

COMP 4900C: Assignment 1  
Due: Tuesday, Feb. 6, 2007

1. (2 points) The rotation matrix in the x-y plane is

$$R(\phi) = \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix}$$

- (a) Verify  $R(\phi_1)R(\phi_2) = R(\phi_1 + \phi_2)$  from the identities for  $\cos(\phi_1 + \phi_2)$  and  $\sin(\phi_1 + \phi_2)$ .  
 (b) What is  $R(\phi)R(-\phi)$ ?

2. (4 points) Scale a vector  $[x \ y]^T$  in the plane can be achieved by

$$x' = sx \text{ and } y' = sy$$

where  $s$  is a scalar.

- (a) Write out the matrix form of this transformation.  
 (b) Write out the transformation matrix for homogeneous coordinates.  
 (c) If the transformation also includes a translation

$$x' = sx + t_x \text{ and } y' = sy + t_y$$

Write out the transformation matrix for homogeneous coordinates.

- (d) What is the equivalent of the above matrix for three-dimensional vectors?

3. (2 points) Find the least square solution  $\bar{x}$  for  $Ax = b$  if

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Verify that the error vector  $b - A\bar{x}$  is orthogonal to the columns of  $A$ .

4. (2 points) A pinhole camera has focal length  $f = 500$ , pixel sizes  $s_x = s_y = 1$ , and its principal point is at  $(o_x, o_y) = (320, 240)$ . The world coordinate frame and the camera coordinate frame can be related by  $X_c = RX_w + T$ , where

$$R = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad T = \begin{bmatrix} 70 \\ 95 \\ 120 \end{bmatrix}$$

- (a) Write out the 3x4 projection matrix that projects a point in the world coordinate frame onto the image plane in pixel coordinate.  
 (b) What are the pixel coordinates of the world point

$$X_w = [150 \ 200 \ 400]^T ?$$