

Image Processing

Lecture 2.2 – Image Pyramids

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Pyramids

- Downsampling (decimation)
- Upsampling (interpolation)
- Pyramids
 - Gaussian Pyramids
 - Laplacian Pyramids (Lecture 2.3)
- Applications
 - Template matching (object detection)
 - Detecting stable points of interest
 - Image Registration
 - Compression
 - Image Blending
 - ...

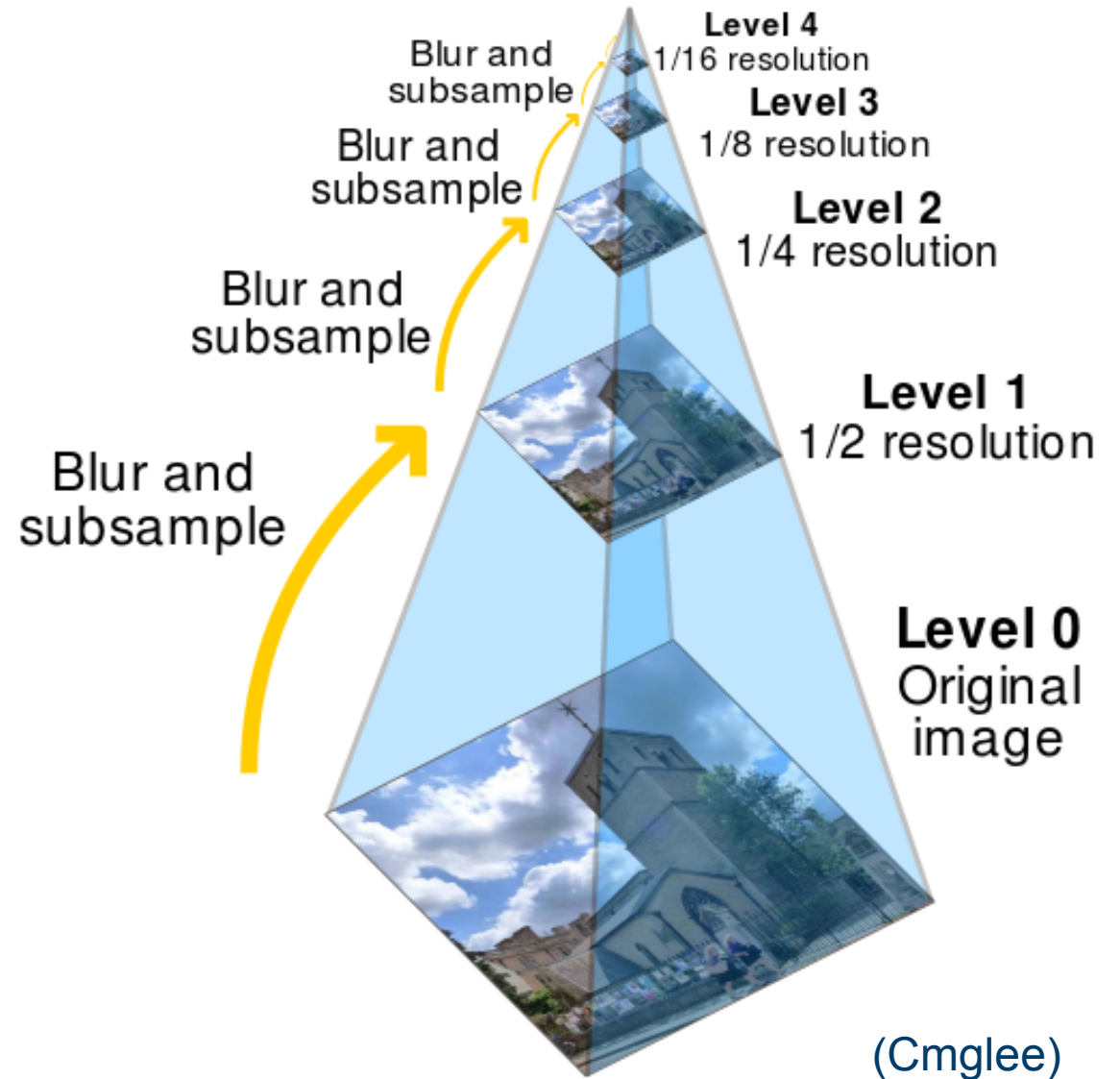


Image Scaling

- Assume that the image is too big for practical use:
 - Requires too much memory
 - Time consuming to process
 - Too big for the screen
 - ...
- A smaller image can be obtained by image sub-sampling



Image Downsampling



1/2



1/4



1/8

Throw away every other row and column → image reduced to $\frac{1}{2}$ size along each dimension.

Image Downsampling



1/2



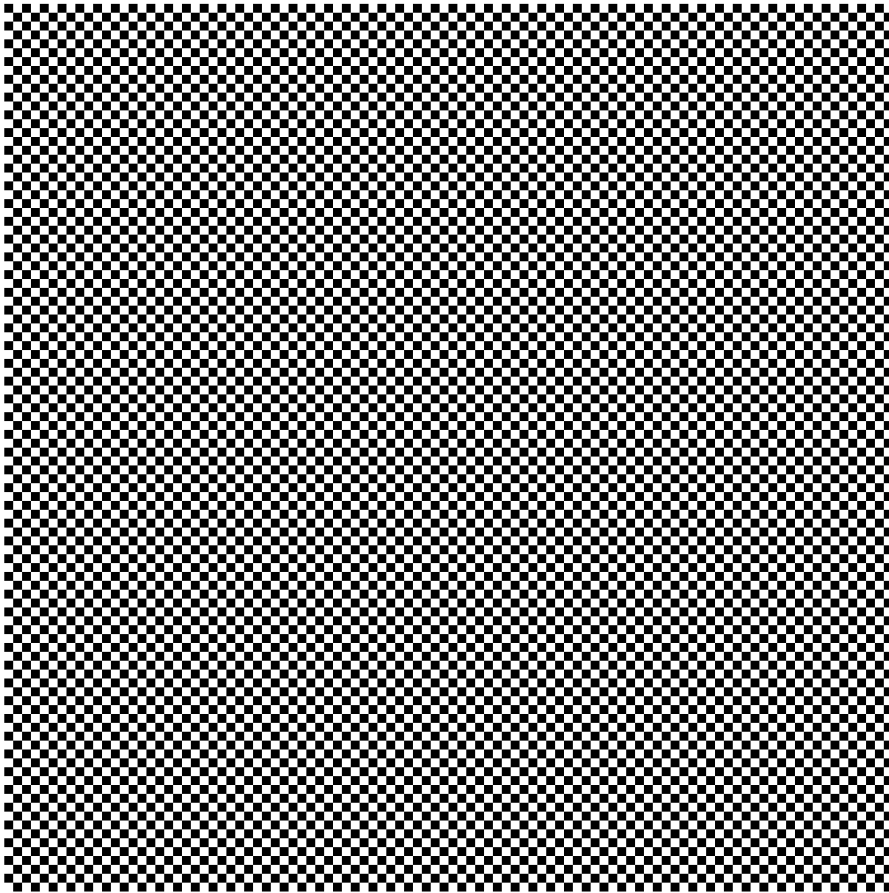
1/4 (2x zoom)



1/8 (4x zoom)

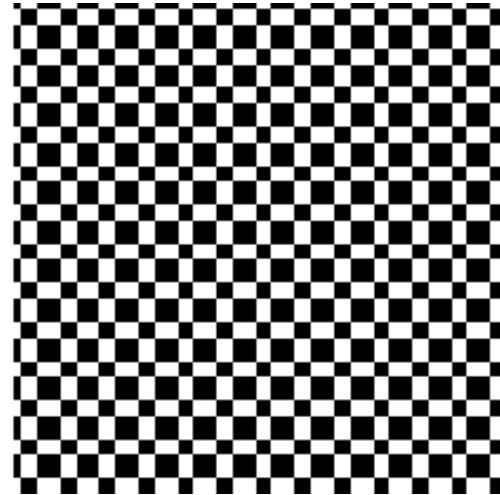
The subsampled images are of low quality. Why?

Spatial undersampling

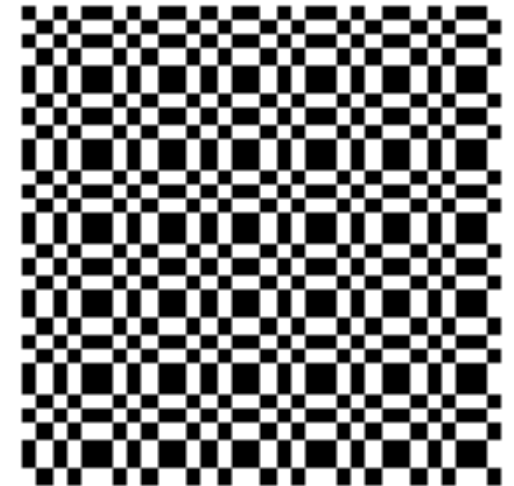


Checkerboard with 10 x 10 pixel squares

Downsampled images



1/10

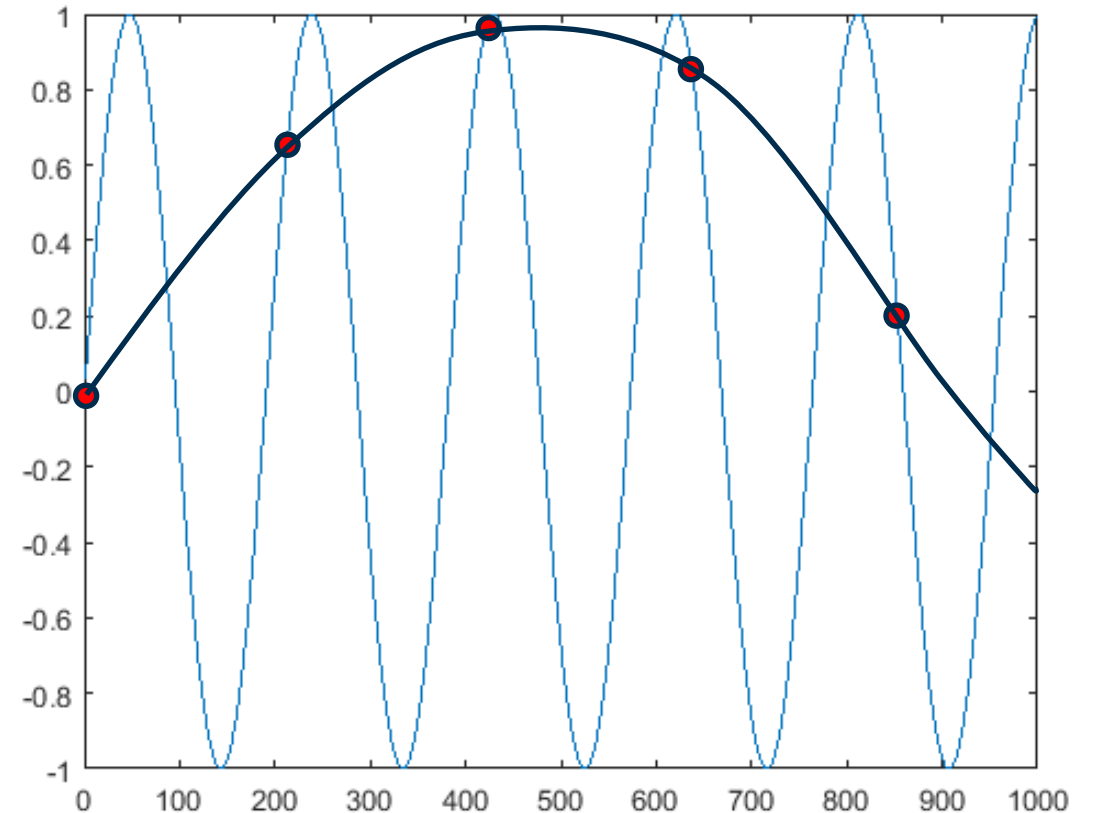


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Aliasing

- Occurs when the (spatial) sampling rate is not high enough to capture the details in the image
- High frequencies are transformed to lower frequencies (i.e. aliases)
- To avoid aliasing the sampling rate must be at least two times the maximum frequency in the image (at least two samples per cycle)
- This minimum sampling rate is called the **Nyquist rate**.

Aliasing can be avoided by low-pass filtering the image before downsampling



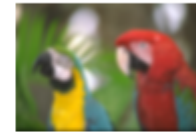
Gaussian pre-filtering (low pass)



Gaussian 1/2



Gaussian 1/4



Gaussian 1/8

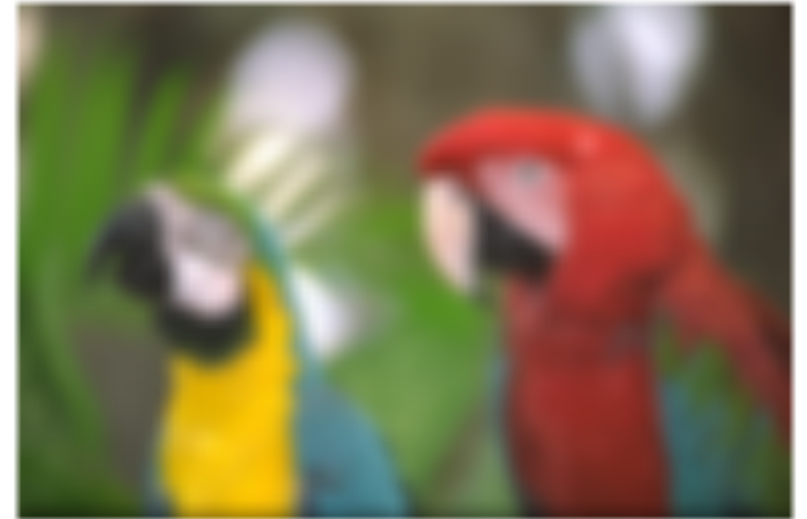
Gaussian pre-filtering (low pass)



Gaussian 1/2



Gaussian 1/4



Gaussian 1/8

Compared to downsampling without low-pass filtering...



1/2



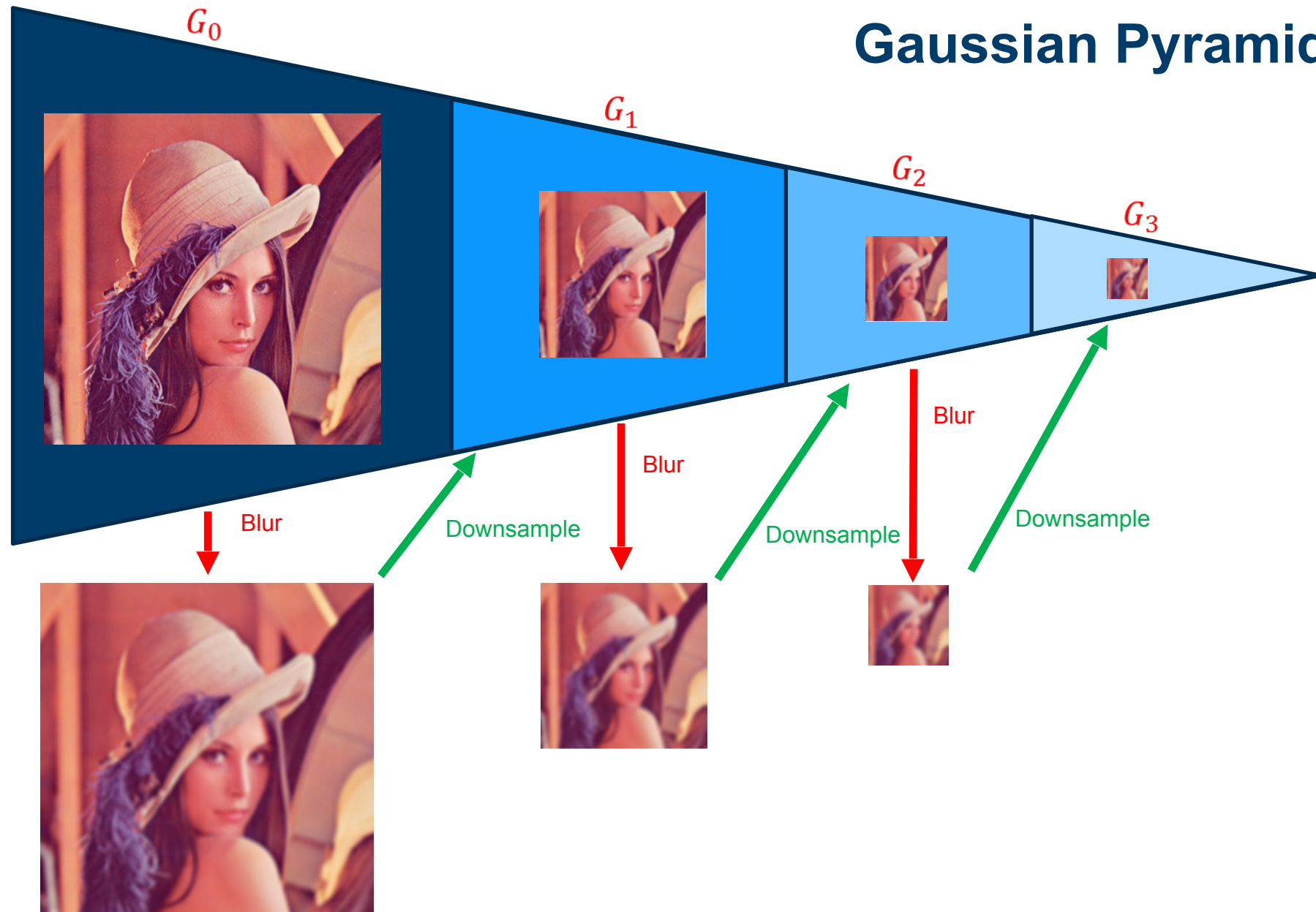
1/4



1/8

Conclusion: Low-pass filtering (i.e. smoothing with a Gaussian kernel) before subsampling the image!

Gaussian Pyramid



Upsampling

10 x magnification



Nearest neighbor interpolation:

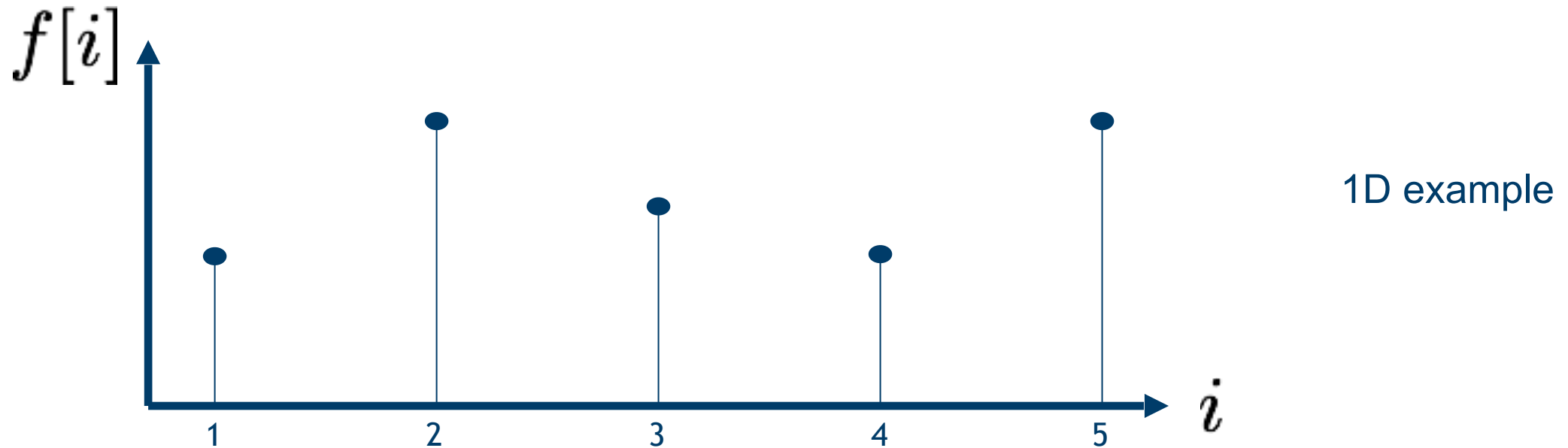
- Repeat each row and column 10 times
- Fast and simple approach.



Image interpolation

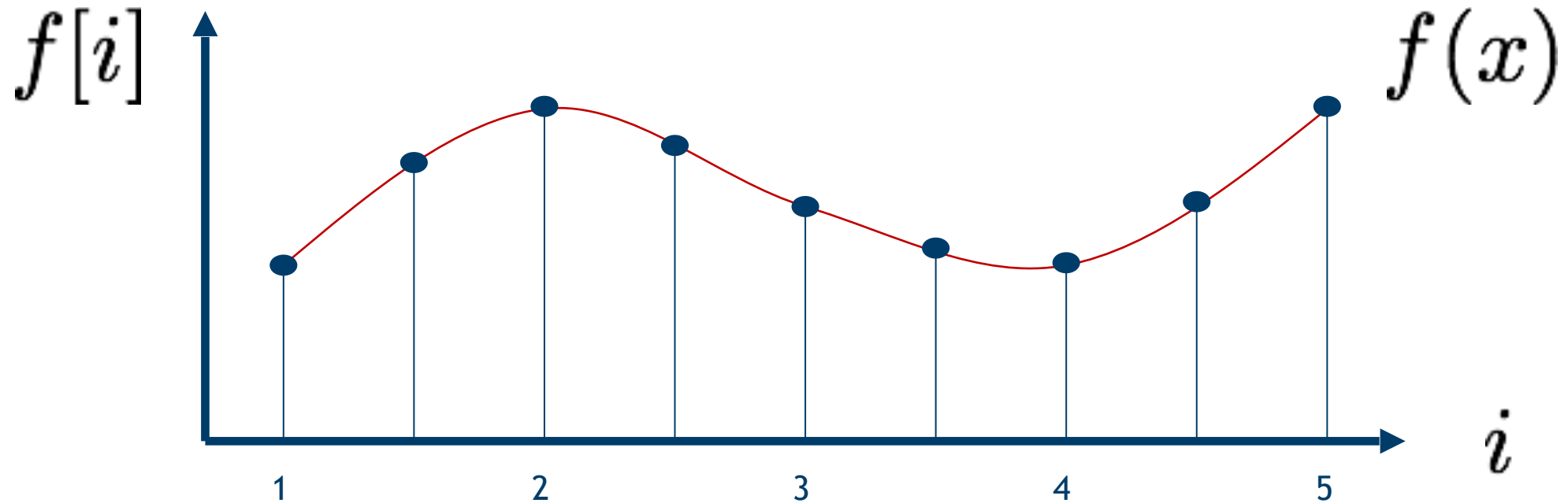
A digital image is a discrete point-sampling of a continuous function:

$$f[i, j] = \text{quantize}\{f(i\Delta x, j\Delta y)\} \text{ where } x = i\Delta x \text{ and } y = j\Delta y$$



A new image could be generated, at any resolution and scale, if the original function could be reconstructed.

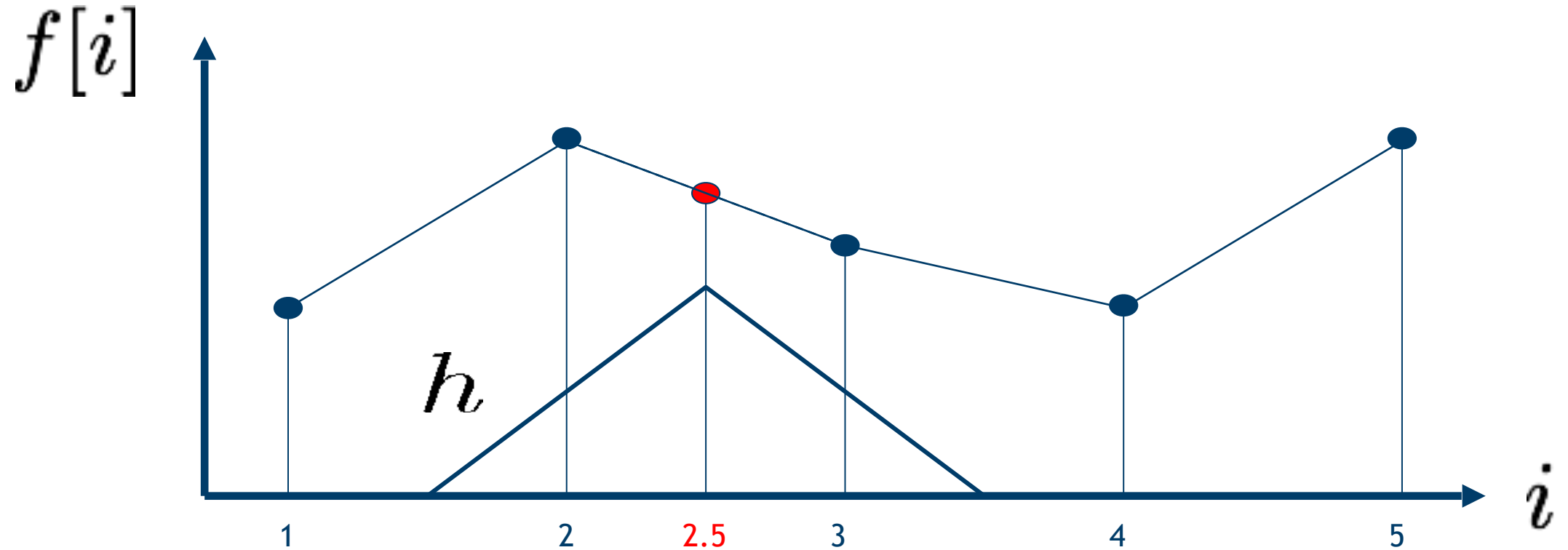
Interpolation by convolution



$$g[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k h[u, v] f[i - ru, j - rv]$$

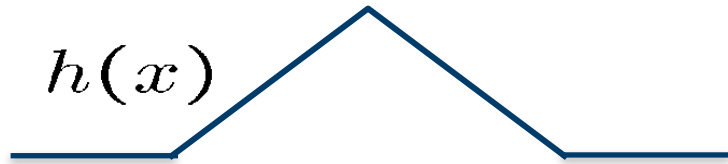
r = scale factor

Linear interpolation (bilinear for images)



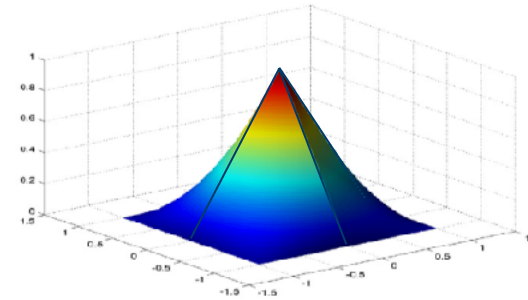
Some kernels for signal and image interpolation

Linear:

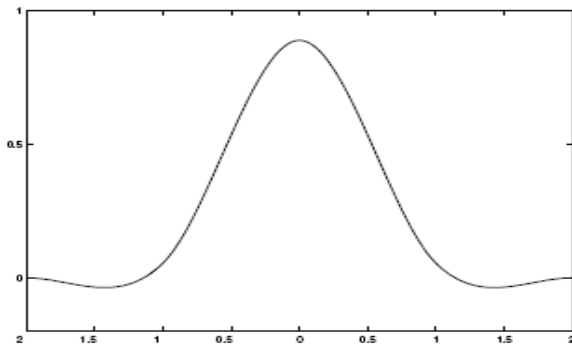


Bilinear:

$h(x, y)$



Bicubic (better choice for images):



Nearest neighbor:

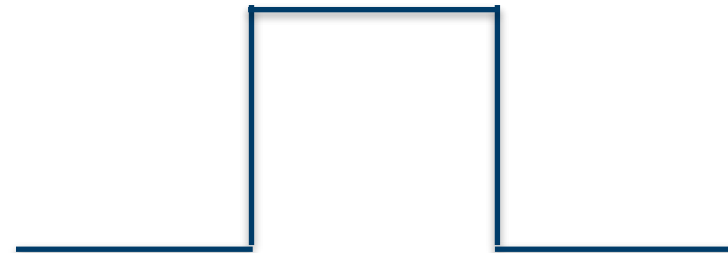
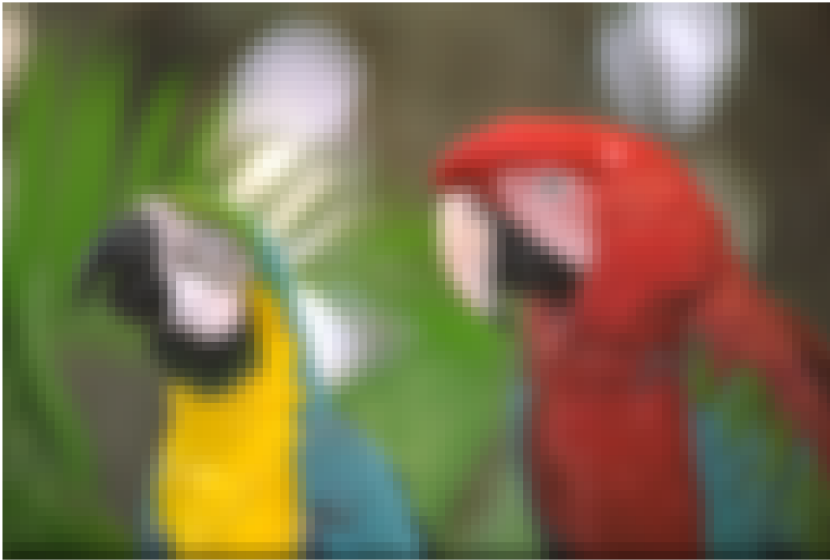


Image interpolation - examples



Nearest neighbor



Bilinear



Bicubic

Application: Template Matching with Image Pyramids

Input: Image, Template

1. Match template at current scale
2. Downsample image
3. Repeat 1-2 until image is very small
4. Take responses above some threshold, perhaps with non-maxima suppression.

Summary

Image Pyramids:

- Downsampling
- Upsampling
- Gaussian Pyramids

More information: Szeliski 3.5

