

1. Give a criterion to automatically estimate k , the number of dimensions of the eigenspace used in eigenspace learn. That is some criteria of choosing k automatically.
2. Assume that the only source of error in a simple stereo system is the error in estimating the disparity. Assume that this error is fixed and is at least 1 pixel. So if the stereo system says the disparity is 5 pixels it is really between 4 and 6 pixels. And if the stereo system says the disparity is 10 pixels then it is really between 9 and 11 pixels. The change in Z (ΔZ) due to this one pixel error in estimating the disparity is called the absolute error of the stereo system. Compute the ratio of $\Delta Z(5 \text{ pixels}) / \Delta Z(10 \text{ pixels})$. From this answer hypothesize a relationship between ΔZ and disparity d . Prove that your hypothesis is true by computing the partial derivative of Z with respect to disparity d .
3. There is a simple stereo system with one camera placed above the other in the y direction (not the x direction as usual) by a distance of b . In such a case there is no rotation between the cameras, only a translation by a vector $T = [0, b, 0]$. First compute the essential matrix E in this case using the formula provided. Assume that both cameras have the same focal length f . Prove that in this case, for this given E that the epipolar lines are vertical. That is for a given point $(p_b)^T$ in the bottom image prove that the epipolar line defined by the equation $(p_t)^T E (p_b)^T = 0$ is a horizontal line. Here p_t is (x_t, y_t, f) and p_b is (x_b, y_b, f) which are the points in the top and bottom image plane. We are given p_b and E , and the unknown is p_t .
4. Assume that there is a 3D point X on a plane that is viewed by two cameras. The projection of this 3D point in camera one is defined by $x = P X$, and in camera two by $x' = P' X$. Here $x = [u, v, 1]$ the pixel co-ordinates in image one of X and $x' = [u', v', 1]$ the pixel co-ordinates in the other images, P and P' are the 3 by 4 projection matrices and X is a point in 3D space $= [x, y, z, 1]$. Prove

that in this case $x = M x'$, where M is a 3 by 3 matrix called a homography. Hint: Define the world co-ordinate frame for the 3D point X so that the x, y axis is on the plane. In other words X is a point on the plane implies that it is defined as $X = [x, y, 0, 1]$ in homogeneous co-ordinates. Now write down the two projection equations with the elements of P being simply $p_{11}, p_{12}, \dots, p_{34}$, and P' being defined as $p'_{11}, p'_{12}, \dots, p'_{34}$.