上海交通大学
SHANGHAI JIAO TONG UNIVERSITY

Lecture 02- Basics of Image processing

EE382-Visual localization & Perception

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What we are about to learn.

- CCD & CMOS
- Global shutter V.S rolling shutter
- Human color system & Bayer pattern
- Image as a 2D function （continuous) /2D matrix (discrete)
- Linear filtering
- Image derivatives
- Histogram equalization/specialization

What is an digital image?

- A digital image is the data read from the image sensor – CCD or CMOS
- CCD (**charge –coupled device**)
- CMOS(**complementary metal-oxide semiconductor**)
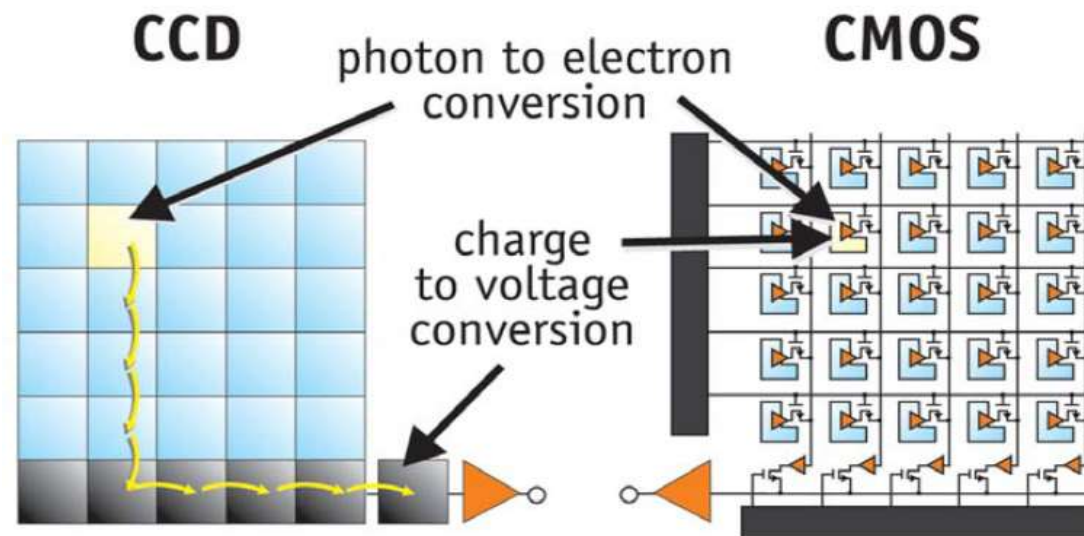
CCDs move photogenerated charge from pixel to pixel and convert it to voltage at an output node. CMOS imagers convert charge to voltage inside each pixel.

CCD V.S. CMOS

Early ages:

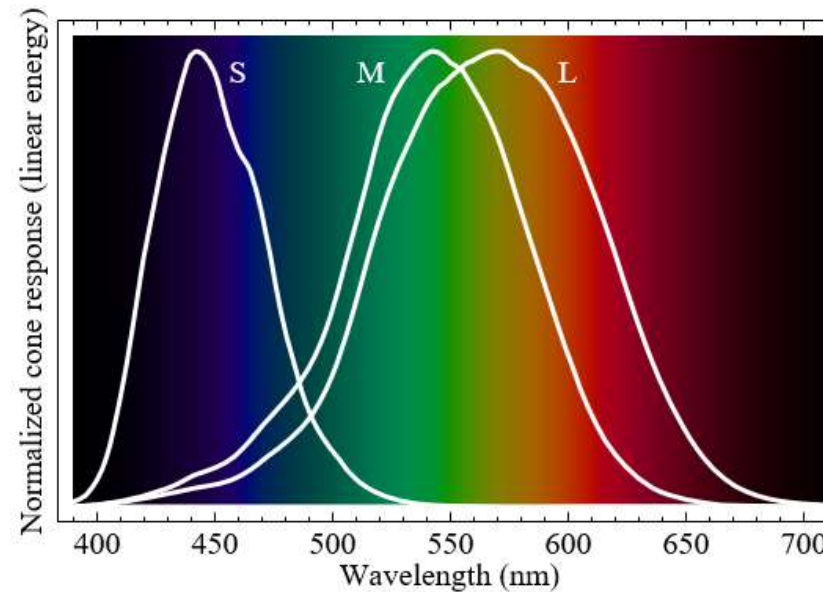| • **CCD**	• **CMOS**
High quality	Lower quality (skew, wobble)
Less noise	High noise
More expensive	Cheaper
More power consumption	Less power consumption
Global Shutter	*Rolling Shutter*

Note that, CMOS becomes the most popular image sensor in the market now.
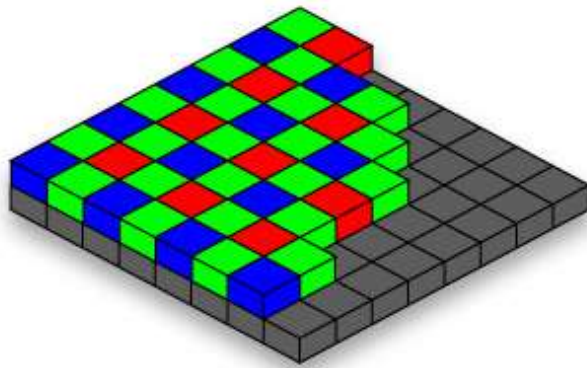
Global Shutter v.s Rolling Shutter

Color image
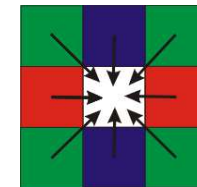
- Color vision of human (daylight)

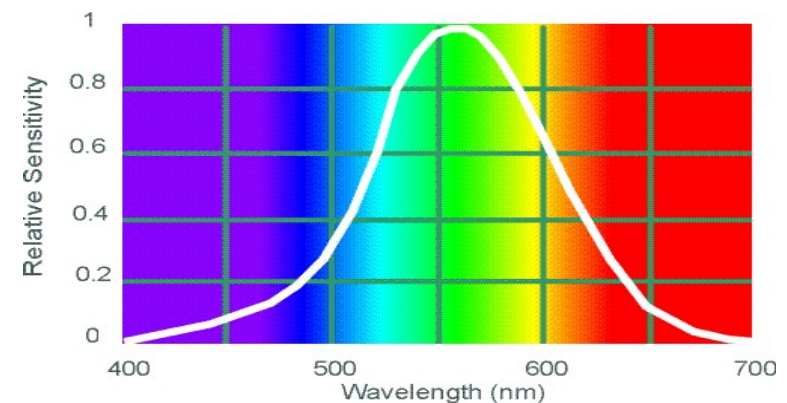| Cone type	Name	Range	Peak wavelength
S (Short)	β	400–500 nm	420–440 nm
M (Medium)	γ	450–630 nm	534–555 nm
L (Long)	ρ	500–700 nm	564–580 nm

Color image

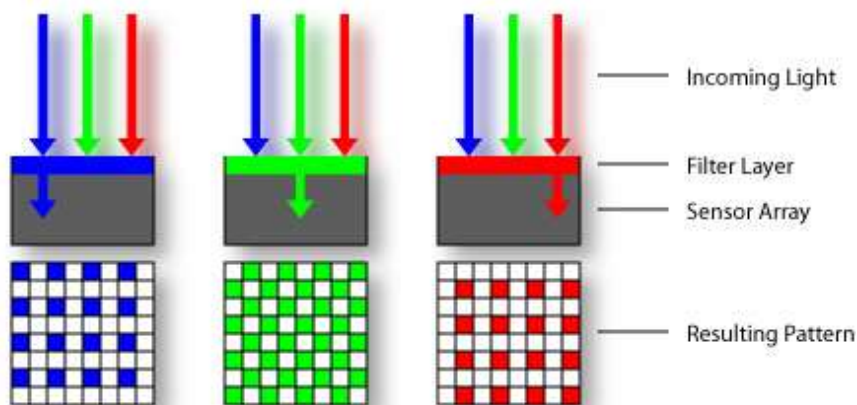- ## Bayer pattern

Demosaicing : interpolating the missing color from neighboring pixels

Why are more green filters?

Incoming Light

Filter Layer

Sensor Array

Resulting Pattern

Human Luminance Sensitivity Function

Summary

- An image is the data from the image sensor (CMOS or CCD)
- Today, CMOS becomes the dominant image sensor.
- Rolling shutter could cause deformation such as skew, wobble.
- Three cone cells receives red, green and blue lights in human vision system.
- Bayer mask is used for the image sensor to receive three colors simultaneously.

Image as a function

- An image can be mathematically thought of as a function, $I : \mathbb{R}^2 \to \mathbb{R}$

  - $I(x, y)$ represents the intensity at the position (x, y)

  - $(x, y) \in \Omega$, where $\Omega : [1, w] \times [1, h]$

  - $I \in [0, 255]$

- A color image is thought as a 'vector-valued' function :

$$I(x, y) = \begin{bmatrix} R(x, y) \\ G(x, y) \\ B(x, y) \end{bmatrix}$$

Image as an array

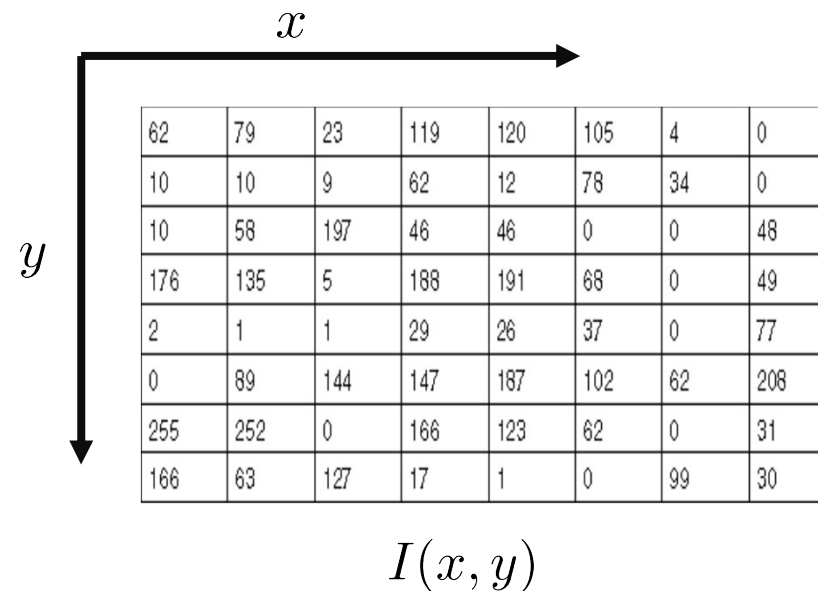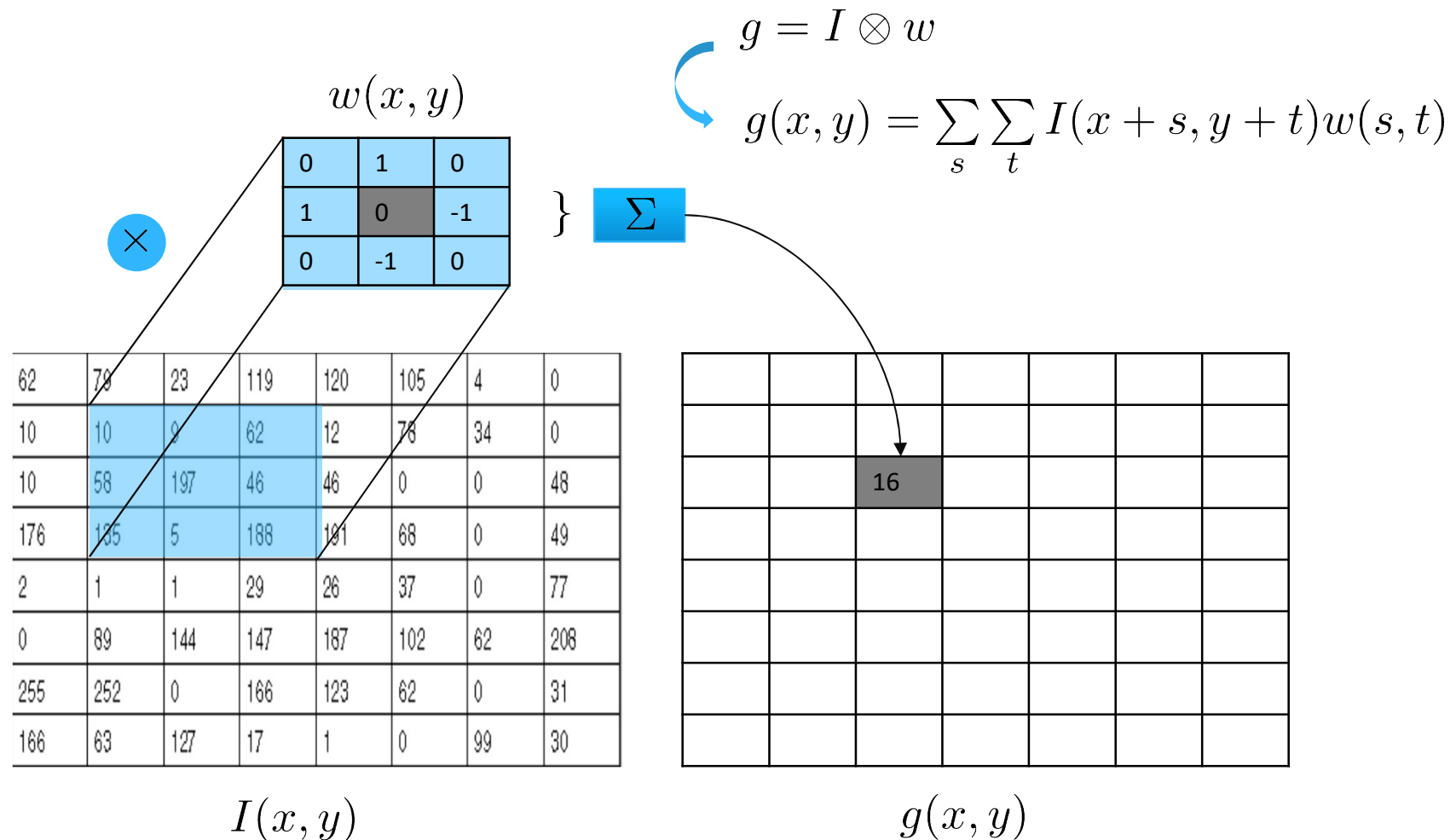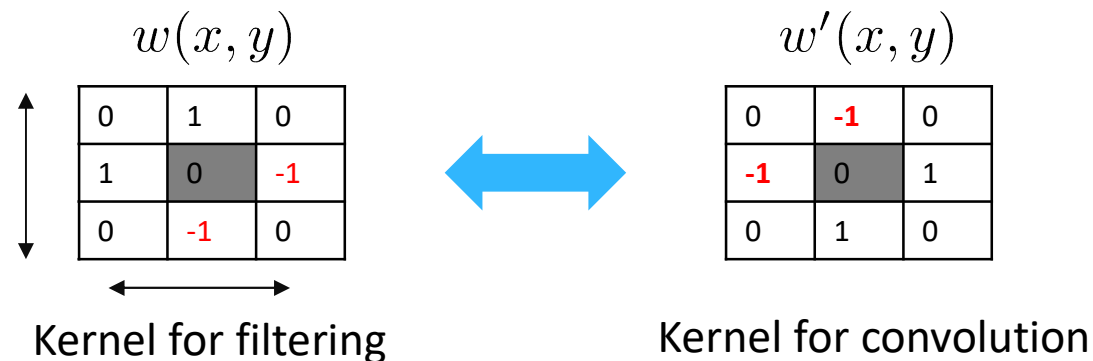- An image can be thought as a matrix of integer values

x

62	79	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
166	63	127	17	1	0	99	30

y

$$I(x, y)$$

Image filtering

- Correlation

$$g = I \otimes w$$

$$g(x, y) = \sum_s \sum_t I(x + s, y + t) w(s, t)$$

$w(x, y)$

0	1	0
1	0	-1
0	-1	0

$\times$

$\}$ Σ

62	78	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
166	63	127	17	1	0	99	30

$I(x, y)$

		16					

$g(x, y)$

Image filtering

- Convolution is the same as correlation except the kernel is flipped both horizontally and vertically

$$g(x, y) = \sum_s \sum_t I(x - s, y - t) w(s, t)$$

$w(x, y)$

0	1	0
1	0	-1
0	-1	0

Kernel for filtering

$w'(x, y)$

0	-1	0
-1	0	1
0	1	0

Kernel for convolution

Box filter

- Kernel/mask – a small matrix determining the filter results

$$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Box blur
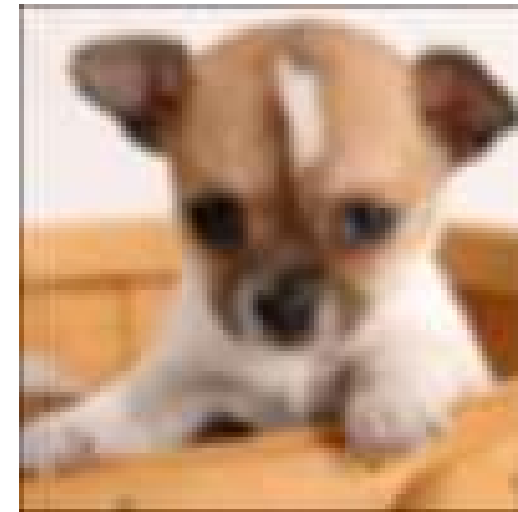
Gaussian filter

$$w(x, y) = \frac{1}{2\pi\sigma^2} \exp - \frac{x^2 + y^2}{2\sigma^2}$$

$$\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Gaussian blur (3x3)

$$\sigma \approx 0.85$$
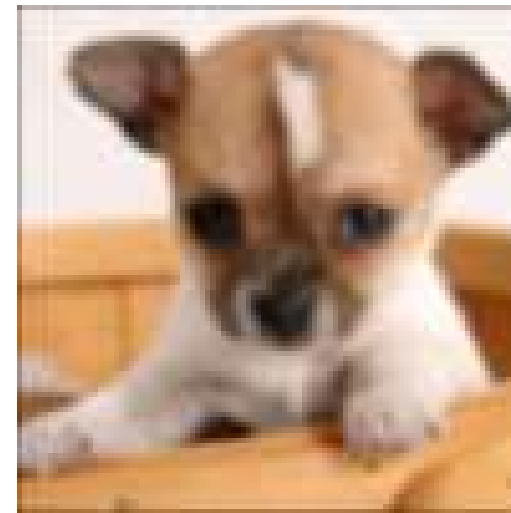
Gaussian filter

$$w(x, y) = \frac{1}{2\pi\sigma^2} \exp -\frac{x^2 + y^2}{2\sigma^2}$$

$$\frac{1}{256} \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1] \end{bmatrix}$$

Gaussian blur (5x5)
$\sigma \approx 1.05$
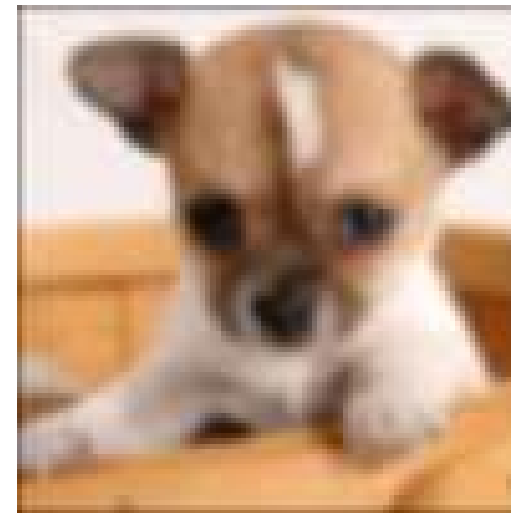
Gradient operator

- Sobel operator (x – direction)
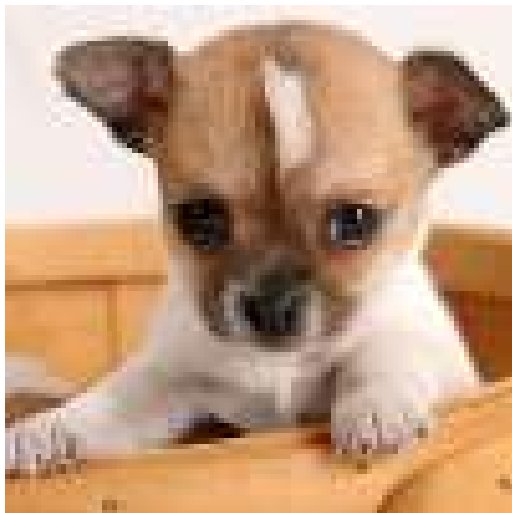
$$w(x, y) = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$
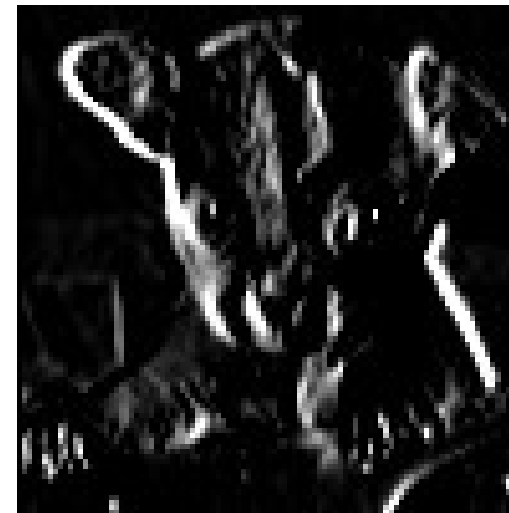
Gradient operator

- Sobel operator (y-direction)

$$w(x, y) = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$
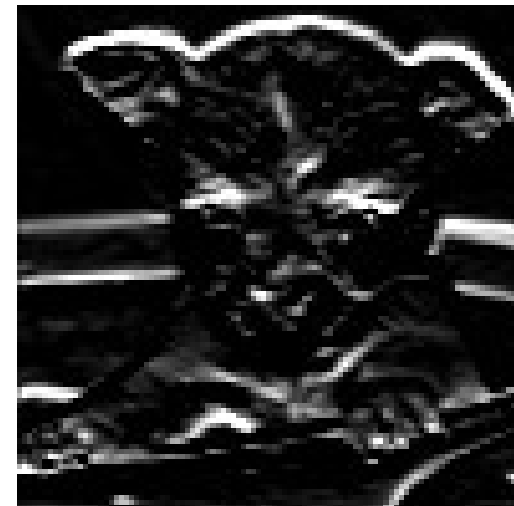
Summary

- An image is described by a 2D function (mathematic) or a matrix/array (algorithm).
- Image filtering is described by the correlation operator
- Convolution is equal to correlation except that the kernel is flipped in both directions.
- Box and Gaussian filter cause image blur
- Gradient operators can be used to detect edges.

Image derivative

Let's consider one-dimensional example:
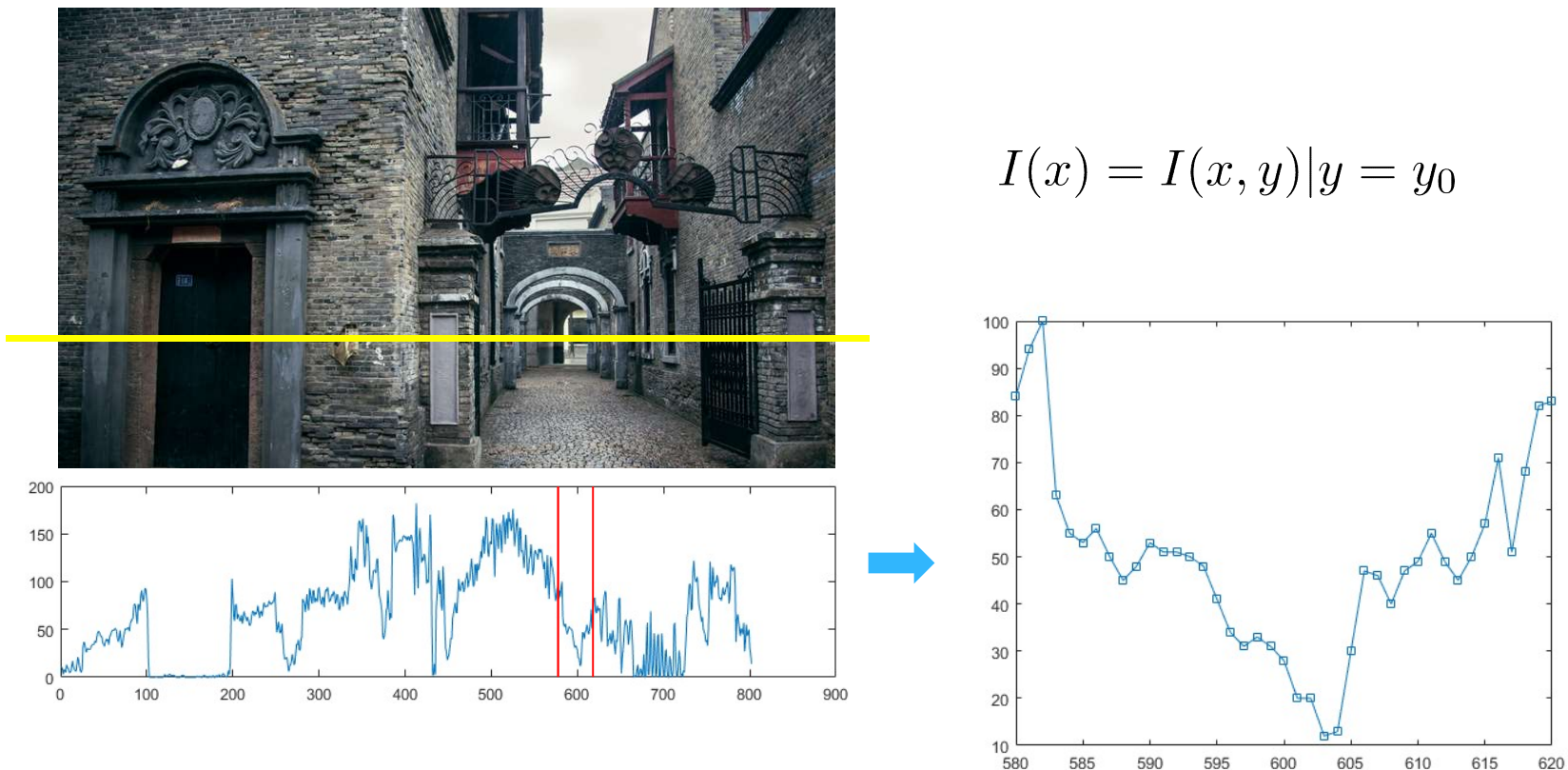
$$I(x) = I(x,y)|y = y_0$$

First derivative

- Sometimes we want to know about value of

$I(x + dx), dx < 1.$ It can be approximated by Taylor series expansion:

$$I(x + dx) \approx I(x) + \frac{\partial I(x)}{\partial x} dx + o(dx^2)$$
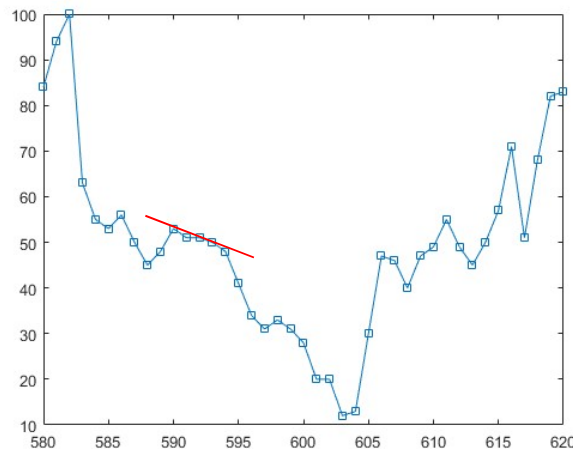
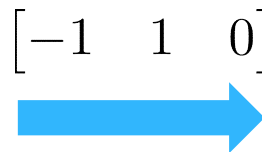$$I_x = \frac{\partial I(x)}{\partial x}$$

First derivative

- Compute derivatives (1D)

$$I_x = \lim_{dx \to 0} \frac{I(x+dx) - I(x)}{dx} \approx I(x+1) - I(x) \qquad \begin{bmatrix} -1 & 1 & 0 \end{bmatrix}$$

$$\approx I(x) - I(x-1) \qquad \begin{bmatrix} 0 & -1 & 1 \end{bmatrix}$$

$$\approx I(x+1) - I(x-1) \qquad \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 0 \end{bmatrix}$$
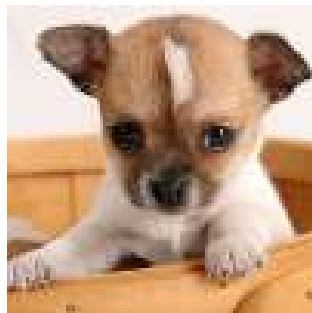
$I(x)$

$I_x(x)$
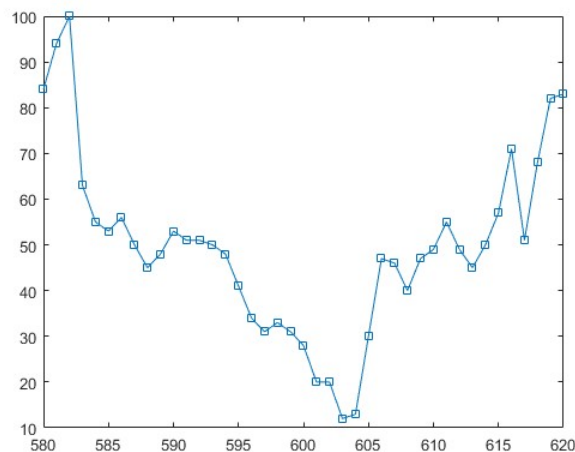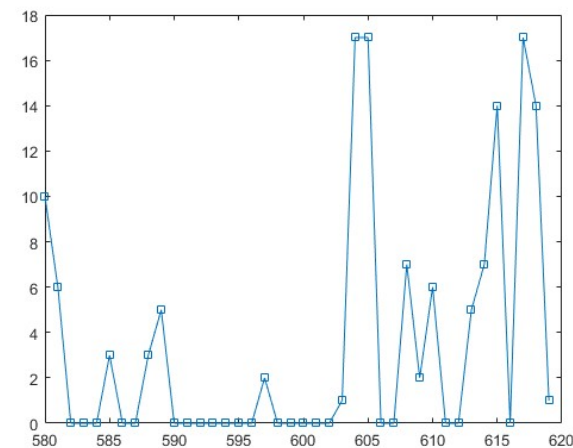
First derivative

- 2D cases, the partial derivatives are computed as

$$\frac{\partial I(x,y)}{\partial x} = I_x \approx I \otimes D_x \qquad \frac{\partial I(x,y)}{\partial y} = I_y \approx I \otimes D_y$$

Sobel operators

$$D_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad D_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$
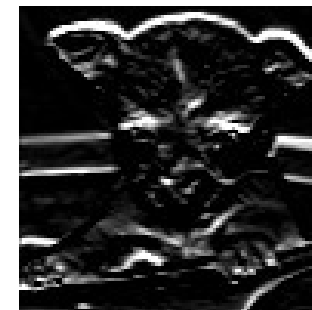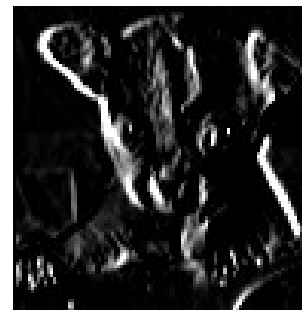
First derivative

- The image can be locally approximated by 2D Taylor series expansion:

$$I(x + dx, y + dy) \approx I(x, y) + \begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix}$$

$$= I(x, y) + \nabla I \begin{bmatrix} dx \\ dy \end{bmatrix}$$

Later, I'll show how this approximation is used to derive the Harris corner detector.

Second derivative

- The second derivative is computed as

$$I_{xx} = \lim_{dx \to 0} \frac{I_x(x+dx,y) - I_x(x,y)}{dx}$$

$$\approx I_x(x+1,y) - I_x(x,y)$$

- Let $I_x(x,y) \approx I(x,y) - I(x-1,y)$, we have

$$I_{xx}(x,y) = I(x+1,y) - I(x,y) - (I(x,y) - I(x-1,y))$$

$$= I(x-1,y) - 2I(x,y) + I(x+1,y)$$

$$I_{xx} = I \otimes D_{xx} \text{, where } D_{xx} = \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$$
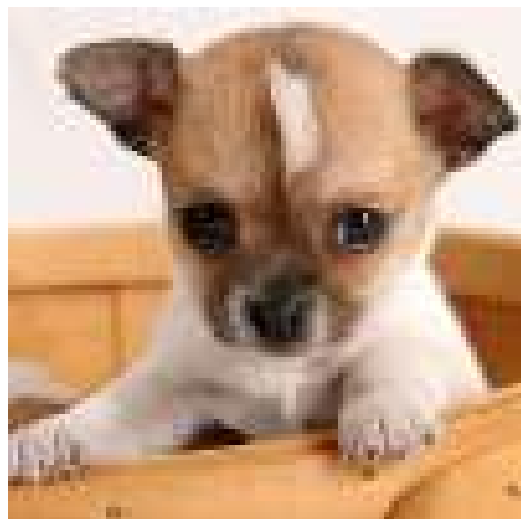
Second derivative

- The second derivative is computed as

$$I_{xy} = \lim_{dy \to 0} \frac{I_x(x, y+dy) - I_x(x,y)}{dy}$$

$$\approx I_x(x, y+1) - I_x(x, y-1)$$

- Let $I_x(x,y) \approx I(x+1,y) - I(x-1,y)$, we have

$$I_{xy}(x,y) = I(x+1, y+1) - I(x-1, y+1)$$
$$-(I(x+1, y-1) - I(x-1, y-1))$$

$$I_{xy} = I \otimes D_{xy} \quad \text{, where } D_{xy} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Second derivatives

$$D_{xx} = \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$$

$$I_{xx}$$

$$D_{xy} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$I_{xy}$$

Summary

- Image can be locally approximated by Taylor series expansion.
- First derivative of an image can be computed by finite difference in x and y directions
- Second derivative can be also computed by finite difference.
- Finite difference is implemented by filtered by a gradient kernel.

Histogram

- The histogram of an image is a distribution of number of pixels at each gray level (for 8-bit pixel, there are 255 gray levels). $h(g), \text{where} \quad g \in [0, 1]$
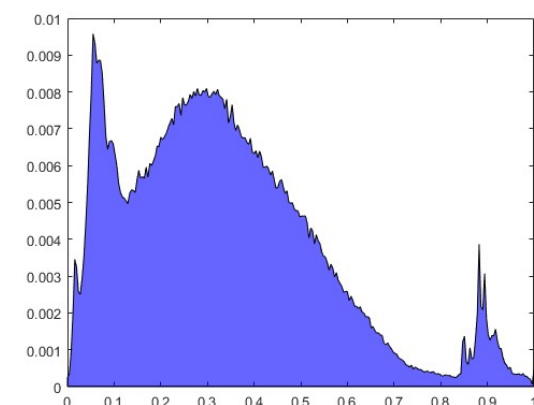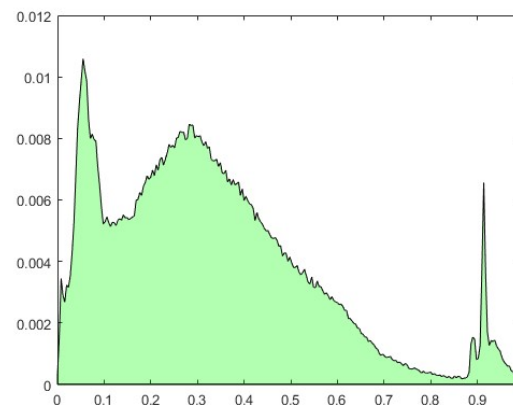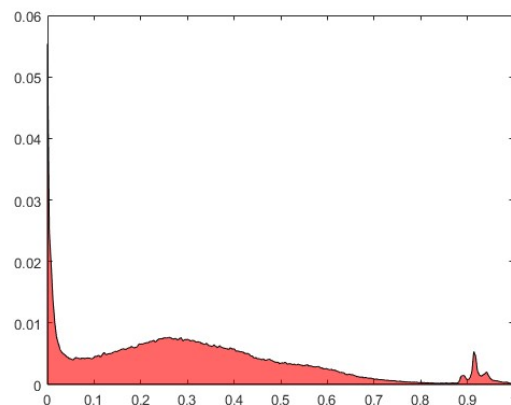
Histogram equalization

- Sometimes we want to find a brightness mapping function that maps $f(I)$ each gray level to a new gray level to change the histogram of the image:

$$h(I) \rightarrow h'(I')$$

$$I' = f(I)$$
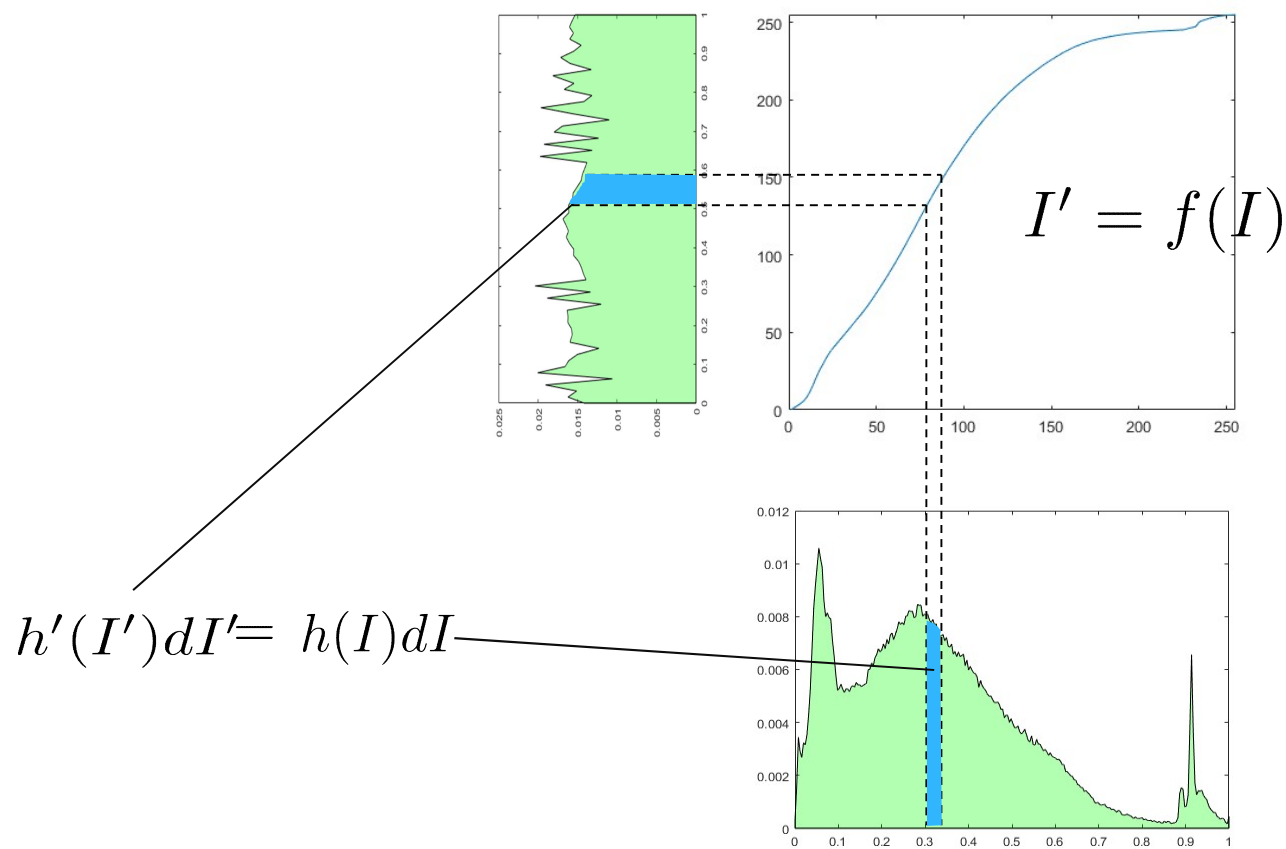
$$h'(I')dI' = h(I)dI$$

Histogram equalization

- The new histogram is a constant

$$h'(I') = const.$$

$$h'(I')dI' = h(I)dI$$

$$\Rightarrow dI' = h(I)dI$$

$$\Rightarrow f'(I)dI = h(I)dI \qquad (I' = f(I))$$

$$\Rightarrow f'(I) = h(I)$$

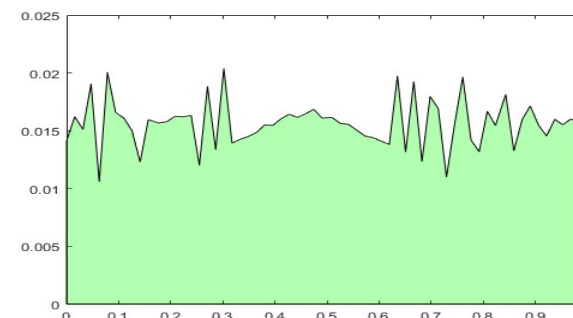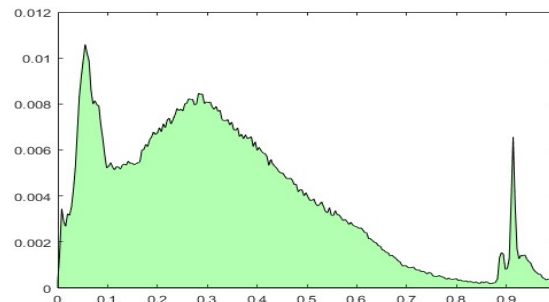$$\Rightarrow f(I) = \int_{t=0}^{t=I} h(t)dt$$

- $f(I)$ is a **c**umulative **d**istribution **f**unction (CDF) of $h(t)$
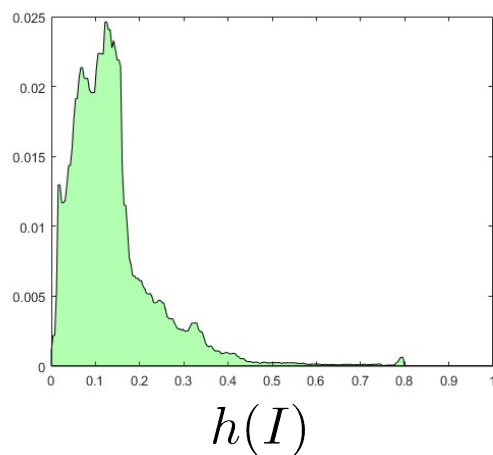
Histogram equalization

- Algorithm:
  - 1.Compute the histogram of an input image $h(I)$
  - 2.Calculate the cumulative distribution function of $h(I)$, namely $f(I)$
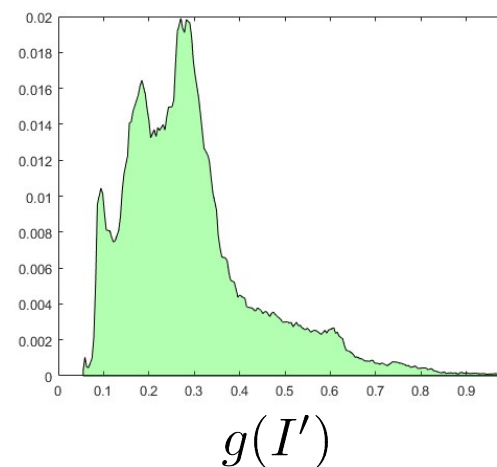 - 3. Map the intensity of each pixel by

$$I'(x, y) = f(I(x, y))$$

Histogram specialization
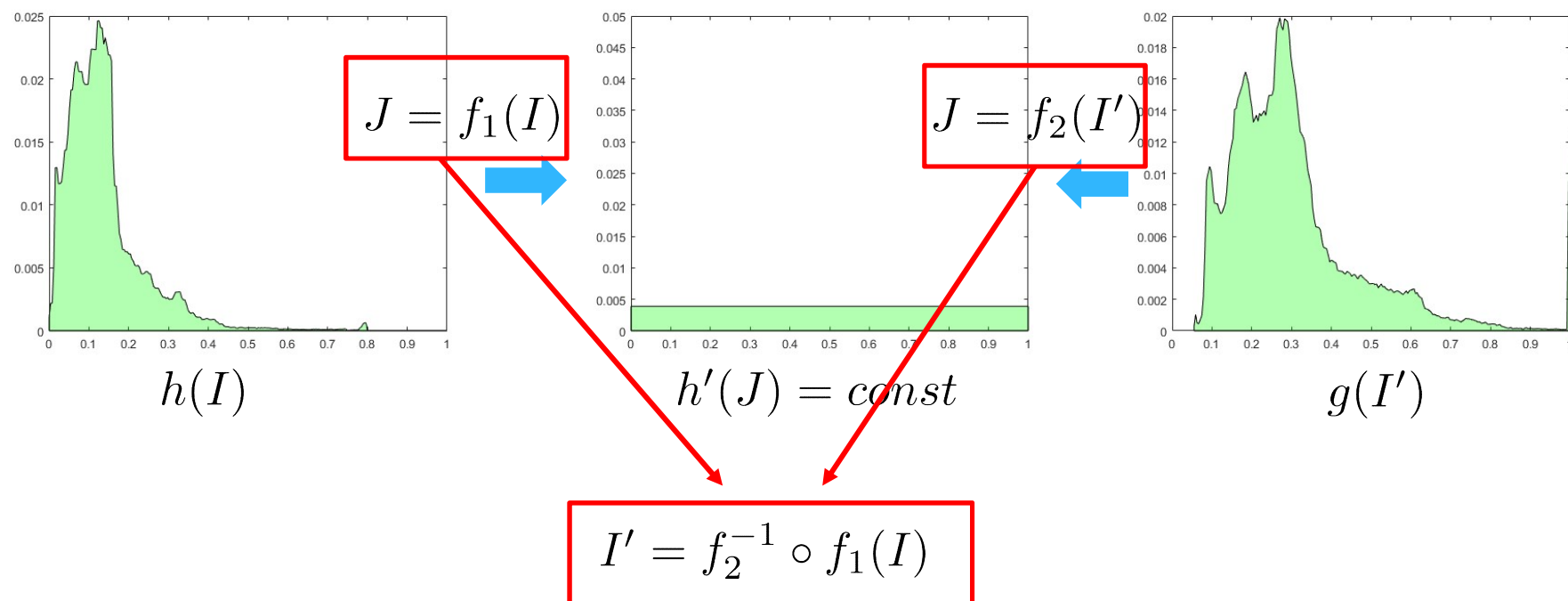
- Case 2: The new histogram is an arbitrary histogram

$$I' = f(I)$$

?

$h(I)$

$g(I')$

Histogram specialization

- Consider two brightness mapping functions transform both histograms into a constant histogram:

$J = f_1(I)$

$J = f_2(I')$

$h(I)$

$h'(J) = const$

$g(I')$

$I' = f_2^{-1} \circ f_1(I)$

Histogram specialization

- Algorithm:
  - 1. Calculate the cumulative histogram of the first image : $f_1(I)$
  - 2. Calculate the cumulative histogram of the second image: $f_2(I')$
  - 3. build a lookup table $I' = f_2^{-1} \circ f_1(I)$ by finding the corresponding gray level I'_j of each gray level I_i, where $I'_j = \arg\min_j |f_1(I_i) - f_2(I'_j)|$
  - 4. map the new intensity of each pixel by finding the lookup table

Histogram specialization

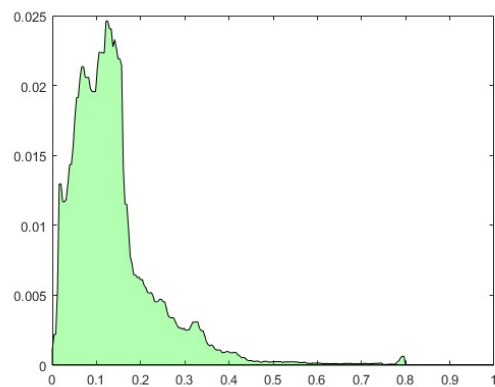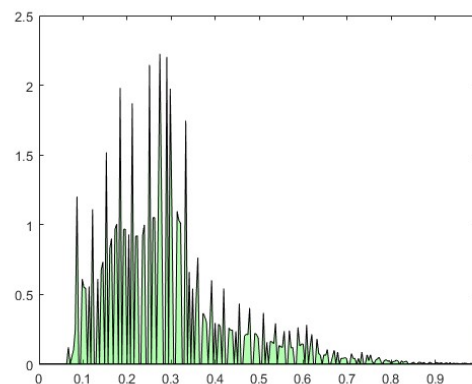- Illustration
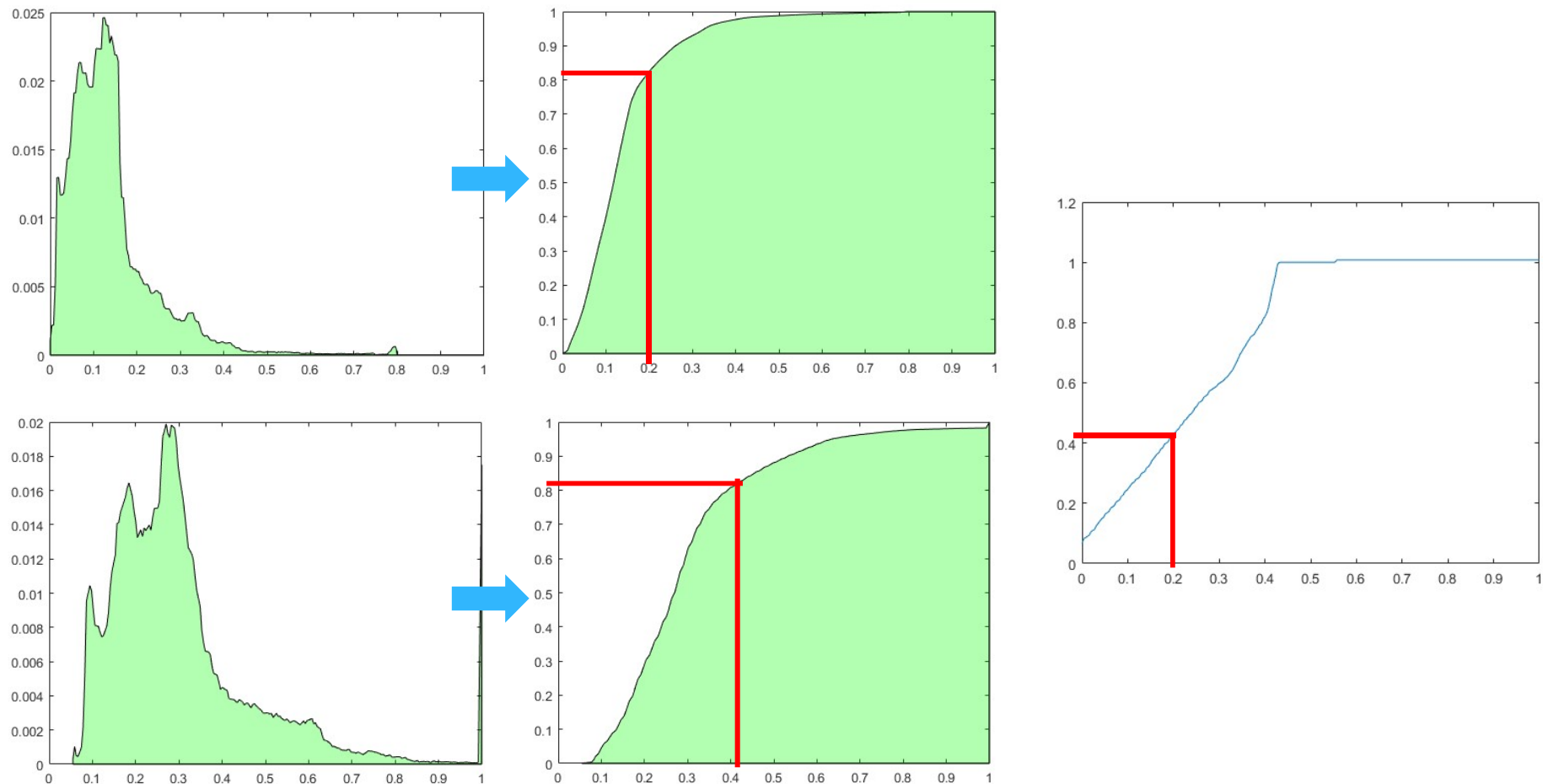
Histogram specialization
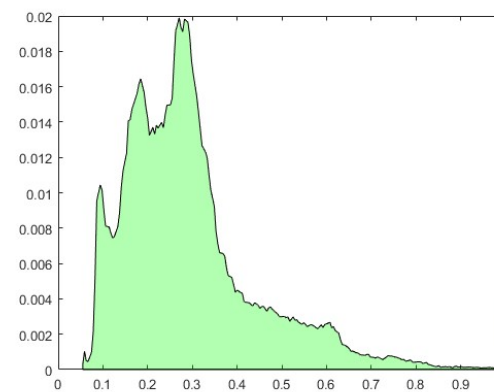
Original

Adjusted

Target

$h(I)$

$h'(I)$

$g(I')$

Summary

- Histogram of an image is a distribution of pixels at different brightness levels.

- Histogram equalization distributes pixels equally at all gray levels and can improve the details of the image appearance.

- Histogram specialization adjust the brightness of the original image to make it be close to the target image.

- Histogram can address the illumination change to some extend and make feature matching easier.