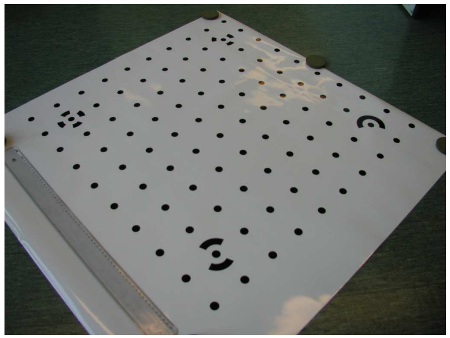
Reference points — targets

- Two types; artificial and natural.
- Artificial or *signalized* points have known shape.
- Have to be attached to the object.
- Enables filtering for pre-defined shapes.
- May be *coded*.
- May have known 3D coordinates.



Natural targets

- Wanted properties for natural targets (from Förstner, 1986):

  Distinctness Points should be different from their neighbourhood.

  Invariance The selection filter should be invariant with respect to the expected image variation, i.e. different lighting and/or projection.

  Stability The selected points should be stable with respect to noise, i.e. if they are selected in one image they should likely be selected in another.

  Seldomness The selected points should be “unique” within the image. This reduces the risk of picking out regions that appear in multiple places in the image.

  Interpretability It is desired that points may be interpreted in some way, e.g. corners, edges, blobs, etc.

- Points satisfying this are called *interest points*.
- The requirements are satisfied by e.g. corners, point objects, line crossings, textures, etc. Sometimes the term “corner” is used as a generic term.
- The requirements are not satisfied by e.g. edges or surfaces with little texture.

Examples

Harris corner detector and the *Förstner interest operator*

- Both are based on the covariance matrix for the gradient at each point in the image.
- Let $g_x \approx \partial I/\partial x$ and $g_y \approx \partial I/\partial y$ be the approximation of the intensity gradient in the x and y directions.
- To reduce noise sensitivity, the g_x and g_y should be lowpass filtered.
- If $A = g_x^2, B = g_y^2, C = g_x g_y$, then the matrix

$$M = \begin{bmatrix} A & C \\ C & B \end{bmatrix},$$

describes the shape of the *autocorrelation function* around a point.

Three cases

- The eigenvalues $\alpha \geq 0$ and $\beta \geq 0$ of M will be proportional to the curvature of the autocorrelation function.
- The eigenvalues are rotationally invariant.
- We have three cases to study:
  1. If both curvatures are small, i.e. the autocorrelation function is locally flat, the region has approximately constant intensity.
  2. If one curvature is large and one is small, i.e. the autocorrelation function is locally ridge-like, the point is on an edge.
  3. If both curvatures are large, the point is on a corner.
- We will want to contruct a measure that is large for corners and small for edges.
- We will use the fact that

$$Tr(M) = \alpha + \beta = A + B$$

and

$$Det(M) = \alpha\beta = AB - C^2$$

to avoid having to calculate the eigenvalues α and β.

The Harris corner detector

- The Harris corner detector calculates the descriptor

$$R = Det(M) - kTr(M)^2 = \alpha\beta - k(\alpha + \beta)^2$$

for some k, e.g. $k = 0.04$.

- The descriptor R is small if any of α and β is small and large if both are large.
- A drawback is that R has no scale.

The Förstner interest operator

- The Förstner interest operator uses several measures.
- The value

$$q = \frac{4Det(M)}{Tr(M)^2} = 1 - \left(\frac{\alpha - \beta}{\alpha + \beta}\right)^2$$

is a measure of isotrophy; q is large (close to 1) when α and β are equal; q is small (close to 0) when α and β are different.

- For pixels with large enough q value a preliminary weight is calculated:

$$w = \begin{cases} \frac{Det(M)}{Tr(M)} = \frac{\alpha\beta}{\alpha+\beta}, & q > q_{min} \\ 0, & \text{otherwise} \end{cases}$$

- The final weight w^* is given by non-maximum suppression:

$$w^*_{rc} = \begin{cases} w_{rc} & w_{rc} \text{ local max} \\ 0 & \text{otherwise} \end{cases}$$

- A high w^* value indicates a window of good locality.

The Förstner interest operator

- For each pair of windows i and j a correlation coefficient r_{ij} is calculated between the windows around the corresponding points.
- From r_{ij} a similarity measure r_i and a seldomness measure S_i are defined as

$$r_i = \max_{j \neq i} r_{ij}$$

$$S_i = \begin{cases} \frac{1-r_i}{r_i} & r_i > 0 \\ \infty & \text{otherwise} \end{cases}$$

- The "uniqueness-corrected" weight $u_i = w^*_i S_i$ combines locality with uniqueness.

Preliminary matching

- Given two sets of windows from two different images, preliminary matching weights between window i in image 1 and window j in image 2 are given by

$$m_{ij} = \begin{cases} \frac{N}{2}\,\frac{t_{ij}}{1-t_{ij}}\,\frac{1}{\sigma_i\sigma_j}\,\sqrt{u_i u_j}, & t_{ij} > t_{min} \\ 0 & \text{otherwise} \end{cases}$$

where t_{ij} is the correlation coefficient between window i in image 1 and window j in image 2 and σ_i and σ_j are the standard deviations within the respective windows.

Example

An image I and the derivative filters h_x and h_y.

$$I = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 2 & 2 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 & 2 & 2 & 2 & 1 & 1 \\ 3 & 3 & 3 & 2 & 2 & 2 & 2 & 1 & 1 \\ 3 & 3 & 3 & 2 & 2 & 2 & 3 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix},$$

$$h_x = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad h_y = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

Example

The gradient images:

$$g_x = \begin{bmatrix} * & * & * & * & * & * & * & * & * \\ * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * \\ * & 0 & -1 & -1 & 0 & 0 & 1 & 1 & * \\ * & 0 & -1 & -1 & 0 & 0 & 1 & 1 & * \\ * & 0 & 1 & 1 & 0 & 0 & 1 & 1 & * \\ * & 0 & 1 & 1 & 0 & -1 & 1 & 2 & * \\ * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * \\ * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * \\ * & * & * & * & * & * & * & * & * \end{bmatrix},$$

$$g_y = \begin{bmatrix} * & * & * & * & * & * & * & * & * \\ * & 0 & 0 & -1 & -1 & -1 & -1 & 0 & * \\ * & 0 & 0 & -1 & -1 & -1 & -1 & 0 & * \\ * & -2 & -2 & 0 & 0 & 0 & 0 & 0 & * \\ * & -2 & -2 & 0 & 0 & 0 & -1 & 0 & * \\ * & 2 & 2 & 1 & 1 & 1 & 1 & 0 & * \\ * & 2 & 2 & 1 & 1 & 1 & 2 & 0 & * \\ * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * \\ * & * & * & * & * & * & * & * & * \end{bmatrix}$$

Second order terms

$$g_x^2 = \begin{bmatrix} * & * & * & * & * & * & * & * & * \\ * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * \\ * & 0 & 1 & 1 & 0 & 0 & 1 & 1 & * \\ * & 0 & 1 & 1 & 0 & 0 & 1 & 1 & * \\ * & 0 & 1 & 1 & 0 & 0 & 1 & 1 & * \\ * & 0 & 1 & 1 & 0 & 1 & 1 & 4 & * \\ * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * \\ * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * \\ * & * & * & * & * & * & * & * & * \end{bmatrix},$$

$$g_y^2 = \begin{bmatrix} * & * & * & * & * & * & * & * & * \\ * & 0 & 0 & 1 & 1 & 1 & 1 & 0 & * \\ * & 0 & 0 & 1 & 1 & 1 & 1 & 0 & * \\ * & 4 & 4 & 0 & 0 & 0 & 0 & 0 & * \\ * & 4 & 4 & 0 & 0 & 0 & 1 & 0 & * \\ * & 4 & 4 & 1 & 1 & 1 & 1 & 0 & * \\ * & 4 & 4 & 1 & 1 & 1 & 4 & 0 & * \\ * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * \\ * & * & * & * & * & * & * & * & * \end{bmatrix},$$

$$g_x g_y = \begin{bmatrix} * & * & * & * & * & * & * & * & * \\ * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * \\ * & 0 & 0 & 1 & 0 & 0 & -1 & 0 & * \\ * & 0 & 2 & 0 & 0 & 0 & 0 & 0 & * \\ * & 0 & -2 & 0 & 0 & 0 & -1 & 0 & * \\ * & 0 & 2 & 1 & 0 & -1 & 1 & 0 & * \\ * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * \\ * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * \\ * & * & * & * & * & * & * & * & * \end{bmatrix},$$

3x3 average of second order terms

$$A = \overline{g_x^2} = \begin{bmatrix} * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & 4 & 4 & 2 & 2 & 4 & * & * \\ * & * & 6 & 6 & 3 & 3 & 6 & * & * \\ * & * & 6 & 6 & 4 & 4 & 10 & * & * \\ * & * & 4 & 4 & 3 & 3 & 8 & * & * \\ * & * & 2 & 2 & 2 & 2 & 6 & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \end{bmatrix},$$

$$B = \overline{g_y^2} = \begin{bmatrix} * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & 10 & 8 & 6 & 6 & 4 & * & * \\ * & * & 17 & 10 & 3 & 4 & 3 & * & * \\ * & * & 25 & 14 & 3 & 4 & 3 & * & * \\ * & * & 26 & 16 & 6 & 10 & 8 & * & * \\ * & * & 18 & 12 & 6 & 9 & 7 & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \end{bmatrix},$$

$$C = \overline{g_x g_y} = \begin{bmatrix} * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & 3 & 3 & 1 & -1 & -1 & * & * \\ * & * & 1 & 1 & 1 & -2 & -2 & * & * \\ * & * & 3 & 3 & 0 & -1 & -1 & * & * \\ * & * & 1 & 1 & 0 & -1 & -1 & * & * \\ * & * & 3 & 3 & 0 & 0 & 0 & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \end{bmatrix},$$

Trace and Det

$$Trace = A + B = \begin{bmatrix} * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & 14 & 12 & 8 & 8 & 8 & * & * \\ * & * & 23 & 16 & 6 & 7 & 9 & * & * \\ * & * & 31 & 20 & 7 & 8 & 13 & * & * \\ * & * & 30 & 20 & 9 & 13 & 16 & * & * \\ * & * & 20 & 14 & 8 & 11 & 13 & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \end{bmatrix},$$

$$Det = AB - C^2 = \begin{bmatrix} * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & 31 & 23 & 11 & 11 & 15 & * & * \\ * & * & 101 & 59 & 8 & 8 & 14 & * & * \\ * & * & 141 & 75 & 12 & 15 & 29 & * & * \\ * & * & 103 & 63 & 18 & 29 & 63 & * & * \\ * & * & 27 & 15 & 12 & 18 & 42 & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \end{bmatrix},$$

Isotrophy q and weights w and w^*

$$q = \frac{4Det}{Tr^2} = \begin{bmatrix} * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & 0.63 & 0.64 & 0.69 & 0.69 & 0.94 & * & * \\ * & * & 0.76 & 0.92 & 0.89 & 0.65 & 0.69 & * & * \\ * & * & 0.59 & 0.75 & 0.98 & 0.94 & 0.69 & * & * \\ * & * & 0.46 & 0.63 & 0.89 & 0.69 & 0.98 & * & * \\ * & * & 0.27 & 0.31 & 0.75 & 0.60 & 0.99 & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \end{bmatrix},$$

$$w = \frac{Det}{Tr} = \begin{bmatrix} * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & 2.21 & 1.92 & 1.38 & 1.38 & 1.88 & * & * \\ * & * & 4.39 & 3.69 & 1.33 & 1.14 & 1.56 & * & * \\ * & * & 4.55 & 3.75 & 1.71 & 1.88 & 2.23 & * & * \\ * & * & 0 & 3.15 & 2.00 & 2.23 & 3.94 & * & * \\ * & * & 0 & 0 & 1.50 & 1.64 & 3.23 & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \end{bmatrix},$$

$$w^* = \begin{bmatrix} * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 & 1.88 & * & * \\ * & * & 0 & 0 & 0 & 0 & 0 & * & * \\ * & * & 4.55 & 0 & 0 & 0 & 0 & * & * \\ * & * & 0 & 0 & 0 & 0 & 3.94 & * & * \\ * & * & 0 & 0 & 0 & 0 & 0 & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \end{bmatrix},$$

Corresponding image regions

$$w^* = \begin{bmatrix} * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 & 1.88 & * & * \\ * & * & 0 & 0 & 0 & 0 & 0 & * & * \\ * & * & 4.55 & 0 & 0 & 0 & 0 & * & * \\ * & * & 0 & 0 & 0 & 0 & 3.94 & * & * \\ * & * & 0 & 0 & 0 & 0 & 0 & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \end{bmatrix},$$

$$I = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 2 & 2 & \mathbf{2} & 1 & 1 \\ 1 & 1 & 1 & 2 & 2 & 2 & 2 & 1 & 1 \\ 3 & 3 & \mathbf{3} & 2 & 2 & 2 & 2 & 1 & 1 \\ 3 & 3 & 3 & 2 & 2 & 2 & \mathbf{3} & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix},$$

Similarity r_i and seldomness S_i

The windows

$$I_1 = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 2 & 2 \\ 3 & 3 & 3 & 2 & 2 \\ 3 & 3 & 3 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \qquad I_2 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 1 & 1 \\ 2 & 2 & 2 & 1 & 1 \\ 2 & 2 & 2 & 1 & 1 \end{bmatrix},$$

$$I_3 = \begin{bmatrix} 2 & 2 & 2 & 1 & 1 \\ 2 & 2 & 2 & 1 & 1 \\ 2 & 2 & 3 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

have correlation coefficients $r_{12} = 0.5$, $r_{13} = 0$, $r_{23} = 0.06$, which gives

$$r_1 = 0.50, S_1 = 1, u_1 = 4.55,$$
$$r_2 = 0.50, S_2 = 1, u_2 = 1.88,$$
$$r_3 = 0.06, S_3 = 16, u_3 = 63.$$

500 best corners, blocked

250 most seldom corners

50 strongest couplings

The optimal point within the window

- The center of the window may not be the optimal point.
- The optimal point may be interpreted as
  - a corner, i.e. the intersection of all gray level edges, or
  - the center of a circle, i.e. the intersection of all gray level slopes (orthogonal to edges).
- Both may be formulated as a weighted center of gravity calculation based on the gradients.

The optimal point within the window

- Edges:

$$\begin{bmatrix} g_{r_1} & g_{c_1} \\ g_{r_2} & g_{c_2} \\ \vdots & \vdots \\ g_{r_n} & g_{c_n} \end{bmatrix} \begin{bmatrix} r_0 \\ c_0 \end{bmatrix} \approx \begin{bmatrix} g_{r_1} r_1 + g_{c_1} c_1 \\ g_{r_2} r_2 + g_{c_2} c_2 \\ \vdots \\ g_{r_n} r_n + g_{c_n} r_n \end{bmatrix} \Rightarrow \begin{bmatrix} \sum g_{r_i}^2 & \sum g_{r_i} g_{c_i} \\ \sum g_{r_i} g_{c_i} & \sum g_{c_i}^2 \end{bmatrix} \begin{bmatrix} r_0 \\ c_0 \end{bmatrix} = \begin{bmatrix} \sum g_{r_i}^2 r_i + \sum g_{r_i} g \\ \sum g_{r_i} g_{c_i} r_i + \sum g \end{bmatrix}$$

- Slopes:

$$\begin{bmatrix} g_{c_1} & -g_{r_1} \\ g_{c_2} & -g_{r_2} \\ \vdots & \vdots \\ g_{c_n} & -g_{r_n} \end{bmatrix} \begin{bmatrix} r_0 \\ c_0 \end{bmatrix} \approx \begin{bmatrix} -g_{c_1} r_1 + g_{r_1} c_1 \\ -g_{c_2} r_2 + g_{r_2} c_2 \\ \vdots \\ -g_{c_n} r_n + g_{r_n} r_n \end{bmatrix} \Rightarrow \begin{bmatrix} \sum g_{c_i}^2 & -\sum g_{r_i} g_{c_i} \\ \sum -g_{r_i} g_{c_i} & \sum g_{r_i}^2 \end{bmatrix} \begin{bmatrix} r_0 \\ c_0 \end{bmatrix} = \begin{bmatrix} \sum g_{r_c}^2 r_i - \sum g_{r_i} g \\ -\sum g_{r_i} g_{c_i} r_i + \sum \end{bmatrix}$$

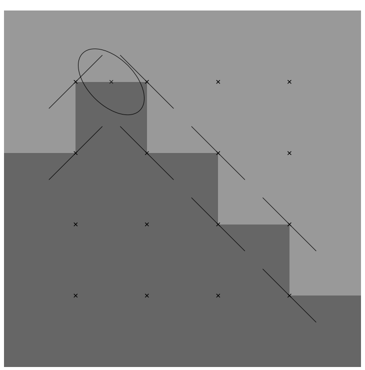
where (r_i, c_i) are the coordinates for each pixel and g_{r_i} and g_{c_i} are the gradient at those coordinates.

- Furthermore, from the solution it is possible to estimate the covariance of the point estimation.

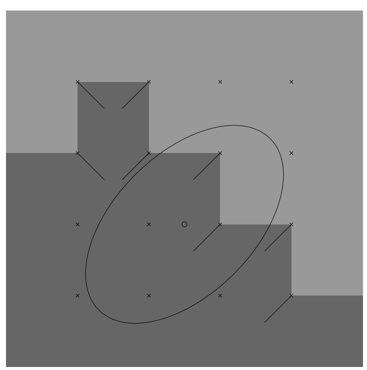
Example 1 from the paper

- Roberts edge filter,

$$h_x = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}, h_y = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}.$$



edge/corner interpretation

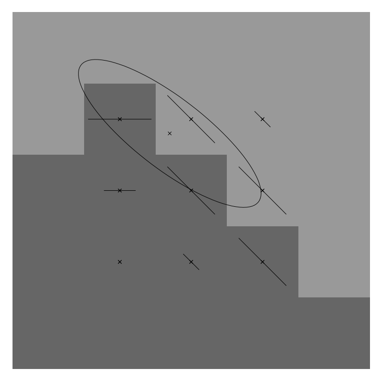


slope/circle interpretation

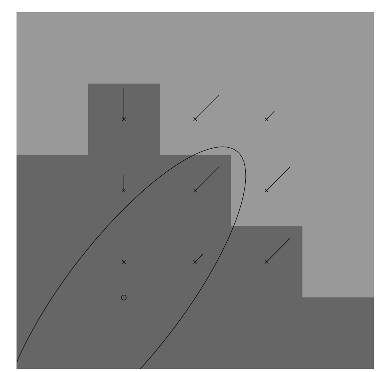
Example 1 from the paper

- Sobel edge filter,

$$h_x = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}, h_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}.$$



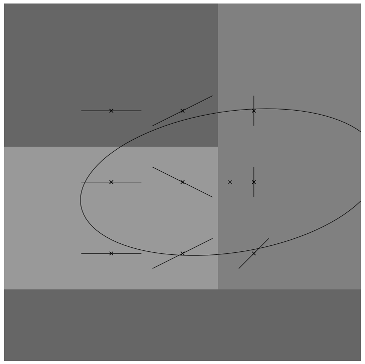
edge/corner interpretation



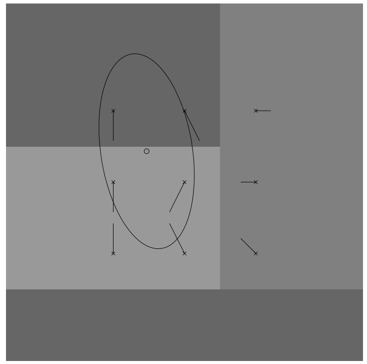
slope/circle interpretation

Region I_1

$$h_x = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}, h_y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}.$$



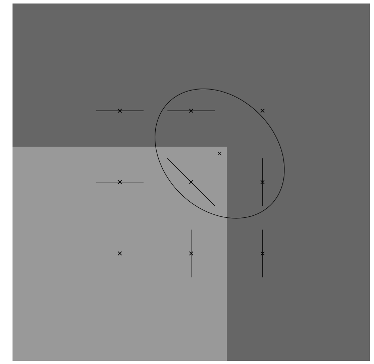
edge/corner interpretation



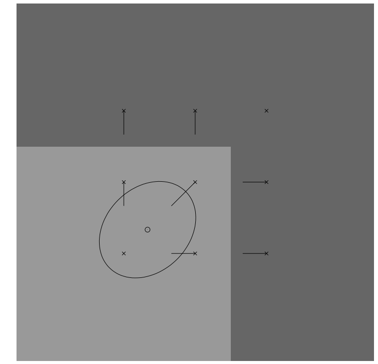
slope/circle interpretation

Region I_2

$$h_x = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}, h_y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}.$$



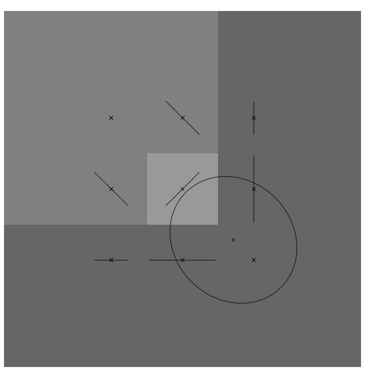
edge/corner interpretation



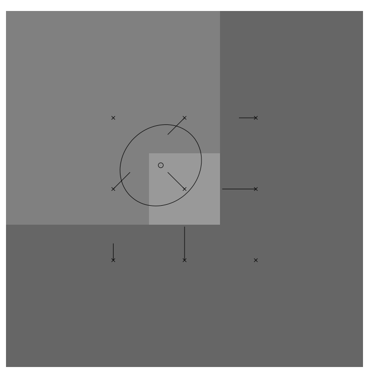
slope/circle interpretation

Region I_3

$$h_x = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}, h_y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}.$$



edge/corner interpretation



slope/circle interpretation