

CompSci 527 Final Exam Sample

Questions in this sample are meant to give you an idea of the flavor of the problems in the final exam for COMPSCI 527, and the style of the desired answers. The amount of space provided under each question is *not* an indication of the length of the answer, and the number of questions on the final exam may be different. Questions will focus on the following new material covered after the midterm. However, some questions may refer to older material when necessary.

- Textbook sections:
 - 7.1, 7.2, 7.3, 7.4
 - 13.1, 13.2, 13.3
 - 20.1, 20.2
- Class notes:
 - Histogram equalization
 - Image filtering
 - Gaussian and Laplacian pyramids
- Papers:
 - Section 6.1 of a paper by David Lowe
 - A paper by Dalal and Triggs
 - A paper by Sivic and Zisserman

All notes and papers mentioned above can be found on the syllabus web page for this class. The paper by Felzenszwalb *et al* will *not* be covered in the exam.

- Homework assignments 3 and 4
- This exam sample

The exam will be closed-book, closed-notes, and you will not be allowed to have anything other than the exam and a pen/pencil and an eraser on your desk. If you need scratch space during the exam, you will have to write in the margins or on the backs of the pages, which will be blank.

1. Let x be a real-valued random variable and let

$$U_x[\mu, w] = \begin{cases} \frac{1}{w} & \text{for } \mu - \frac{w}{2} \leq x \leq \mu + \frac{w}{2} \\ 0 & \text{elsewhere} \end{cases}$$

be the uniform distribution of width w centered at μ . One can define a mixture of uniform distributions as

$$\Pr(x|\boldsymbol{\theta}) = \sum_{k=1}^K \lambda_k U_x[\mu_k, w_k]$$

where the values of λ_k define a categorical distribution over the values $1, \dots, K$, and

$$\boldsymbol{\theta} = (\lambda_1, \mu_1, w_1, \dots, \lambda_K, \mu_K, w_K) .$$

- (a) If you are given $\boldsymbol{\theta}$, how do you draw from $\Pr(x|\boldsymbol{\theta})$? Describe briefly but precisely an algorithm that draws a single sample x from this distribution. Assume that the only random generator at your disposal is a function `rand` that takes no arguments and returns a single scalar drawn from $U_x[0, 1]$, and explain precisely how you use `rand` to draw from the distribution(s) you need. You may use MATLAB notation if you like, but a clear, plain-language explanation or pseudo-code listing are OK as well.
- (b) Explain clearly and succinctly why using the idea of Expectation Maximization (EM) for parameter estimation may be problematic for mixtures of uniform distributions.

2. Let $h_I(u)$ and $h_J(v)$ for $u, v \in \{0, \dots, 255\}$ be the histograms of a gray-level image I and of the gray-level image J obtained by applying histogram equalization to I . Histogram equalization maps pixel value u of I to pixel value

$$v = f(u) = \left\lceil \frac{256}{n} H_I(u) - 1 \right\rceil$$

of J . In this expression, n is the number of pixels in the image, H_I is the cumulative histogram of I ,

$$H_I(u) = \sum_{i=0}^u h_I(i) ,$$

and the brackets $\lceil \cdot \rceil$ denote rounding to the nearest integer.

Construct a histogram h_I for an image I with $n > 256$ pixels such that

$$h_I(u) \neq h_J(v) \quad \text{for all} \quad u \in \{0, \dots, 255\} \quad \text{and} \quad v \in \{0, \dots, 255\} .$$

That is, no value in the output histogram h_J is a copy of any value in the input histogram h_I .

3. The image C on the right below was obtained by convolving the image I on the left with a 2×2 kernel H whose origin (we also called this the “hot spot”) is in the bottom right pixel (marked by the dot). The ‘same’ option was used in Matlab, so the sizes of I and C are the same.

$$I = \begin{bmatrix} 1 & 0 & 5 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 9 \\ 0 & 7 & 0 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 2 & 0 & 2 & 0 & 6 & 0 \end{bmatrix} \quad H = \begin{bmatrix} & \\ & \\ & \\ & \bullet \end{bmatrix} \quad C = \begin{bmatrix} 3 & 8 & 5 & 2 & 20 & 27 \\ 15 & 22 & 6 & 9 & 9 & 9 \\ 10 & 7 & 3 & 3 & 2 & 3 \\ 7 & 4 & 6 & 12 & 19 & 1 \\ 2 & 2 & 2 & 6 & 6 & 0 \end{bmatrix}$$

Fill in the four values of the kernel. You may want to briefly explain your reasoning if you are not sure about your answer. [Hint: if you are doing a lot of calculations, think again.]

4. Is the following convolution kernel separable? If so, separate it. If not, prove that it is not.

$$H = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$$

5. What is the gradient of the following function at $x = y = 0$?

$$f(x, y) = (x - 2)^3 \sin y$$

6. A SIFT descriptor for a 16×16 image window W is a vector of 128 numbers, which can be thought of as being grouped in 16 consecutive groups of 8 numbers each. What does each group of 8 numbers represent? Explain in one or two brief, clear, and accurate sentences.

7. The K-means algorithm takes as input N points and a positive integer K . It returns a partition of the N input points into K sets with certain properties, and the centroids of the points in each of these sets. State two different ways in which you could initialize the K means algorithm. Just describe briefly in English, no formulas are needed.

8. In a 2003 paper, Sivic and Zisserman describe an image retrieval system based on SIFT descriptors and visual words. In your answers to the questions below, you may use terminology either from computer vision (images, features, ...) or from document retrieval (documents, words, ...).

- (a) Give a formula for t_{id} , the so-called “term-frequency–inverse document frequency” weight used to evaluate the contribution of a word to a bag-of-words descriptor. Use the following variables in your formula:

- n_{id} is the number of occurrences of word i in document d
- n_d is the total number of words in document d
- n_i is the number of occurrences of word i in the whole training set
- N is the number of documents in the whole training set

If you do not remember the formula, describe clearly and succinctly the criteria for it.

- (b) If applied blindly, the formula for t_{id} above gives unsatisfactory results. Describe some of the pre-processing Sivic and Zisserman recommend before the weights t_{id} are computed, and explain their rationale.

- (c) What is the principle of spatial consistency? Explain clearly and succinctly, and in broad lines. You need not explain the details of how the principle is used in the paper.