COMP 4900D: Assignment 1 Solutions

1. (a) $\cos(\phi_1+\phi_2)=\cos\phi_1\cos\phi_2-\sin\phi_1\sin\phi_2$, $\sin(\phi_1+\phi_2)=\sin\phi_1\cos\phi_2+\cos\phi_1\sin\phi_2$

$$R(\phi_1+\phi_2)=\begin{bmatrix}\cos(\phi_1+\phi_2) & \sin(\phi_1+\phi_2)\\ -\sin(\phi_1+\phi_2) & \cos(\phi_1+\phi_2)\end{bmatrix}=\begin{bmatrix}\cos\phi_1\cos\phi_2-\sin\phi_1\sin\phi_2 & \sin\phi_1\cos\phi_2+\cos\phi_1\sin\phi_2\\ -\sin\phi_1\cos\phi_2-\cos\phi_1\sin\phi_2 & \cos\phi_1\cos\phi_2-\sin\phi_1\sin\phi_2\end{bmatrix}$$

$$R(\phi_1)R(\phi_2)=\begin{bmatrix}\cos\phi_1 & \sin\phi_1\\ -\sin\phi_1 & \cos\phi_1\end{bmatrix}\begin{bmatrix}\cos\phi_2 & \sin\phi_2\\ -\sin\phi_2 & \cos\phi_2\end{bmatrix}=\begin{bmatrix}\cos\phi_1\cos\phi_2-\sin\phi_1\sin\phi_2 & \cos\phi_1\sin\phi_2+\sin\phi_1\cos\phi_2\\ -\sin\phi_1\cos\phi_2-\cos\phi_1\sin\phi_2 & \cos\phi_1\cos\phi_2-\sin\phi_1\sin\phi_2\end{bmatrix}$$

(b) $$R(\phi)R(-\phi)=\begin{bmatrix}\cos\phi & \sin\phi\\ -\sin\phi & \cos\phi\end{bmatrix}\begin{bmatrix}\cos\phi & -\sin\phi\\ \sin\phi & \cos\phi\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

2. (a) $x'=sx$, $y'=sy$ $\qquad \begin{bmatrix}x'\\ y'\end{bmatrix}=\begin{bmatrix}s & 0\\ 0 & s\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$

(b) $$\begin{bmatrix}x'\\ y'\\ 1\end{bmatrix}=\begin{bmatrix}s & 0 & 0\\ 0 & s & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ 1\end{bmatrix}$$

(c) $$\begin{bmatrix}x'\\ y'\\ 1\end{bmatrix}=\begin{bmatrix}s & 0 & t_x\\ 0 & s & t_y\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ 1\end{bmatrix}$$

(d) $$\begin{bmatrix}x'\\ y'\\ z'\\ 1\end{bmatrix}=\begin{bmatrix}s & 0 & 0 & t_x\\ 0 & s & 0 & t_y\\ 0 & 0 & s & t_z\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\\ 1\end{bmatrix}$$

3. normal equation $A^TAx=A^Tb$

$$\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\ 1\\ 0\end{bmatrix}$$

$$\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}1\\ 1\end{bmatrix}\qquad \bar{x}=\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}1/3\\ 1/3\end{bmatrix}$$

4. (a) $$P=\begin{bmatrix}-500 & 0 & 320\\ 0 & -500 & 240\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 1 & 0 & 70\\ -1 & 0 & 0 & 95\\ 0 & 0 & 1 & 120\end{bmatrix}=\begin{bmatrix}0 & -500 & 320 & 3400\\ 500 & 0 & 240 & -18700\\ 0 & 0 & 1 & 120\end{bmatrix}$$

(b) $$X_h=P\begin{bmatrix}150\\ 200\\ 400\\ 1\end{bmatrix}=\begin{bmatrix}31400\\ 152300\\ 520\end{bmatrix}$$

$$X_{im}=\begin{bmatrix}X_h[1]/X_h[3]\\ X_h[2]/X_h[3]\end{bmatrix}=\begin{bmatrix}60.38\\ 292.88\end{bmatrix}$$