



# Multi-View Geometry: Small Motion and Epipolar Constraints

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# Motion Models (Review)

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \approx \begin{bmatrix} 1 & -\gamma & \beta \\ \gamma & 1 & -\alpha \\ -\beta & \alpha & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}$$

3D Rigid Motion

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \approx \left( \begin{bmatrix} 0 & -\gamma & \beta \\ \gamma & 0 & -\alpha \\ -\beta & \alpha & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}$$

$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}$  = Velocity Vector

$$\begin{bmatrix} X' - X \\ Y' - Y \\ Z' - Z \end{bmatrix} \approx \begin{bmatrix} 0 & -\gamma & \beta \\ \gamma & 0 & -\alpha \\ -\beta & \alpha & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}$$

$\begin{bmatrix} V_{T_x} \\ V_{T_y} \\ V_{T_z} \end{bmatrix}$  = Translational Component of Velocity

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} \approx \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} V_{T_x} \\ V_{T_y} \\ V_{T_z} \end{bmatrix}$$

$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$  = Angular Velocity



# Small Motions

$$t = \delta t v$$

$$R = I + \delta t [\omega_{\times}]$$

$$p' = p + \delta t \dot{p}$$

$$p^T \mathcal{E} p' = 0$$

$$p^T [v_{\times}] (I + \delta t [\omega_{\times}]) (p + \delta t \dot{p}) = 0$$

$$p^T ([v_{\times}] [\omega_{\times}]) p - (p \times \dot{p}).v = 0$$

Exercise 7.2

$$\dot{p} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \text{Velocity Vector}$$

$$v = \begin{bmatrix} V_{T_x} \\ V_{T_y} \\ V_{T_z} \end{bmatrix} = \text{Translational Component of Velocity}$$

$$\omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \text{Angular Velocity}$$



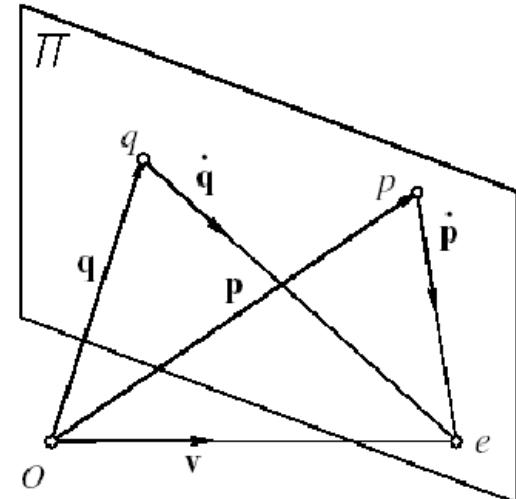
# Translating Camera

$$p^T ([v_x] \llbracket \omega_x]) p - (p \times \dot{p}) \cdot v =$$

$$\omega = 0$$

$$(p \times \dot{p}) \cdot v = 0$$

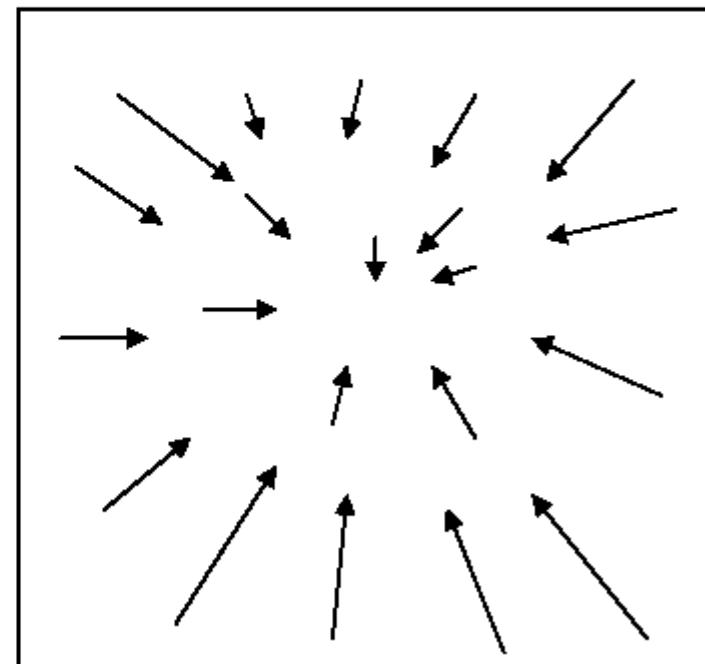
$p$ ,  $\dot{p}$ , and  $v$  are coplanar  
and  $\dot{p}$  lies in image plane



Focus of expansion (FOE): Under pure translation, the motion field at every point in the image points toward the focus of expansion.



# FOE for Translating Camera





# FOE from Basic Equations of Motion (see later optical flow)

$$\dot{p}_x = \frac{V_{T_z}x - V_{T_x}f}{Z} - \omega_y f + \omega_z y + \frac{\omega_x xy}{f} - \frac{\omega_y x^2}{f}$$

$$\dot{p}_y = \frac{V_{T_z}y - V_{T_y}f}{Z} + \omega_x f - \omega_z x - \frac{\omega_y xy}{f} + \frac{\omega_x y^2}{f}$$

$$\omega = 0$$

$$\dot{p}_x = \frac{V_{T_z}x - V_{T_x}f}{Z}$$

$$\dot{p}_y = \frac{V_{T_z}y - V_{T_y}f}{Z}$$

$$x_0 = f \frac{V_{T_x}}{V_{T_z}}$$

$$y_0 = f \frac{V_{T_y}}{V_{T_z}}$$



$$\dot{p}_x = (x - x_0) \frac{V_{T_z}}{Z}$$

$$\dot{p}_y = (y - y_0) \frac{V_{T_z}}{Z}$$

