

Lecture 3.2.1 Corner features

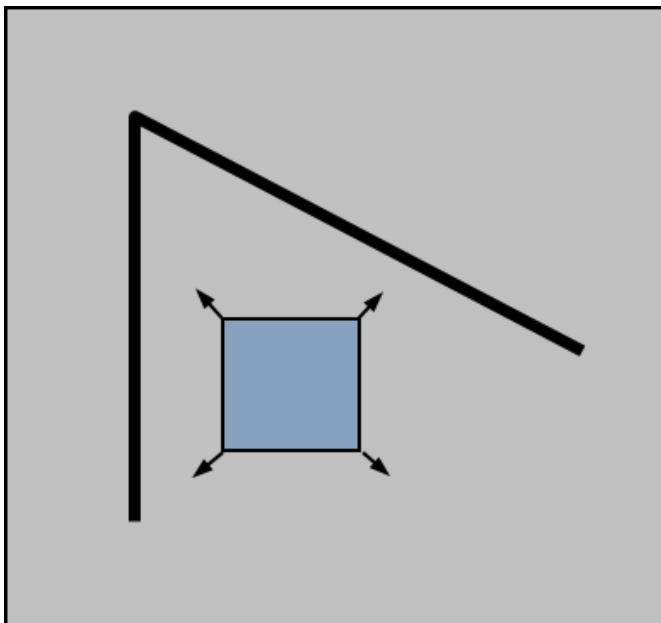
Trym Vegard Haavardsholm

Slides from Rick Szeliski, S. Seitz, Svetlana Lazebnik, Derek Hoiem, Grauman&Leibe, James Hayes and Noah Snavely

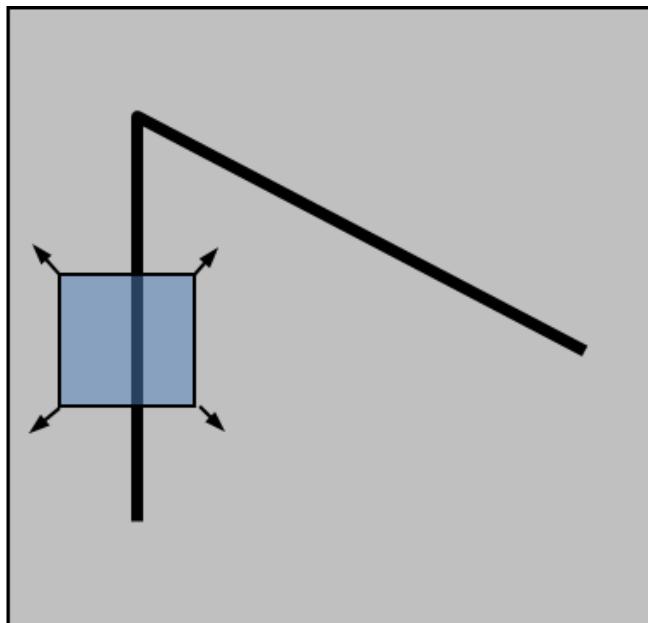


Local measure of feature distinctiveness

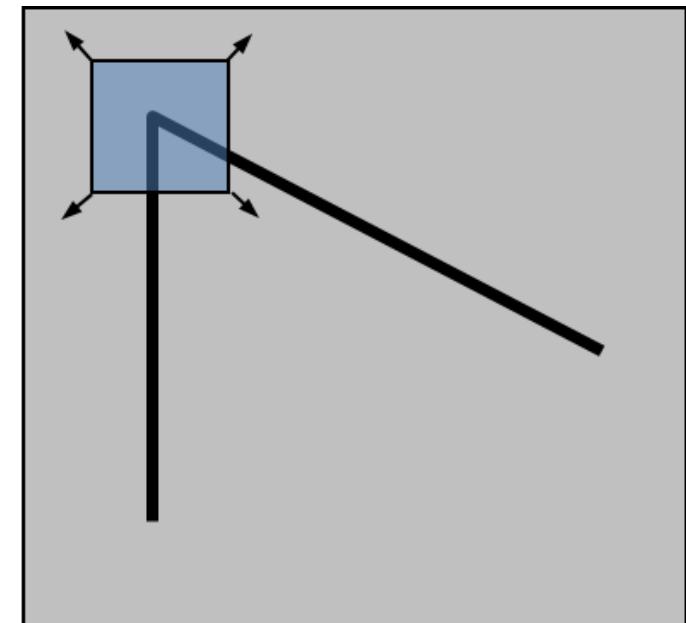
- Consider a small window of pixels around a feature
- How does the window change when you shift it?



“Flat” region:
No change in all directions



“Edge”:
No change along edge

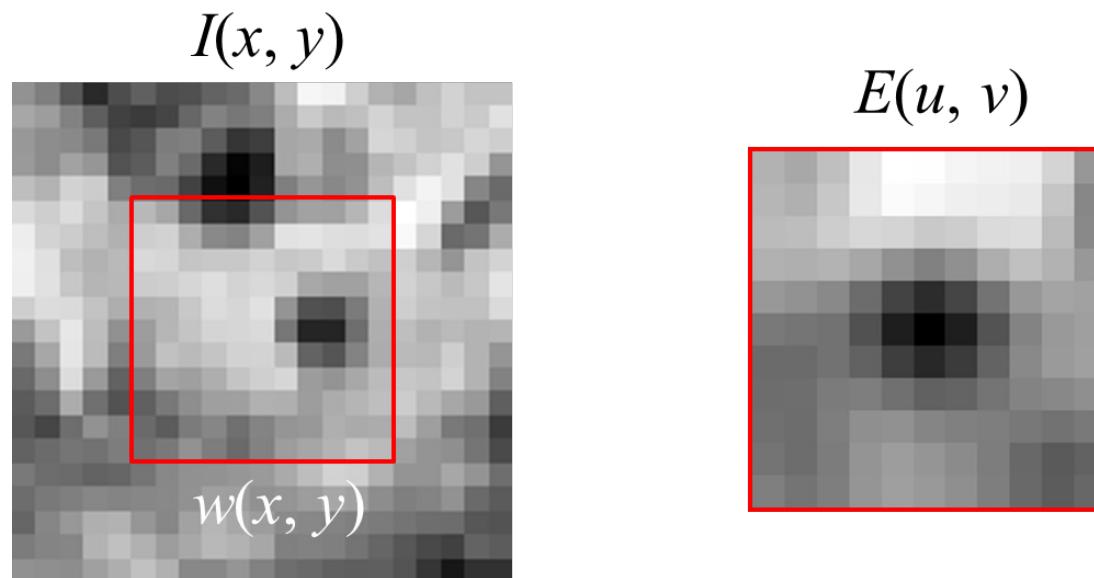


“Corner”:
Change in all directions

Local measure of feature distinctiveness

- Change in appearance of window $w(x,y)$ for the shift $[u,v]$:

$$E(u, v) = \sum_{x, y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

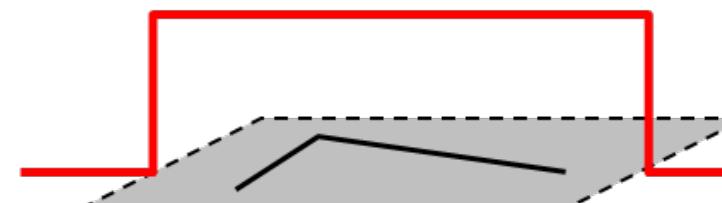


Local measure of feature distinctiveness

- Change in appearance of window $w(x,y)$ for the shift $[u,v]$:

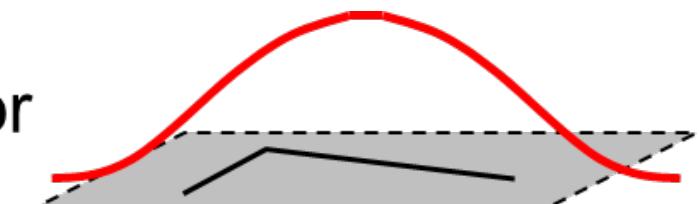
$$E(u, v) = \sum_{x, y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

Window function $w(x, y) =$



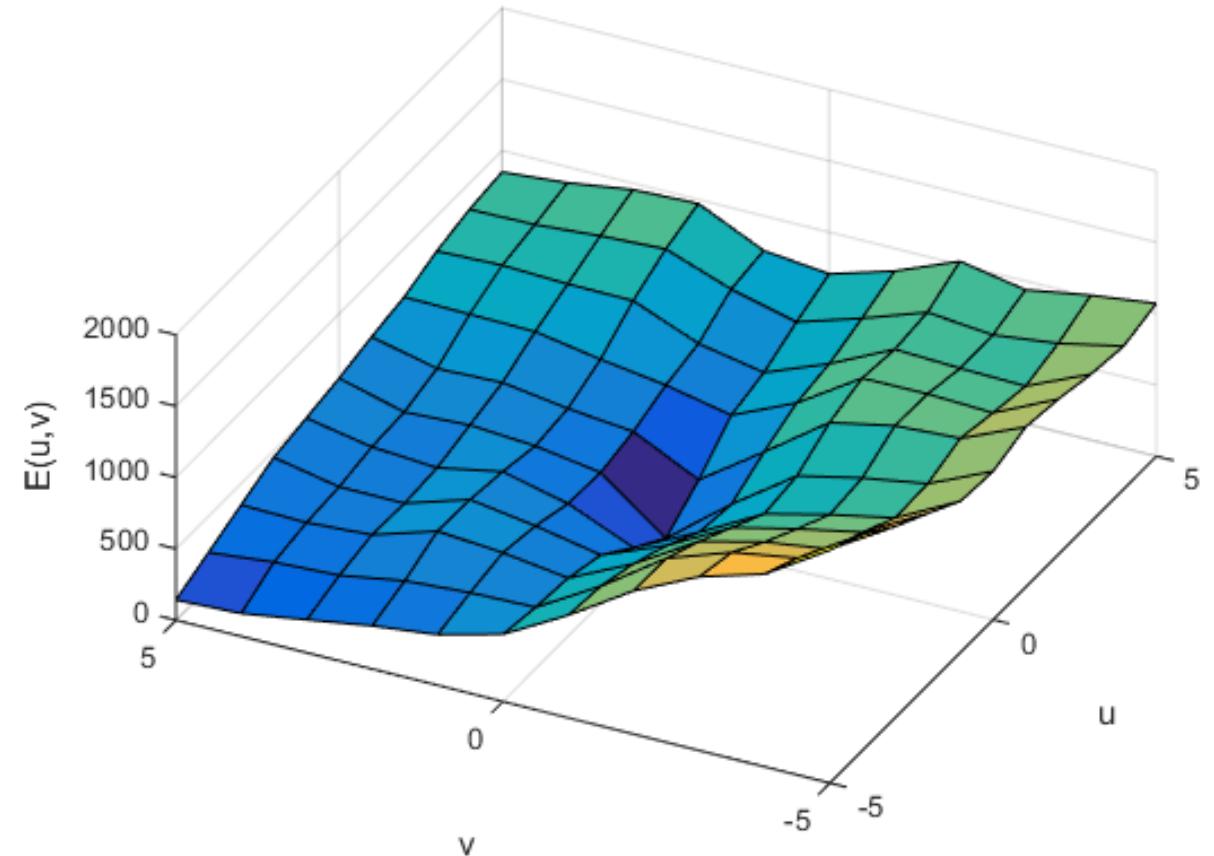
1 in window, 0 outside

or

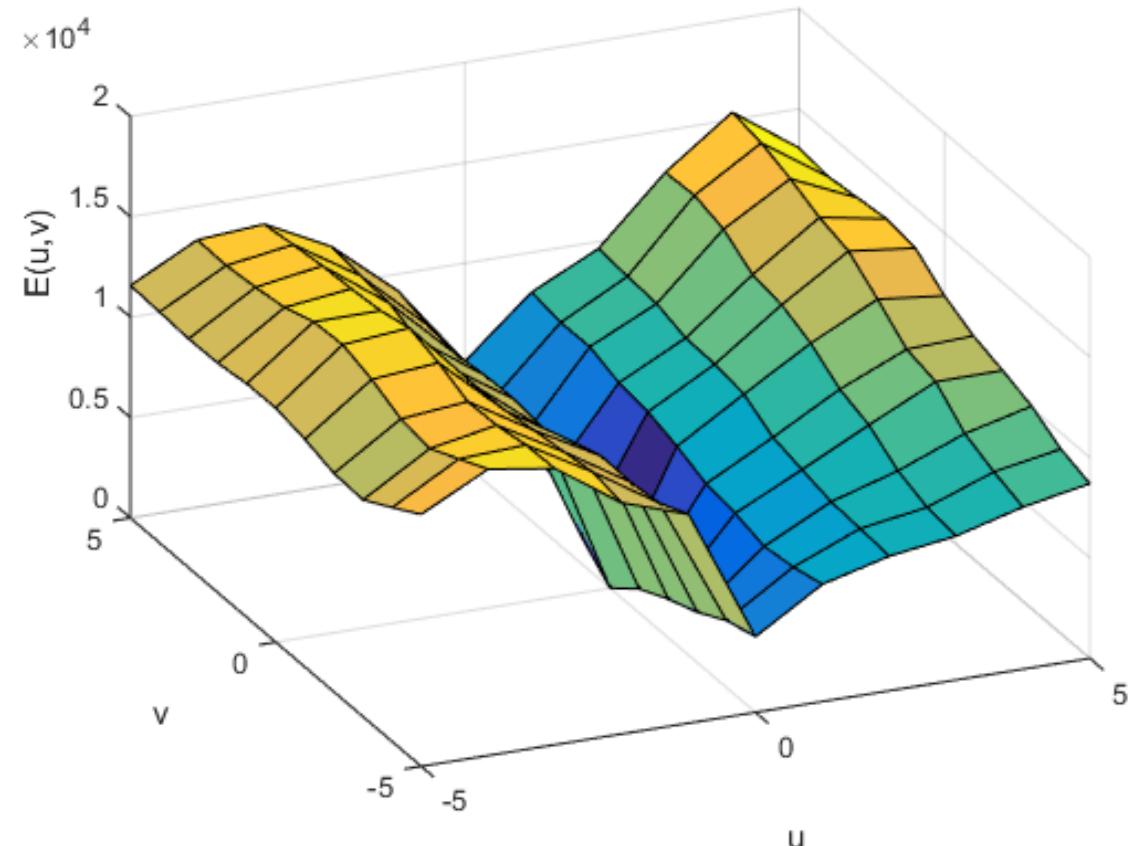


Gaussian

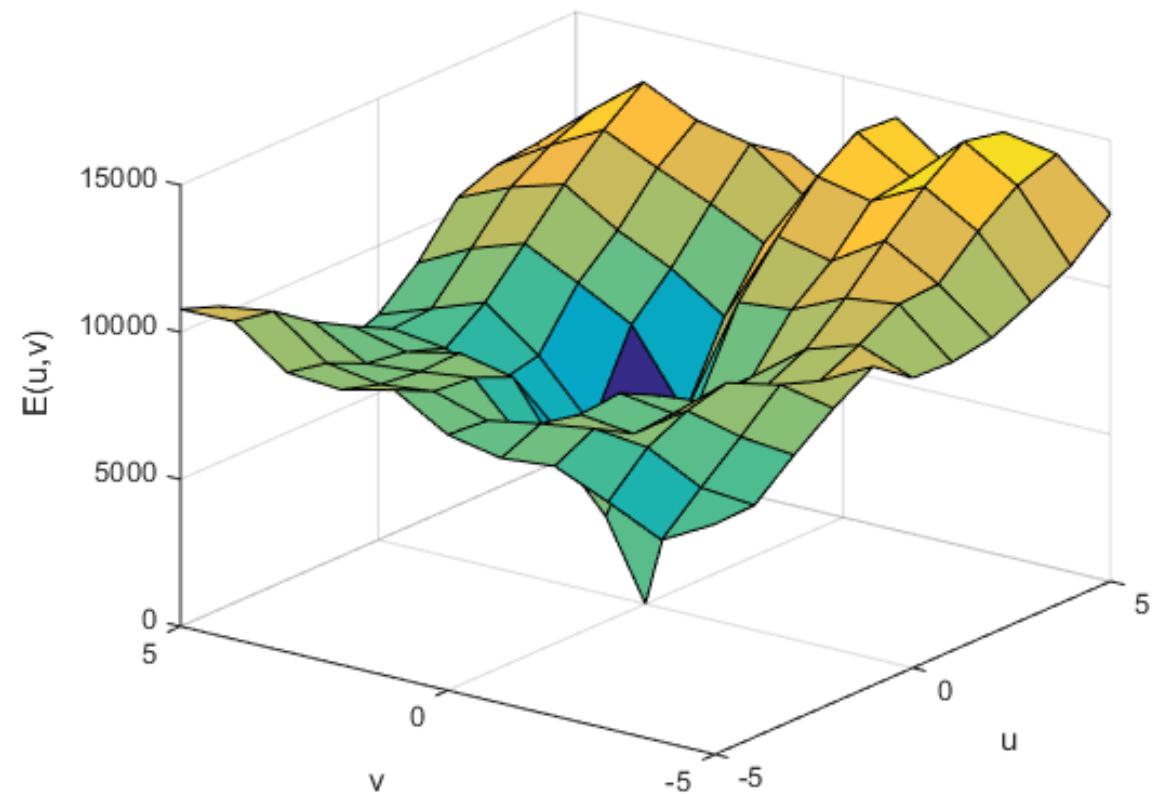
Examples Holmenkollen



Examples Holmenkollen



Examples Holmenkollen



Simplifying the measure

- Local first order Taylor Series expansion of $I(x,y)$:

$$I(x+u, y+v) \approx I(x, y) + I_x u + I_y v$$

- Local quadratic approximation of $E(u,v)$:

$$\begin{aligned} E(u, v) &= \sum_{x,y} w(x, y) [I(x+u, y+v) - I(x, y)]^2 \\ &\approx \sum_{x,y} w(x, y) [I_x u + I_y v]^2 \\ &\approx A u^2 + 2Buv + C v^2 \end{aligned}$$

Simplifying the measure

- Local quadratic approximation of the surface $E(u,v)$:

$$E(u,v) \approx [u \ v] \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = [u \ v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

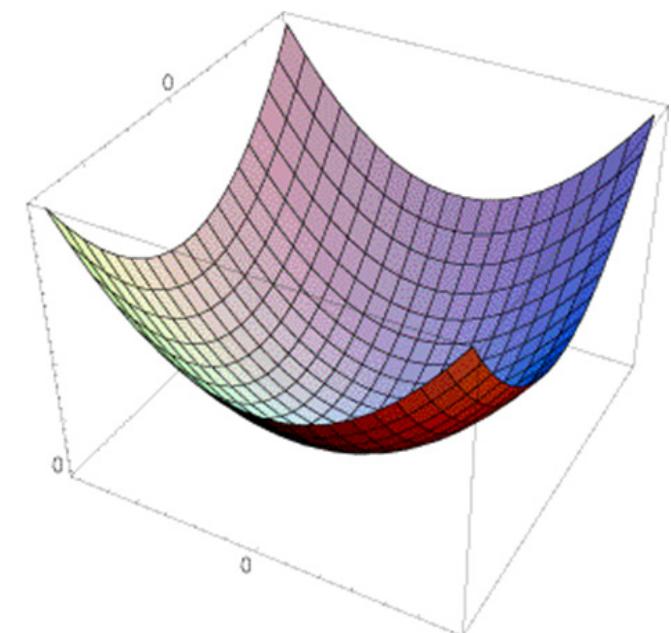
$$A = \sum_{x,y} w(x,y) I_x^2$$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

$$B = \sum_{x,y} w(x,y) I_x I_y$$

$$= \sum_{x,y} w(x,y) \begin{bmatrix} I_x \\ I_y \end{bmatrix} \begin{bmatrix} I_x & I_y \end{bmatrix}$$

$$C = \sum_{x,y} w(x,y) I_y^2$$



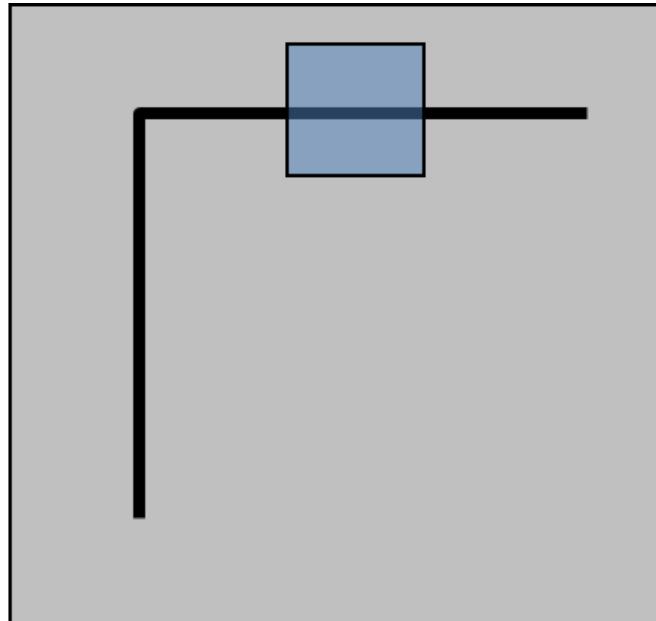
Interpreting the second moment matrix

$$E(u, v) \approx [u \quad v] \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = [u \quad v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{x,y} w(x, y) I_x^2$$

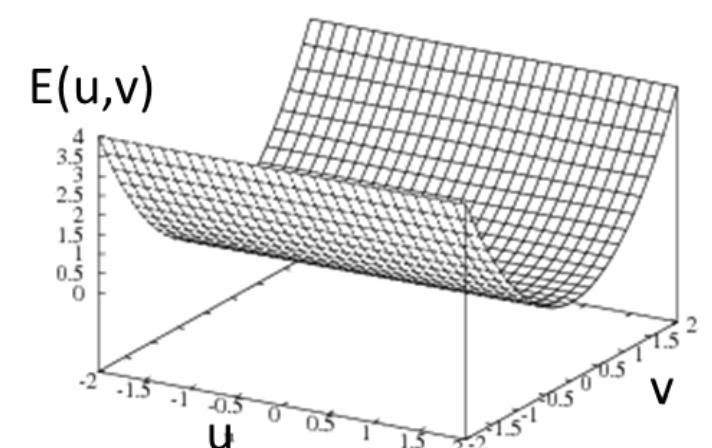
$$B = \sum_{x,y} w(x, y) I_x I_y$$

$$C = \sum_{x,y} w(x, y) I_y^2$$



Horizontal edge: $I_x = 0$

$$M = \begin{bmatrix} 0 & 0 \\ 0 & C \end{bmatrix}$$



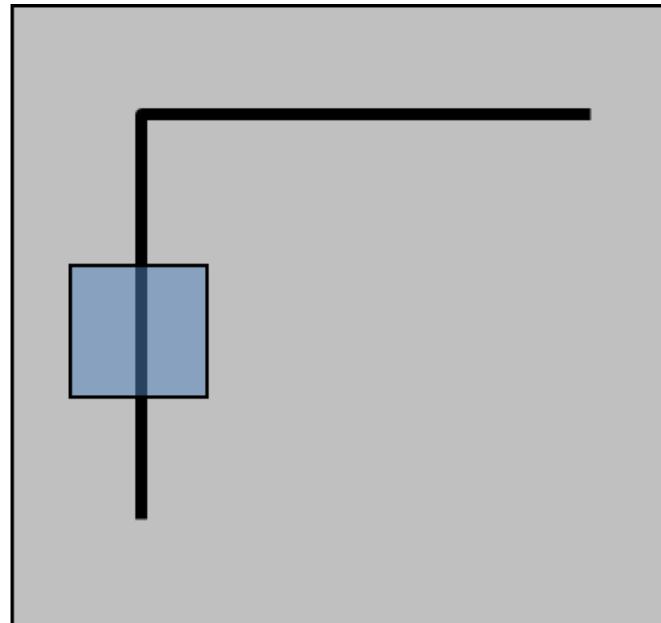
Interpreting the second moment matrix

$$E(u, v) \approx [u \quad v] \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = [u \quad v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{x,y} w(x, y) I_x^2$$

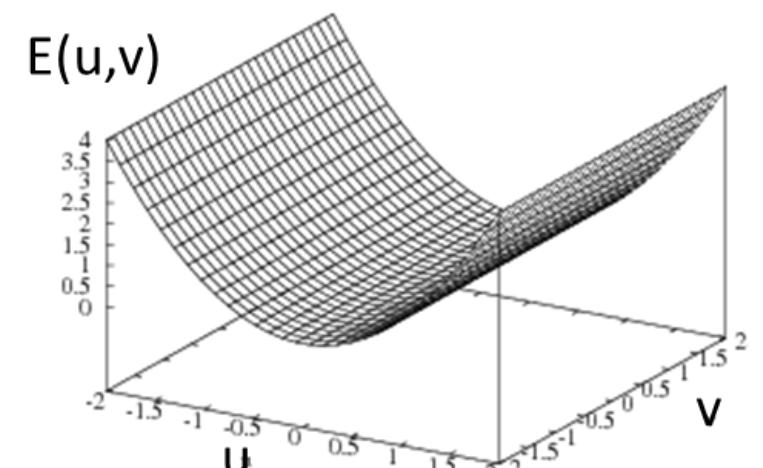
$$B = \sum_{x,y} w(x, y) I_x I_y$$

$$C = \sum_{x,y} w(x, y) I_y^2$$

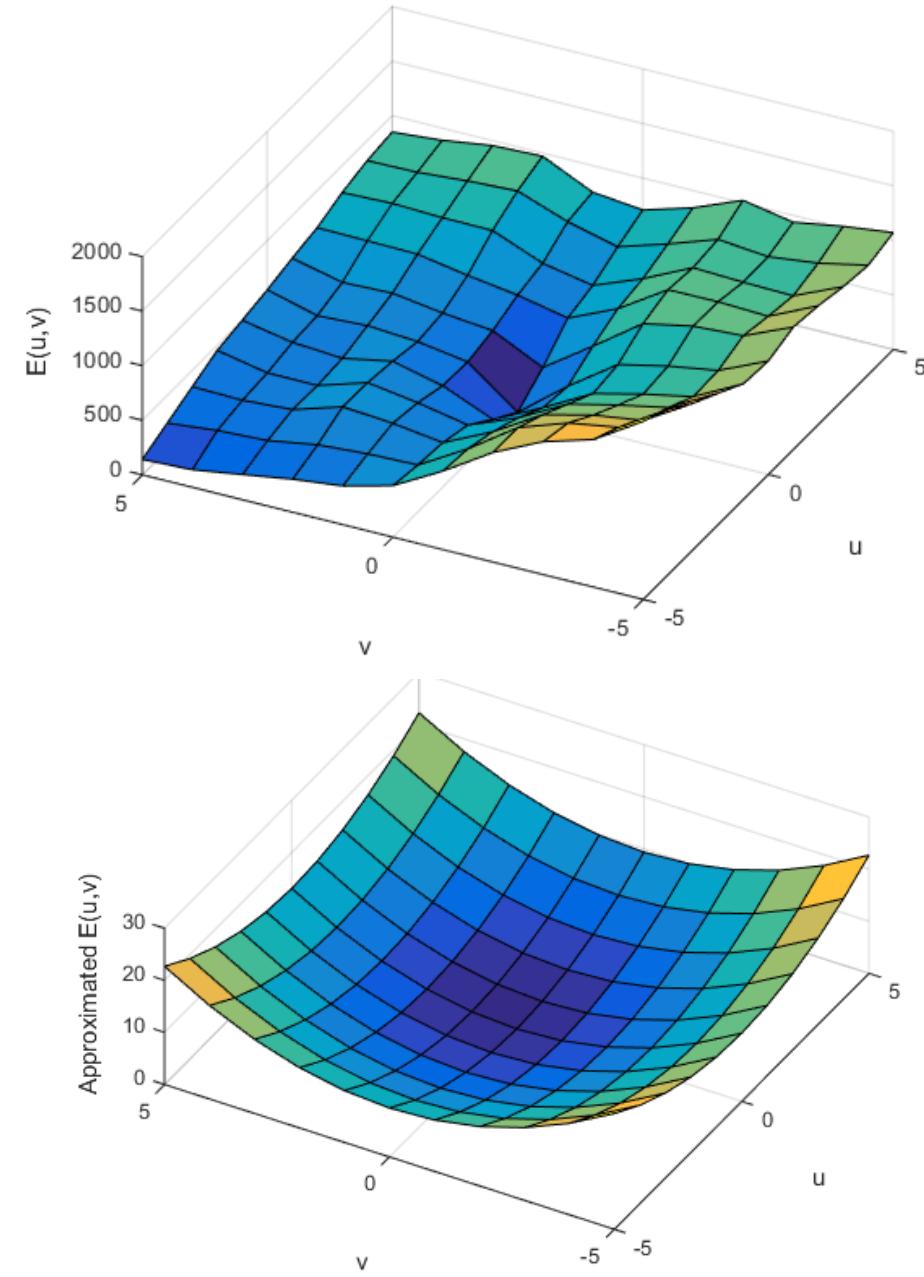


Vertical edge: $I_y = 0$

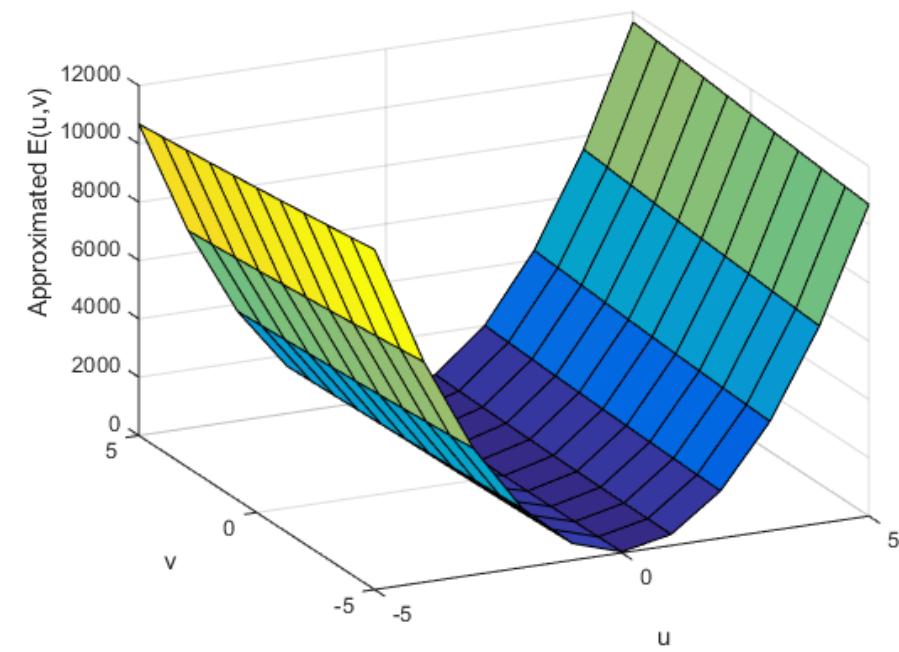
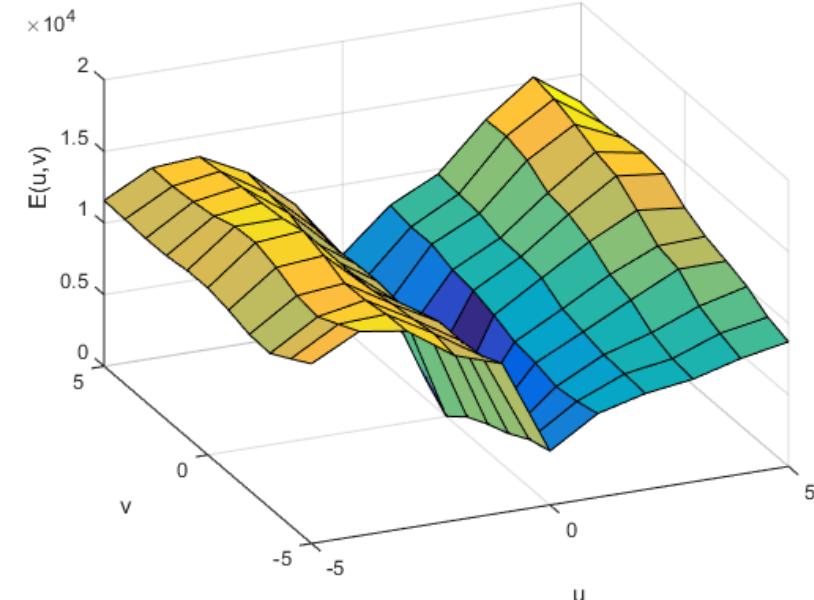
$$M = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}$$



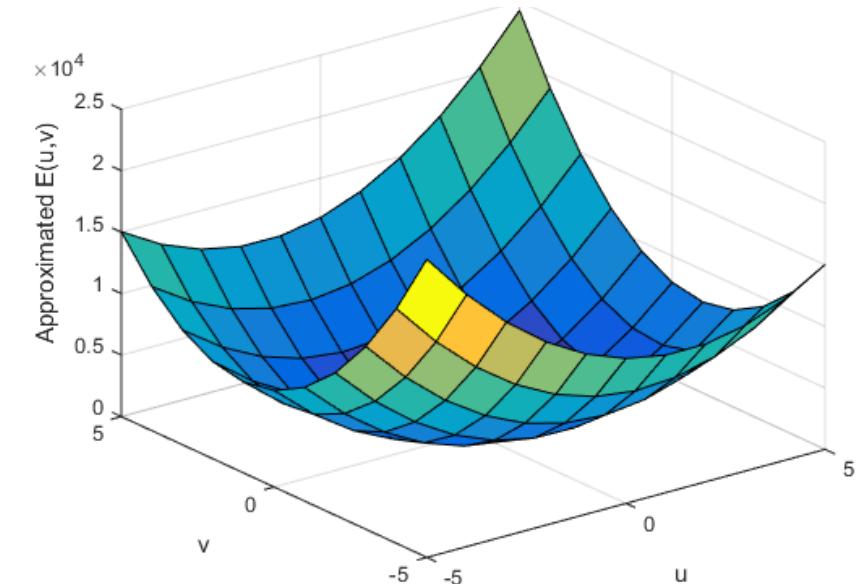
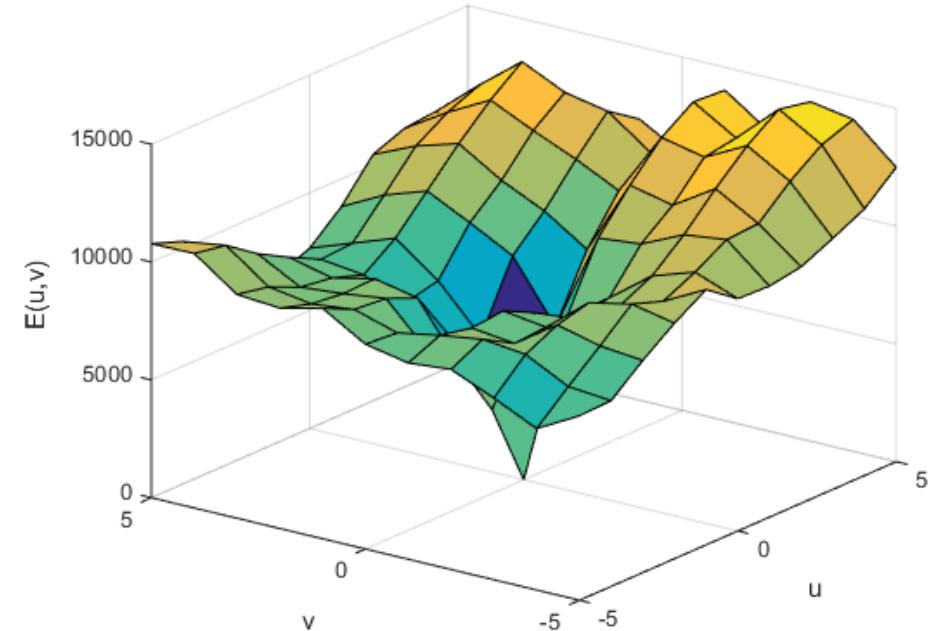
Examples Holmenkollen



Examples Holmenkollen



Examples Holmenkollen

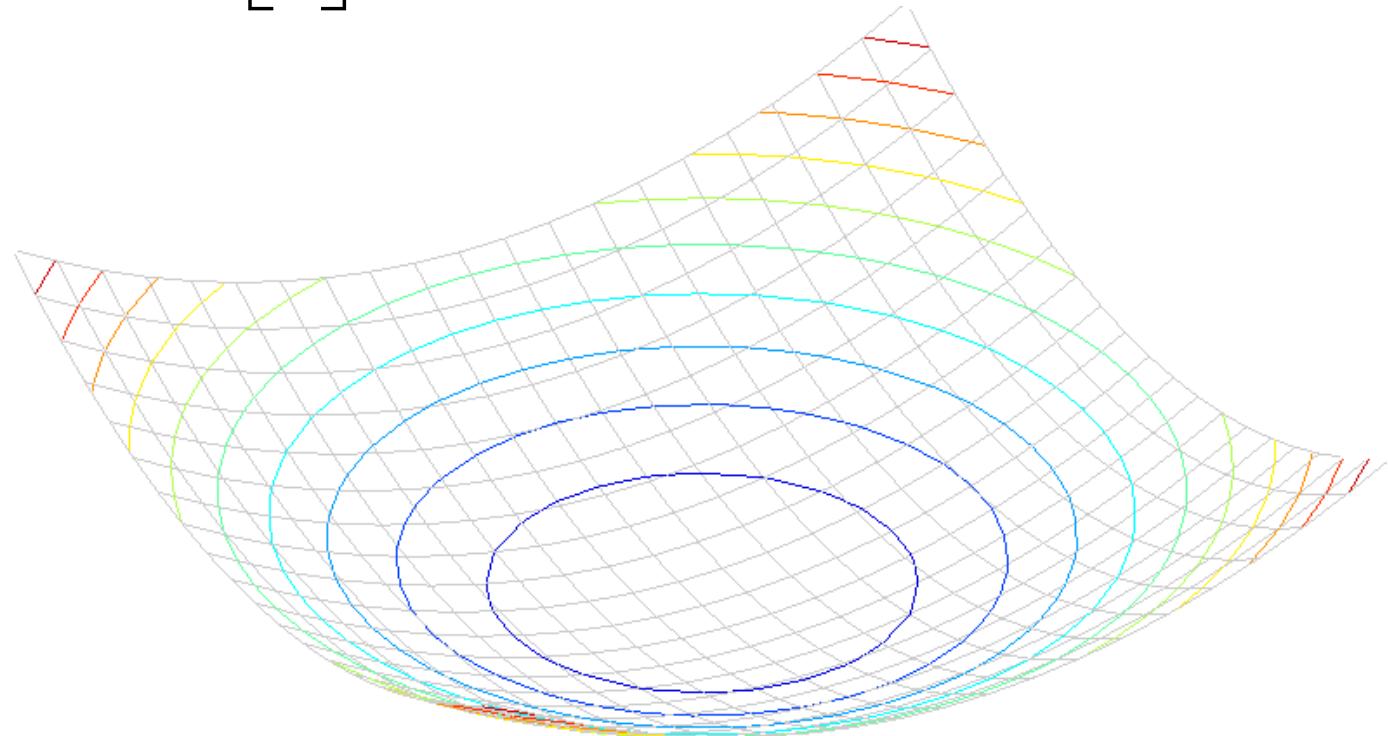


Simplifying the measure even further

- Consider a horizontal “slice” of $E(u,v)$:

$$E(u,v) \approx [u \quad v] M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$

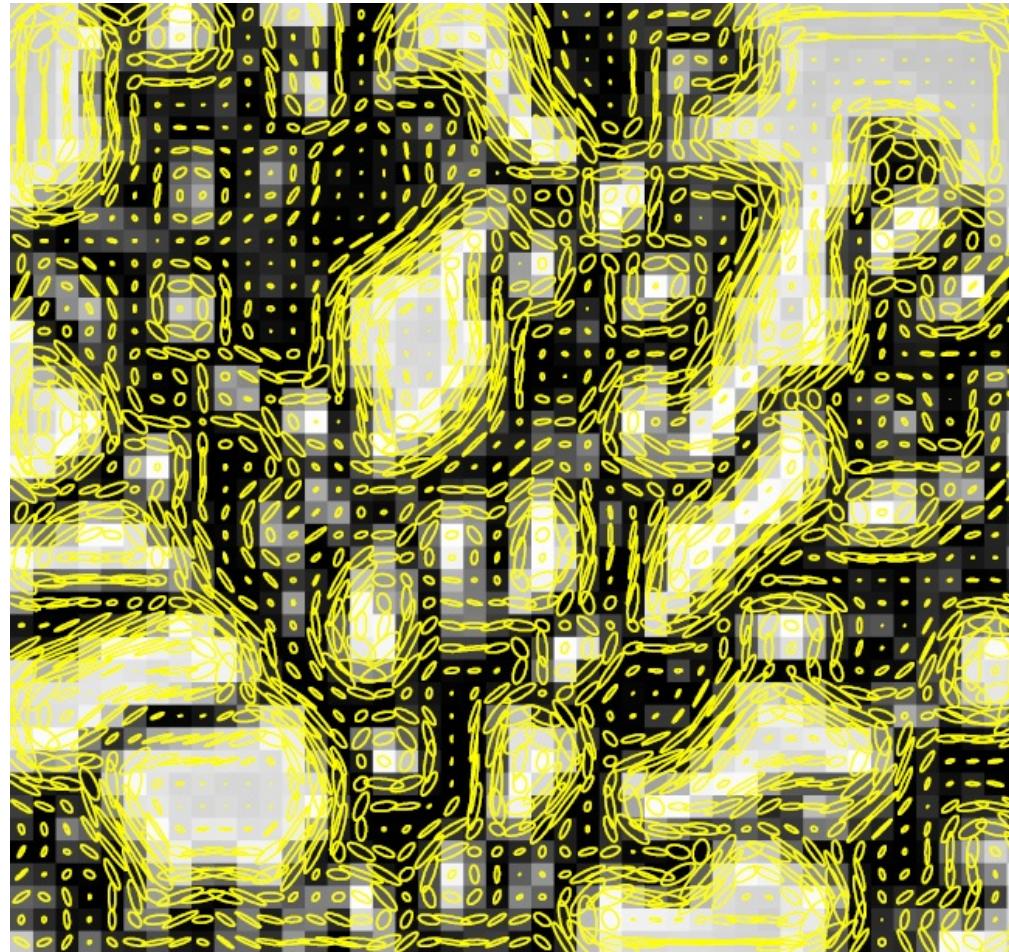
- This is the equation of an ellipse



Visualization of second moment matrices



Visualization of second moment matrices

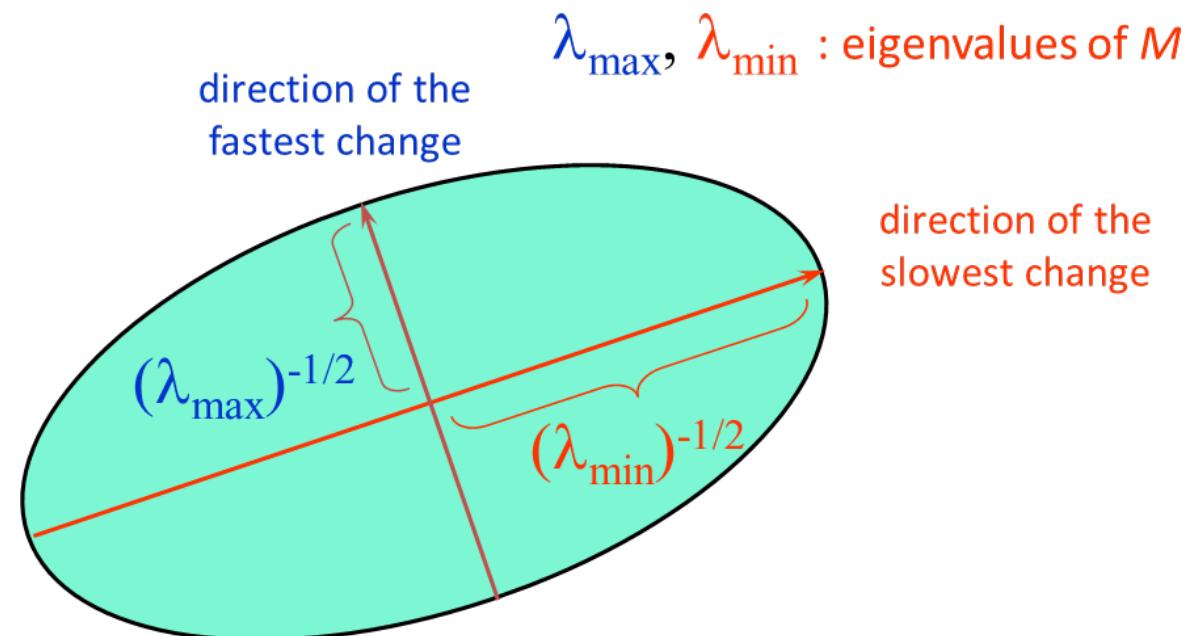


Simplifying the measure even further

- Consider a horizontal “slice” of $E(u,v)$:

$$E(u,v) \approx [u \quad v] M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$

- This is the equation of an ellipse
 - Describe the surface using the eigenvalues of M



The eigenvalues and eigenvectors of M

- The eigenvalues:

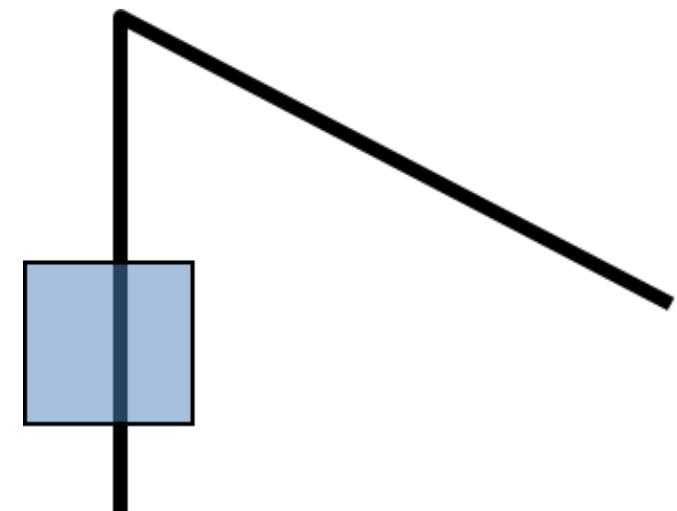
$$\lambda = \frac{1}{2} \left[(A+C) \pm \sqrt{4B^2 + (A-C)^2} \right]$$

- Once you know λ , you find the eigenvectors \mathbf{x} by solving

$$\begin{bmatrix} A - \lambda & B \\ B & C - \lambda \end{bmatrix} \mathbf{x} = 0$$

The eigenvalues and eigenvectors of \mathbf{M}

- Define shift directions with the smallest and largest change in error
- \mathbf{x}_{\max} = direction of largest increase in E
- λ_{\max} = amount of increase in direction \mathbf{x}_{\max}
- \mathbf{x}_{\min} = direction of smallest increase in E
- λ_{\min} = amount of increase in direction \mathbf{x}_{\min}

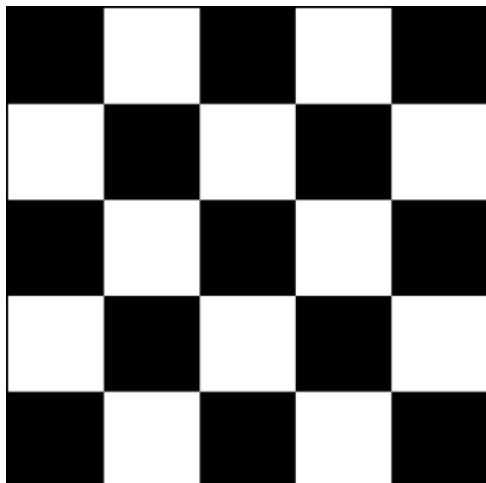


Local measure of feature distinctiveness

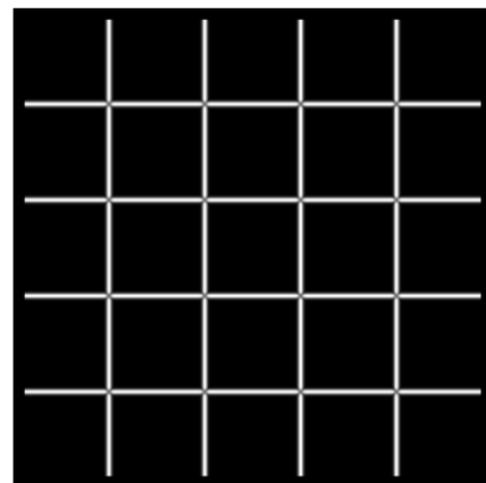
- How are λ_{\max} , \mathbf{x}_{\max} , λ_{\min} , \mathbf{x}_{\min} relevant for feature detection?
 - What is our feature scoring function?

Local measure of feature distinctiveness

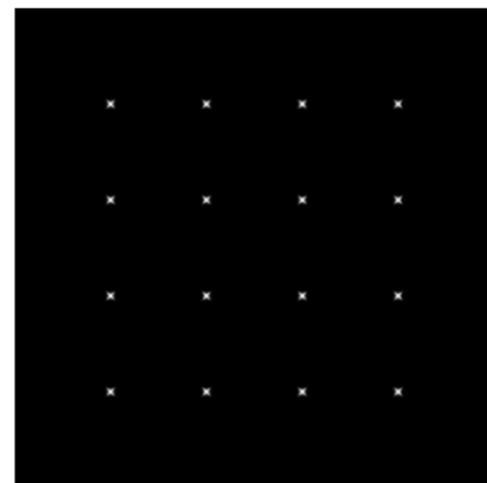
- How are λ_{\max} , \mathbf{x}_{\max} , λ_{\min} , \mathbf{x}_{\min} relevant for feature detection?
 - What is our feature scoring function?
- Want $E(u,v)$ to be large for small shifts in all directions
 - the minimum of $E(u,v)$ should be large, over all unit vectors $[u \ v]$
 - this minimum is given by the smaller eigenvalue (λ_{\min}) of M



I

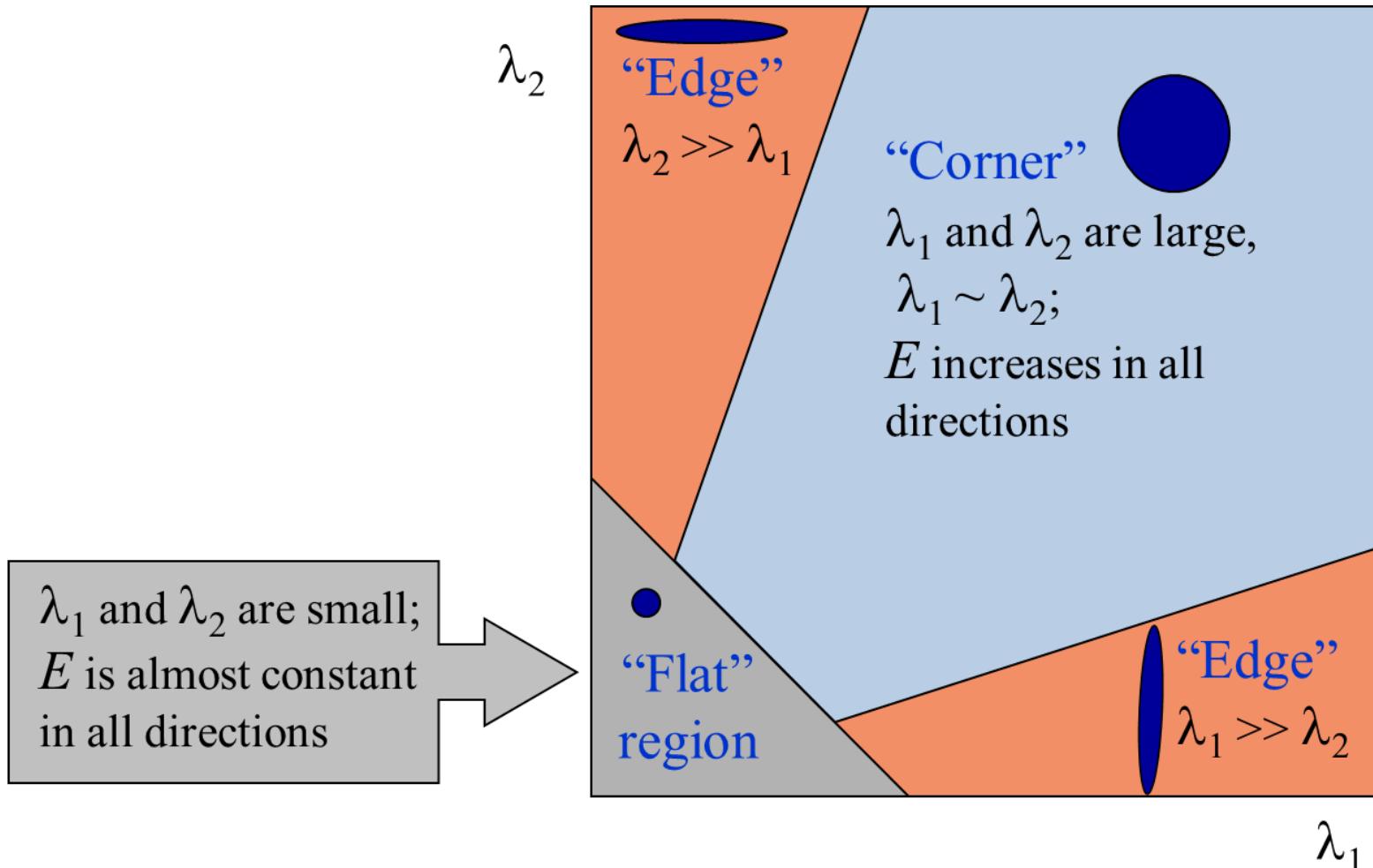


λ_{\max}



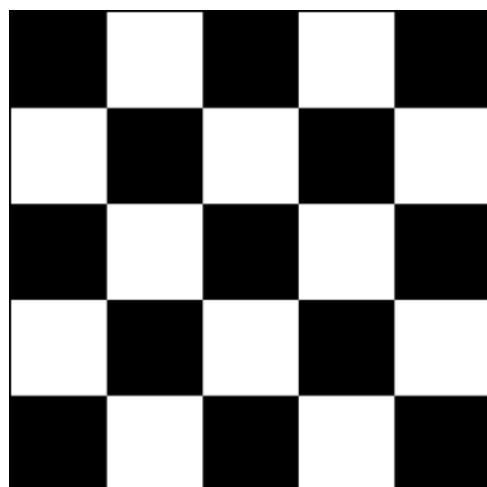
λ_{\min}

Interpreting the eigenvalues

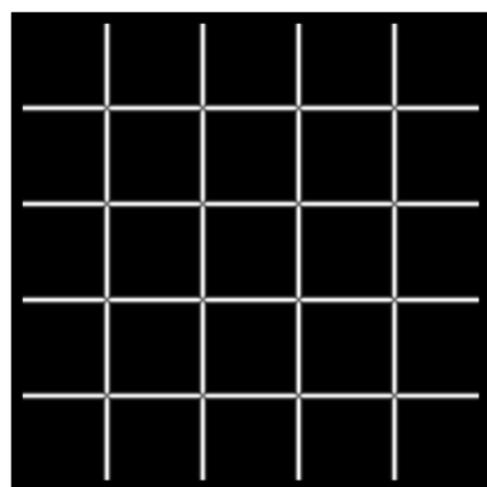


Corner detection summary

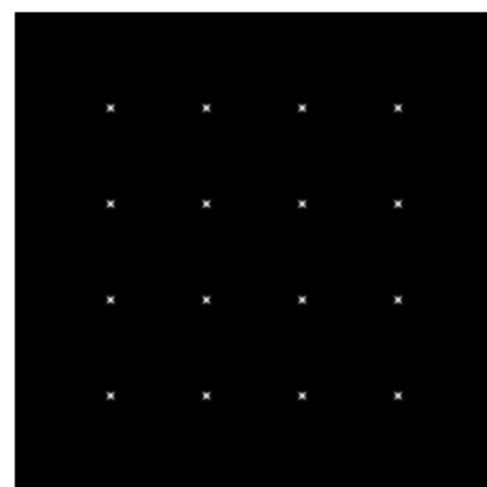
- Compute the gradient at each point in the image using derivatives of Gaussians
- Create the second moment matrix M from the entries in the gradient
- Compute the eigenvalues



I



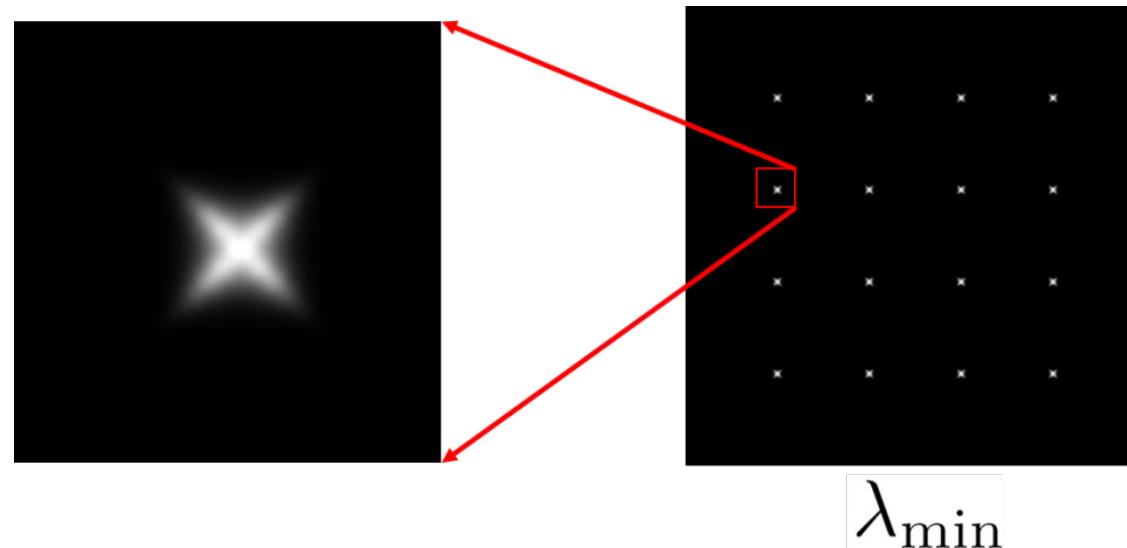
λ_{\max}



λ_{\min}

Corner detection summary

- Compute the gradient at each point in the image using derivatives of Gaussians
- Create the second moment matrix M from the entries in the gradient
- Compute the eigenvalues
- Find points with large response ($\lambda_{\min} > \text{threshold}$)
- Choose those points where λ_{\min} is a local maximum as features



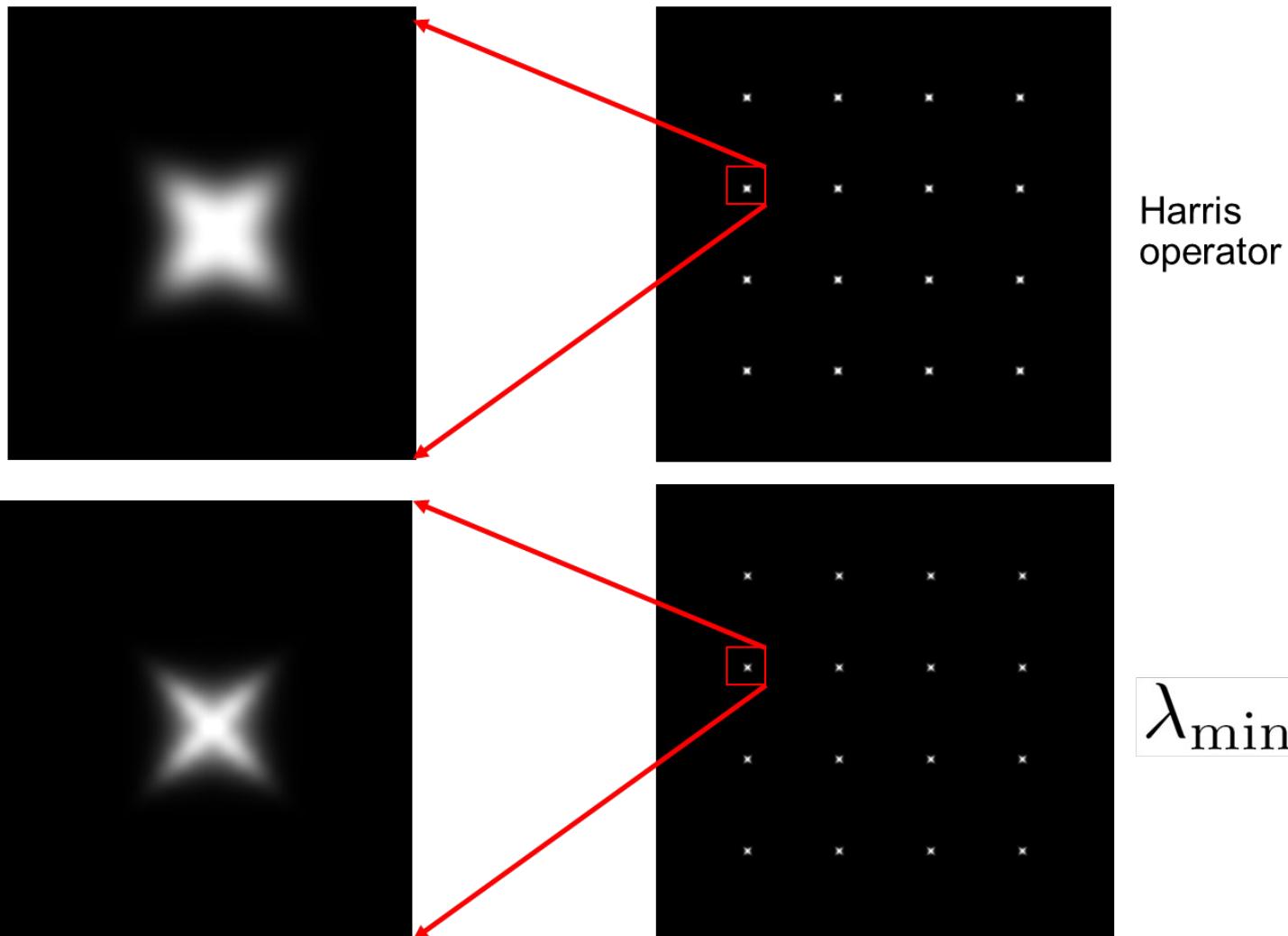
The Harris operator

- An alternative to λ_{\min} :

$$f = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2 = \det(M) - \alpha \text{trace}(M)^2$$

- $\alpha = 0.06$
- Very similar to λ_{\min} but less expensive (no square root)
- Called the “Harris Corner Detector” or “Harris Operator”
- Lots of other detectors, this is one of the most popular

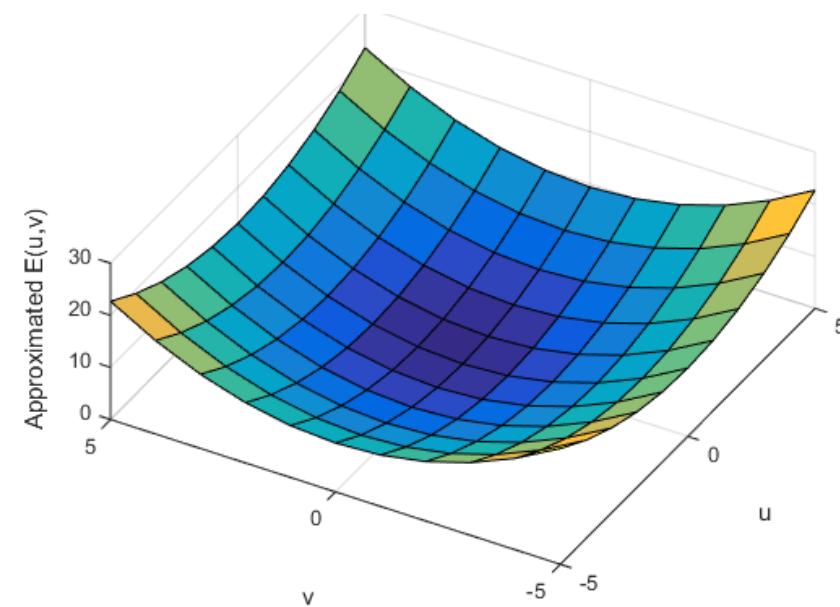
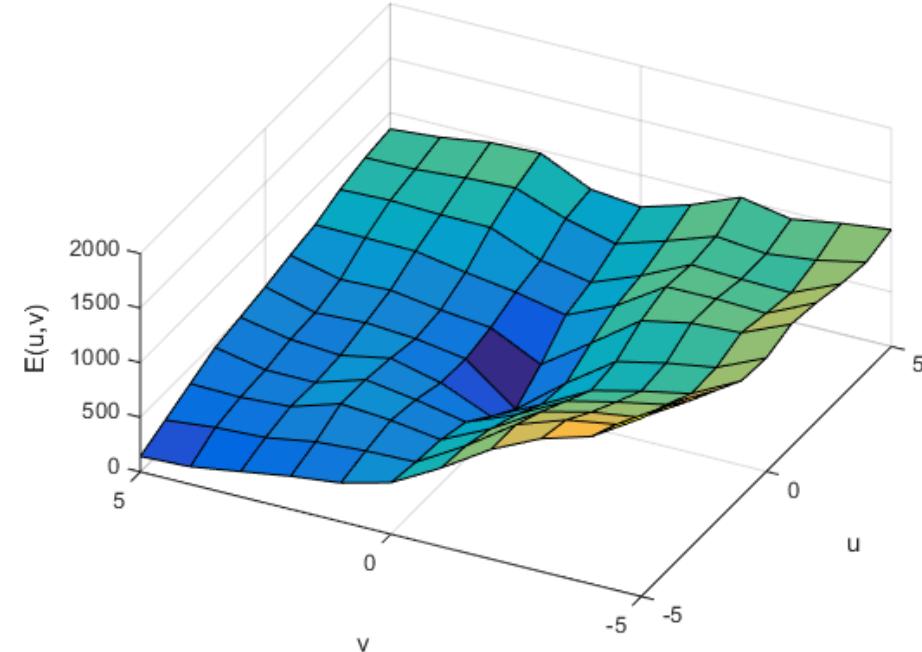
The Harris operator



Examples Holmenkollen



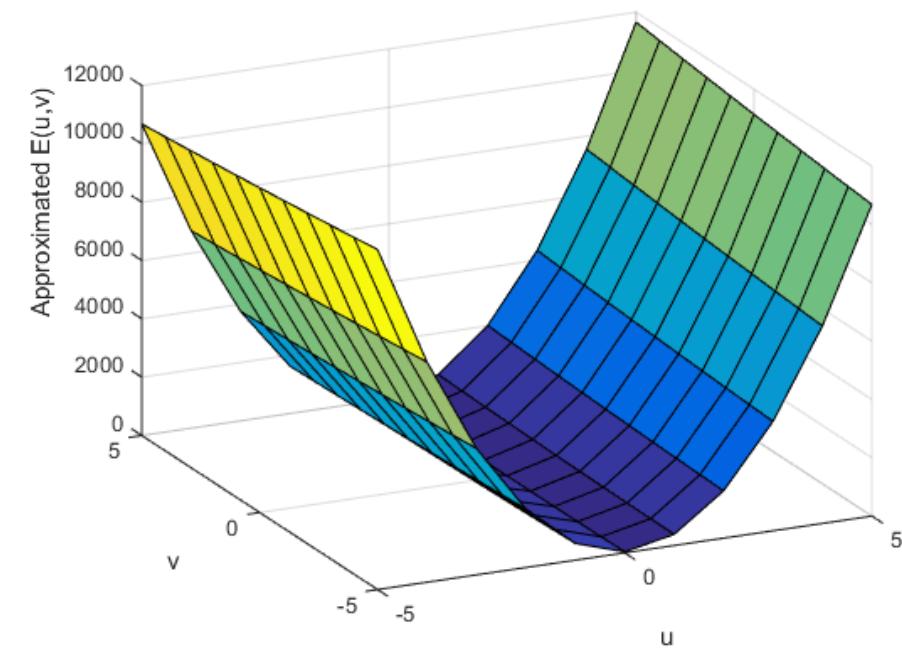
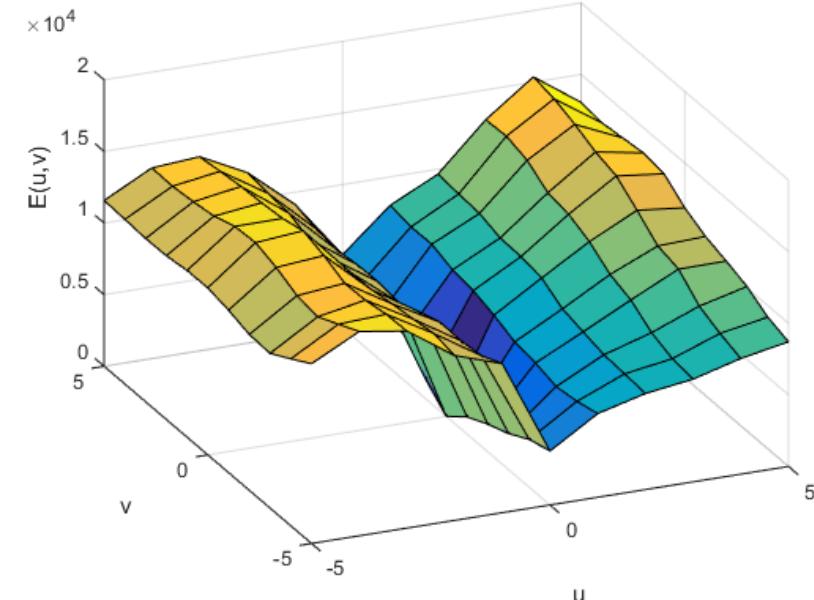
$$\lambda_{\min} = 0.4$$



Examples Holmenkollen



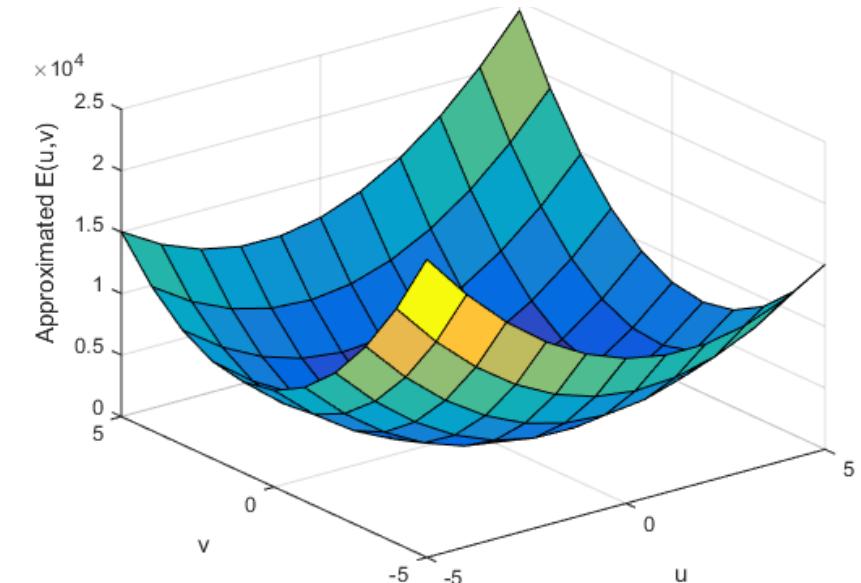
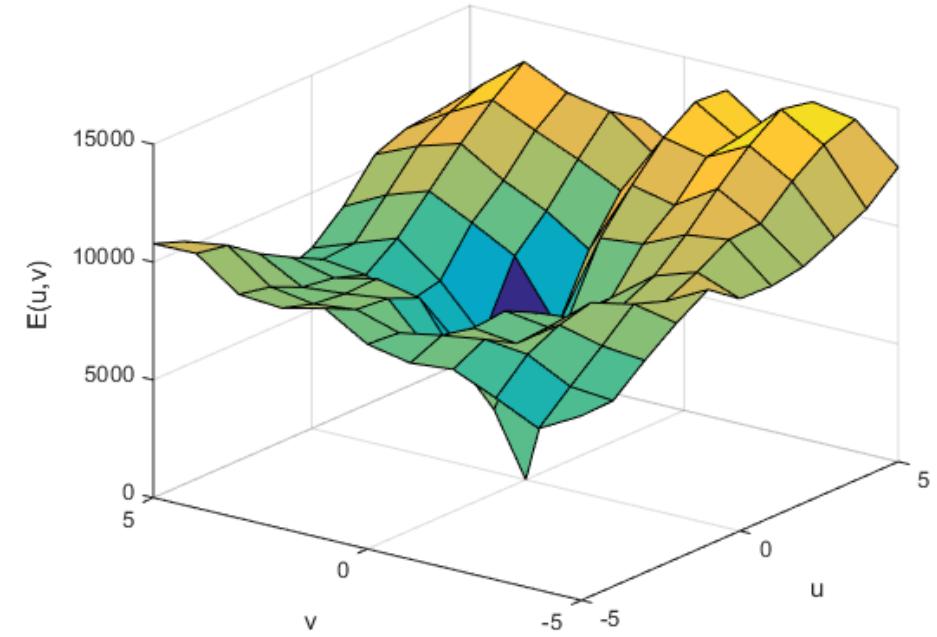
$$\lambda_{\min} = 1.2$$



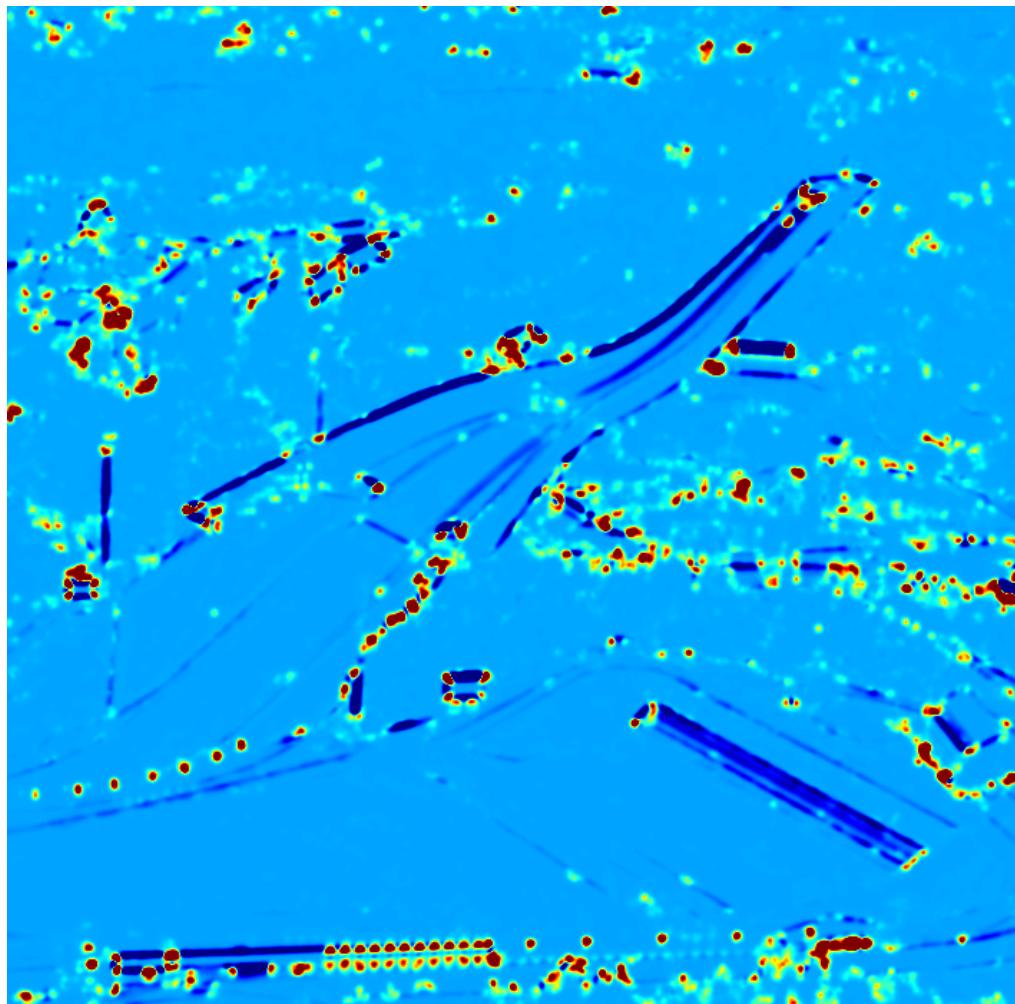
Examples Holmenkollen



$$\lambda_{\min} = 272$$



Holmenkollen example

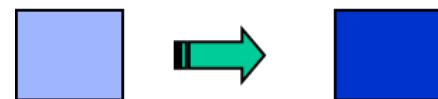


Invariance and covariance

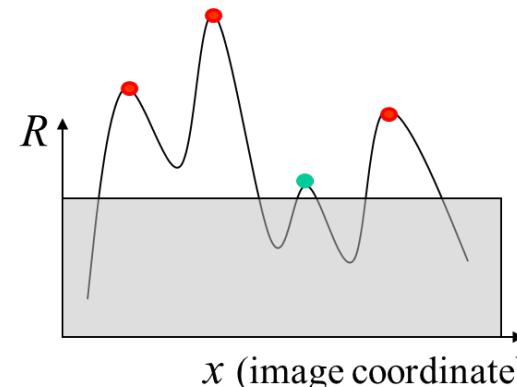
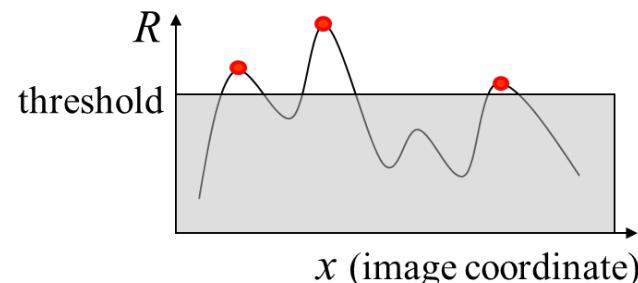
- We want corner locations to be *invariant* to photometric transformations and *covariant* to geometric transformations
 - **Invariance:** image is transformed and corner locations do not change
 - **Covariance:** if we have two transformed versions of the same image, features should be detected in corresponding locations

Harris detector properties

- **Affine intensity change**


$$I \rightarrow a I + b$$

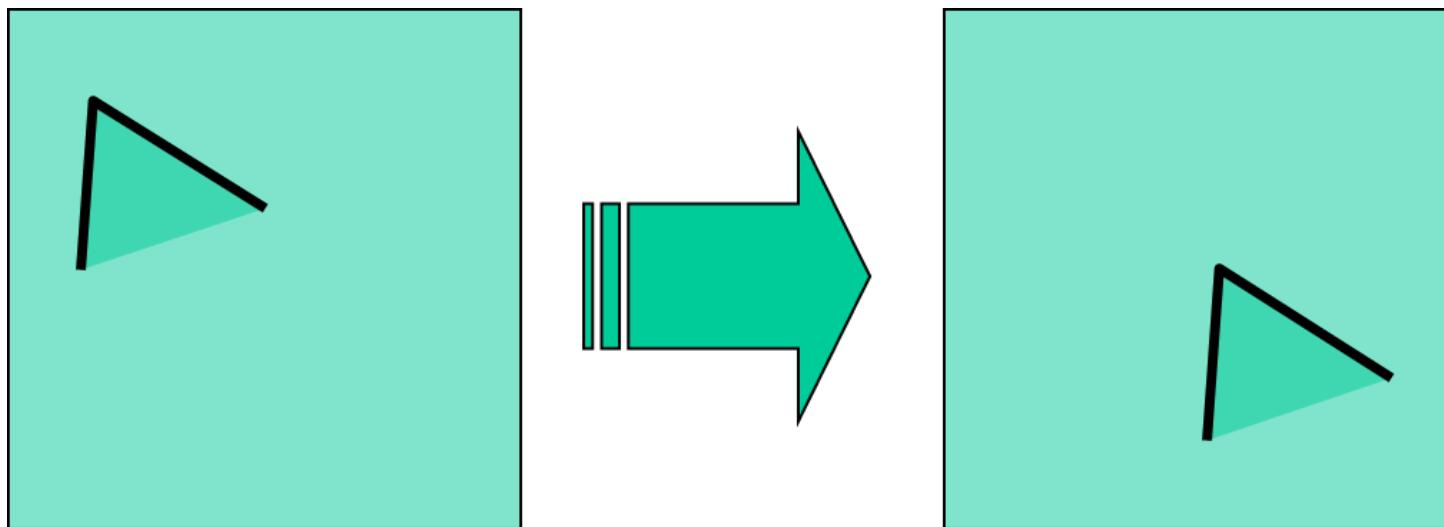
- Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$
- Intensity scaling: $I \rightarrow a I$



Partially invariant to affine intensity change

Harris detector properties

- Image translation

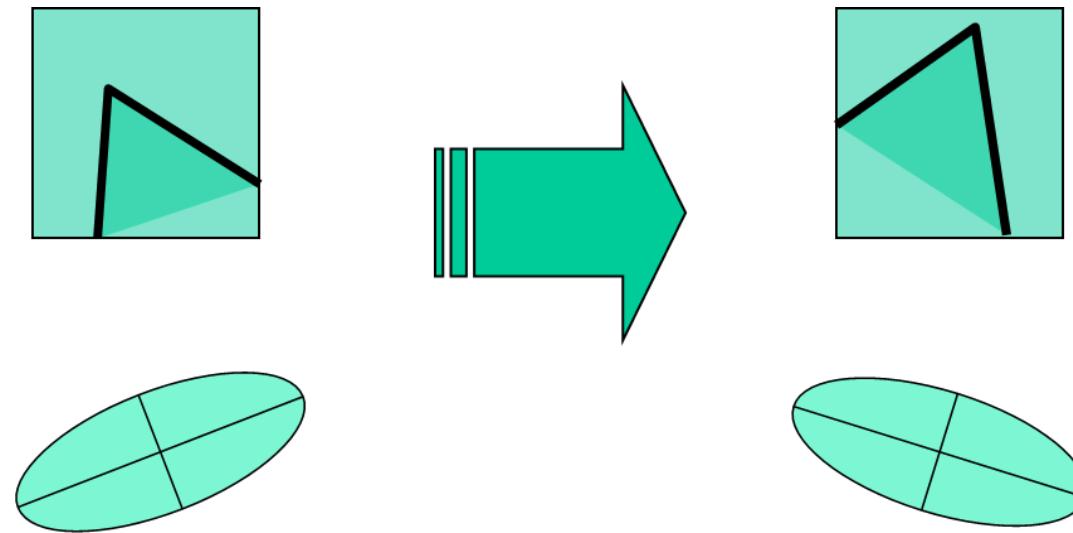


- Derivatives and window function are shift-invariant

Corner location is covariant w.r.t. translation

Harris detector properties

- Image rotation

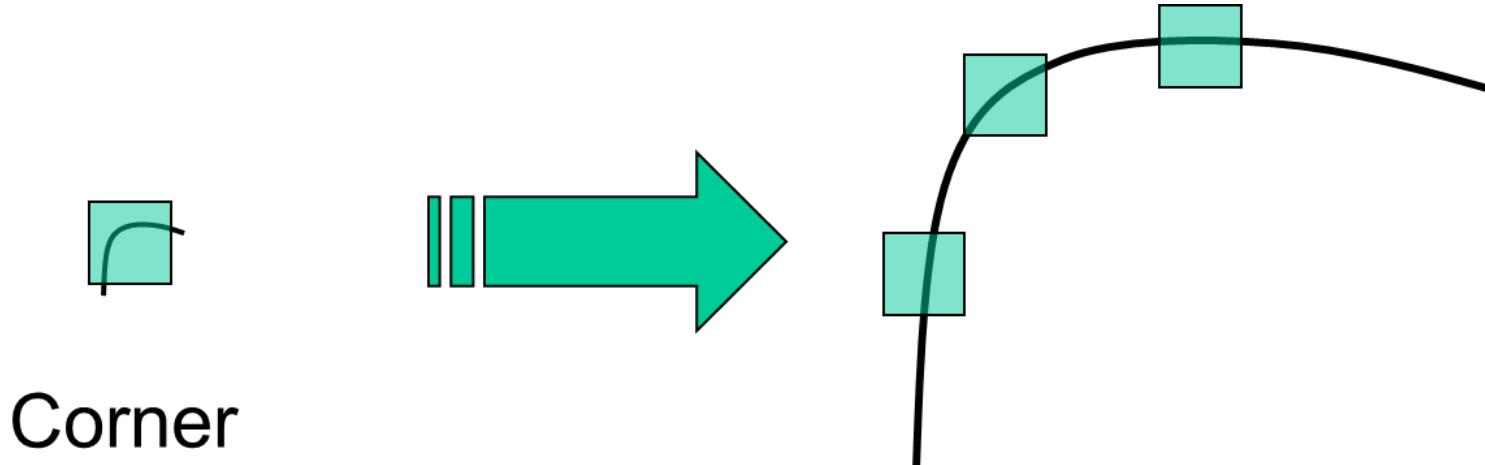


- Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner location is covariant w.r.t. rotation

Harris detector properties

- Scaling

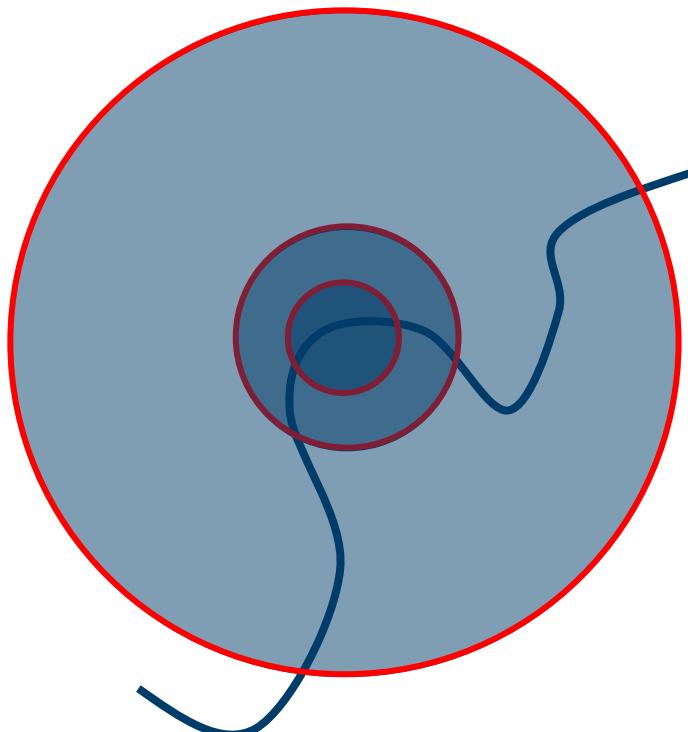


All points will
be classified
as edges

Corner location is not covariant to scaling!

Scale robust corner detection

- Find scale that gives local maximum of score f
 - In both position and scale



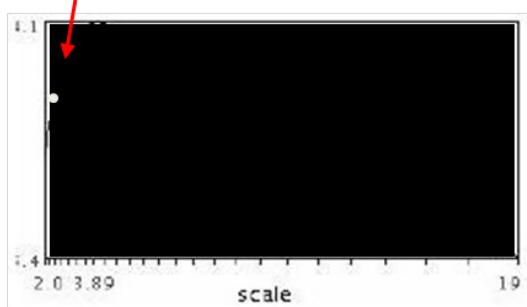
Automatic scale selection



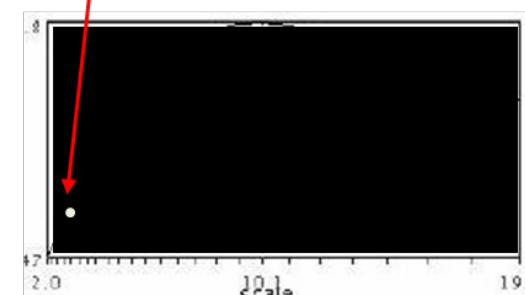
$$f(I_{i_1 \dots i_m}(x, \sigma)) = f(I_{i_1 \dots i_m}(x', \sigma'))$$



Automatic scale selection

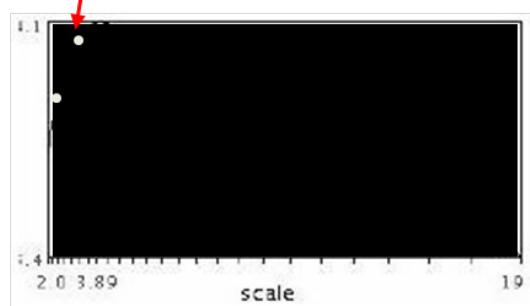


$$f(I_{i_1 \dots i_m}(x, \sigma))$$

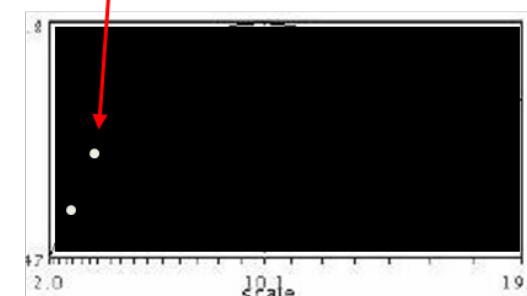


$$f(I_{i_1 \dots i_m}(x', \sigma))$$

Automatic scale selection

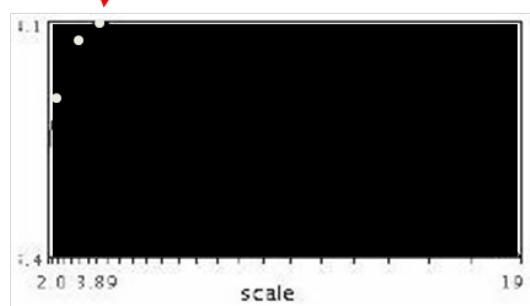


$$f(I_{i_1 \dots i_m}(x, \sigma))$$

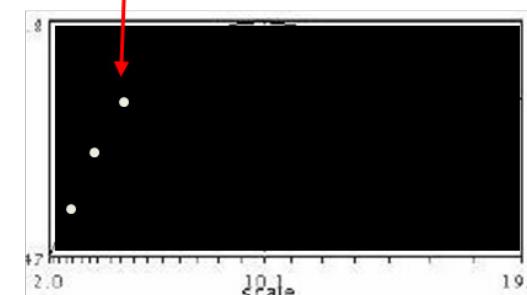


$$f(I_{i_1 \dots i_m}(x', \sigma))$$

Automatic scale selection

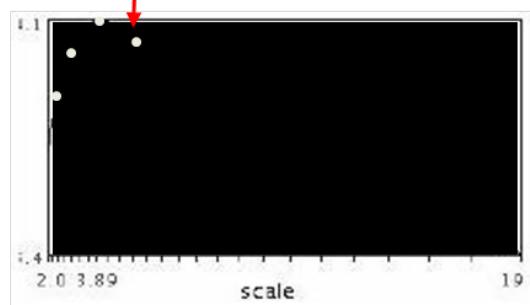


$$f(I_{i_1 \dots i_m}(x, \sigma))$$

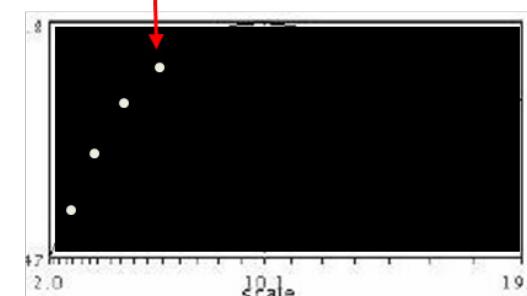


$$f(I_{i_1 \dots i_m}(x', \sigma))$$

Automatic scale selection

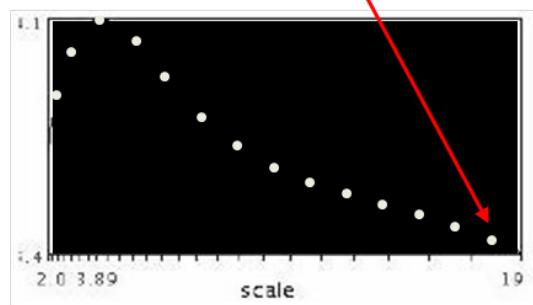
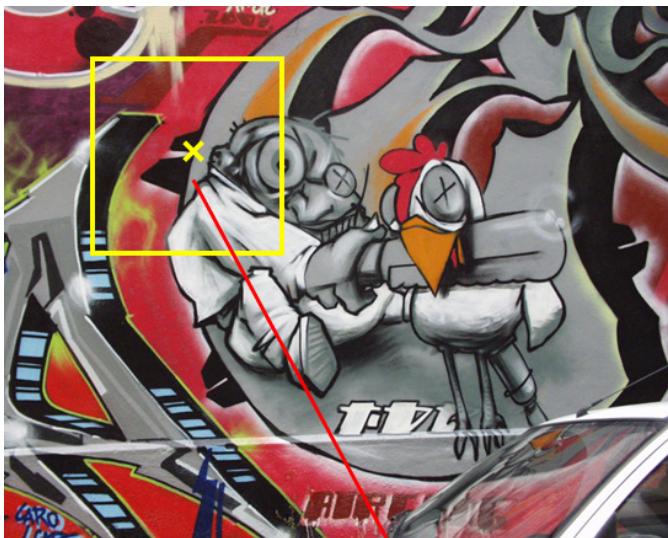


$$f(I_{i_1 \dots i_m}(x, \sigma))$$

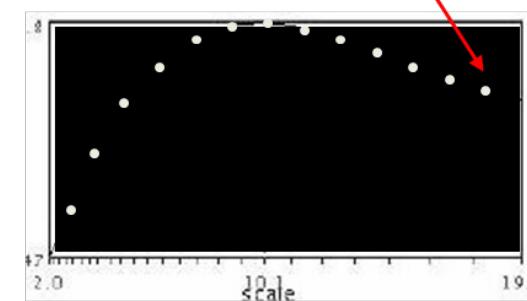


$$f(I_{i_1 \dots i_m}(x', \sigma))$$

Automatic scale selection

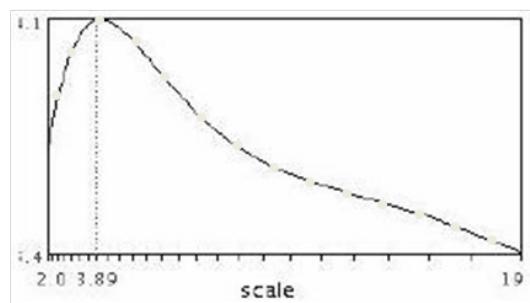
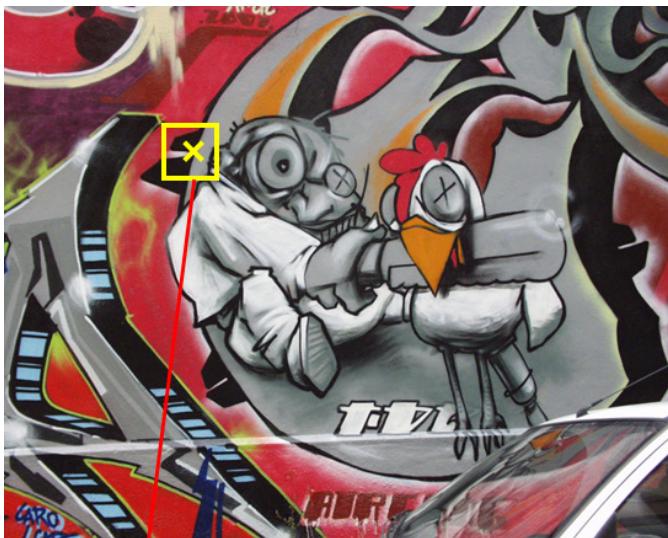


$$f(I_{i_1 \dots i_m}(x, \sigma))$$

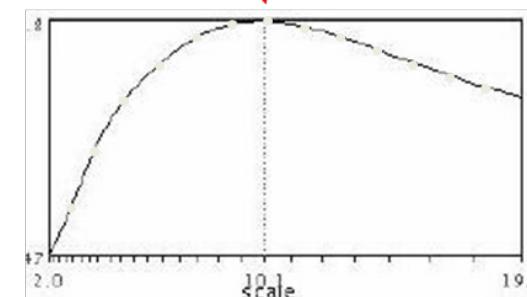


$$f(I_{i_1 \dots i_m}(x', \sigma))$$

Automatic scale selection



$$f(I_{i_1 \dots i_m}(x, \sigma))$$



$$f(I_{i_1 \dots i_m}(x', \sigma'))$$

Next

- Blob detector: stable in space and scale