Two Methods for Proving Japanese Theorem II
Using Maxima and KeTCindy
:An Application of the MNR Method

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Wasan and Maxima

Problems of Wasan(Sangaku)

- They are beautiful but hard to solve.

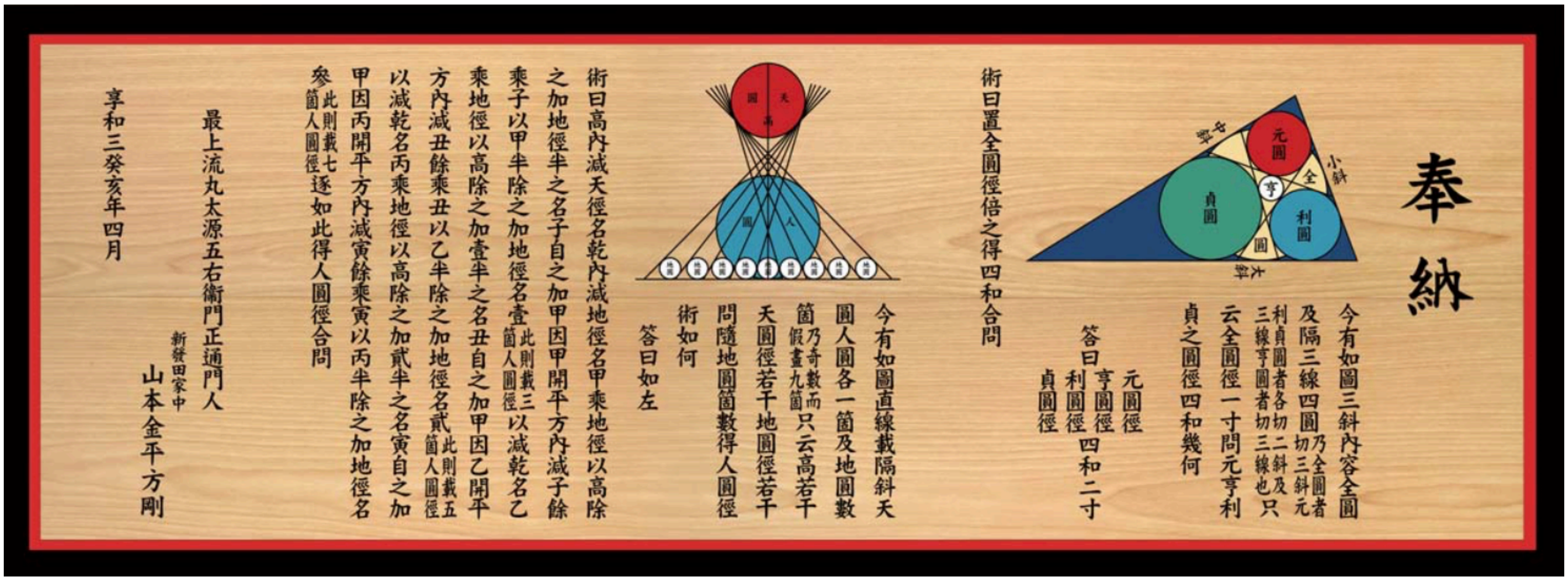

Fig.1 Hakusan Shrine [*1,2]

*1:Wakuta,Togawa:The Sangaku Lost from the Hakusan Shrine in Niigata

*2:The left problem is called ‘Japanese Theorem II’

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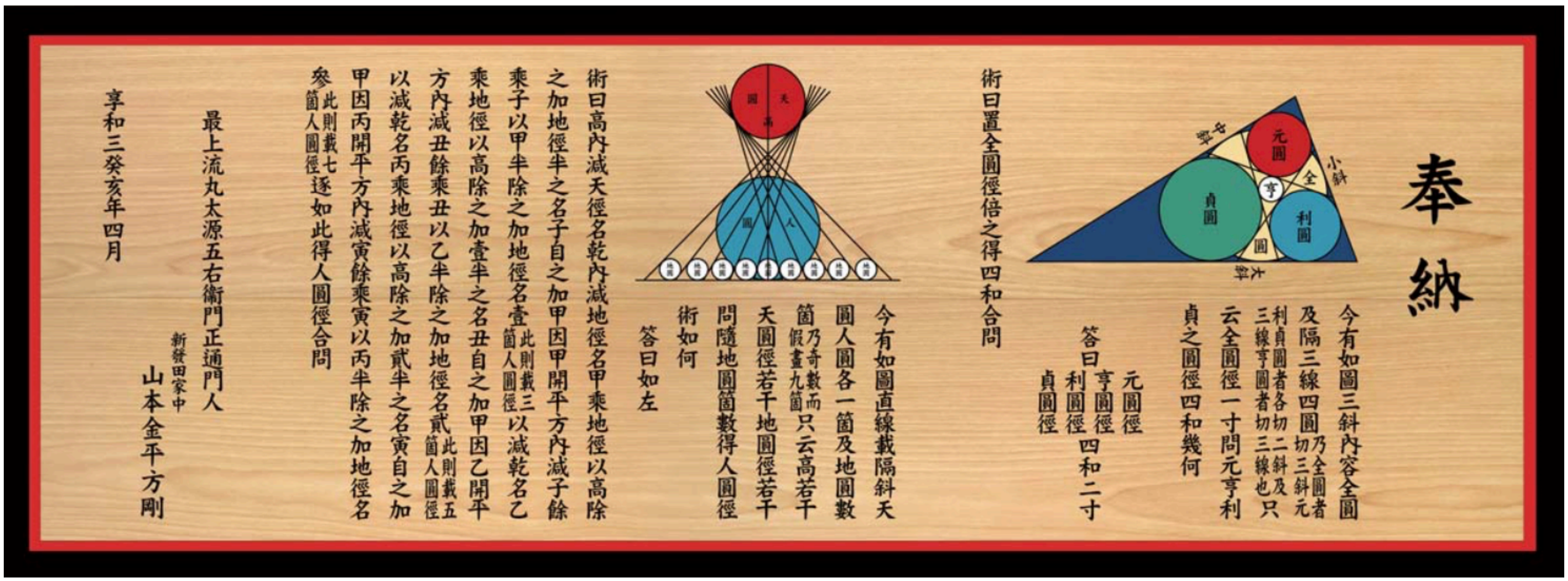


Fig.1 Hakusan Shrine [*1,2]

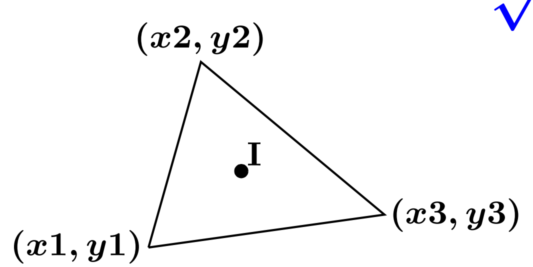
*1:Wakuta,Togawa:The Sangaku Lost from the Hakusan Shrine in Niigata

*2:The left problem is called ‘Japanese Theorem II’

- We want to solve them with Maxima.

To solve a simple geometry problem

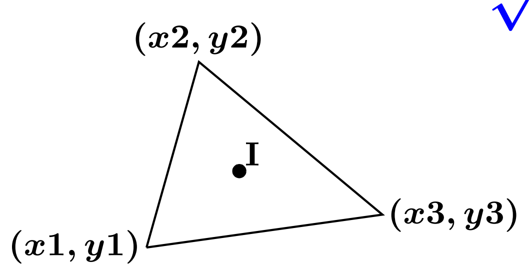
Distance point and a line $d = \frac{|ax+by+c|}{\sqrt{a^2+b^2}}$



- **Find the incenter of a triangle**

To solve a simple geometry problem

Distance point and a line $d = \frac{|ax+by+c|}{\sqrt{a^2+b^2}}$



- **Find the incenter of a triangle**
- **Create simultaneous equations**

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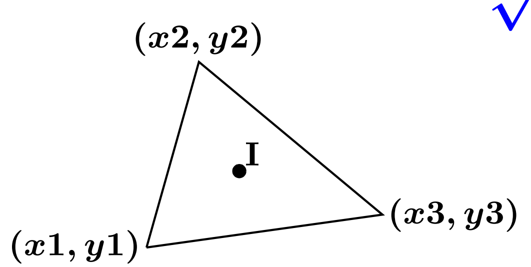
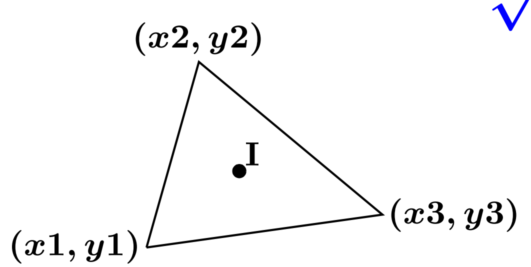


- **Find the incenter of a triangle**
- **Create simultaneous equations**

```
tmp1:((y2-y1)*x-(x2-x1)*y-(y2-y1)*x1+(x2-x1)*y1)^2;
tmp2:(y2-y1)^2+(x2-x1)^2;
tmp3:((y3-y1)*x-(x3-x1)*y-(y3-y1)*x1+(x3-x1)*y1)^2;
tmp4:(y3-y1)^2+(x3-x1)^2;
tmp5:((y3-y2)*x-(x3-x2)*y-(y3-y2)*x2+(x3-x2)*y2)^2;
tmp6:(y3-y2)^2+(x3-x2)^2;
eq1:factor(tmp1*tmp4-tmp2*tmp3);
eq2:factor(tmp1*tmp6-tmp5*tmp2);
sol:algsys([eq1,eq2],[x,y]);
```

To solve a simple geometry problem

Distance point and a line $d = \frac{|ax+by+c|}{\sqrt{a^2+b^2}}$



- Find the incenter of a triangle
- Create simultaneous equations

tmp1:((y2-y1)*x-(x2-x1)*y-(y2-y1)*x1+(x2-x1)*y1)^2;

tmp2:(y2-y1)^2+(x2-x1)^2;

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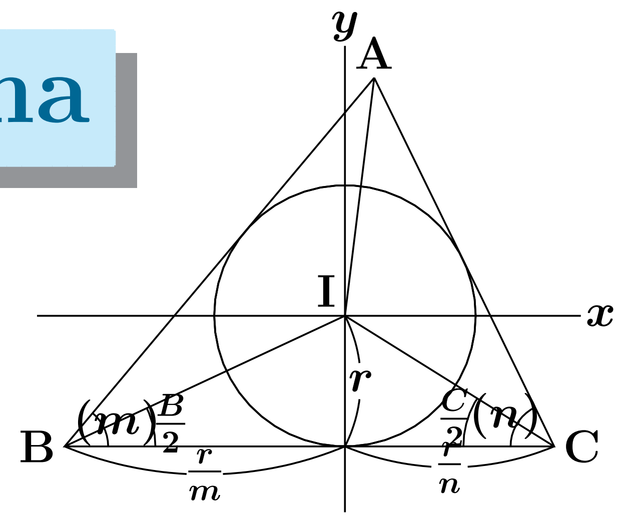
tmp4:(y3-y1)^2+(x3-x1)^2;

tmp5:((y3-y2)*x-(x3-x2)*y-(y3-y2)*x2+(x3-x2)*y2)^2;
tmp6:(y3-y2)^2+(x3-x2)^2;
eq1:factor(tmp1*tmp4-tmp2*tmp3);
eq2:factor(tmp1*tmp6-tmp5*tmp2);
sol:algsys([eq1,eq2],[x,y]);

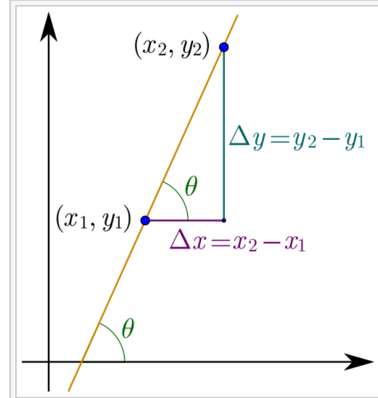
$\Longrightarrow$ We can not find the solution.

MNR package for Maxima



- $m = \tan\frac{B}{2}, n = \tan\frac{C}{2}$
  $r =$ radius of the inscribed circle

MNR package for Maxima

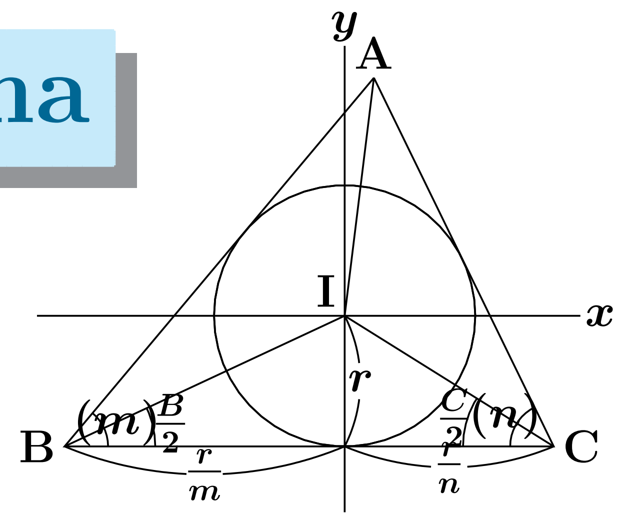


Slope: $m = \frac{\Delta y}{\Delta x} = \tan(\theta)$

- $m = \tan\frac{B}{2}, n = \tan\frac{C}{2}$

  $r =$ radius of the inscribed circle

MNR package for Maxima



- $m = \tan\frac{B}{2}, n = \tan\frac{C}{2}$
  r = radius of the inscribed circle

  We write $\angle B = (m),\ \angle C = (m)$

- All quantities of the triangle can be expressed as rational expressions of m, n, and r.

vtx $\mathrm{B}(-\frac{\mathrm{r}}{\mathrm{m}},\ -\mathrm{r}), \mathrm{C}(\frac{\mathrm{r}}{\mathrm{n}},\ -\mathrm{r}), \mathrm{A}\left(\frac{\mathrm{r(n-m)}}{\mathrm{1-mn}}, \frac{\mathrm{1+mn}}{\mathrm{1-mn}}\right)$

edg $\mathrm{BC} = \frac{\mathrm{r}}{\mathrm{m}} + \frac{\mathrm{r}}{\mathrm{n}}, \mathrm{AB} = \frac{\mathrm{r(1+m^2)}}{\mathrm{m(1-mn)}}, \mathrm{AC} = \frac{\mathrm{r(1+n^2)}}{\mathrm{n(1-mn)}}$

inC,inR, cirC,cirR

excircle exCa, exRa, exCb, exRb, exCc,exRc

MNR Commands 1

- Basic commands

  putT(m,n,r) Place a triangle T
  with incenter is [0,0]
  slideT(p1,p2) Translate T so that p1 to p2
  rotateT(m,p) Rotate T around p by (m)

- Main commands for transformation

numer(f),denom(f)	Simp. numer/denom
frfactor(f)	Simp. fraction
frev(eq,rep)	Subst. and simp.

MNR Commands 2

- **Main commands for figures**

  supA(m) Supplementary angles

  comA(m) Complementary angles

  dotProd(v1,v2),crossProd(v1,v2)

  lenSeg2(p1,p2) Square of the distance

  comTan1 Common tangent of Circles

  comTan1C Common tangent (Cross type)

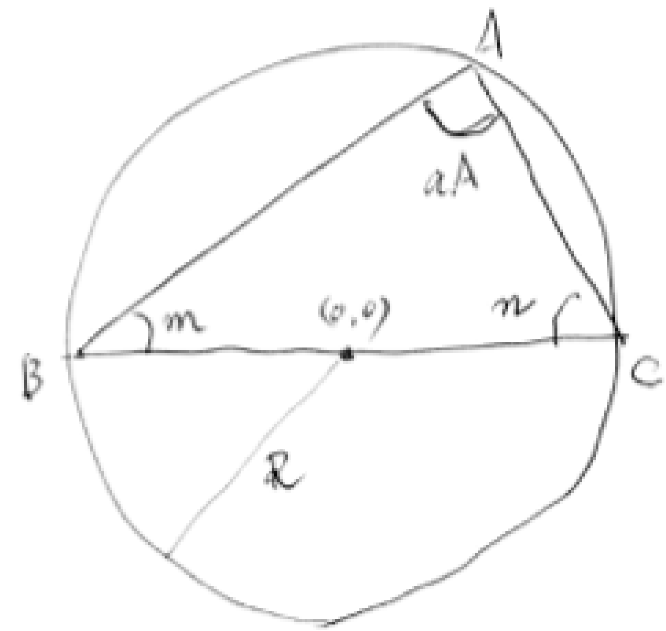
Practice of MNR

Overall flow

(1) Rename the sample file of ketcindy.
(2) Readmnr(1,1,1) in the ketlib slot and go
(3) Copy (file+)ketlib.txt to ketlib slot
(4) Copy (file+)figures.txt to figures slot
(5) Draw a rought sketch
(6) Write scripts in mkcmd.txt
(7) Remove // in figures slot
(8) Click buttons in the screen
(9) Then the formulas and figures are displayed

Circular angle on diameter

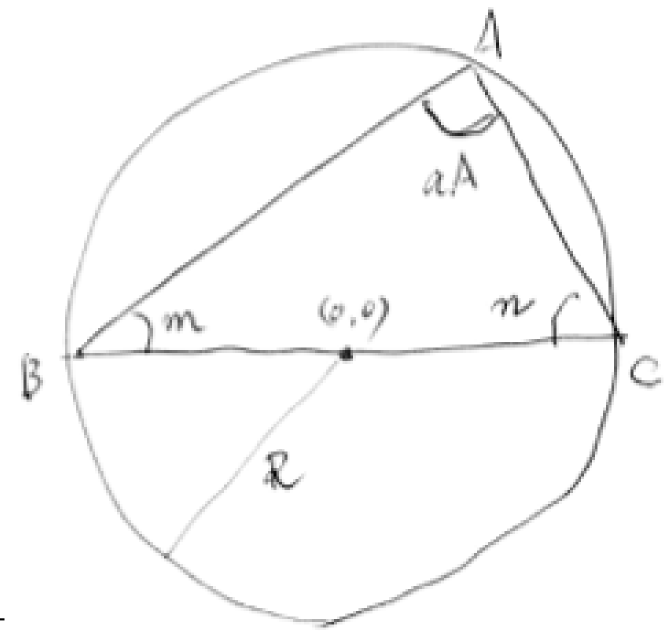
Demo:1circularangle.cdy



Circular angle on diameter

Demo:1circularangle.cdy

```
mkcmd1():=(
 cmdL1=concat(Mxbatch("mnr"),[
 "putT(m,n,r)",
 "eq:vtxL[2]-cirC[2]",
 "sol:solve(eq,m)",
 "fe:frevL([vtxT,vtxL,vtxR,cirC,cirR,angT],sol[1])",
 "A:fe[1]; B:fe[2]; C:fe[3]; O:fe[4]; R:fe[5]; aA:fe[6]",
 "end"
 ]);
 var1="sol::A::B::C::O::R::aA";
 Pos=NE.xy+[0.5,-0.5]; Dy=1;
);
Dispfig1(r,n):=(
  Setwindow([-5,5],[-5.5,3]);
//   r=1.5; n=tanhalf(40);
  Parsevv(var1);
  Listplot("1",[A,B,C,A]);
  Circledata("1",[O,R]);
  Letter([A,"n","A",B,"w","B",C,"e","C"]);
);
```



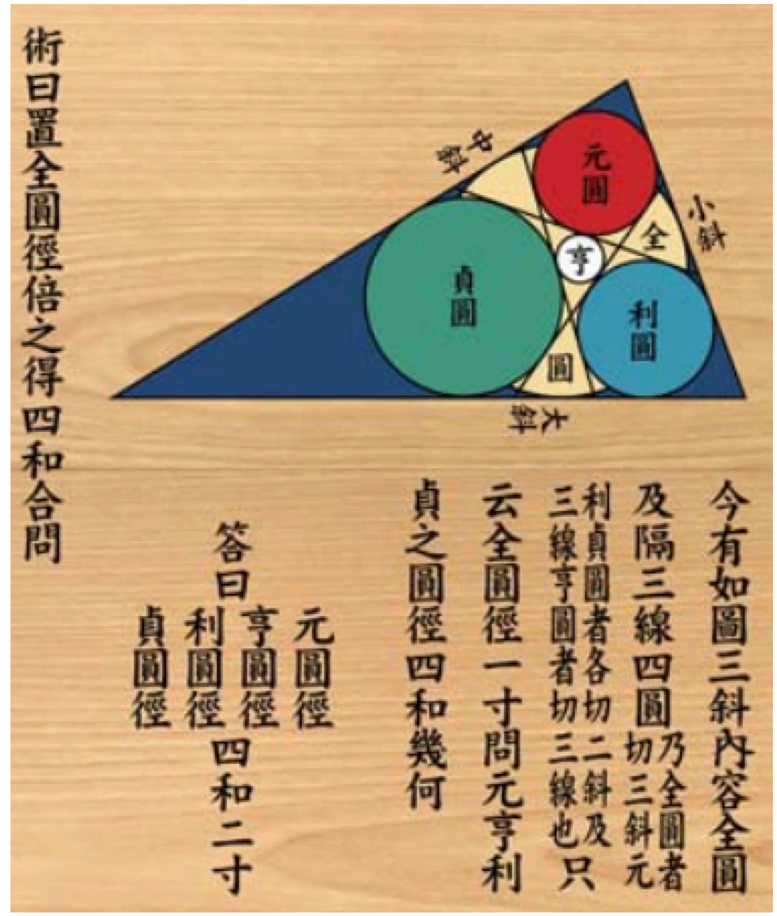
Solving Japanese Th.II

Question, Answer, Solution

Q Let the radius of the inscribed circle of the large triangle be R, the radius of the inscribed circle, the radii of the three circumscribed circles of the small triangle be r,r1,r2,r3. Find the sum r+r1 +r2 +r3.

A $r + r1 + r2 + r3 = 2R$

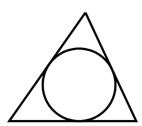
S Multiply R by 2 to get the answer.

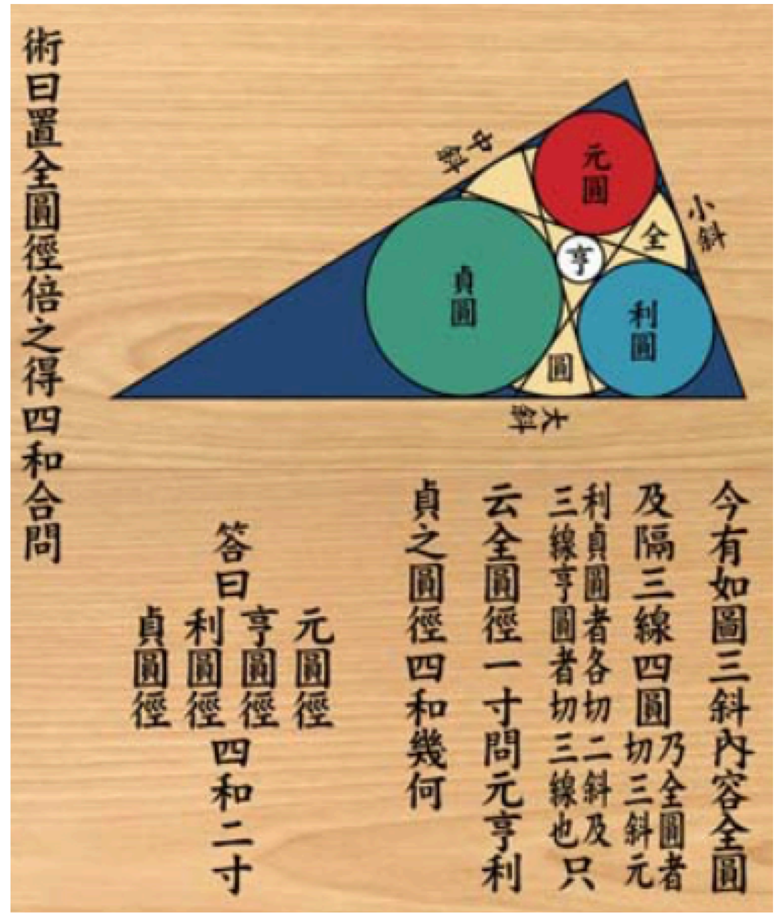


Solution1 (from inside)

Demo:Japanesetheorem2in.cdy

(1) r is of the smallest T



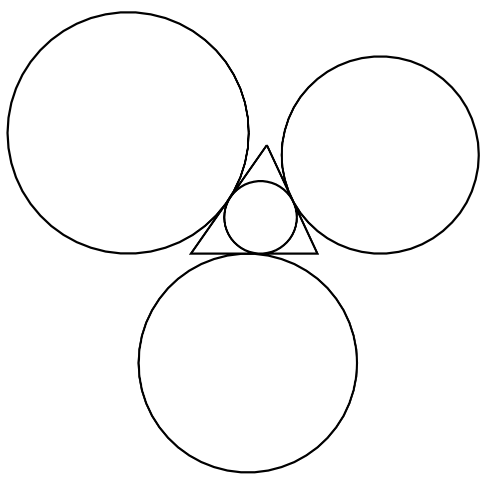


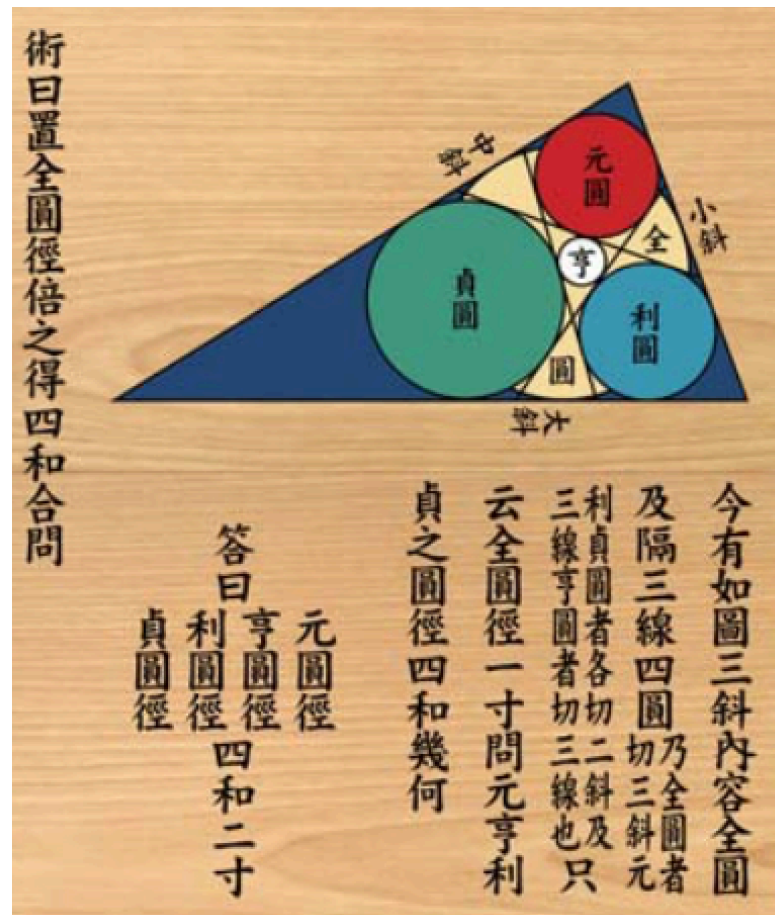
Solution1 (from inside)

Demo:Japanesetheorem2in.cdy

(1) r is of the smallest T

(2) $r1, r2, r3$ are exCircle





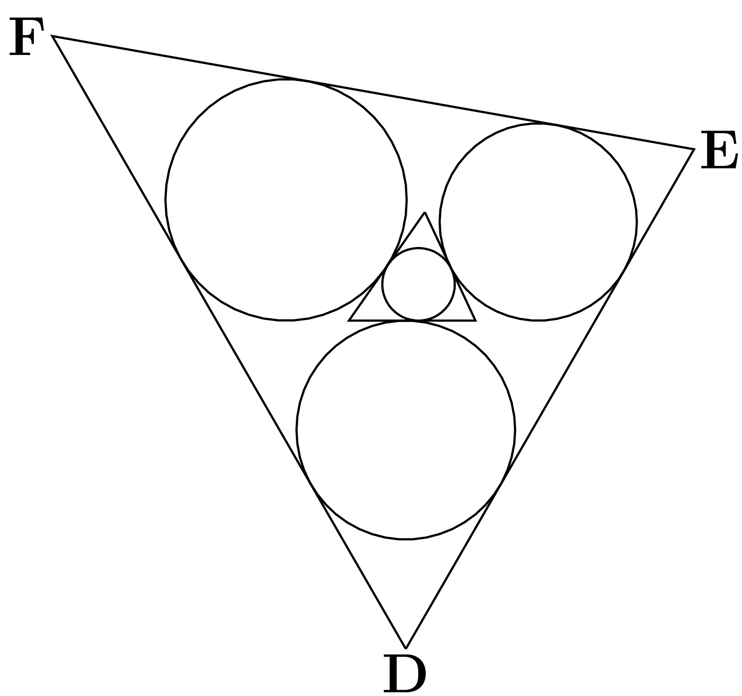
Solution1 (from inside)

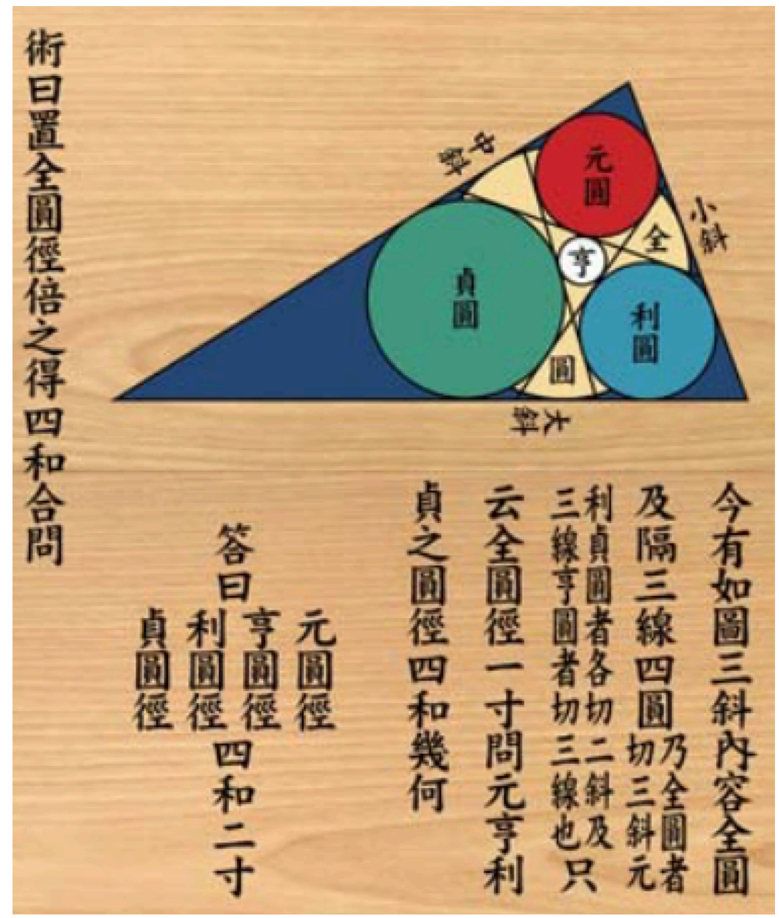
Demo:Japanesetheorem2in.cdy

(1) r is of the smallest T

(2) $r1, r2, r3$ are exCircle

(3) Outer T is formed
by common tangents





Solution1 (from inside)

Demo:Japanesetheorem2in.cdy

(1) r is of the smallest T

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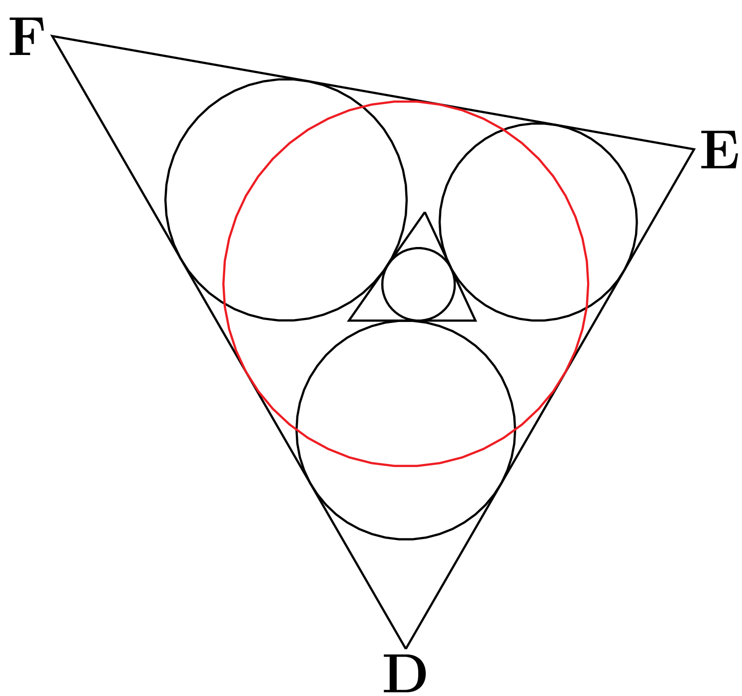
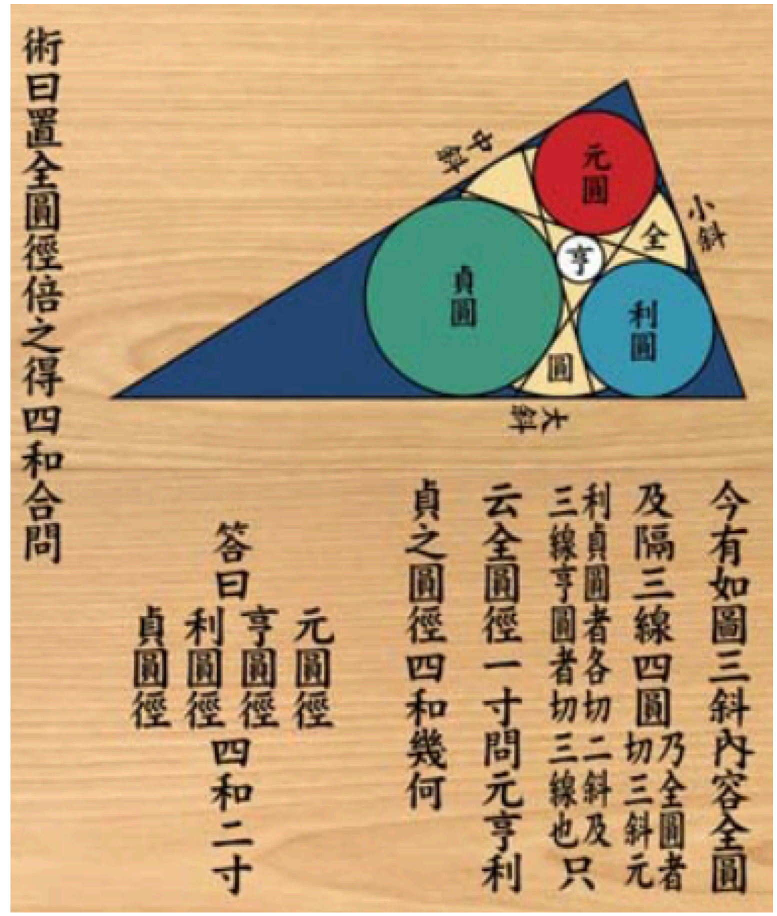
(4) R is inR of DEF





Solution1 (from inside)

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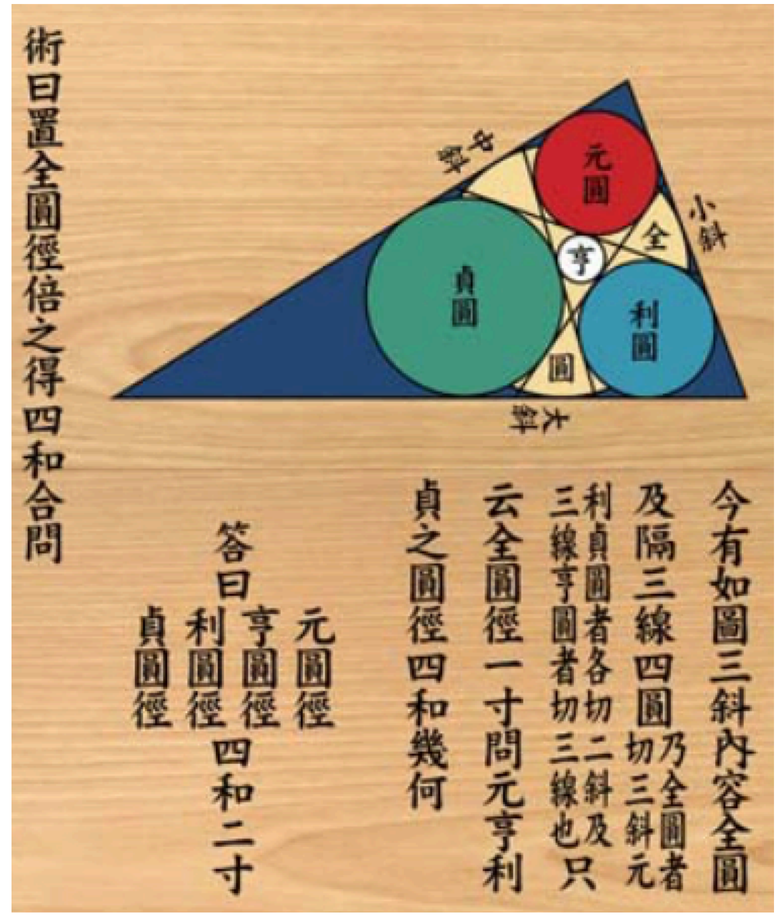
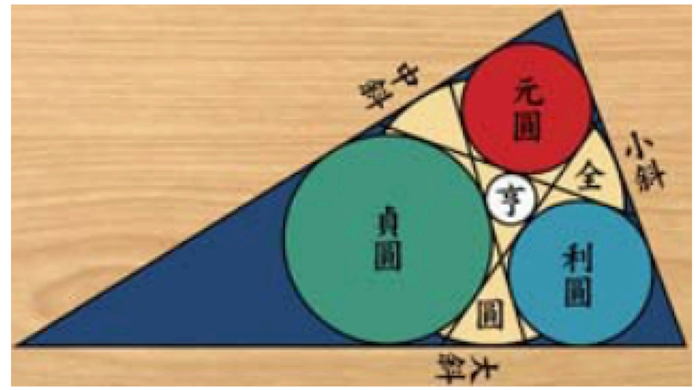


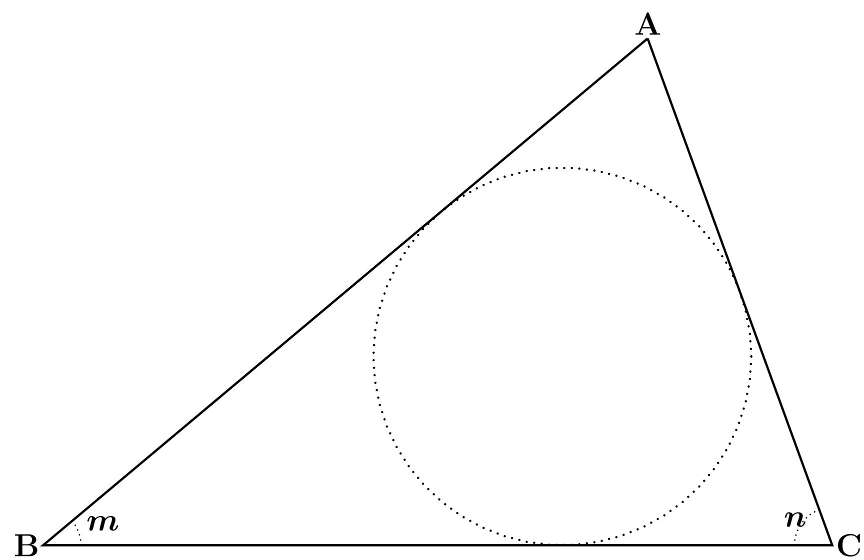
Solution2(from outside)

Demo:Japanesetheorem2out.cdy

Q **Let the radius of the inscribed circle of the large triangle be R, the radius of the inscribed circle, the radii of the three circumscribed circles of the small triangle be r,r1,r2,r3. Find the sum r+r1+r2+r3.**



(1) PutT ABC and R



Solution2(from outside)

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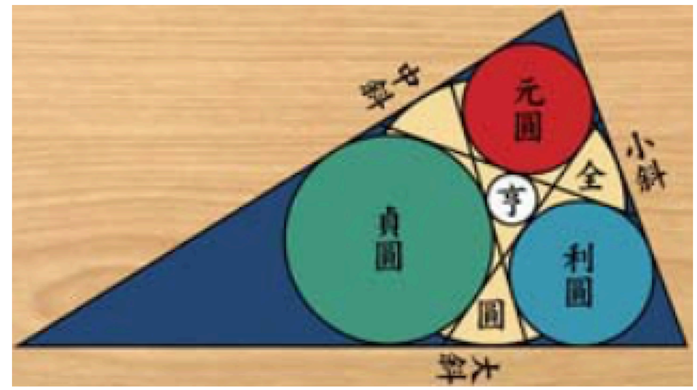
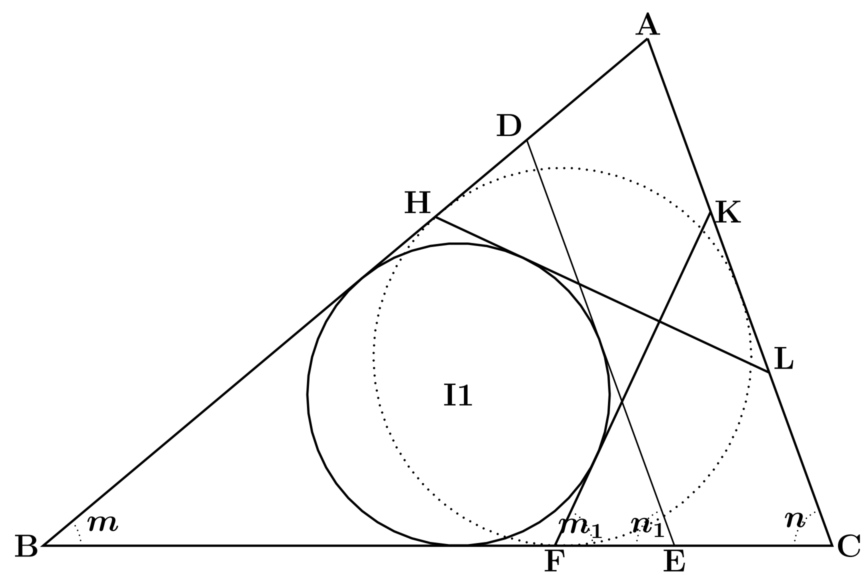
(1) PutT ABC and R

(2) Put DE,FK,HL and I1





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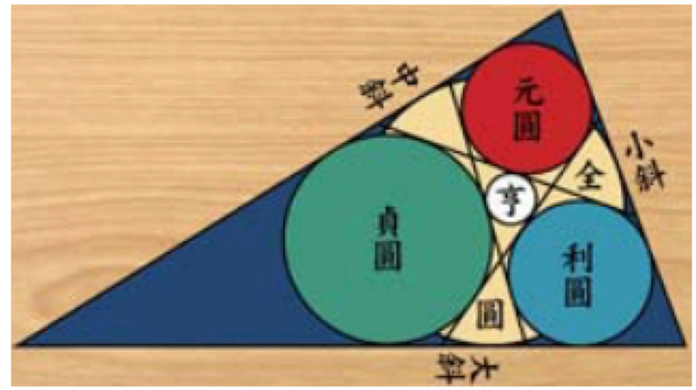

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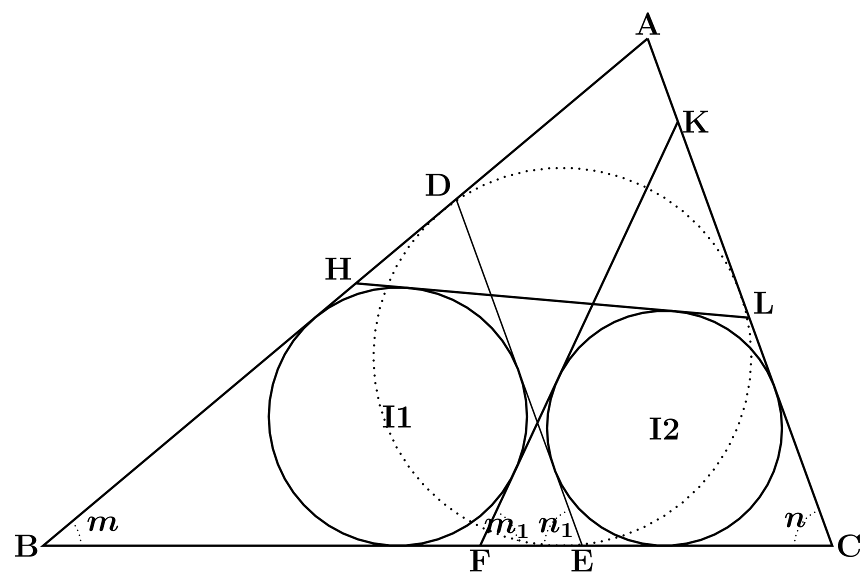
(1) PutT ABC and R

(2) Put DE,FK,HL and I1

(3) Put I2



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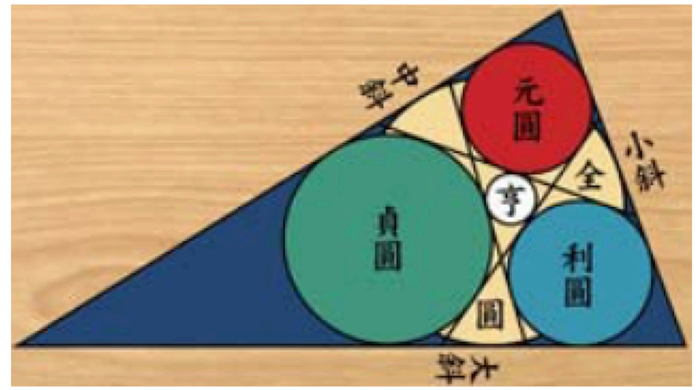

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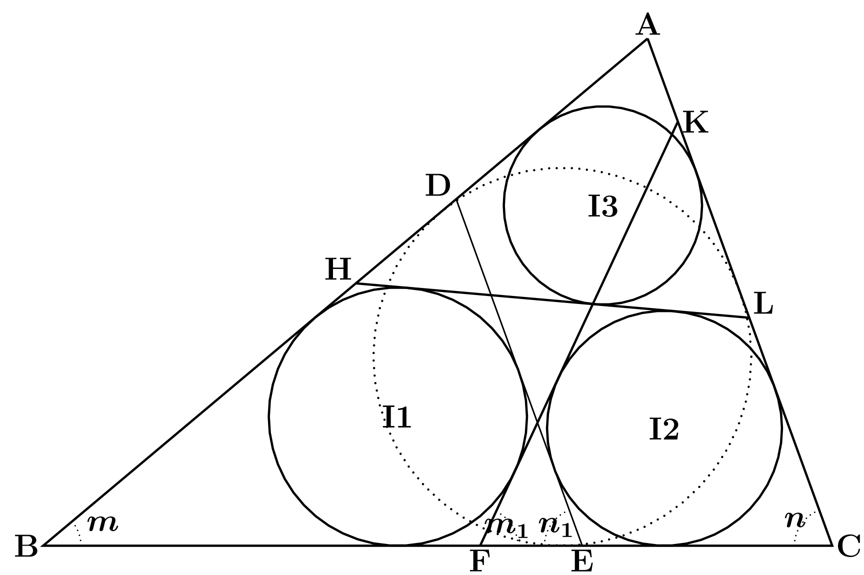
(4) Put I3



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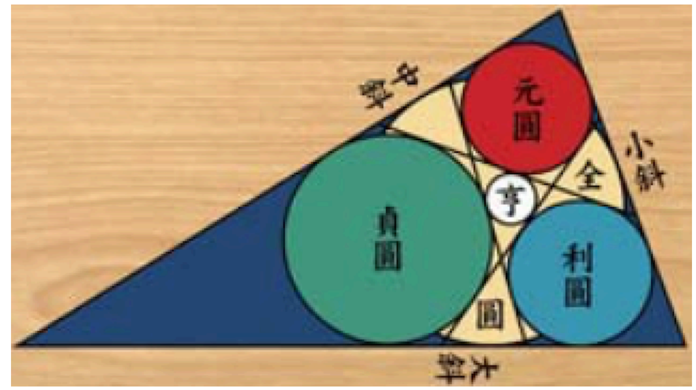

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(1) PutT ABC and R

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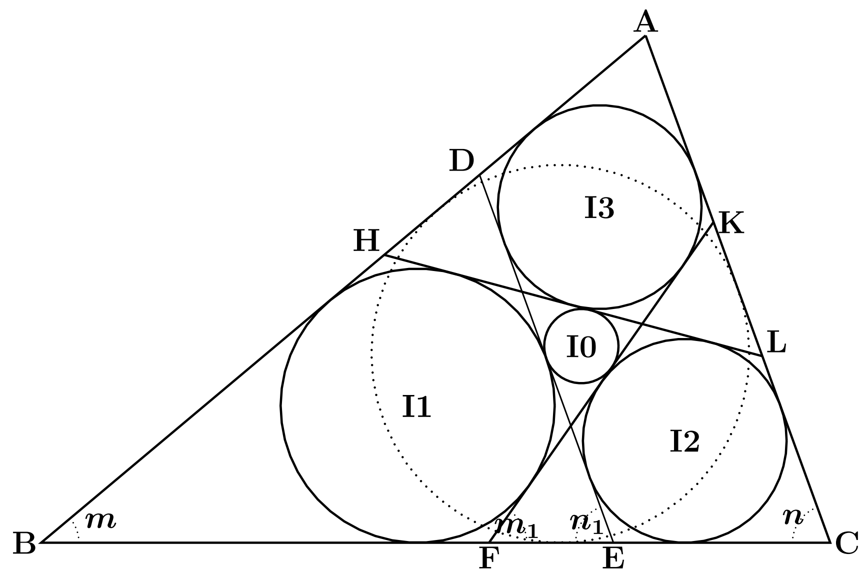
(3) Put I2

(4) Put I3

(5) Make I3 tangent to FK



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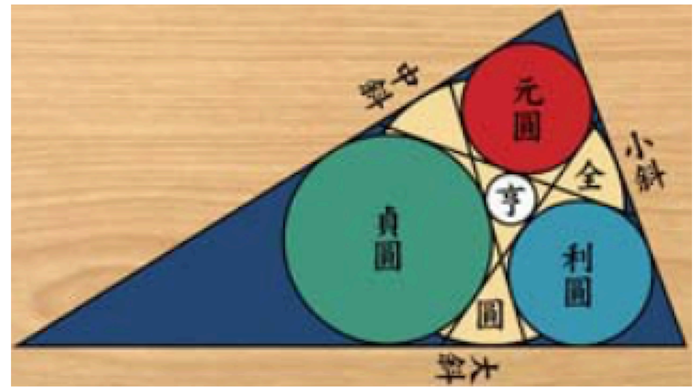

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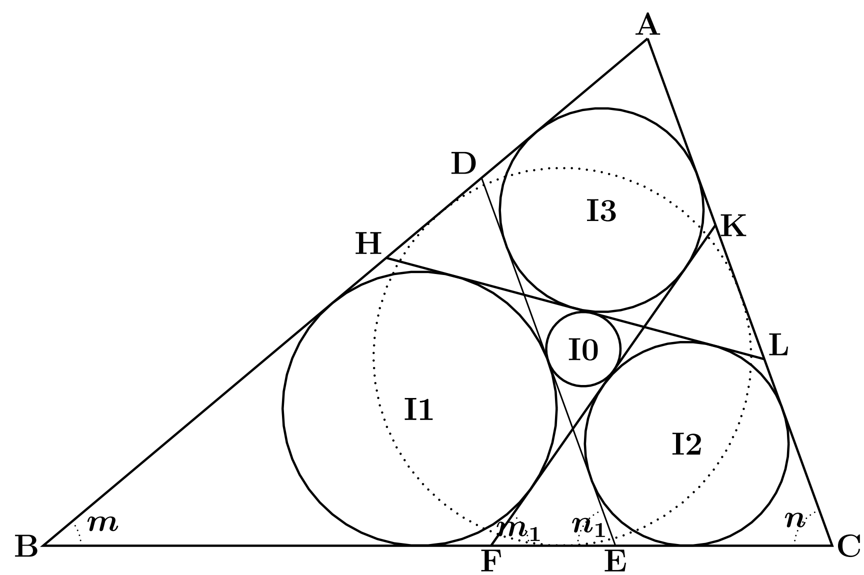

(1) PutT ABC and R

(2) Put DE,FK,HL and I1

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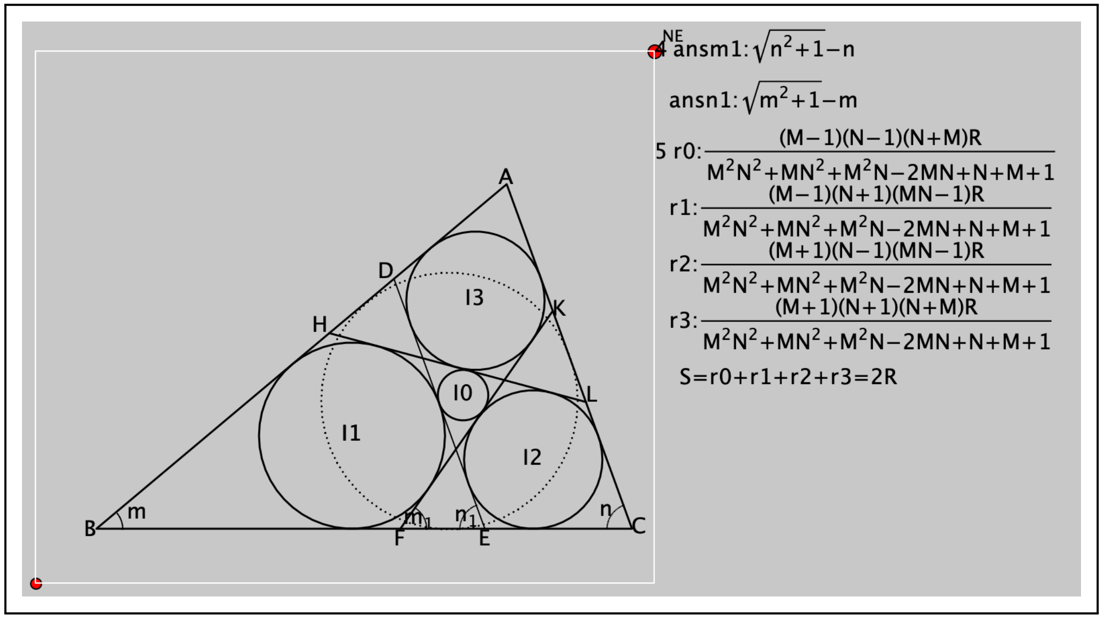
(5) Make I3 tangent to FK

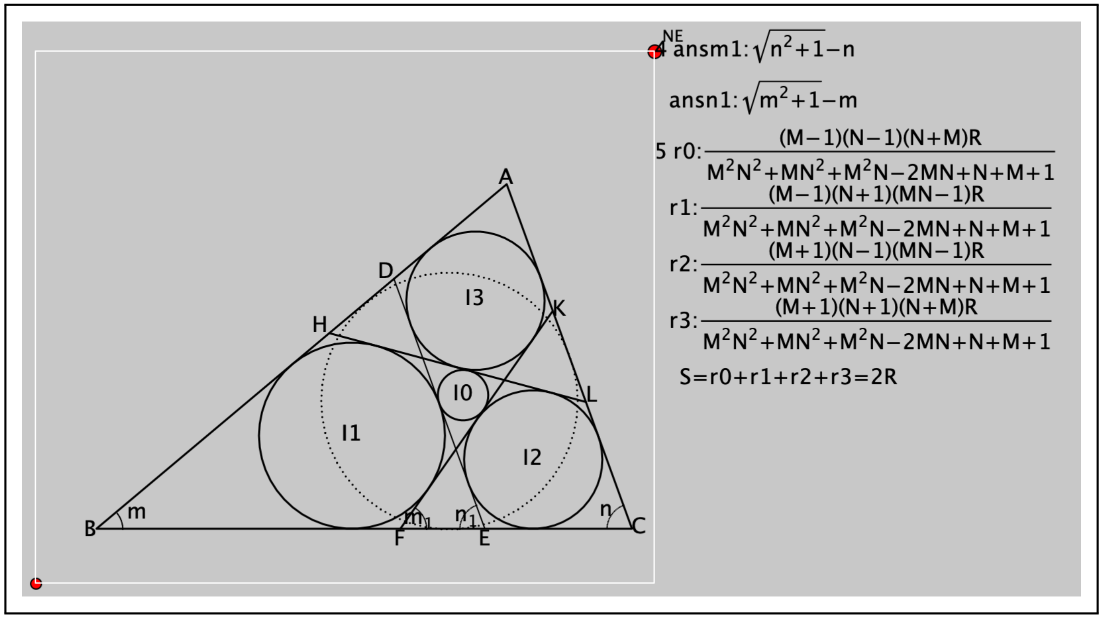
(6) Put I0

Solution(from outside) : Completed



- $m_1 = \sqrt{n^2+1} - n, n_1 = \sqrt{m^2+1} - m$ **are not rational expressions, so the calculation in Maxima fails.**

Solution(from outside) : Completed



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- However, since $\sqrt{n^2+1} = \frac{1}{\cos\frac{C}{2}}$, the tangent of the quarter angle M, N can be expressed as a rational formula.

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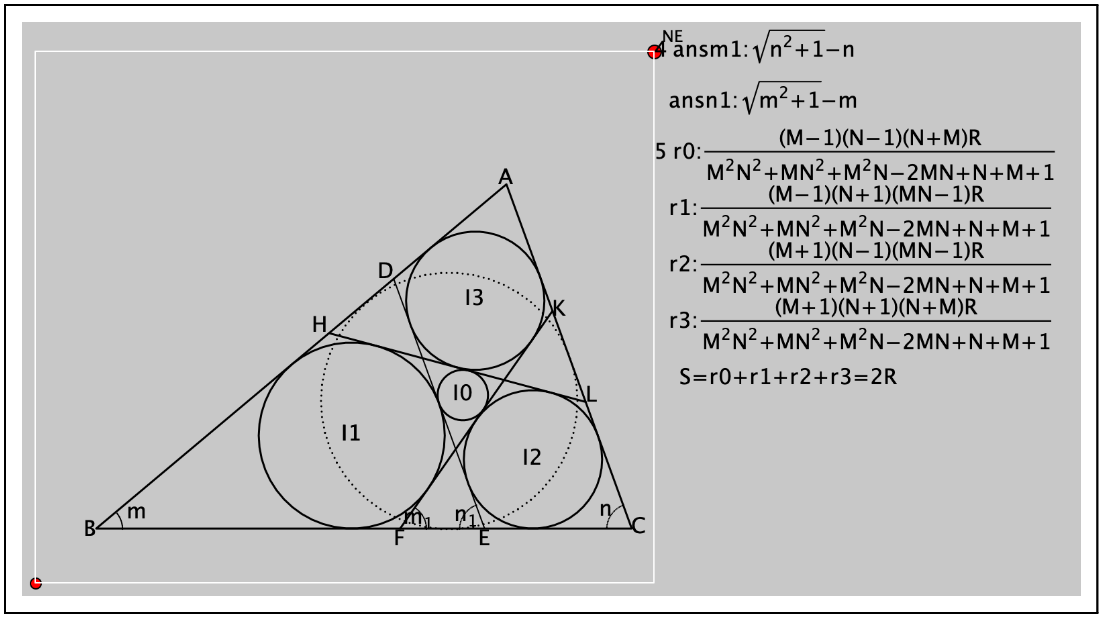

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- By expressing r_0, r_1, r_2, r_3 in this way, rational calculations can be performed.

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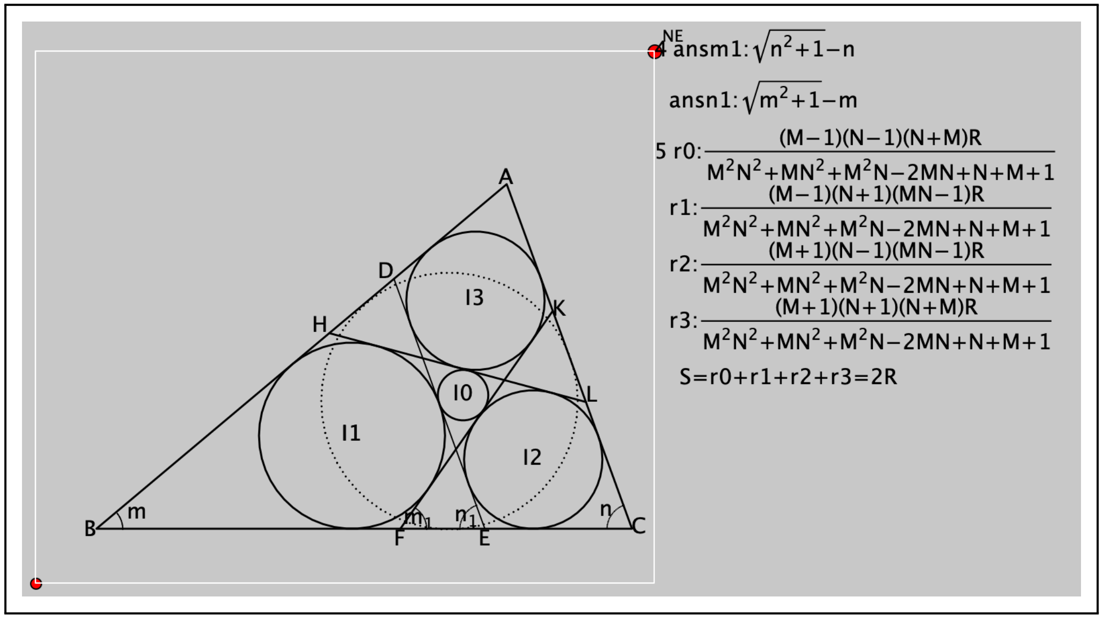

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- However, since $\sqrt{n^2+1} = \frac{1}{\cos\frac{C}{2}}$, the tangent of the quarter angle M, N can be expressed as a rational formula.
- By expressing r_0, r_1, r_2, r_3 in this way, rational calculations can be performed.
- From this, we get $r_0 + r_1 + r_2 + r_3 = 2R$

Joy of Solving Problems

- **Problem-solving requires hard work and inspiration.**

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- **This is why solving them is so satisfying.**

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- I asked Gemini and ChatGPT to draw illustrations.

Joy of Solving Problems

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(G) Archimedes discovered Archimedes' principle while taking a bath and was so excited that he jumped out of the bath without clothes.

Joy of Solving Prob

- Problem-solving requires hard
- This is why solving them is so
- I asked Gemini and ChatGPT

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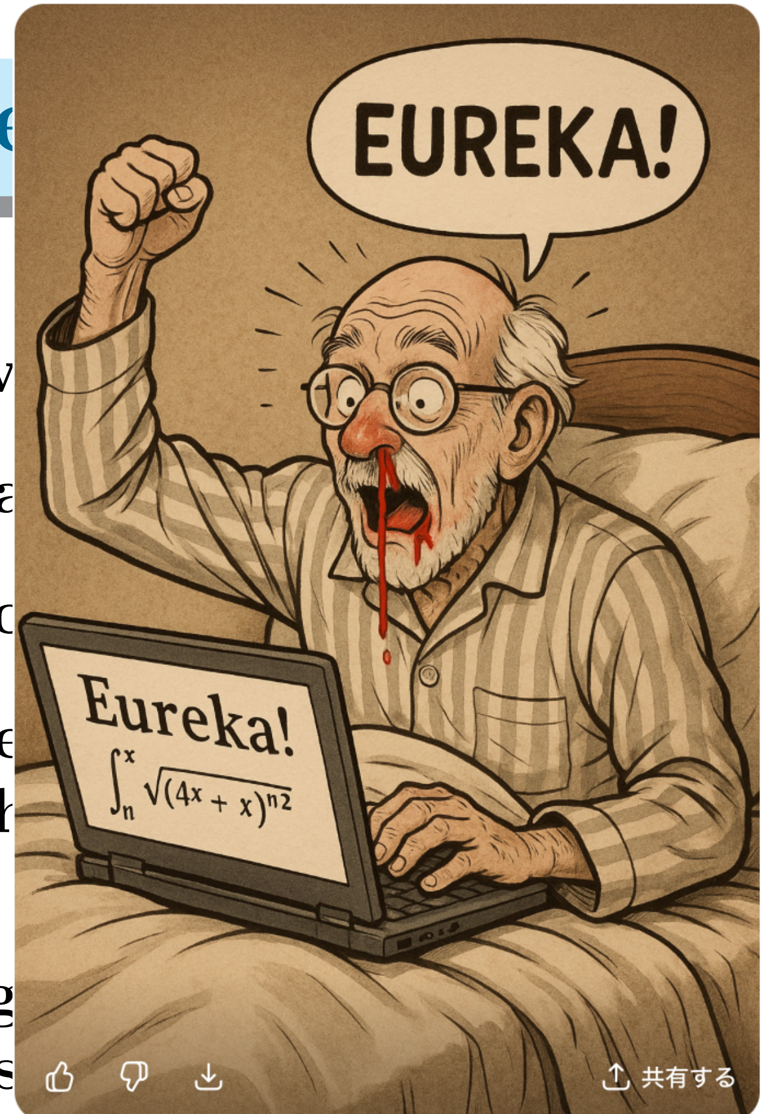
Joy of Solving Problems

- **Problem-solving requires hard work and inspiration.**
- **This is why solving them is so satisfying.**
- **I asked Gemini and ChatGPT to draw illustrations.**

(G) Archimedes discovered Archimedes' principle while taking a bath and was so excited that he jumped out of the bath without clothes.

(C) An elderly researcher was trying to solve a difficult problem using a computer algebra system every night after going to bed, and one day he finally succeeded. He excitedly shouted, 'Eureka!' and immediately his nose started bleeding.

Joy of Solving Proble



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Conclusions

Conclusion1

(1) Use a CAS to solve WASAN

- It bridges the gap between historical mathematics and modern technology.
- It provides a powerful tool for a deeper understanding of Wasan, while also serving as a catalyst for the ongoing development and improvement of computer algebra systems themselves.

Conclusion2

(2) Use of the "MNR method"

- It is a specialized approach designed to solve complex geometry problems, particularly those found in Wasan.
- Its importance lies in its ability to overcome a specific, significant challenge that arises when using modern CAS to tackle these historical problems.

Acknowledgements

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- **I would especially like to express my sincere gratitude to the organizers.**