

# Image Analysis

## Lecture 9.2 - Image Feature Extraction

Idar Dyrdal

# Image Analysis

## Typical steps:

- Pre-processing
- Segmentation (object detection)
- *Feature extraction*
- Feature selection
- Classifier training
- Evaluation of classifier performance.



# Features for image analysis

## Applications:

- Remote sensing
- Medical imaging
- Character recognition
- Robot Vision
- ...

## Major goal of image feature extraction:

*Given an image, or a region within an image, generate the features that will subsequently be fed to a classifier in order to classify the image in one of the possible classes.*

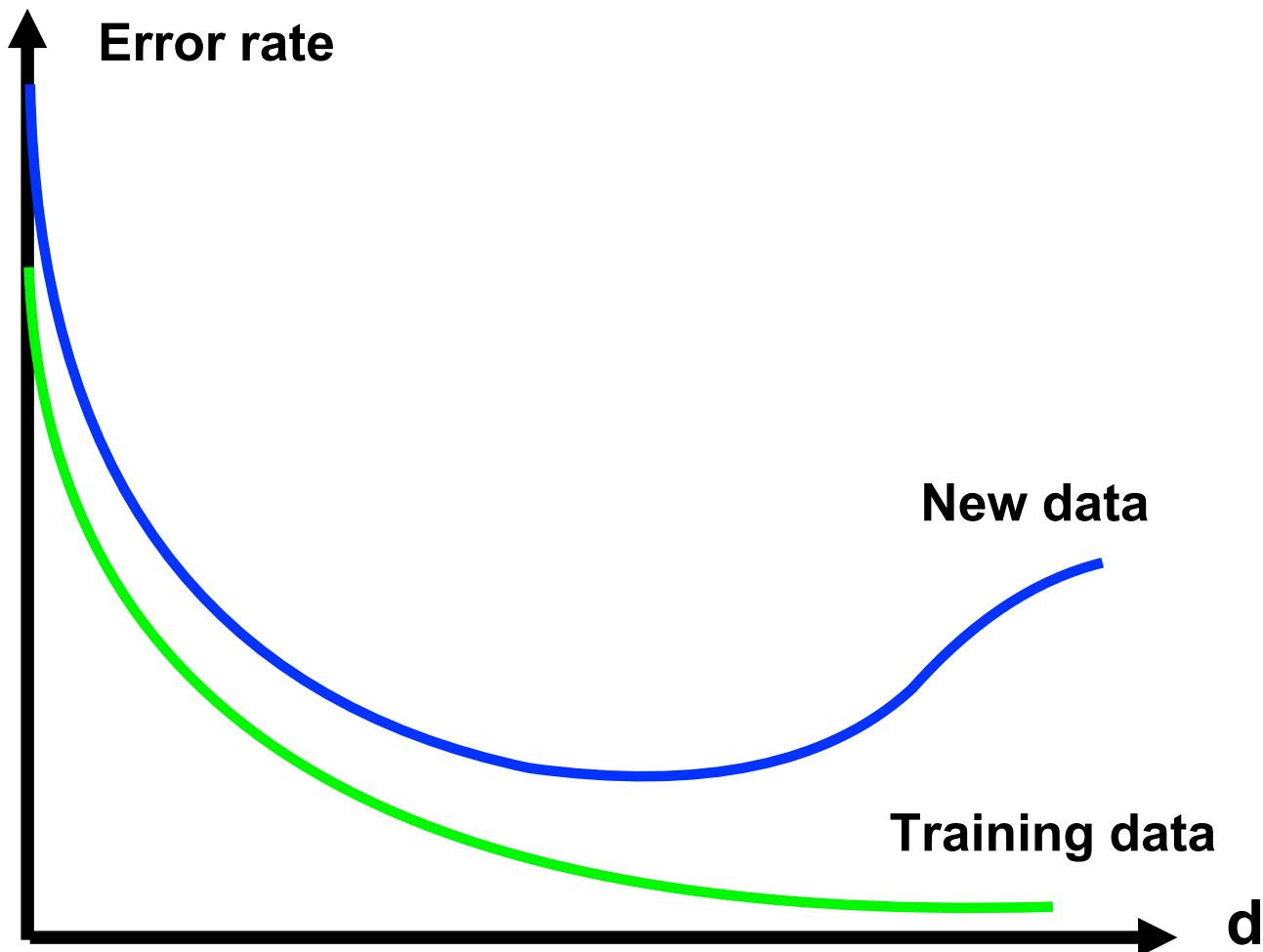
(Theodoridis & Koutroumbas: «Pattern Recognition», Elsevier 2006).

# Feature extraction

*The goal is to generate features that exhibit high information-packing properties:*

- Extract the information from the raw data that is most relevant for discrimination between the classes
  - Extract features with low within-class variability and high between class variability
  - Discard redundant information.
- 
- The information in an image  $f[i,j]$  must be reduced to enable reliable classification (generalization)
  - A 64x64 image → 4096-dimensional feature space!

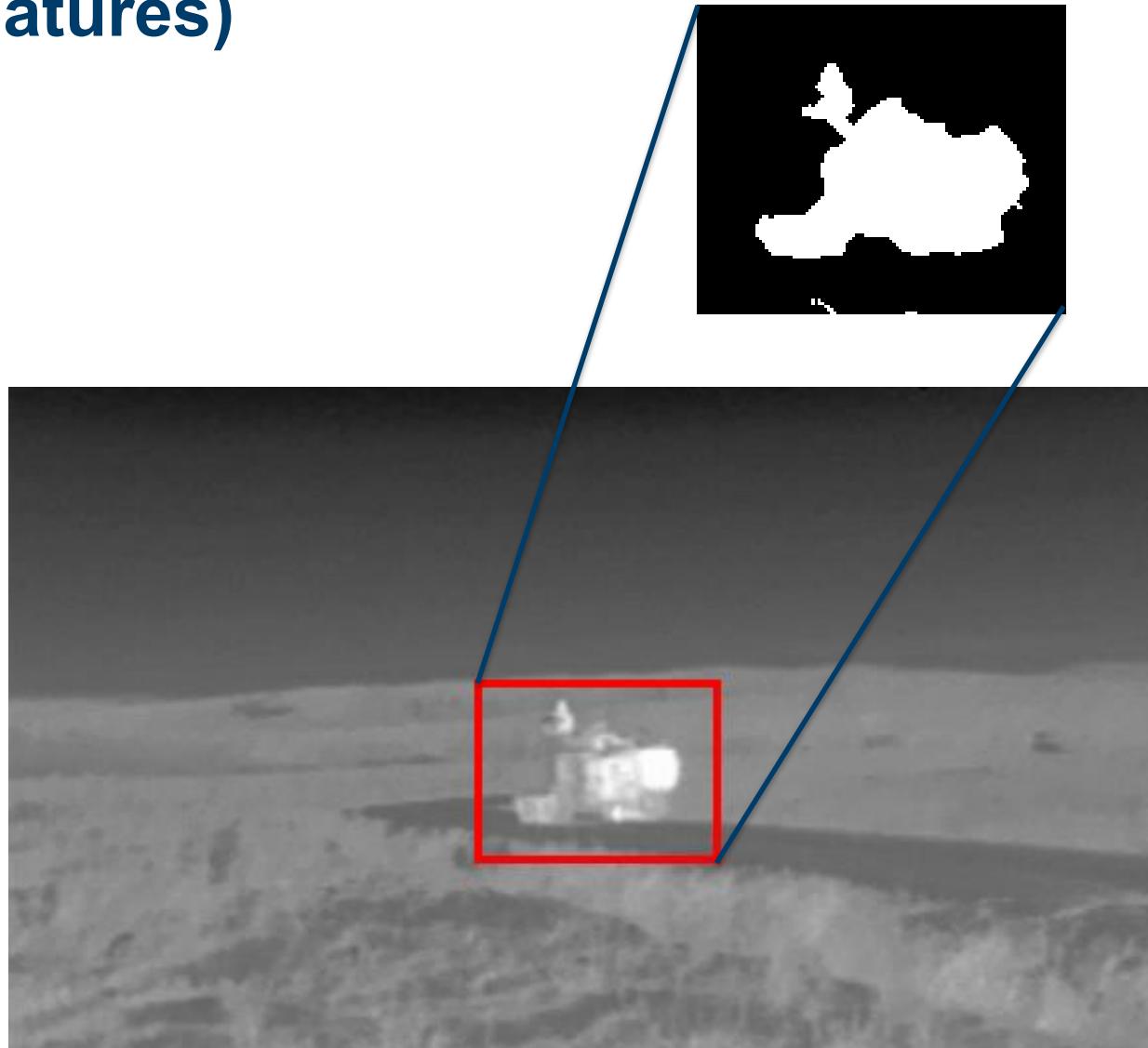
## “Curse of dimensionality”



$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_d \end{bmatrix}$$

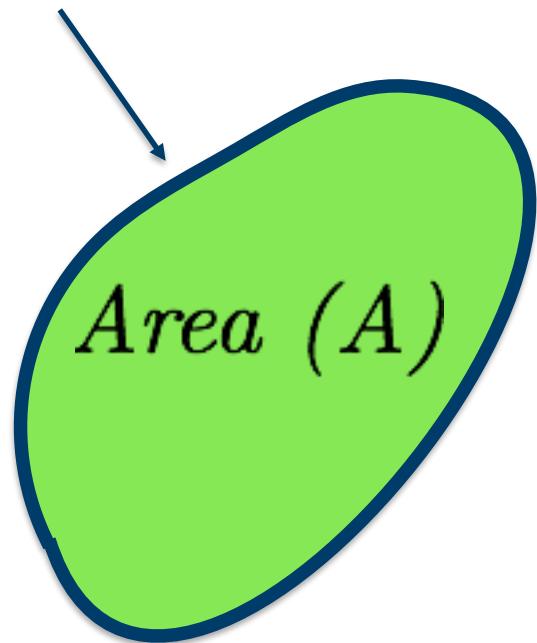
## Feature types (regional features)

- Colour features
- Gray level features
- Shape features
- Histogram (texture) features



## Shape features - example

*Perimeter (P)*



*Area (A)*

Possible shape feature:

$$\frac{P^2}{A}$$

## Moments

Geometric moments (order p, q):

$$m_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p y^q f(x, y) dx dy \approx \sum_i \sum_j i^p j^q f[i, j]$$

Central moments:

$$\mu_{pq} = \sum_i \sum_j (i - \tilde{i})^p (j - \tilde{j})^q f[i, j] \text{ where } \begin{cases} \tilde{i} = \frac{m_{10}}{m_{00}} \\ \tilde{j} = \frac{m_{01}}{m_{00}} \end{cases}$$

## Binary images

$$f[i, j] = \begin{cases} 1 & \Rightarrow \text{Object pixel} \\ 0 & \Rightarrow \text{Background pixel} \end{cases}$$



Area:  $m_{00} = \sum_i \sum_j f[i, j]$

Center of mass:  $\begin{cases} m_{10} = \sum_i \sum_j i f[i, j] \Rightarrow \tilde{i} = \frac{m_{10}}{m_{00}} \\ m_{01} = \sum_i \sum_j j f[i, j] \Rightarrow \tilde{j} = \frac{m_{01}}{m_{00}} \end{cases}$

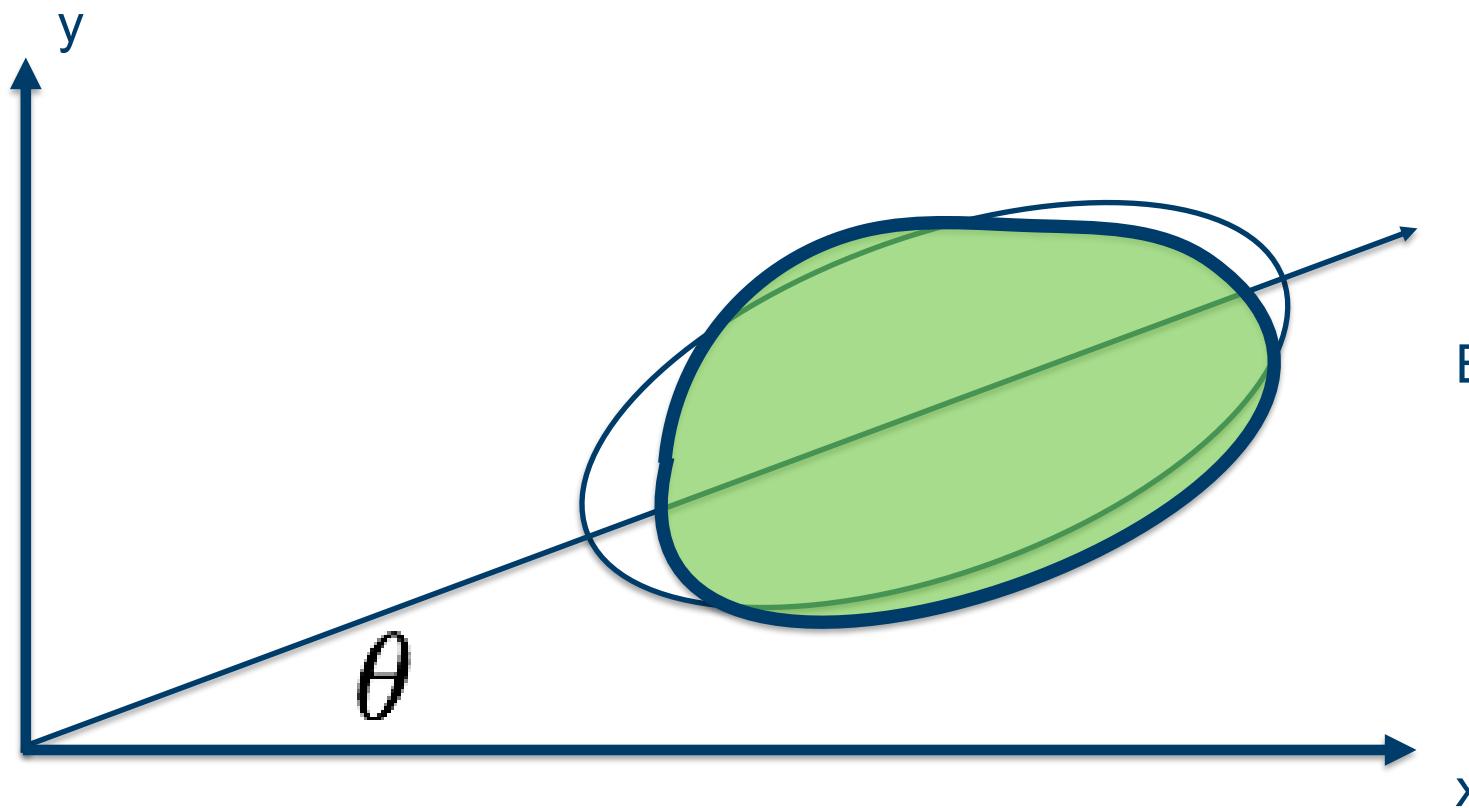
## Moments of inertia

$$\mu_{20} = \sum_i \sum_j (i - \tilde{i})^2 f[i, j]$$

$$\mu_{02} = \sum_i \sum_j (j - \tilde{j})^2 f[i, j]$$

$$\mu_{11} = \sum_i \sum_j (i - \tilde{i})(j - \tilde{j}) f[i, j]$$

## Closest fitting ellipse



Orientation:

$$\theta = \frac{1}{2} \tan^{-1} \left[ \frac{2\mu_{11}}{\mu_{20} - \mu_{02}} \right]$$

Eccentricity:

$$\epsilon = \frac{(\mu_{20} - \mu_{02})^2 + 4\mu_{11}}{A}$$

## Major and minor axes

$$a^2 = \frac{2(\mu_{20} + \mu_{02} + \sqrt{(\mu_{20} + \mu_{02})^2 + 4\mu_{11}^2})}{\mu_{00}}$$

$$b^2 = \frac{2(\mu_{20} + \mu_{02} - \sqrt{(\mu_{20} + \mu_{02})^2 + 4\mu_{11}^2})}{\mu_{00}}$$

# Histogram (texture) features

- First order statistics (information related to the gray level distribution)
- Second order statistics (information related to spatial/relative distribution of gray level), i.e. second order histogram, co-occurrence matrix

Histogram:

$$P(I) = \frac{\text{Number of pixels with gray level } I}{\text{Total number of pixels in the region}}$$

Moments from gray level histogram:

$$m_p = E\{I^p\} = \sum_{I=0}^{L-1} I^p P(I), \quad p = 1, 2, \dots$$

$$m_1 = E(I) = \text{Mean value of } I$$

Entropy:

$$H = -E\{\ln P(I)\} = - \sum_{I=0}^{L-1} P(I) \ln P(I)$$

# Histogram (texture) features

Central moments:

$$\mu_p = E\{(I - E(I))^p\} = \sum_{I=0}^{L-1} (I - m_1)^p P(I), \quad p = 1, 2, \dots$$

Features:

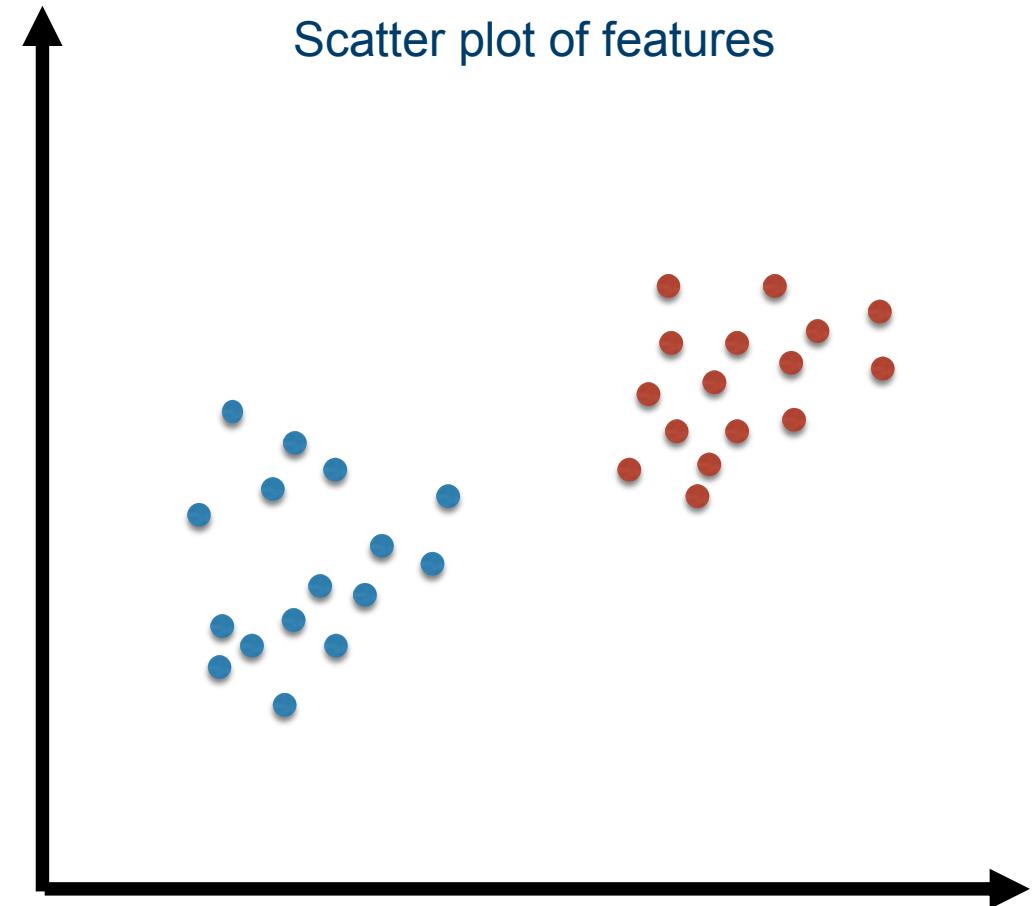
$$\mu_2 = \sigma^2 = \text{variance}$$

$$\mu_3 = \text{skewness}$$

$$\mu_4 = \text{kurtosis}$$

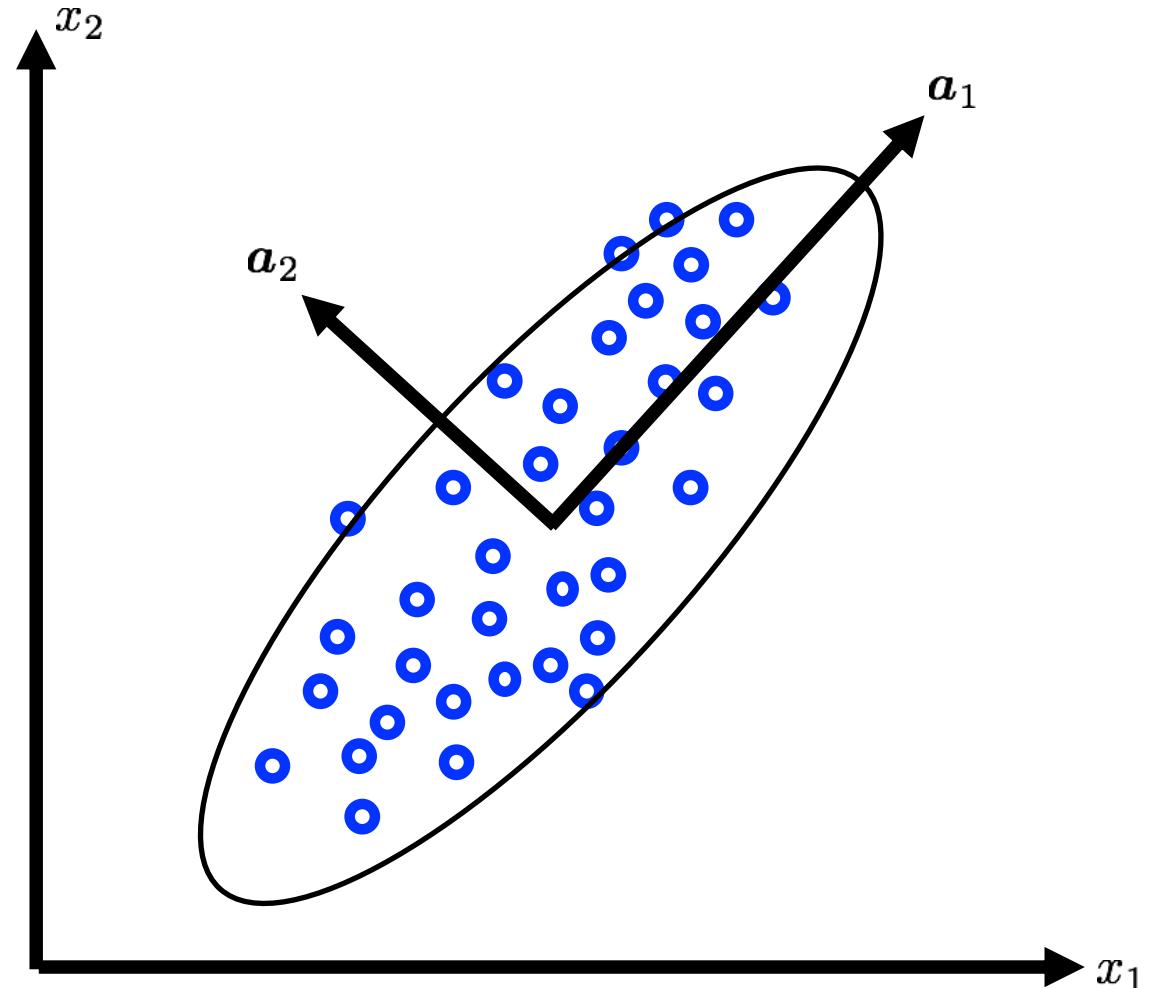
# Feature selection

- A number of feature candidates may have been generated.
- Using all candidates will easily lead to over training (unreliable classification of new data).
- Dimensionality reduction is required, i.e. feature selection!
- Exhaustive search impossible!
- Trial and error (select feature combination, train classifier, estimate error rate).
- Suboptimal search.
- «Branch and Bound» search.
- Linear or non-linear mappings to lower dimensional feature space.

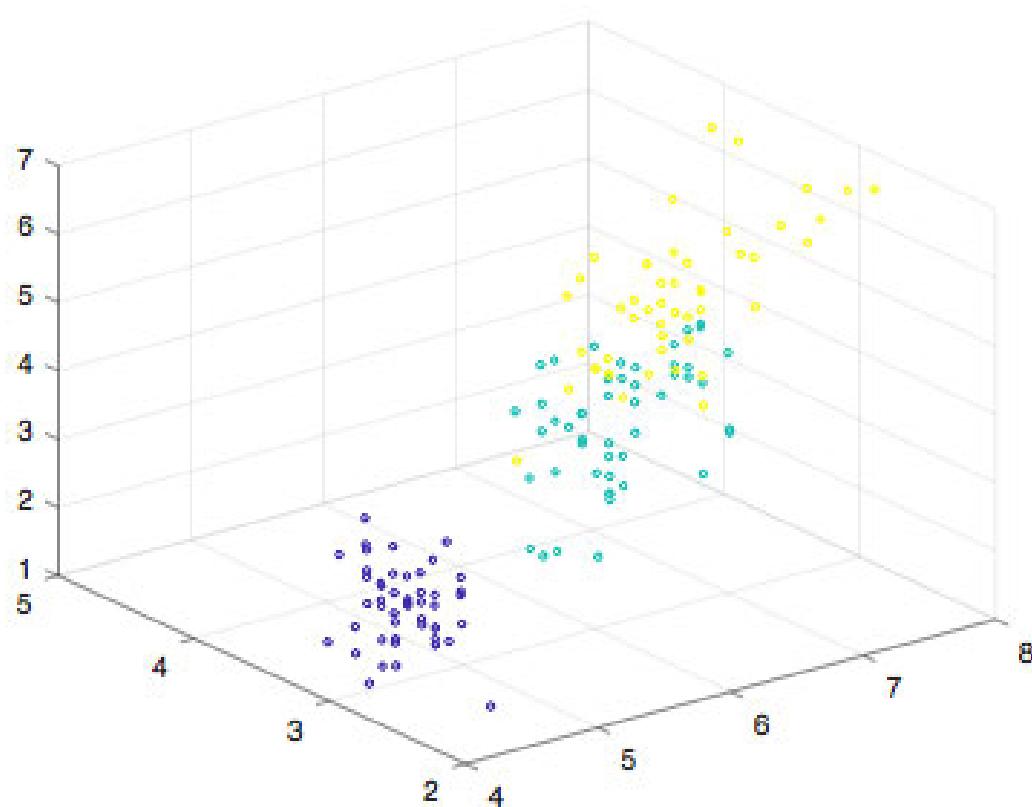


# Dimensionality reduction - linear transformations

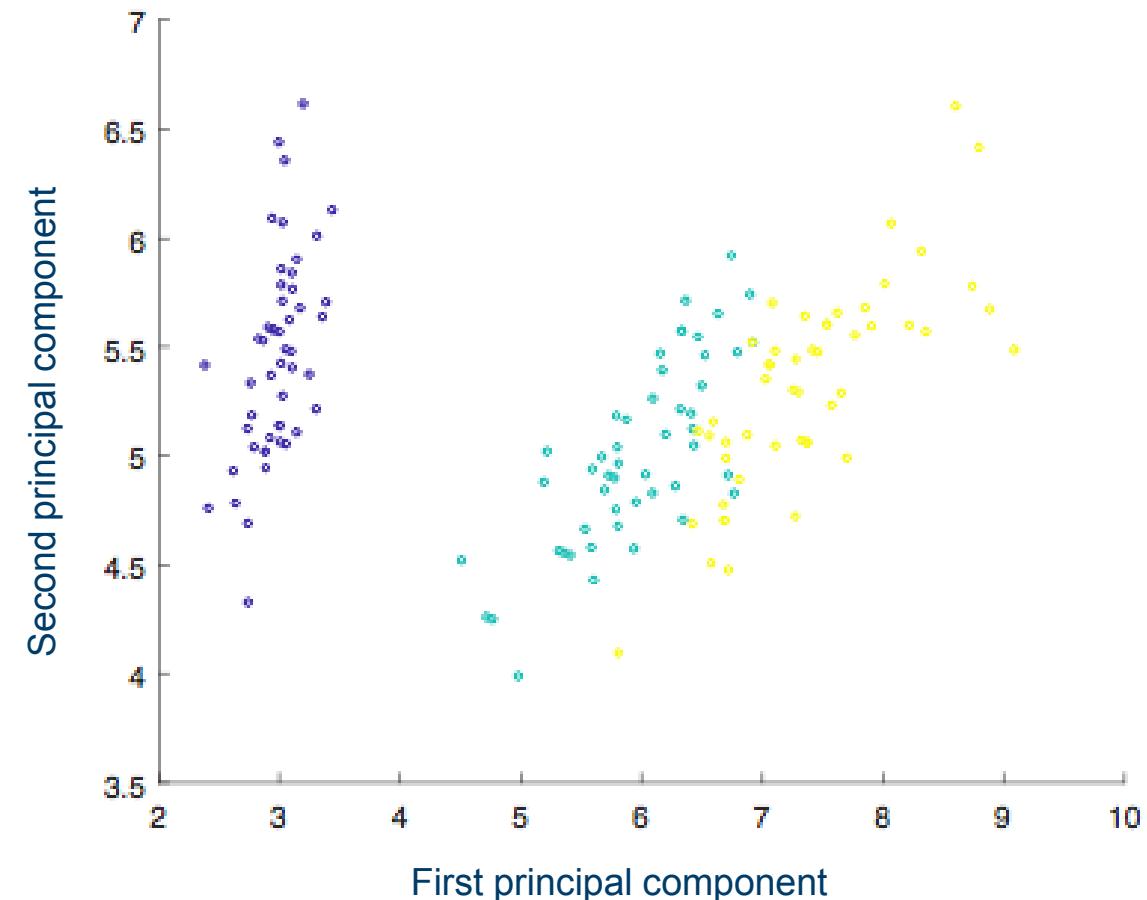
- PCA, ICA, LLE, Isomap
- PCA (Principal Components Analysis) is one of the most important techniques for dimensionality reduction
- It takes advantage of correlations between the features to produce the best possible lower dimensional representation of the data with respect to reconstruction error
- The eigenvectors of the lumped covariance matrix defines the new features in the transformed feature space.



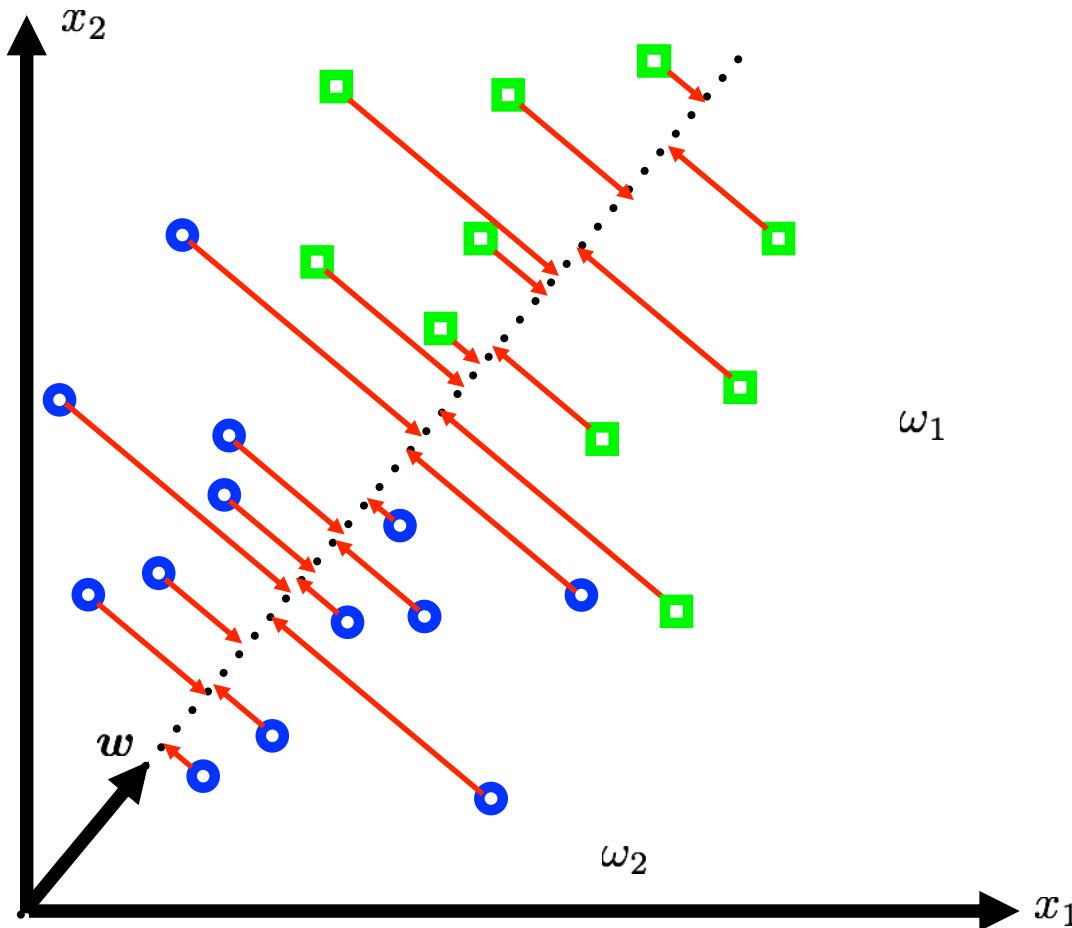
# Principle Components Analysis - example



Fisher's Iris dataset (features 1, 2 and 3)



# Dimensionality reduction – linear transformations



- Projection of multidimensional feature vectors to a lower-dimensional feature space.
- Example: *Fishers linear discriminant* provides a projection from a d-dimensional space ( $d > 1$ ) to a one-dimensional space in such a way that the separation between classes are maximized.

# Summary

## Image feature extraction:

- Feature extraction
- Feature selection

More information: Szeliski 14.4

