



Image Formation III

Chapter 1 (Forsyth&Ponce)

Cameras “*Lenses*”

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CS 6320 S2013

(slides modified from Marc Pollefeys,
UNC Chapel Hill)

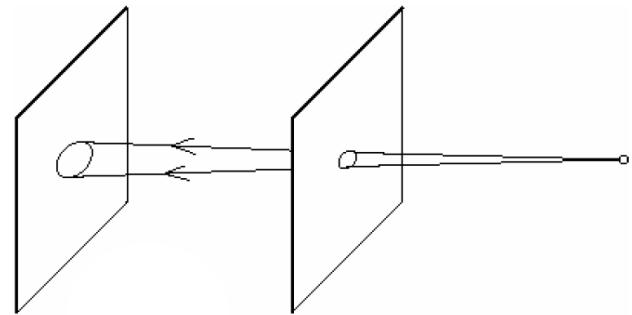


Pinhole size / aperture

How does the size of the aperture affect the image we'd get?



Larger

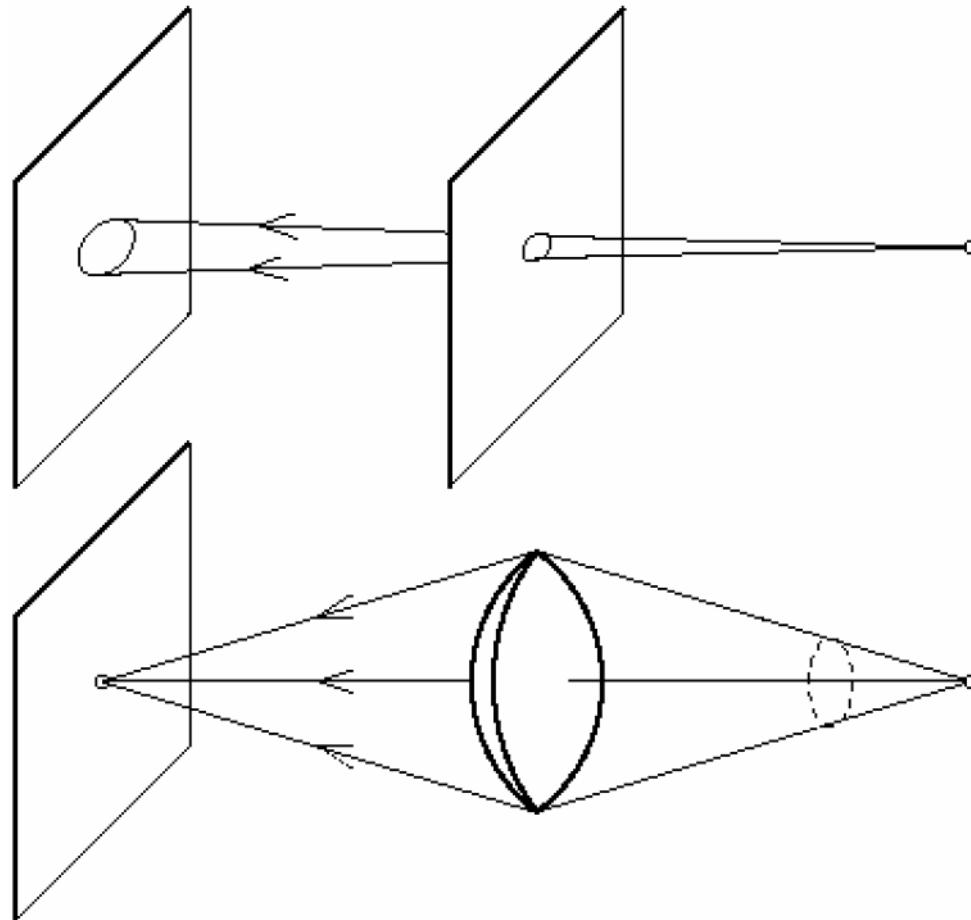


Smaller

Fig. 5.96 The pinhole camera. Note the variation in image clarity as the hole diameter decreases. [Photos courtesy Dr. N. Joel, UNESCO.]

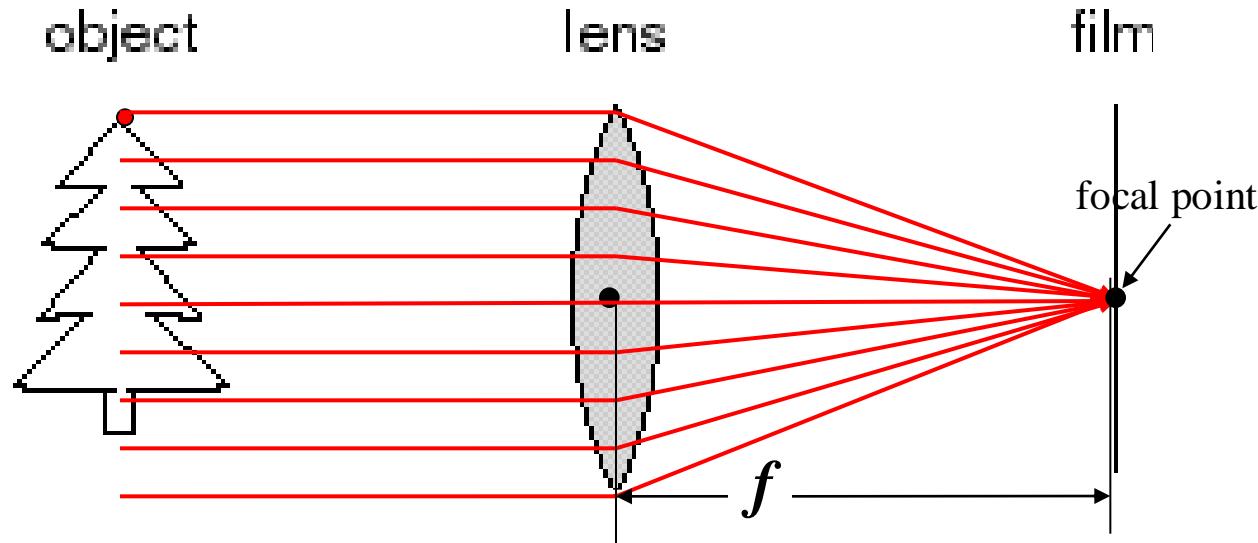


Pinhole vs. lens





Adding a lens

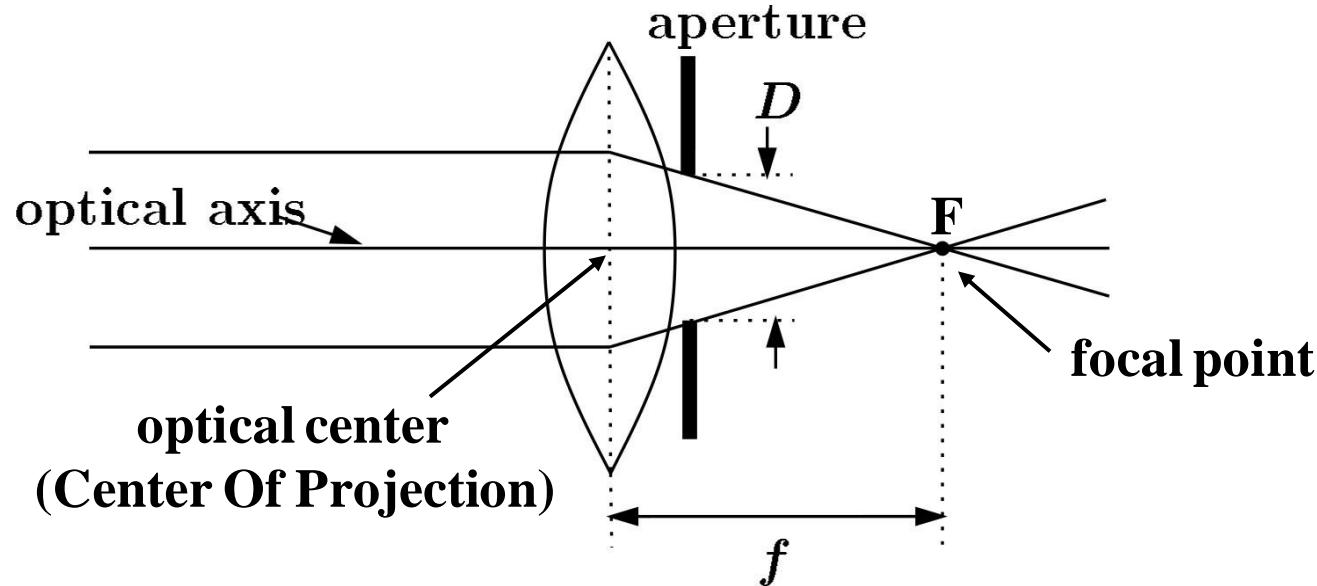


A lens focuses light onto the film

- Rays passing through the center are not deviated
- All parallel rays converge to one point on a plane located at the *focal length f*



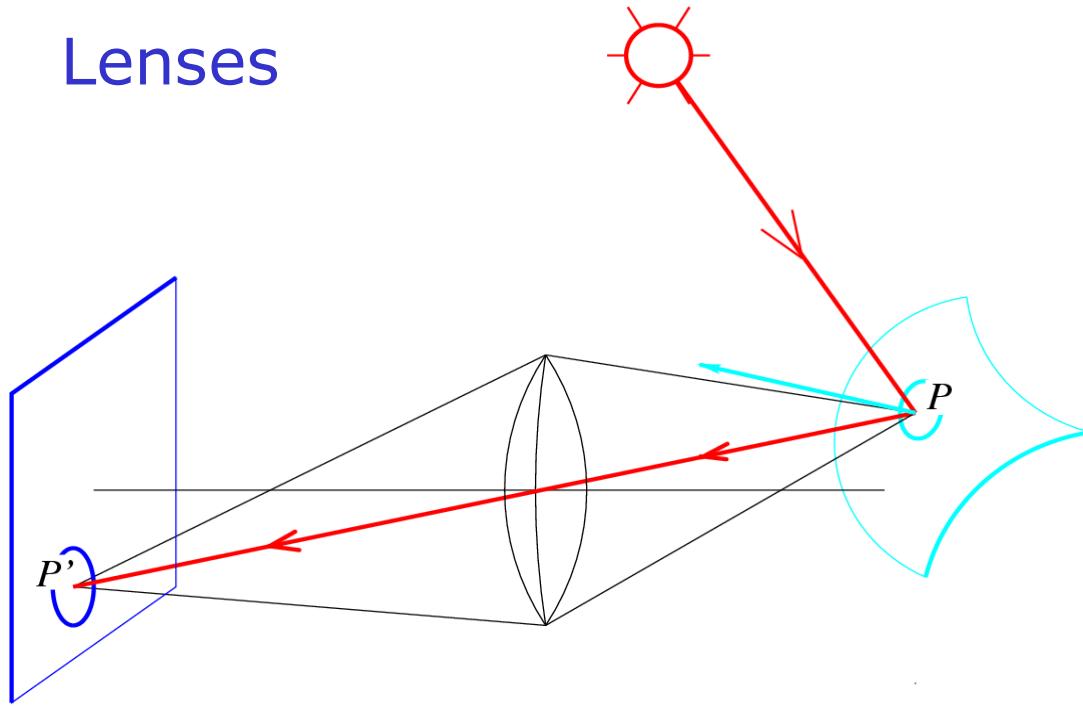
Cameras with lenses



- A lens focuses parallel rays onto a single focal point
- Gather more light, while keeping focus; make pinhole perspective projection practical

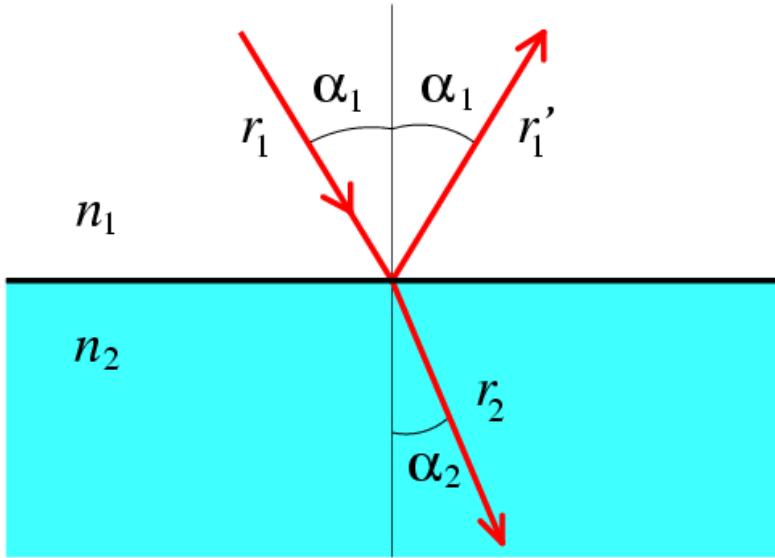


Lenses

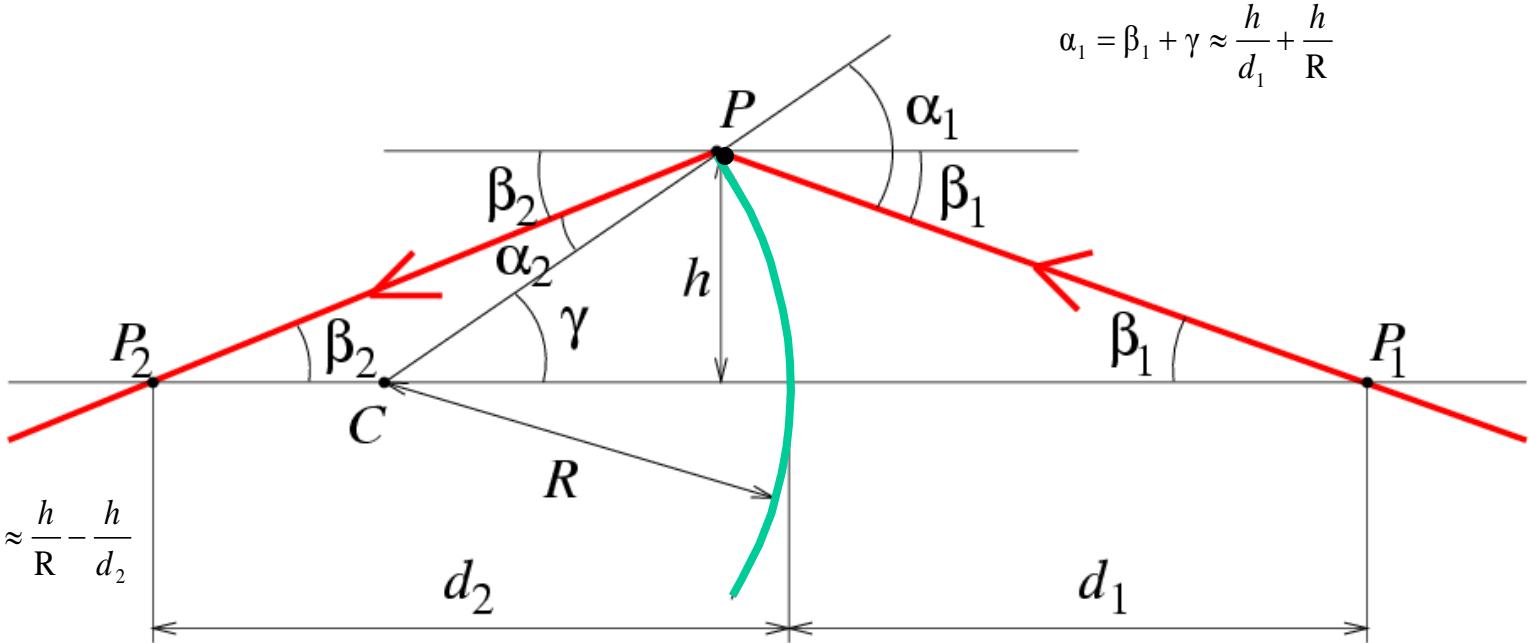


Snell's law

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$



Paraxial (or first-order) optics



Snell's law:

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

Small angles:

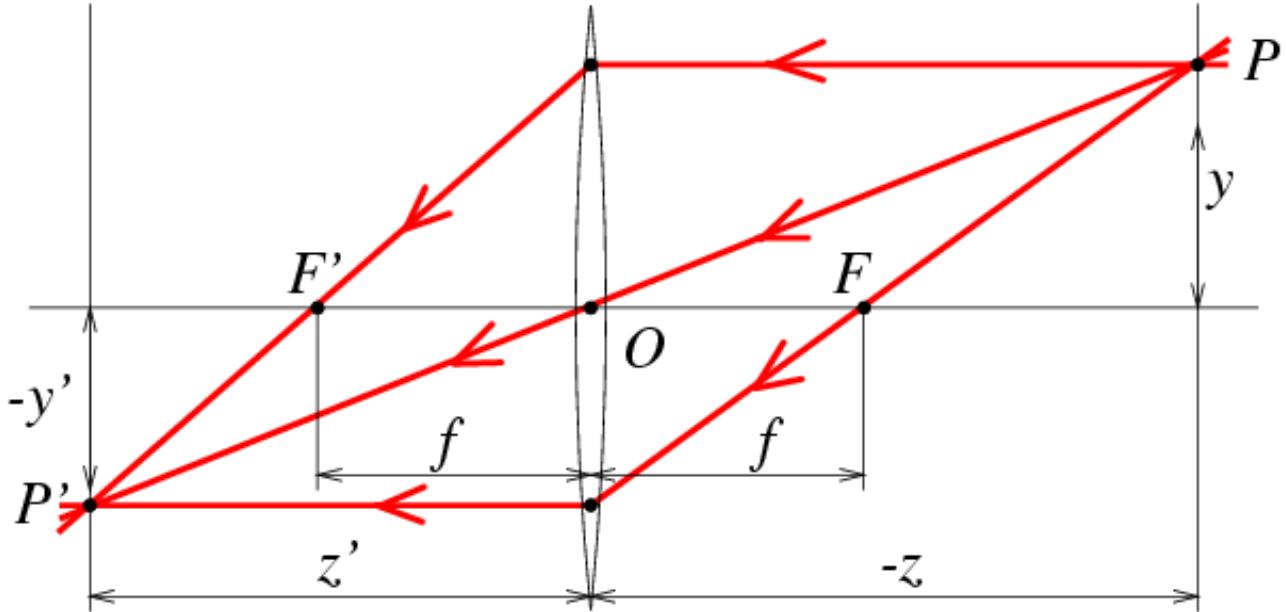
$$n_1 \alpha_1 \approx n_2 \alpha_2$$

$$n_1 \left(\frac{h}{d_1} + \frac{h}{R} \right) = n_2 \left(\frac{h}{R} - \frac{h}{d_2} \right)$$

$$\frac{n_1}{d_1} + \frac{n_2}{d_2} = \frac{n_2 - n_1}{R}$$

Thin Lenses

spherical lens surfaces; thickness \ll radii; same refractive index on both sides; all rays emerging from P and passing through the lens are focused at P'. Let $n_1=1$ (vacuum) and $n_2=n$.



$$\begin{cases} x' = z' \frac{x}{z} \\ y' = z' \frac{y}{z} \end{cases}$$

where

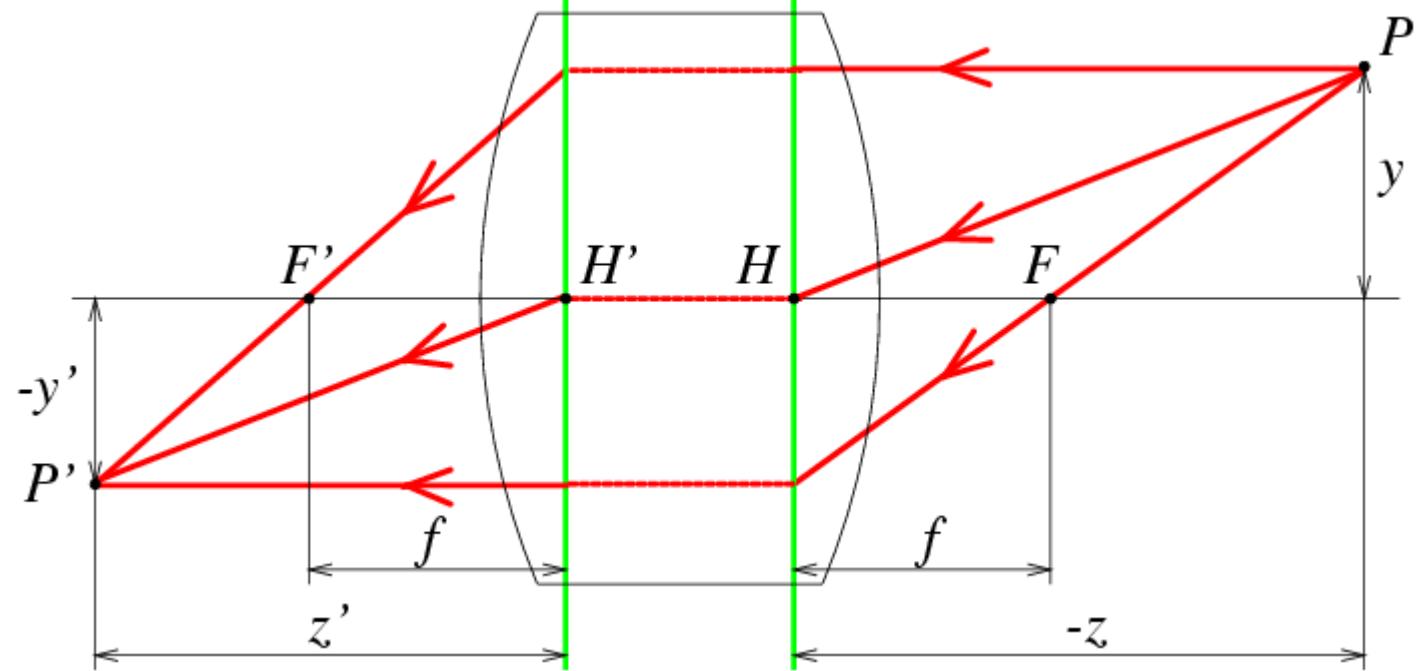
$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

and

$$f = \frac{R}{2(n-1)}$$



Thick Lens

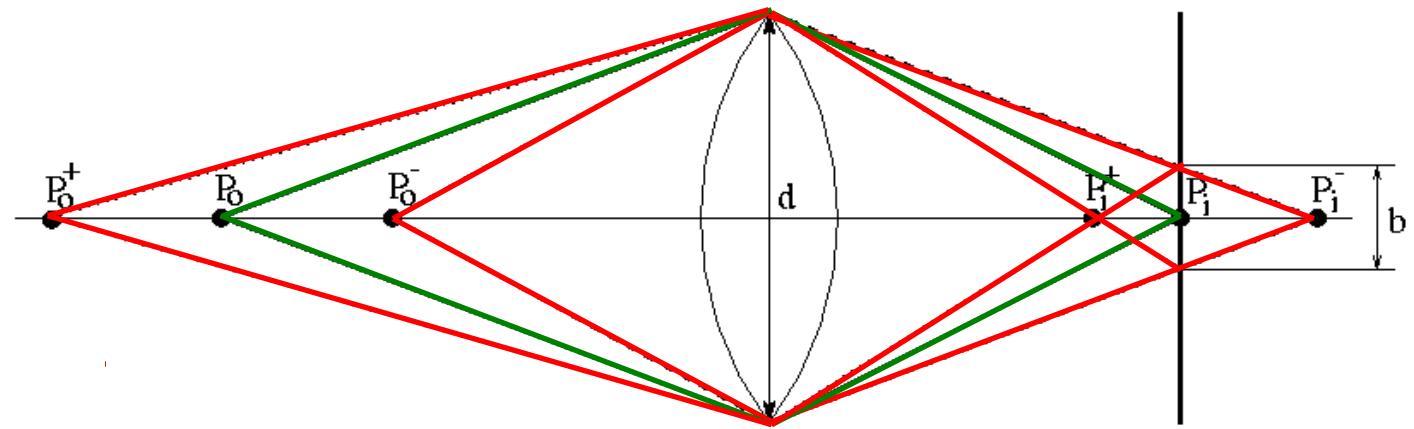


Focus and depth of field





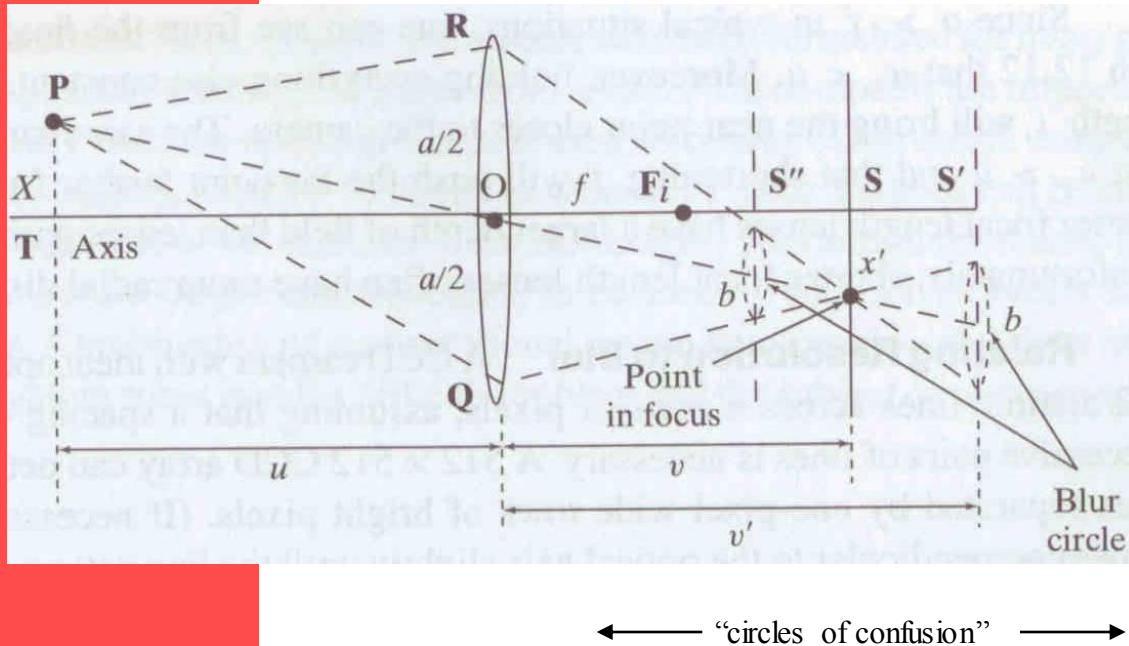
The depth-of-field





Focus and depth of field

- Depth of field: distance between image planes where blur is tolerable



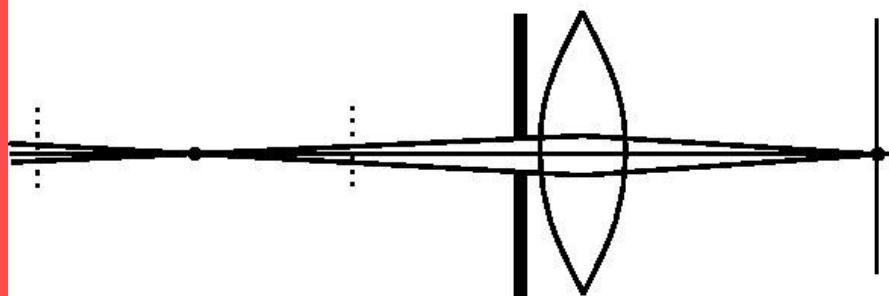
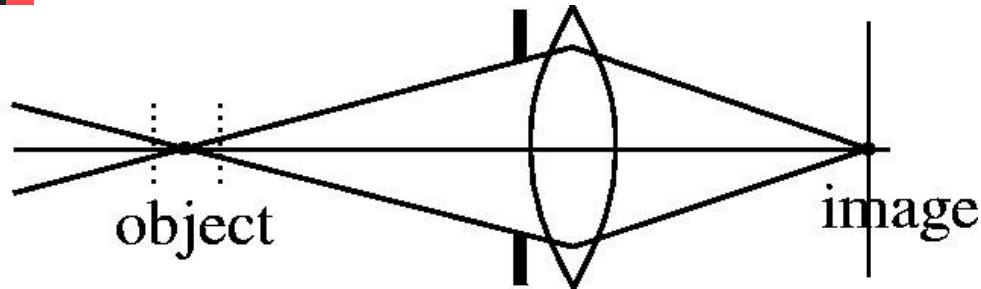
Thin lens: scene points at distinct depths come in focus at different image planes.

(Real camera lens systems have greater depth of field.)



Focus and depth of field

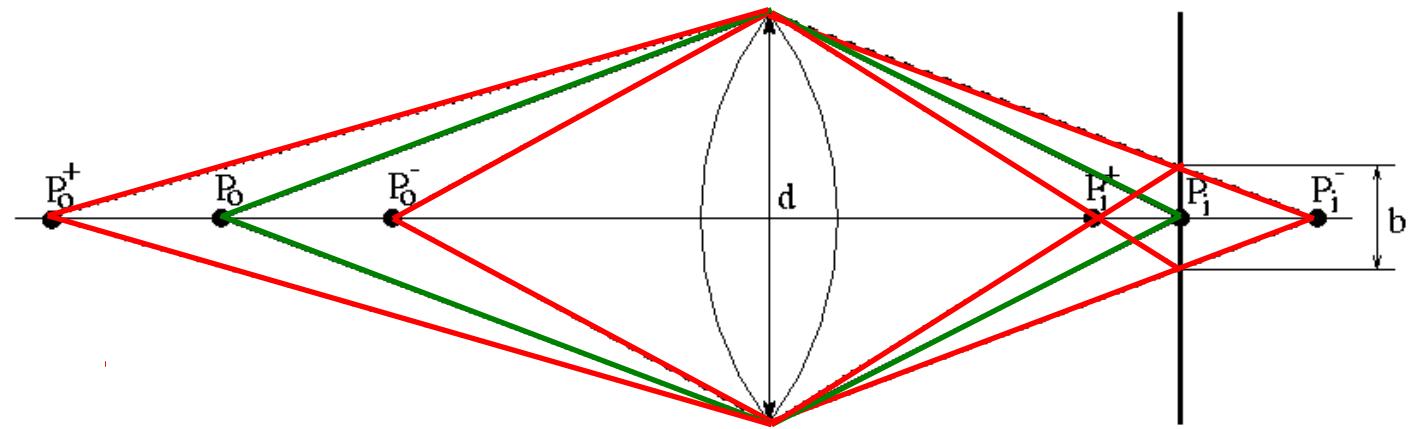
- How does the aperture affect the depth of field?



- A smaller aperture increases the range in which the object is approximately in focus



The depth-of-field





The depth-of-field

yields

$$Z_o^- = f \frac{|Z_i^-|}{|Z_i^-| - f}$$

⋮

$$Z_o^- = f \frac{d Z_o}{b Z_o + f (d - b)}$$

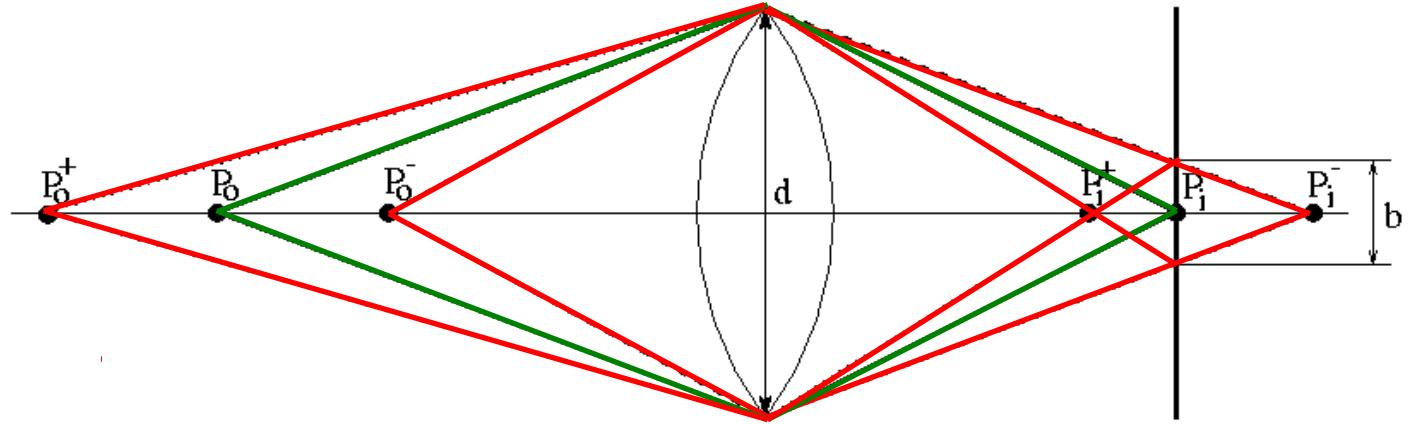
$$\Delta Z_o^- = Z_o - Z_o^- = \frac{Z_o (Z_o - f)}{Z_o + f d / b - f}$$

Similar formula for $\Delta Z_o^+ = Z_o^+ - Z_o$





The depth-of-field



$$\Delta Z_0^- = Z_0 - Z_0^- = \frac{Z_0(Z_0 - f)}{Z_0 + f \ d / b - f}$$

decreases with $d+$, increases with Z_0+
strike a balance between incoming light and
sharp depth range





Deviations from the lens model

3 assumptions :

1. all rays from a point are focused onto 1 image point
2. all image points in a single plane
3. magnification is constant

deviations from this ideal are *aberrations*





Aberrations

2 types :

1. geometrical
2. chromatic

geometrical : small for paraxial rays

study through 3rd order optics $\sin(\theta) \approx \theta - \frac{\theta^3}{6}$

chromatic : refractive index function of wavelength

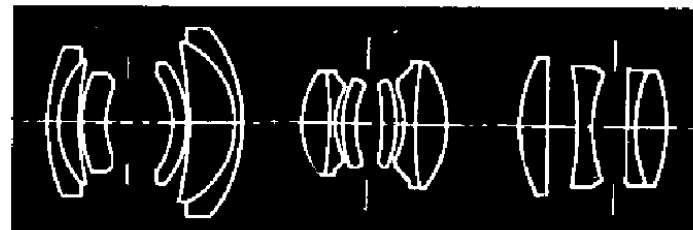




Geometrical aberrations

- spherical aberration
- astigmatism
- distortion
- coma

aberrations are reduced by combining lenses

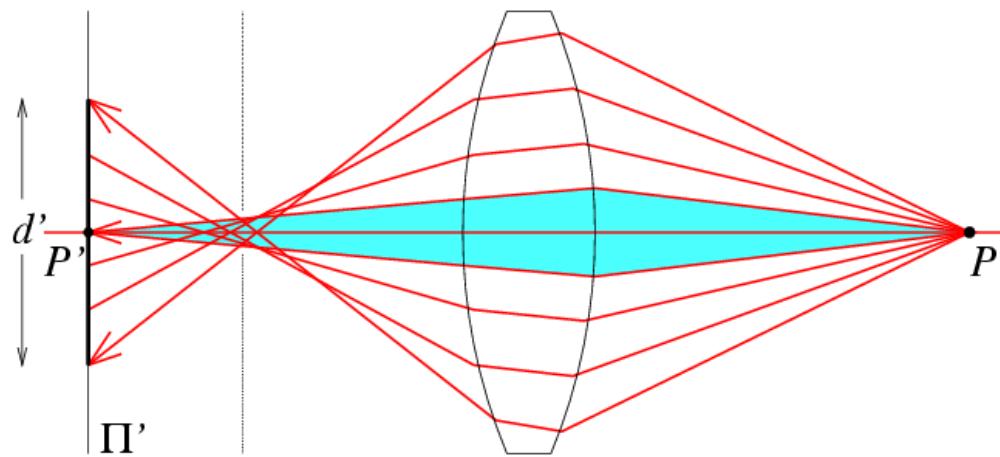




Spherical aberration

rays parallel to the axis do not converge

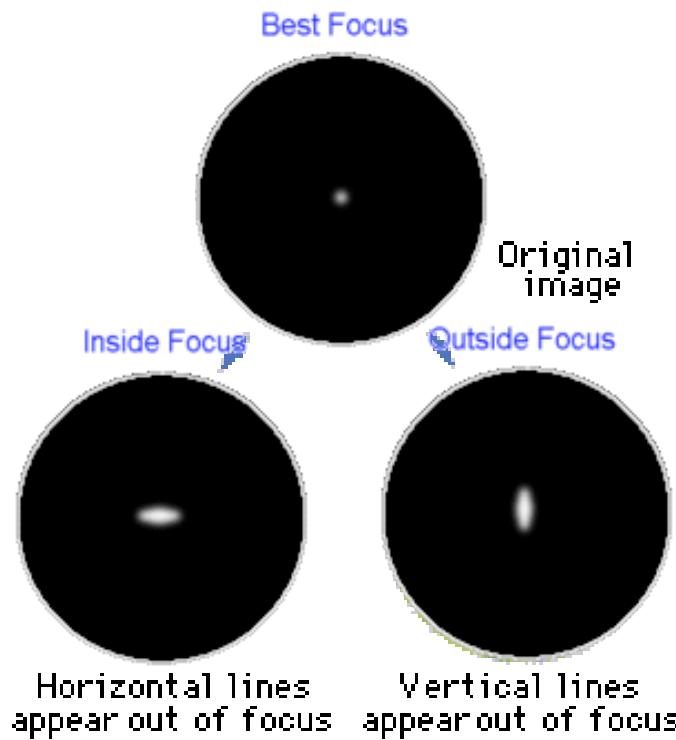
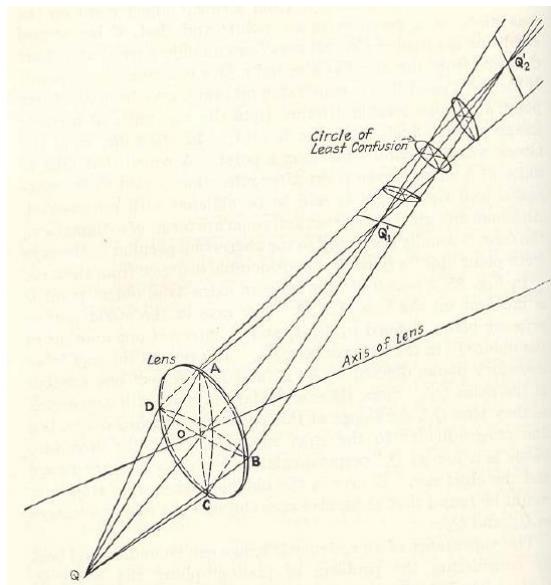
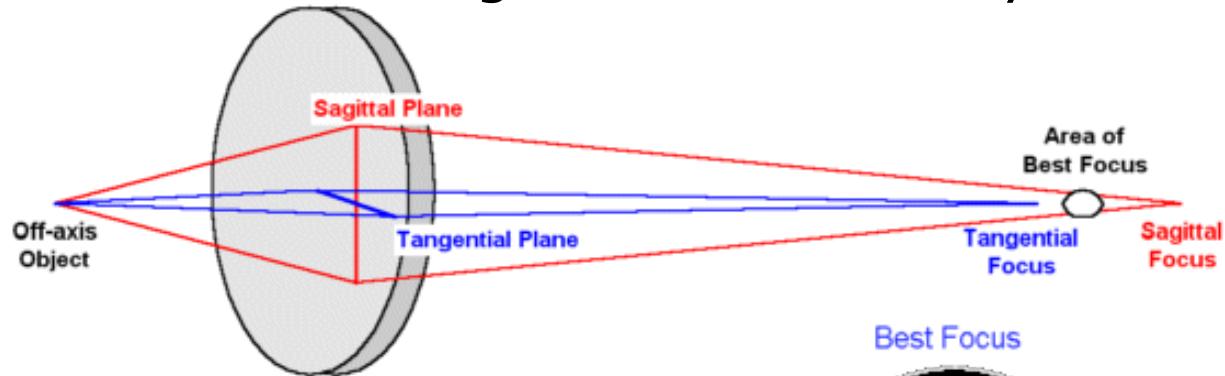
outer portions of the lens yield smaller focal lengths





Astigmatism

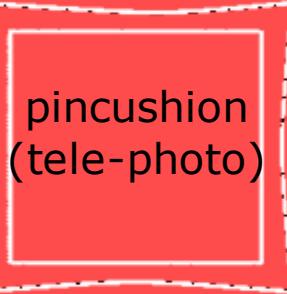
Different focal length for inclined rays





Distortion

magnification/focal length different
for different angles of inclination

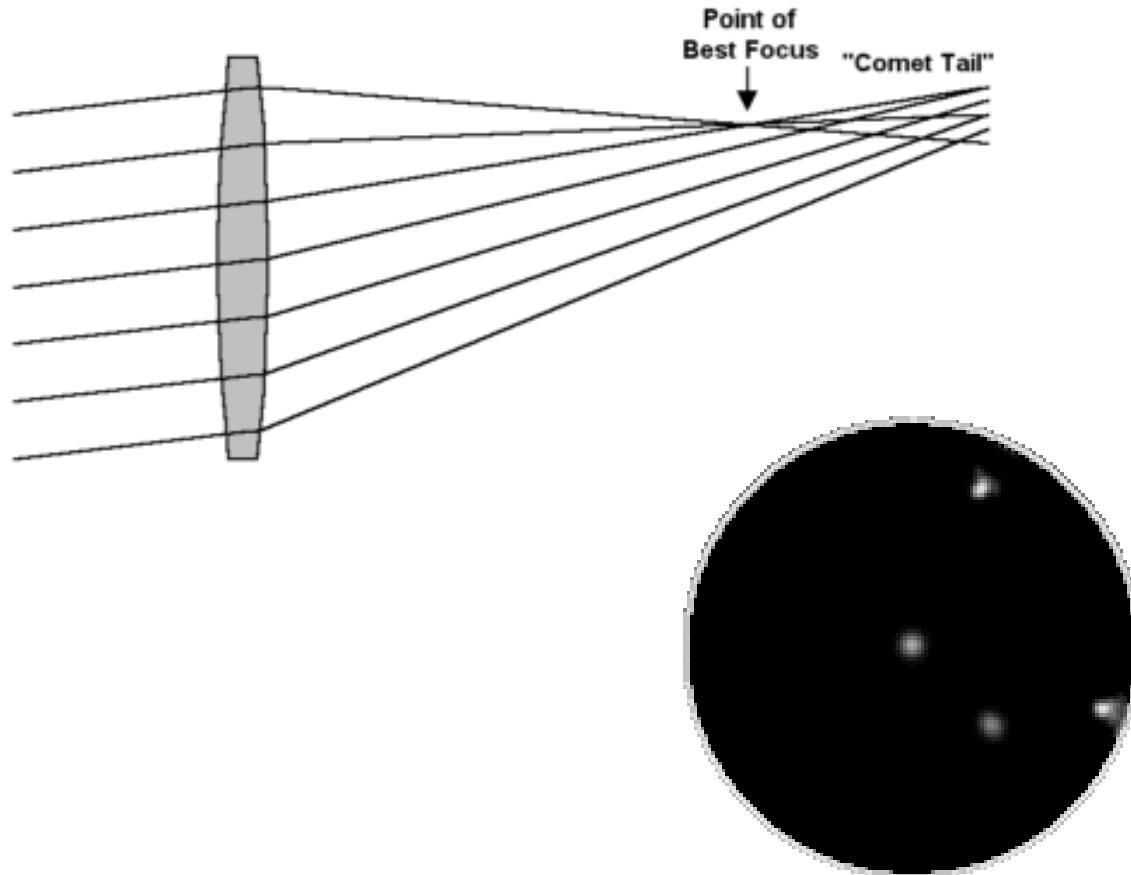


Can be corrected! (if parameters are known)



Coma

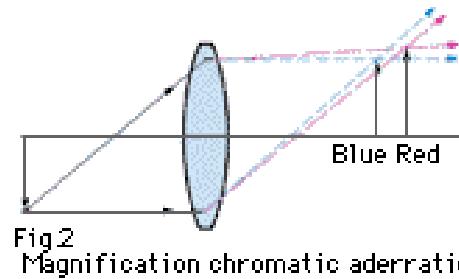
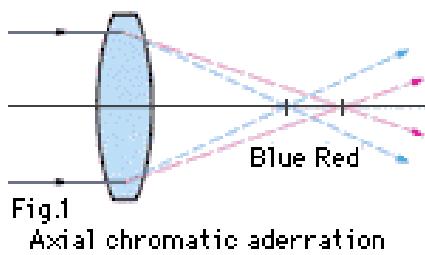
point off the axis depicted as comet shaped blob





Chromatic aberration

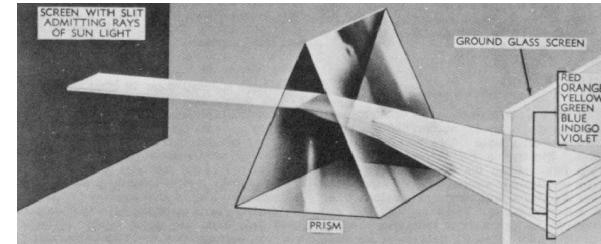
rays of different wavelengths focused in different planes



The image is blurred and appears colored at the fringe.

cannot be removed completely

sometimes *achromatization* is achieved for more than 2 wavelengths





Vignetting

