Some Groups of Geometric Transformations

Name	Symbol	Formula	Notes	DoF		Preserves	Preserves Handedness	Transformation of a Square
				2D	3D			
Translation	$T(n)$	$\mathbf{x} + \mathbf{t}$	Commutes	2	3	orientation	Y	
Rotation (Special Orthogonal)	$SO(n)$	$R\mathbf{x}$	$RR^T = R^T R = I$, $\det(R) = +1$	1	3		Y	
Orthogonal	$O(n)$	$U\mathbf{x}$	$UU^T = U^T U = I$, $\det(U) = \pm 1$	1	3		N	
Rigid (Special Euclidean)	$SE(n)$	$R\mathbf{x} + \mathbf{t}$		3	6		Y	
Euclidean	$E(n)$	$U\mathbf{x} + \mathbf{t}$		3	6	lengths	N	
Similarity	$S(n)$	$sR\mathbf{x} + \mathbf{t}$	$s > 0$	4	7	angles	Y	
General Linear	$GL(n)$	$A\mathbf{x}$	$\det(A) \neq 0$	4	9		N	
Affine	$A(n)$	$A\mathbf{x} + \mathbf{t}$		6	12	parallelism	N	
Collineation (Homography)	$P\Gamma L(n)$	$\frac{A\mathbf{x}+\mathbf{t}}{\mathbf{b}^T\mathbf{x}+c}$		8	15	straight lines	N	

Transformations in each group preserve the "**Preserves**" property on their line and all the ones below it, but not the ones above it.