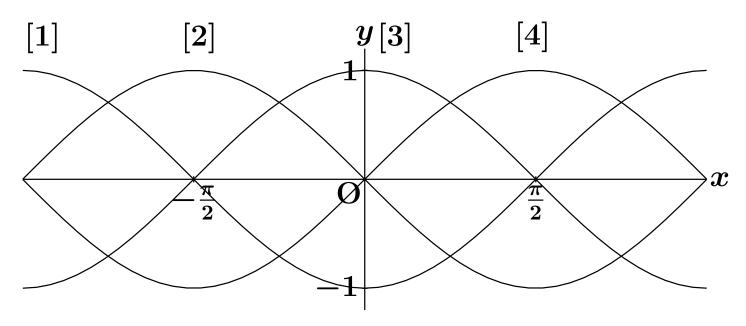
$$[1] \,\,\, y \,\,=\,\, x^{rac{1}{4}}$$

$$[2] y = x^{-2}$$

$$[1] \,\, y \, = \, x^{rac{1}{4}} \qquad \qquad [2] \,\, y \, = \, x^{-2} \qquad \qquad [3] \,\, y \, = \, x^{-rac{1}{2}} \, .$$

三角関数の微分

三角関数のグラフ



問 0626-9 図は

 $y = \sin x, y = \cos x, y = -\sin x, y = -\cos x$ のグラフである. アプリを用いて,関数の番号を答えよ.

$\sin x,\cos x$ の微分

問 0626-10 アプリを用いて導関数を求めよ.

$$[1] \,\, y = \sin x$$

$$[1] y = \sin x \qquad [2] y = \cos x$$

$\sin x, \cos x$ の微分

問 0626-10 アプリを用いて導関数を求めよ.

$$[1] y = \sin x$$

$$[2] y = \cos x$$

• 微分公式

$$(\sin x)' = \cos x, \ (\cos x)' = -\sin x$$

$\sin x,\cos x$ の微分

問 0626-10 アプリを用いて導関数を求めよ.

$$[1] y = \sin x$$

$$[2] y = \cos x$$

• 微分公式

$$(\sin x)' = \cos x, \ (\cos x)' = -\sin x$$

問 0626-11 次の関数を微分せよ

$$y = 2\sin x - 3\cos x$$

tan x の微分

$$\bullet \left| (\tan x)' = \frac{1}{\cos^2 x} \right|$$

$$\tan x = \frac{\sin x}{\cos x}$$
$$\cos^2 x = (\cos x)^2$$

tan x の微分

$$\bullet \left| (\tan x)' = \frac{1}{\cos^2 x} \right|$$

$$(\tan x)' = (\frac{\sin x}{\cos x})'$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cos^2 x = (\cos x)^2$$

an x の微分

$$\begin{array}{ll}
\bullet & \left| (\tan x)' = \frac{1}{\cos^2 x} \right| & \tan x = \frac{\sin x}{\cos x} \\
(\tan x)' = \left(\frac{\sin x}{\cos x} \right)' & \cos^2 x = (\cos x)^2 \\
& = \frac{(\sin x)'(\cos x) - (\sin x)(\cos x)'}{\cos^2 x}
\end{array}$$

an x の微分

tan x の微分

$$\bullet \left[(\tan x)' = \frac{1}{\cos^2 x} \right] \quad \tan x = \frac{\sin x}{\cos x}
(\tan x)' = \left(\frac{\sin x}{\cos x} \right)'
= \frac{(\sin x)'(\cos x) - (\sin x)(\cos x)'}{\cos^2 x}
= \frac{(\cos x \cos x) - \sin x(-\sin x)'}{\cos^2 x}
= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

質問

問 0626-12 次の関数を微分せよ

- $[1] y = \sin x \cos x$
- $[2] y = \sin^2 x (= \sin x \sin x)$
- $[3] y = x \tan x$
- $[4] y = \tan x x$