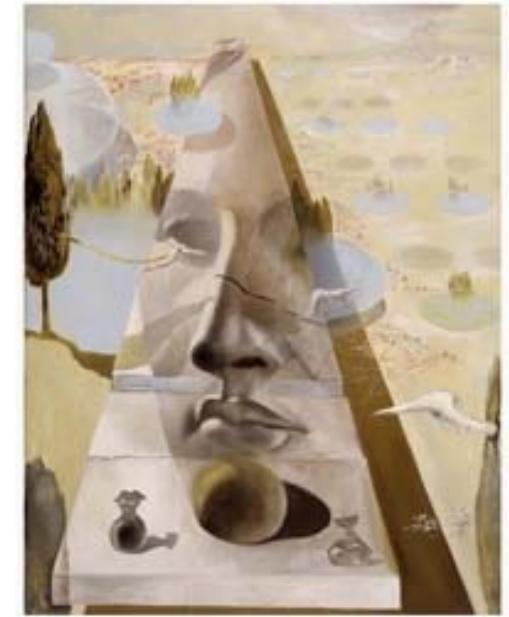


Lecture 7

Multi-view geometry

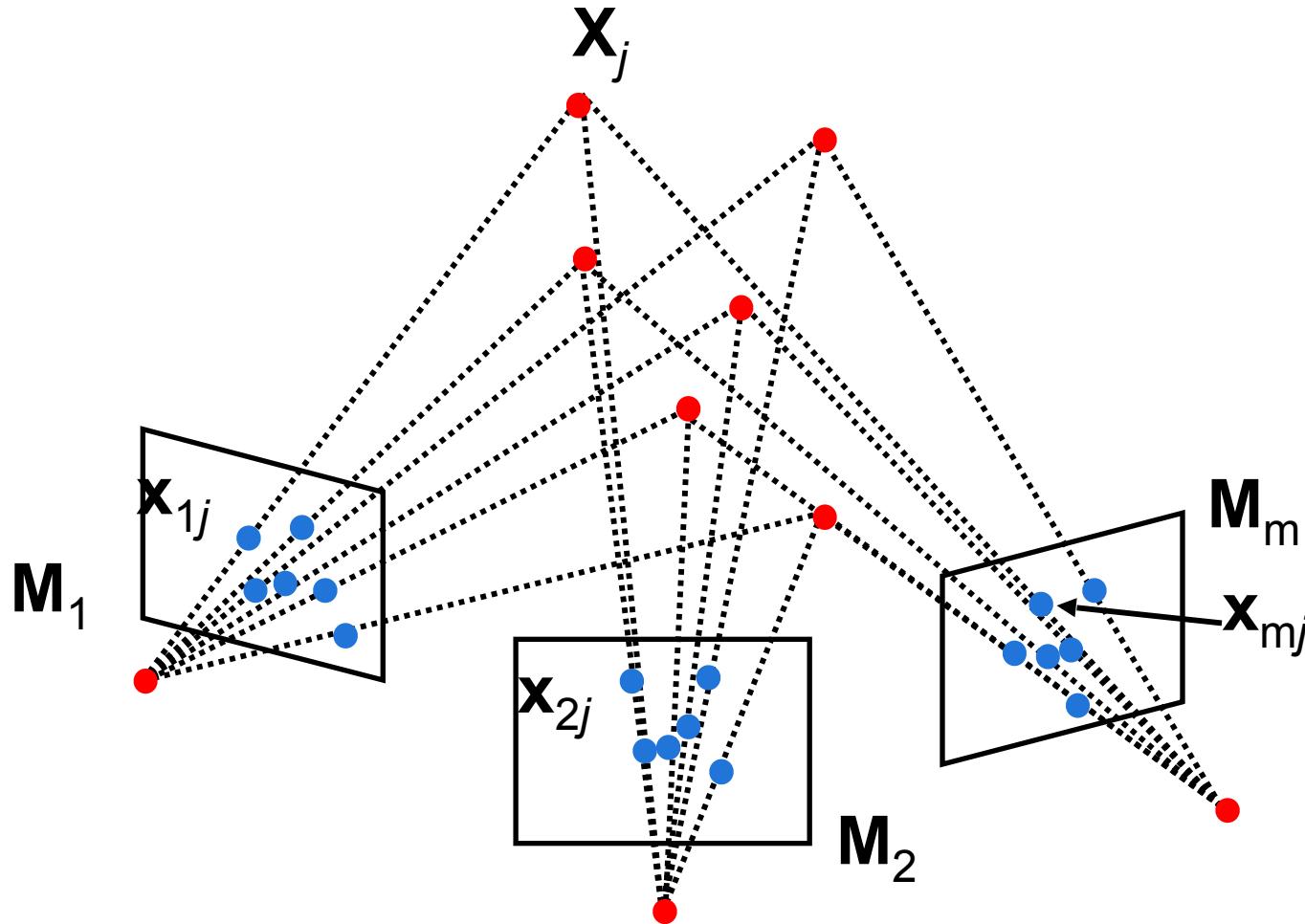


- The SFM problem
- Affine SFM
- Perspective SFM
- Self-calibration
- Applications

Reading:

- [HZ] Chapter 10 “3D reconstruction of cameras and structure”
Chapter 18 “N-view computational methods”
Chapter 19 “Auto-calibration”
- [FP] Chapter 13 “projective structure from motion”
- [Szelisky] Chapter 7 “Structure from motion”

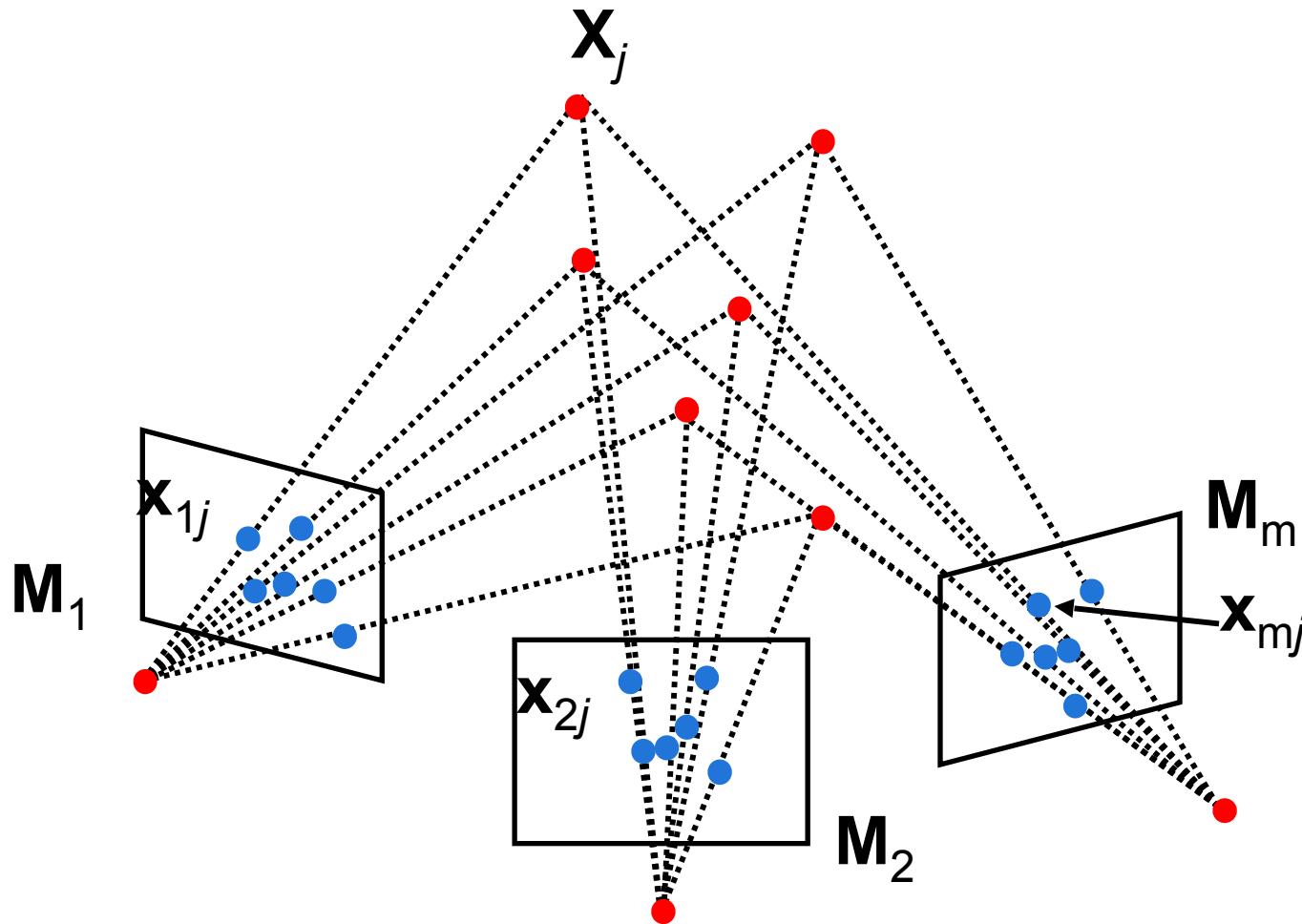
Structure from motion problem



Given m images of n fixed 3D points

- $\mathbf{x}_{ij} = \mathbf{M}_i \mathbf{X}_j, \quad i = 1, \dots, m, \quad j = 1, \dots, n$

Structure from motion problem



From the $m \times n$ correspondences \mathbf{x}_{ij} , estimate:

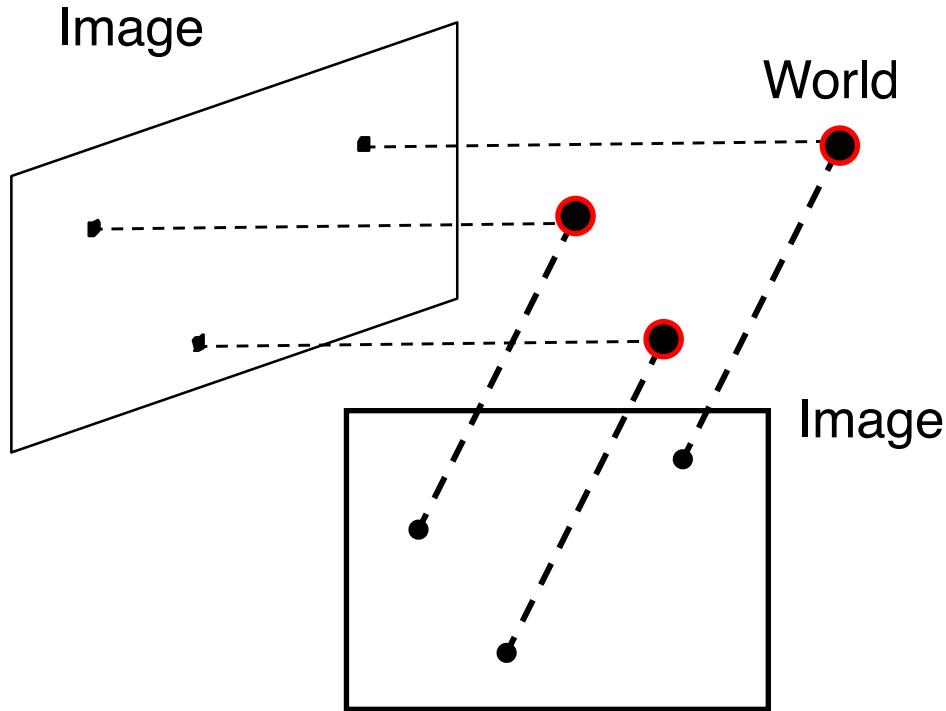
- m projection matrices \mathbf{M}_i
- n 3D points \mathbf{X}_j

motion

structure

Affine structure from motion

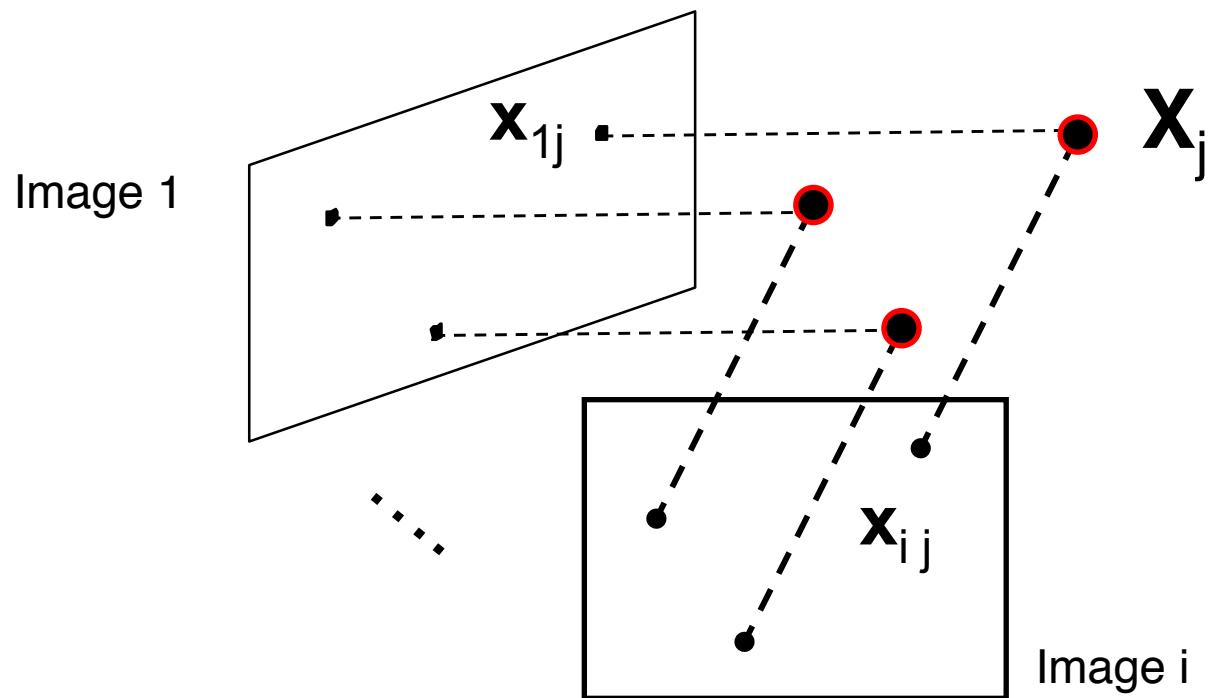
(simpler problem)



From the $m \times n$ correspondences \mathbf{x}_{ij} , estimate:

- m projection matrices \mathbf{M}_i (affine cameras)
- n 3D points \mathbf{X}_j

Affine cameras



Camera matrix \mathbf{M} for the affine case

$$\mathbf{x}_{ij} = \mathbf{A}_i \mathbf{x}_j + \mathbf{b}_i = \mathbf{M}_i \begin{bmatrix} \mathbf{x}_j \\ 1 \end{bmatrix}; \quad \mathbf{M} = \begin{bmatrix} \mathbf{A} & \mathbf{b} \end{bmatrix}$$

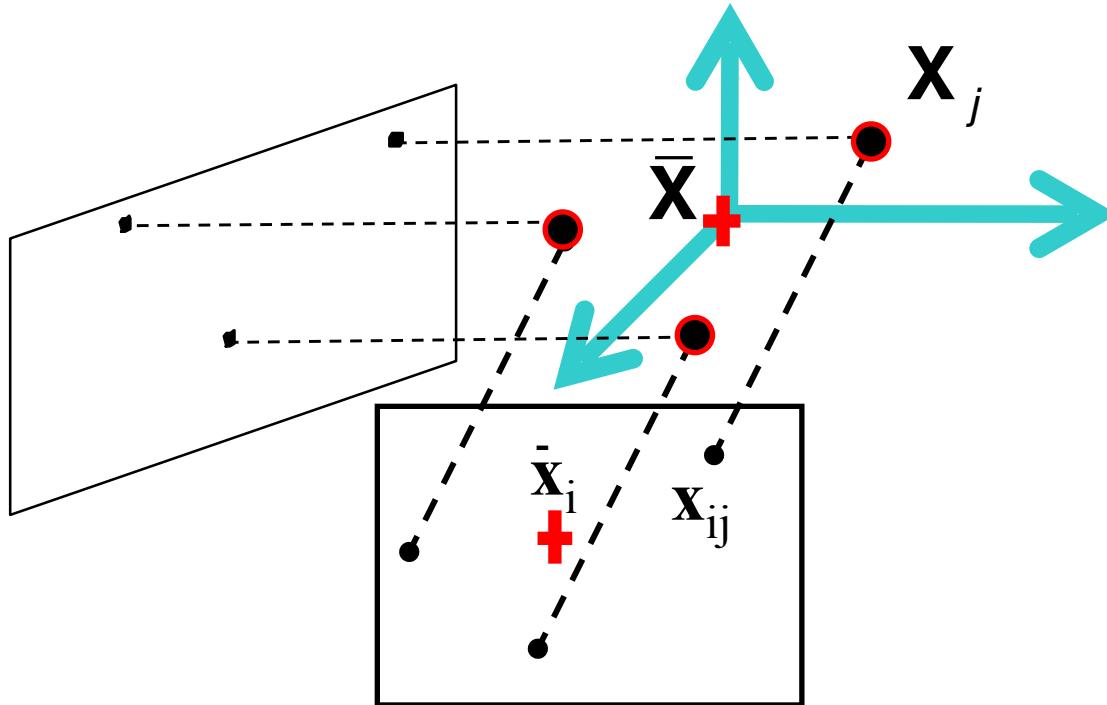
[Eq. 1]

A factorization method – Tomasi & Kanade algorithm

C. Tomasi and T. Kanade [Shape and motion from image streams under orthography: A factorization method.](#) *IJCV*, 9(2):137-154, November 1992.

- Data centering
- Factorization

A factorization method - Centering the data



If the centroid of points in 3D = center of the world reference system

$$\hat{\mathbf{X}}_{ij} = \mathbf{A}_i \hat{\mathbf{X}}_j = \mathbf{A}_i \mathbf{X}_j \quad [\text{Eq. 2}]$$

A factorization method - factorization

Let's create a $2m \times n$ data (measurement) matrix:

$$\mathbf{D} = \begin{bmatrix} \hat{\mathbf{x}}_{11} & \hat{\mathbf{x}}_{12} & \cdots & \hat{\mathbf{x}}_{1n} \\ \hat{\mathbf{x}}_{21} & \hat{\mathbf{x}}_{22} & \cdots & \hat{\mathbf{x}}_{2n} \\ \vdots & \ddots & & \\ \hat{\mathbf{x}}_{m1} & \hat{\mathbf{x}}_{m2} & \cdots & \hat{\mathbf{x}}_{mn} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \vdots \\ \mathbf{A}_m \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \cdots & \mathbf{X}_n \end{bmatrix}$$

points ($3 \times n$) \mathbf{S}

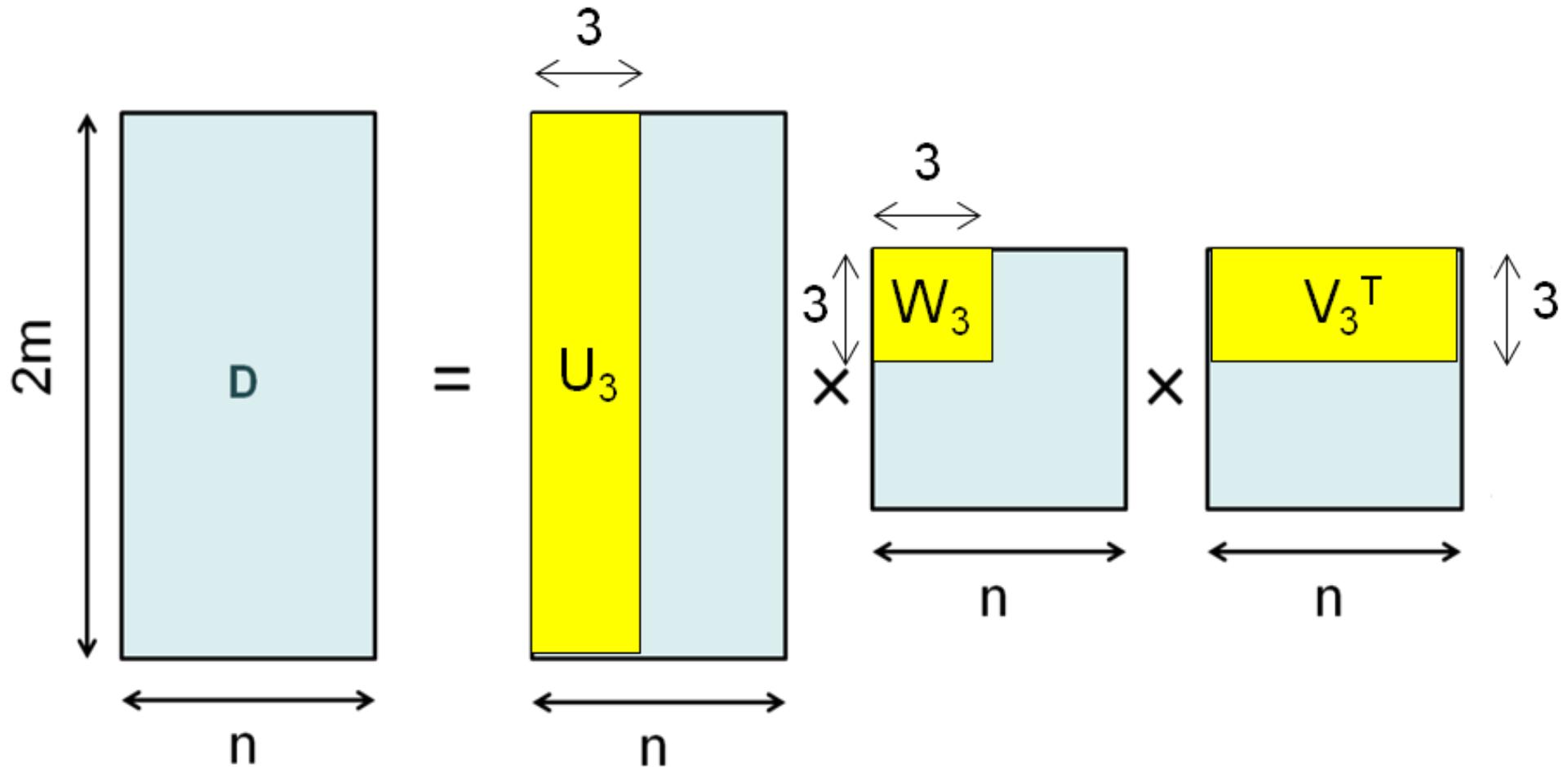
$(2m \times n)$

cameras
 $(2m \times 3)$ \mathbf{M}

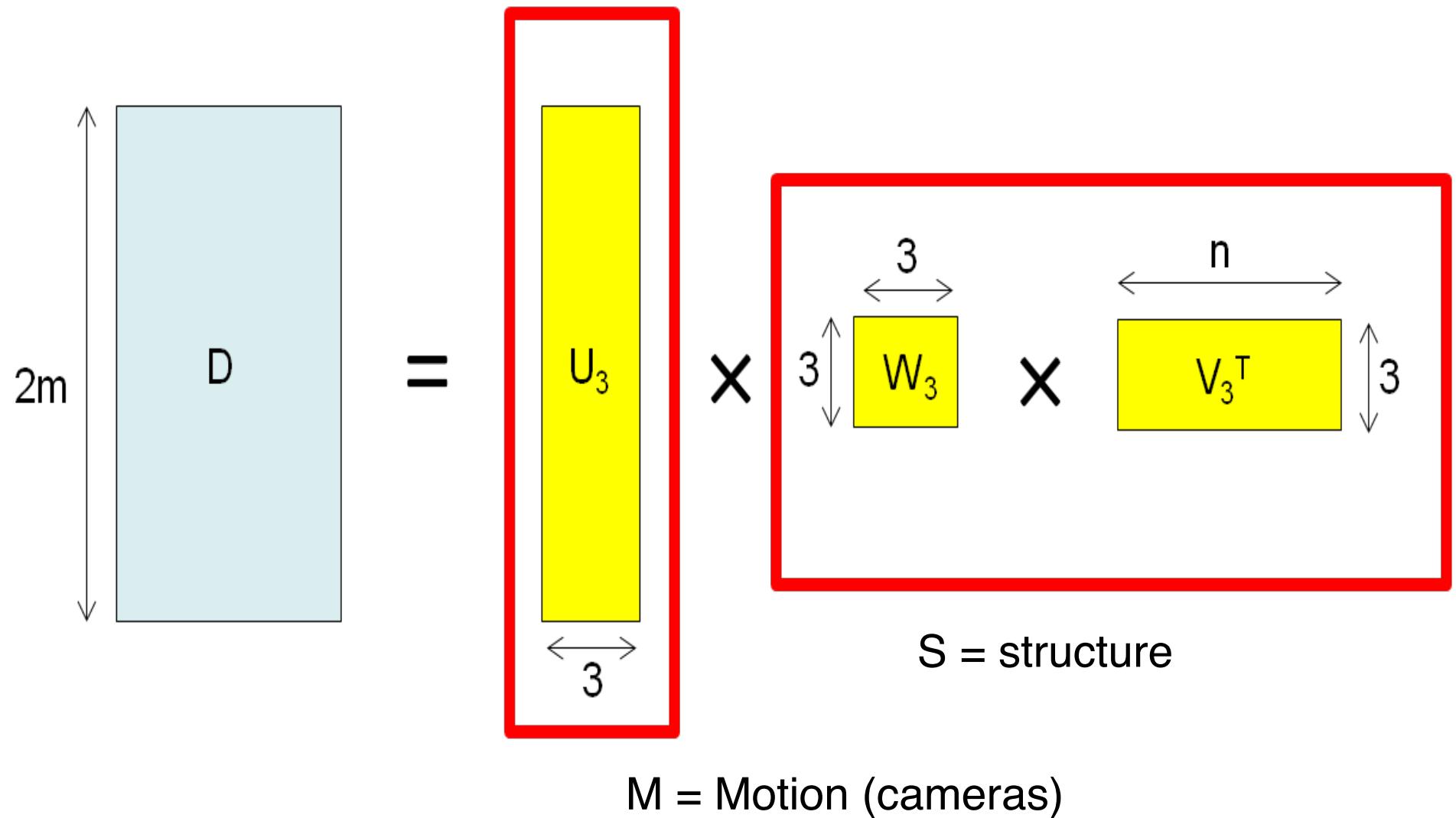
The measurement matrix $\mathbf{D} = \mathbf{M} \mathbf{S}$ has rank 3
(it's a product of a $2m \times 3$ matrix and $3 \times n$ matrix)

Factorizing the Measurement Matrix by SVD

Since rank (D)=3, there are only 3 non-zero singular values



Factorizing the Measurement Matrix by SVD



Affine Ambiguity

$$\mathbf{D} = \mathbf{M} \mathbf{c} \times \mathbf{c}^{-1} \mathbf{s}$$

The diagram illustrates the decomposition of a matrix \mathbf{D} into four components: \mathbf{M} , \mathbf{c} , \mathbf{c}^{-1} , and \mathbf{s} . The matrix \mathbf{D} is represented by a green rectangle on the left. An equals sign follows it. To the right of the equals sign is a vertical yellow rectangle labeled \mathbf{M} . To the right of \mathbf{M} is a small yellow square labeled \mathbf{c} . To the right of \mathbf{c} is a small yellow square labeled \mathbf{c}^{-1} . To the right of \mathbf{c}^{-1} is a horizontal yellow rectangle labeled \mathbf{s} . Below the \mathbf{M} and \mathbf{c} terms is a brace labeled \mathbf{M}' . Below the \mathbf{c}^{-1} and \mathbf{s} terms is a brace labeled \mathbf{s}' .

- The decomposition is not unique. We get the same \mathbf{D} by applying the transformations:

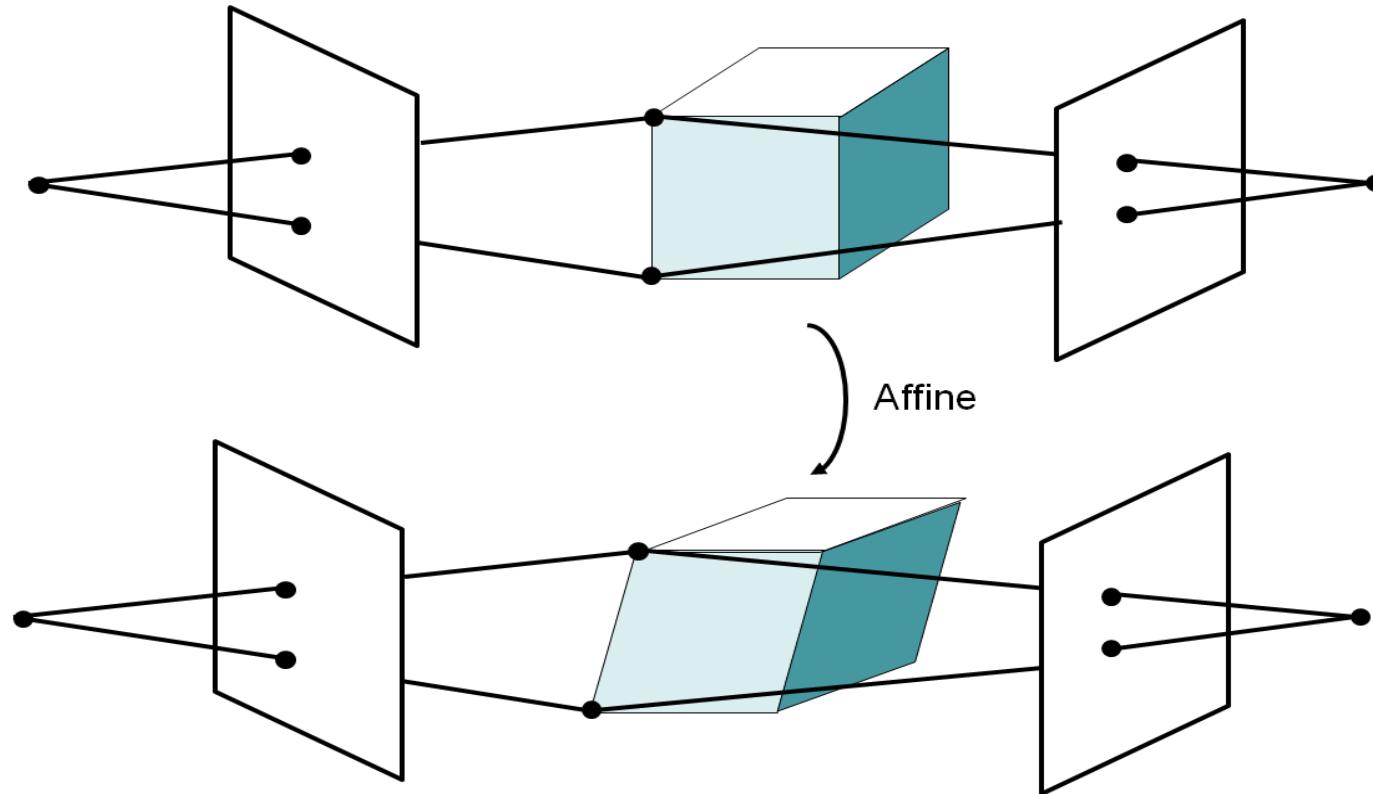
$$\mathbf{M} \rightarrow \mathbf{MC}$$

$$\mathbf{s} \rightarrow \mathbf{C}^{-1}\mathbf{s}$$

where \mathbf{C} is an arbitrary 3×3 matrix describing an affine transformation

- Additional constraints must be enforced to resolve this ambiguity

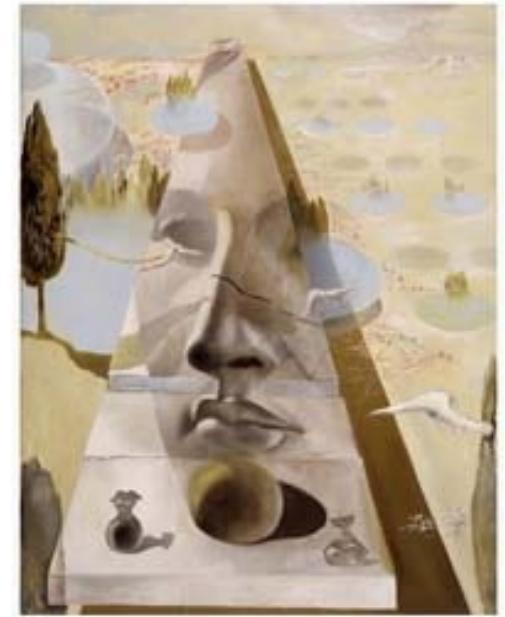
Affine Ambiguity



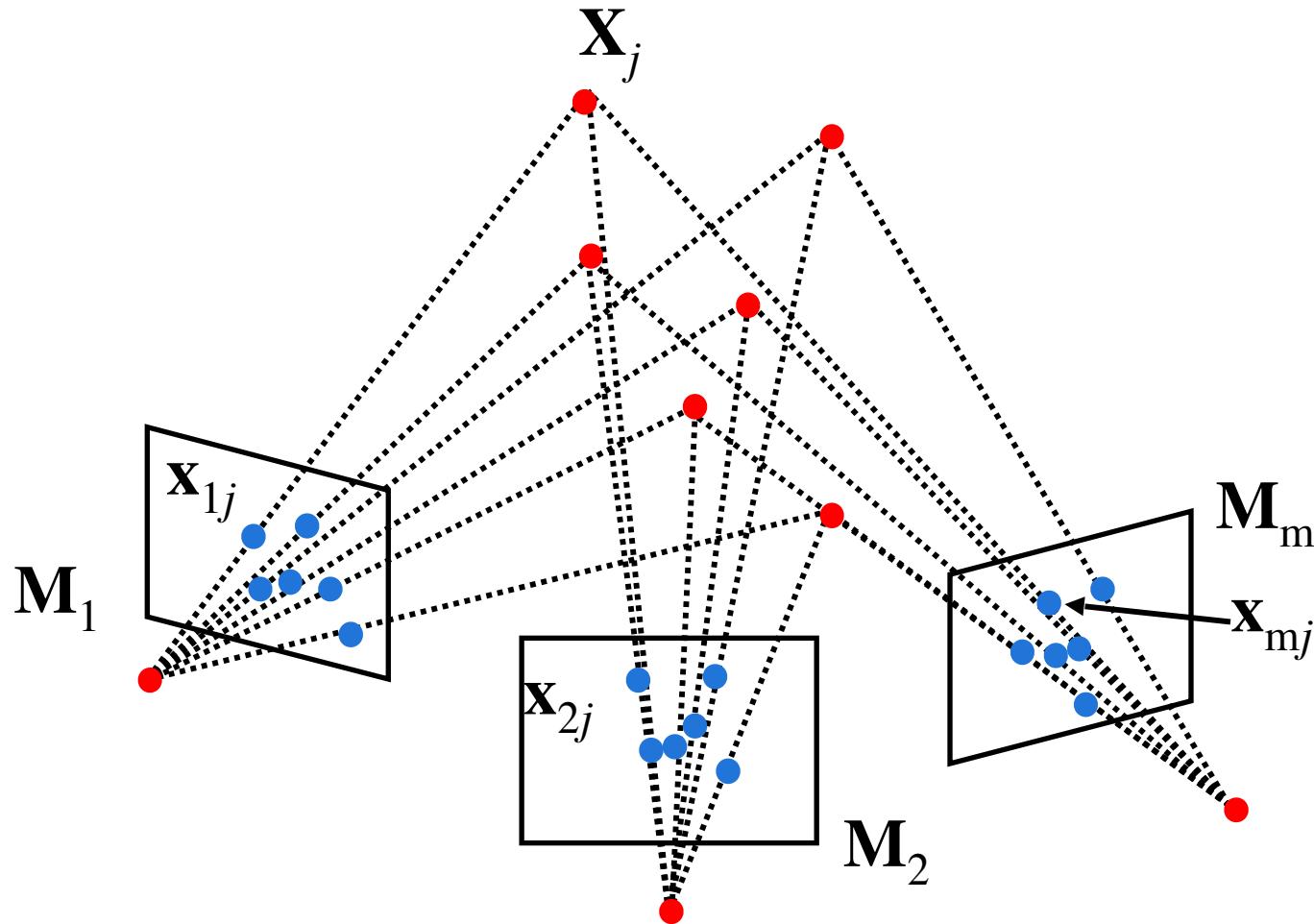
Lecture 7

Multi-view geometry

- The SFM problem
- Affine SFM
- Perspective SFM
- Self-calibration
- Applications



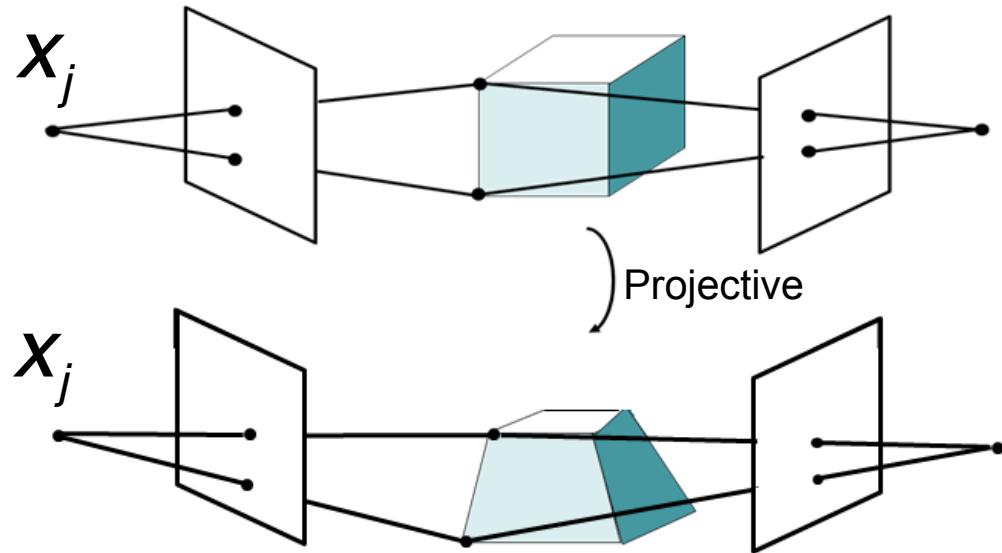
Structure from motion problem



From the $m \times n$ correspondences \mathbf{x}_{ij} , estimate:

- m projection matrices \mathbf{M}_i = **motion**
- n 3D points \mathbf{X}_j = **structure**

Structure from Motion Ambiguities



- In the general case (nothing is known) the ambiguity is expressed by an arbitrary **affine** or **projective transformation**

$$x_j = M_i \boxed{X_j}$$

↓

$$\boxed{H X_j}$$

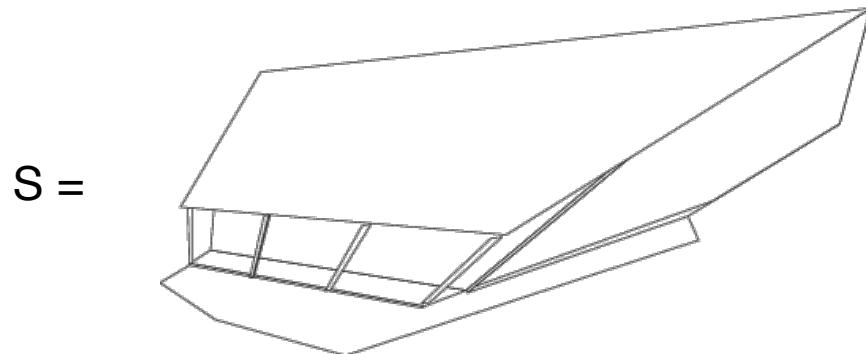
$$\boxed{M_i} = K_i [R_i \ T_i]$$

↓

$$\boxed{M_j H^{-1}}$$

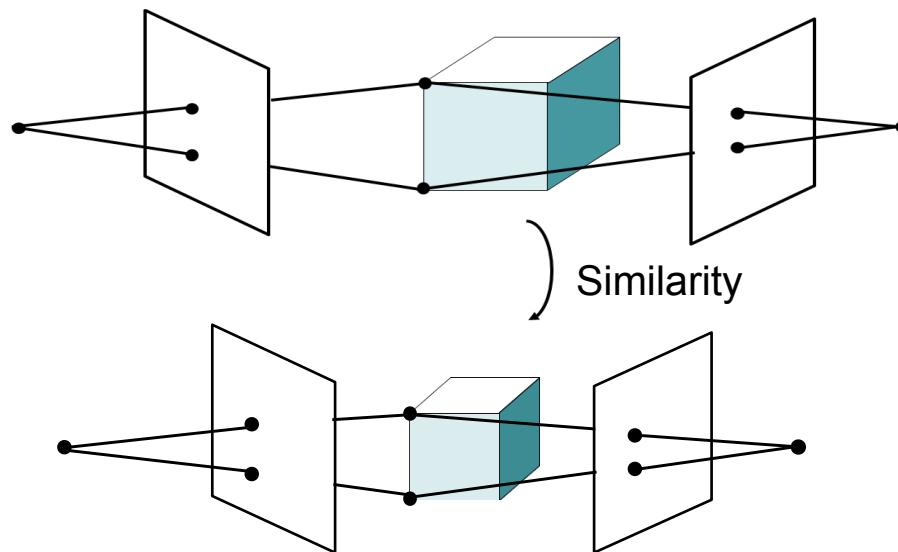
$$x_j = M_i X_j = (M_i H^{-1})(H X_j)$$

Projective Ambiguity



Similarity Ambiguity

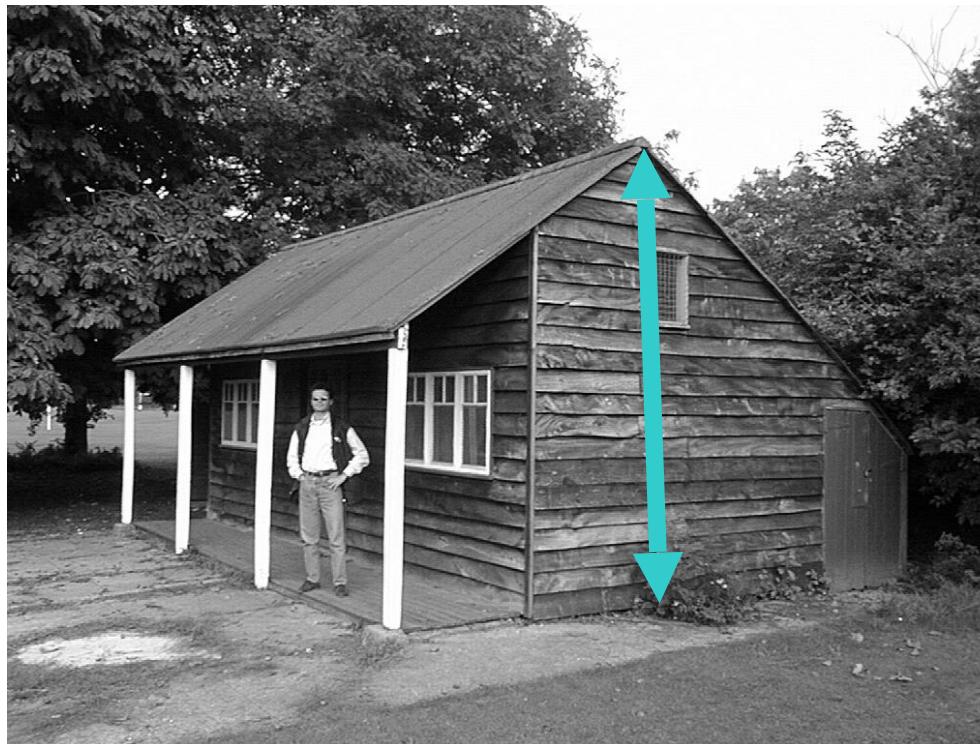
- The scene is determined by the images only up a **similarity transformation** (rotation, translation and scaling)
- This is called **metric reconstruction**



- The ambiguity exists even for (intrinsically) calibrated cameras
- For calibrated cameras, the similarity ambiguity is the **only** ambiguity

Similarity Ambiguity

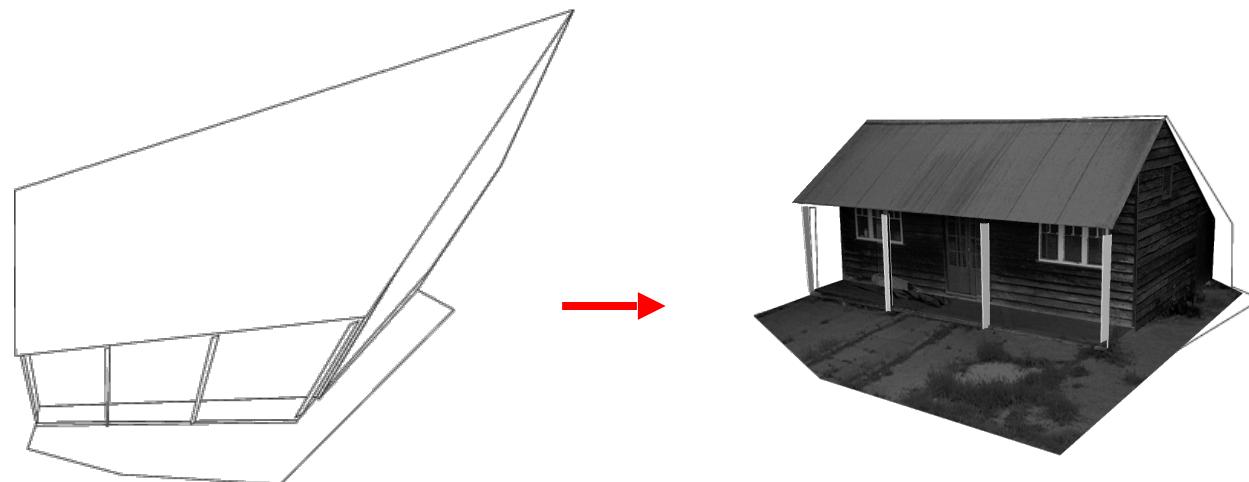
- It is impossible, based on the images alone, to estimate the absolute scale of the scene (i.e. house height)



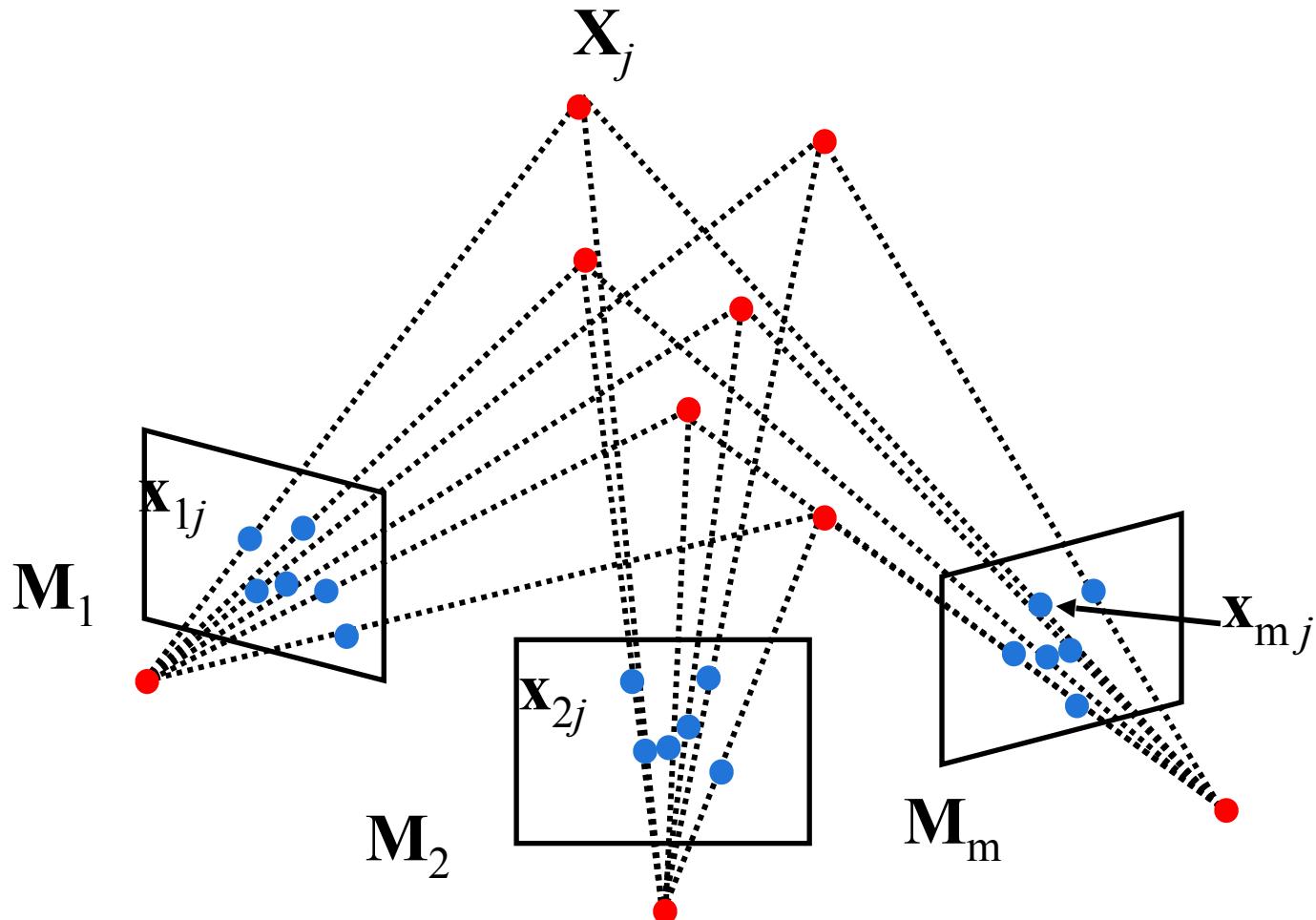
<http://www.robots.ox.ac.uk/~vgg/projects/SingleView/models/hut/hutme.wrl>

Metric reconstruction (upgrade)

- The problem of recovering the metric reconstruction from the perspective one is called **self-calibration**



Structure from motion problem



m cameras $M_1 \dots M_m$

$$M_i = \begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & 1 \end{bmatrix}$$

The Structure-from-Motion Problem

Given m images of n fixed points X_j we can write

$$x_{ij} = M_i X_j \quad \text{for } i = 1, \dots, m \quad \text{and } j = 1, \dots, n.$$

Problem: estimate m 3×4 matrices M_i and n positions X_j from $m \times n$ correspondences x_{ij} .

- If the cameras are not calibrated, cameras and points can only be recovered up to a 4×4 projective (16 parameters)
- Given two cameras, how many points are needed?
- How many equations and how many unknown?

$2m \times n$ equations in $11m + 3n - 16$ unknowns

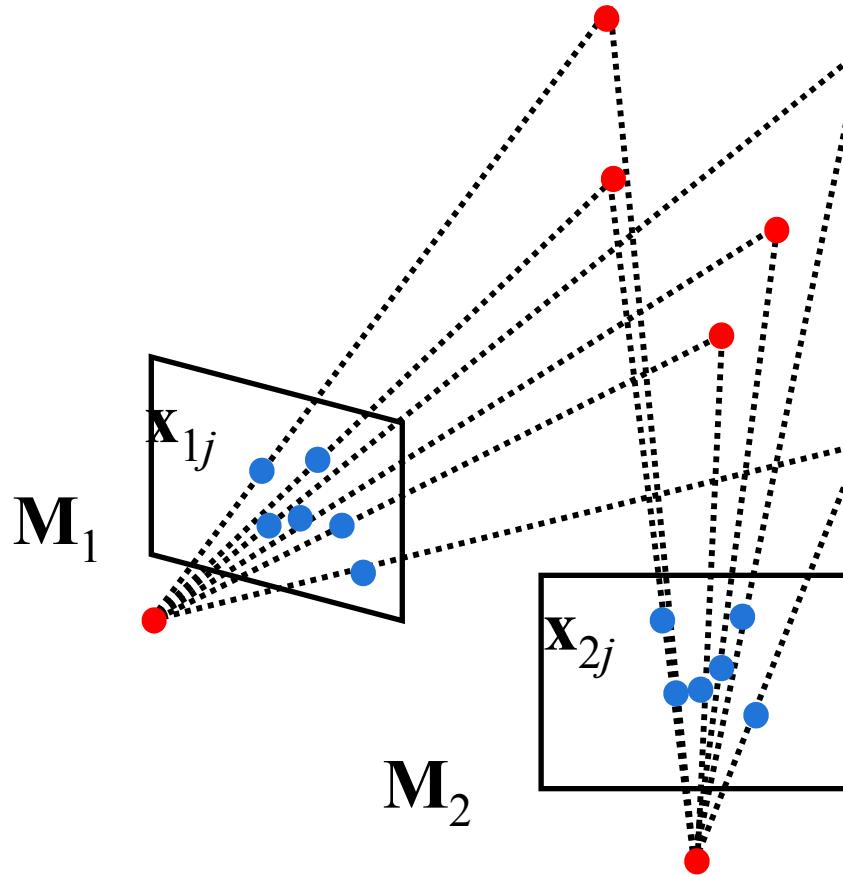
Structure-from-Motion Algorithms

- Algebraic approach (by fundamental matrix)
- Factorization method (by SVD)
- Bundle adjustment

Algebraic approach (2-view case)

1. Compute the fundamental matrix F from two views
(eg. 8 point algorithm)
2. Use F to estimate projective cameras
3. Use these cameras to triangulate and estimate points in 3D

Algebraic approach (2-view case)



$$x_{ij} = M_i X_j$$

Because of the projective ambiguity, we can always apply a projective transformation H such that:

$$M_1 H^{-1} = [I \quad 0]$$

[Eq. 3]

Canonical perspective
camera

$$M_2 H^{-1} = [A \quad b]$$

[Eq. 4]

Algebraic approach (2-view case)

- Call \mathbf{X} a generic 3D point \mathbf{X}_{ij}
- Call \mathbf{x} and \mathbf{x}' the corresponding observations to camera 1 and respectively

$$\left\{ \begin{array}{l} \tilde{M}_1 = M_1 H^{-1} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \quad \mathbf{x} = M_1 H^{-1} H \mathbf{X} = [\mathbf{I} | \mathbf{0}] \tilde{\mathbf{X}} \\ \tilde{M}_2 = M_2 H^{-1} = \begin{bmatrix} \mathbf{A} & \mathbf{b} \end{bmatrix} \quad \mathbf{x}' = M_2 H^{-1} H \mathbf{X} = [\mathbf{A} | \mathbf{b}] \tilde{\mathbf{X}} \\ \tilde{\mathbf{X}} = H \mathbf{X} \end{array} \right. \quad [Eq. 6]$$

$$\mathbf{x}' = [\mathbf{A} | \mathbf{b}] \tilde{\mathbf{X}} = [\mathbf{A} | \mathbf{b}] \begin{bmatrix} \tilde{X}_1 \\ \tilde{X}_2 \\ \tilde{X}_3 \\ 1 \end{bmatrix} = \mathbf{A} [\mathbf{I} | \mathbf{0}] \begin{bmatrix} \tilde{X}_1 \\ \tilde{X}_2 \\ \tilde{X}_3 \\ 1 \end{bmatrix} + \mathbf{b} = \boxed{\mathbf{A} [\mathbf{I} | \mathbf{0}] \tilde{\mathbf{X}}} + \mathbf{b} = \mathbf{Ax} + \mathbf{b} \quad [Eq. 7]$$

$$\mathbf{x}' \times \mathbf{b} = (\mathbf{Ax} + \mathbf{b}) \times \mathbf{b} = \mathbf{Ax} \times \mathbf{b} \quad [Eq. 8]$$

$$\mathbf{x}'^T \cdot (\mathbf{x}' \times \mathbf{b}) = \mathbf{x}'^T \cdot (\mathbf{Ax} \times \mathbf{b}) = 0 \quad [Eq. 9]$$

$$\mathbf{x}'^T (\mathbf{b} \times \mathbf{Ax}) = 0 \quad [Eq. 10]$$

Cross product as matrix multiplication

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [\mathbf{a}_\times] \mathbf{b}$$

Algebraic approach (2-view case)

[Eqs. 5]

$$\left\{ \begin{array}{l} \tilde{M}_1 = M_1 H^{-1} = \begin{bmatrix} I & 0 \end{bmatrix} \quad \mathbf{x} = M_1 H^{-1} H \mathbf{X} = [\mathbf{I} | \mathbf{0}] \tilde{\mathbf{X}} \\ \tilde{M}_2 = M_2 H^{-1} = \begin{bmatrix} A & b \end{bmatrix} \quad \mathbf{x}' = M_2 H^{-1} H \mathbf{X} = [\mathbf{A} | \mathbf{b}] \tilde{\mathbf{X}} \\ \tilde{\mathbf{X}} = H \mathbf{X} \end{array} \right.$$

[Eq. 6]

⋮

$$\mathbf{x}'^T (\mathbf{b} \times \mathbf{A} \mathbf{x}) = 0 \quad [\text{Eq. 10}]$$

$$\mathbf{x}'^T [\mathbf{b}_x] \mathbf{A} \mathbf{x} = 0 \quad \text{is this familiar?}$$

$$\mathbf{F} = [\mathbf{b}_x] \mathbf{A}$$

$$\mathbf{x}'^T \mathbf{F} \mathbf{x} = 0$$

fundamental matrix!

Algebraic approach (2-view case)

1. Compute the fundamental matrix F from two views
(eg. 8 point algorithm)

1. Use F to estimate projective cameras
 - Compute b and A from F
 - Use b and A to estimate projective cameras

2. Use these cameras to triangulate and estimate points in 3D

Compute cameras

$$\mathbf{x}'^T \mathbf{F} \mathbf{x} = 0 \quad \mathbf{F} = [\mathbf{b}_x] \mathbf{A} \quad [\text{Eq. 11}]$$

Compute \mathbf{b} :

- Let's consider the product $\mathbf{F} \mathbf{b}$

$$\mathbf{F} \cdot \mathbf{b} = [\mathbf{b}_x] \mathbf{A} \cdot \mathbf{b} = \mathbf{b} \times \mathbf{A} \cdot \mathbf{b} = 0 \quad [\text{Eq. 12}]$$

- Since \mathbf{F} is singular, we can compute \mathbf{b} as least sq. solution of $\mathbf{F} \mathbf{b} = 0$, with $|\mathbf{b}|=1$ using SVD
- Using a similar derivation, we have that $\mathbf{b}^T \mathbf{F} = 0$

Compute cameras

$$\mathbf{x}'^T \mathbf{F} \mathbf{x} = 0 \quad \mathbf{F} = [\mathbf{b}_x] \mathbf{A} \quad \begin{cases} \mathbf{F} \mathbf{b} = 0 \\ \mathbf{b}^T \mathbf{F} = 0 \end{cases}$$

[Eq. 11]

Compute \mathbf{A} :

- Define: $\mathbf{A}' = -[\mathbf{b}_x]^T \mathbf{F}$
- Let's verify that $[\mathbf{b}_x] \mathbf{A}'$ is equal to \mathbf{F} :

Indeed: $[\mathbf{b}_x] \mathbf{A}' = -[\mathbf{b}_x][\mathbf{b}_x]^T \mathbf{F} = -(\mathbf{b} \mathbf{b}^T - |\mathbf{b}|^2 \mathbf{I}) \mathbf{F} = -\mathbf{b} \mathbf{b}^T \mathbf{F} + |\mathbf{b}|^2 \mathbf{F} = 0 + 1 \cdot \mathbf{F} = \mathbf{F}$

[Eq. 13]

- Thus, $\mathbf{A} = \mathbf{A}' = -[\mathbf{b}_x]^T \mathbf{F}$

[Eqs. 14]

$$\tilde{\mathcal{M}}_1 = \begin{bmatrix} I & 0 \end{bmatrix} \quad \tilde{\mathcal{M}}_2 = \begin{bmatrix} -[\mathbf{b}_x]^T \mathbf{F} & \mathbf{b} \end{bmatrix}$$

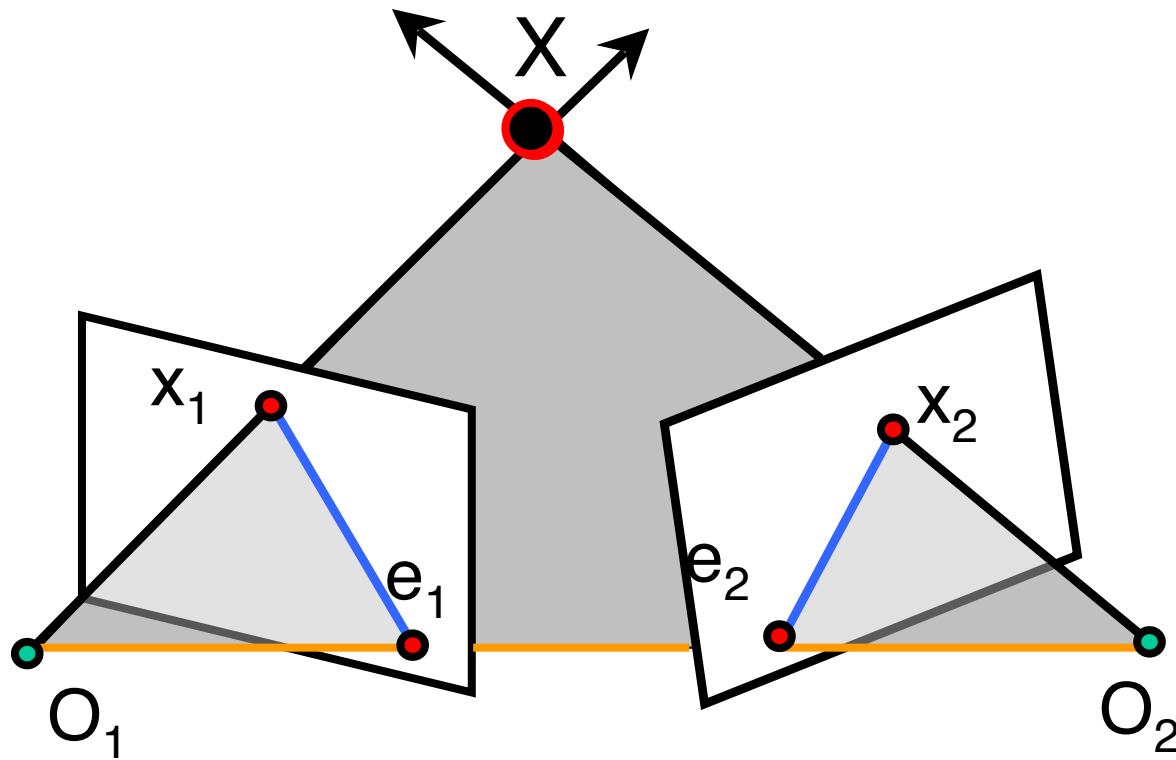
Interpretation of \mathbf{b}

$$\mathbf{x}'^T \mathbf{F} \mathbf{x} = 0 \quad \mathbf{F} = [\mathbf{b}_x] \mathbf{A} \quad \left\{ \begin{array}{l} \mathbf{F} \mathbf{b} = 0 \quad [\text{Eq. 12}] \\ \mathbf{b}^T \mathbf{F} = 0 \end{array} \right.$$

[Eq. 11]

What's $\mathbf{b}??$

Epipolar Constraint [lecture 5]



$F x_2$ is the epipolar line associated with x_2 ($l_1 = F x_2$)

$F^T x_1$ is the epipolar line associated with x_1 ($l_2 = F^T x_1$)

F is singular (rank two)

$$F e_2 = 0 \quad \text{and} \quad F^T e_1 = 0$$

F is 3×3 matrix; 7 DOF

Interpretation of \mathbf{b}

$$\mathbf{x}'^T \mathbf{F} \mathbf{x} = 0 \quad \mathbf{F} = [\mathbf{b}_x] \mathbf{A} \quad \begin{cases} \mathbf{F} \mathbf{b} = 0 \\ \mathbf{b}^T \mathbf{F} = 0 \end{cases}$$

[Eq. 11]

\mathbf{b} is an epipole!

$$\tilde{\mathcal{M}}_1 = \begin{bmatrix} I & 0 \end{bmatrix} \quad \tilde{\mathcal{M}}_2 = \begin{bmatrix} - & [\mathbf{b}_x] \mathbf{F} & \mathbf{b} \end{bmatrix}$$



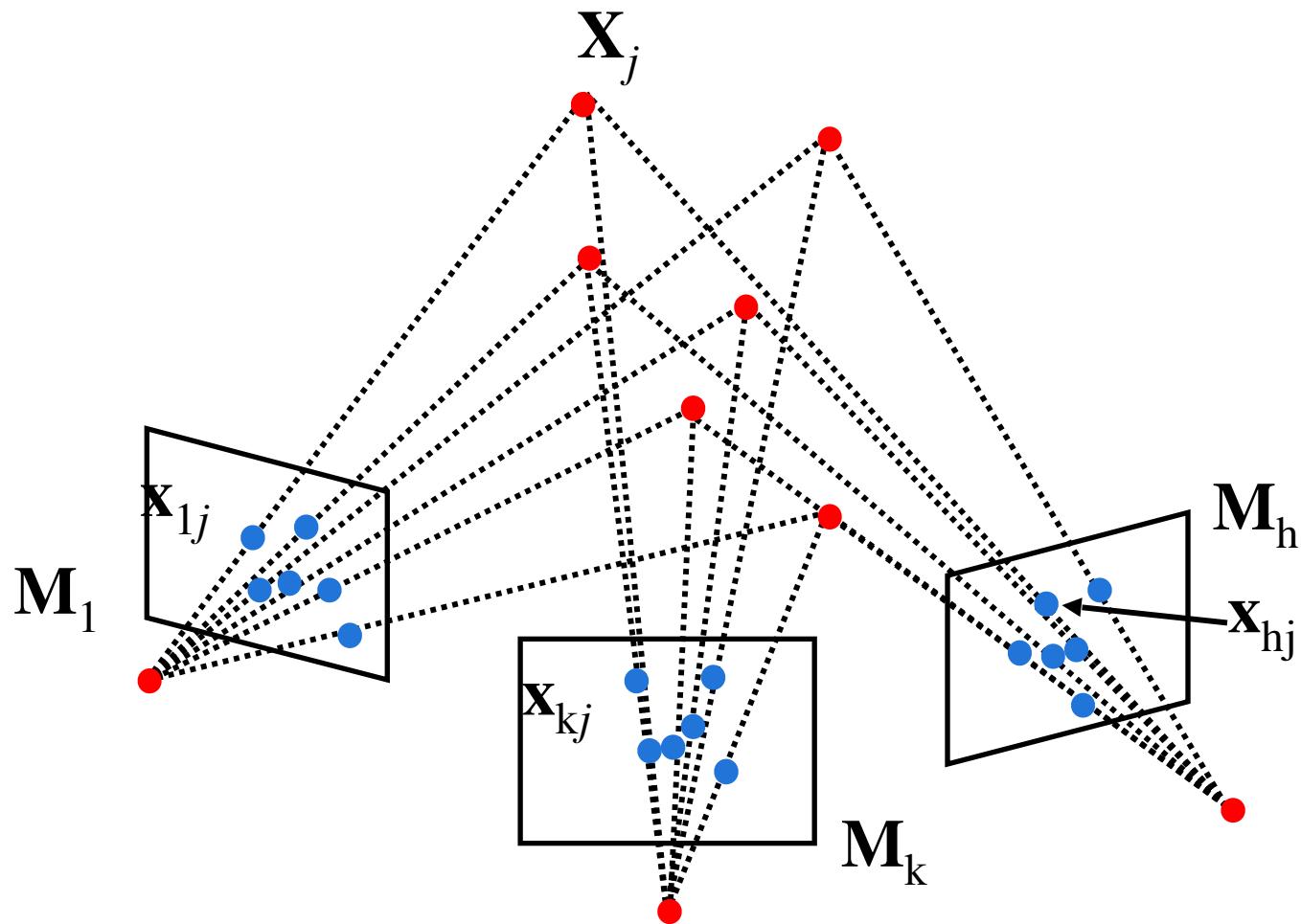

$$\tilde{\mathcal{M}}_1 = \begin{bmatrix} I & 0 \end{bmatrix} \quad \tilde{\mathcal{M}}_2 = \begin{bmatrix} - & [\mathbf{e}_x] \mathbf{F} & \mathbf{e} \end{bmatrix}$$

[Eq. 15] [Eq. 16]

Algebraic approach (2-view case)

1. Compute the fundamental matrix F from two views
(eg. 8 point algorithm)
1. Use F to estimate projective cameras
 - Compute b and A from F
 - Use b and A to estimate projective cameras
2. Use these cameras to triangulate and estimate points in 3D

Algebraic approach for SFM problem



Solutions (motion and structure) for each image pairs I_k and I_h

From I_k and $I_h \rightarrow \tilde{M}_k, \tilde{M}_h, \tilde{X}_{[k,h]}$

3D points associated to point correspondences available between I_k and I_h

Structure-from-Motion Algorithms

- Algebraic approach (by fundamental matrix)
- Factorization method (by SVD)
- Bundle adjustment

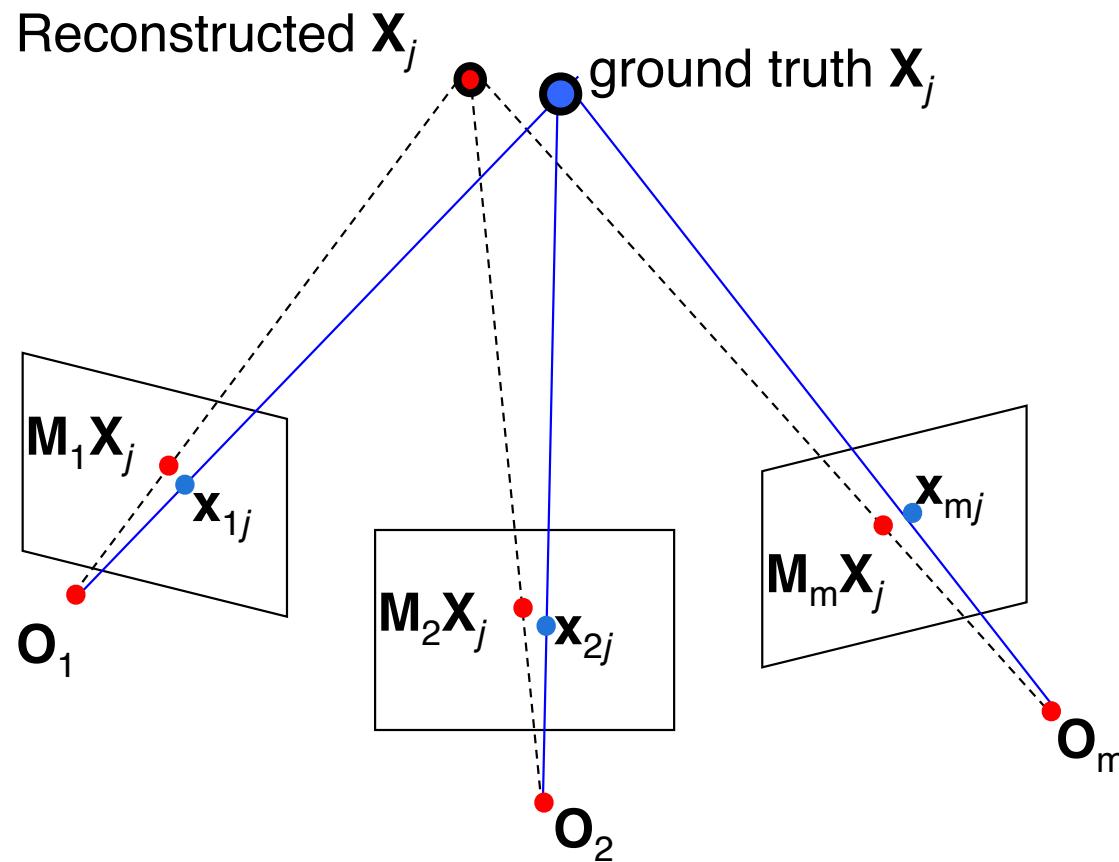
Limitations of the approaches so far

- Factorization methods assume all points are visible.
This not true if:
 - occlusions occur
 - failure in establishing correspondences
- Algebraic methods work with 2 views

Bundle adjustment

- Non-linear method for refining structure and motion
- Minimizes re-projection error

$$E(M, X) = \sum_{i=1}^m \sum_{j=1}^n D(x_{ij}, M_i X_j)^2$$



General Calibration Problem

$$E(M, X) = \sum_{i=1}^m \sum_{j=1}^n D(x_{ij}, M_i X_j)^2$$

↑
measurements ↑
parameters

D is the nonlinear mapping

- Newton Method
- Levenberg-Marquardt Algorithm
 - Iterative, starts from initial solution
 - May be slow if initial solution far from real solution
 - Estimated solution may be function of the initial solution
 - Newton requires the computation of J, H
 - Levenberg-Marquardt doesn't require the computation of H

Bundle adjustment

- **Advantages**
 - Handle large number of views
 - Handle missing data
- **Limitations**
 - Large minimization problem (parameters grow with number of views)
 - Requires good initial condition
- Used as the final step of SFM (i.e., after the factorization or algebraic approach)
- Factorization or algebraic approaches provide a initial solution for optimization problem

Lecture 7

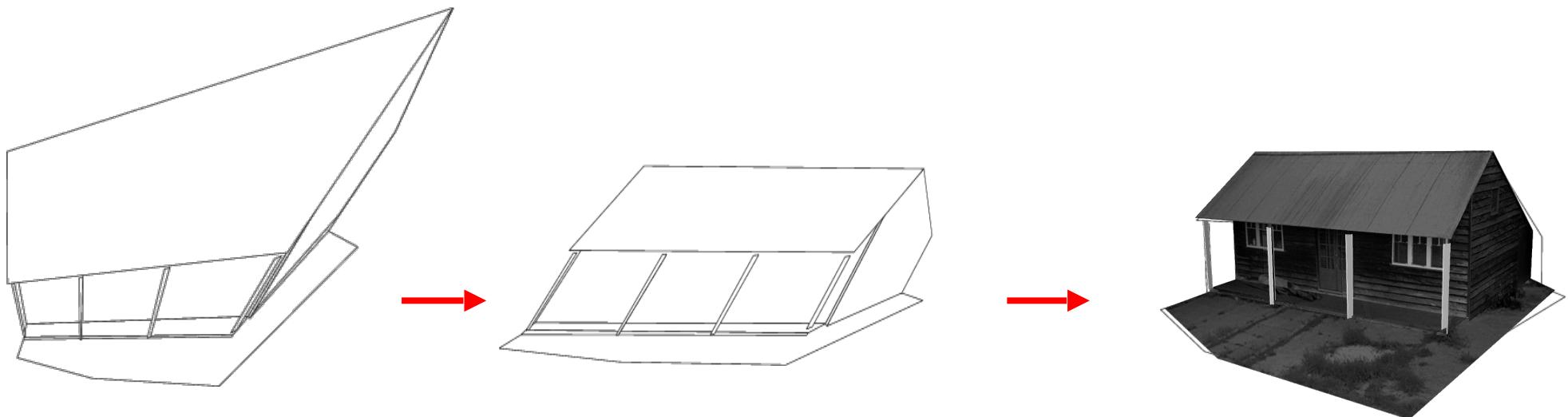
Multi-view geometry



- The SFM problem
- Affine SFM
- Perspective SFM
- Self-calibration
- Applications

Self-calibration

- **Self-calibration** is the problem of recovering the metric reconstruction from the perspective (or affine) reconstruction
- We can self-calibrate the camera by making some assumptions about the cameras



Self-calibration

[HZ] Chapters 19 “Auto-calibration”

Several approaches:

- Use single-view metrology constraints (lecture 4)
- Direct approach (Kruppa Eqs) for 2 views
- Algebraic approach
- Stratified approach

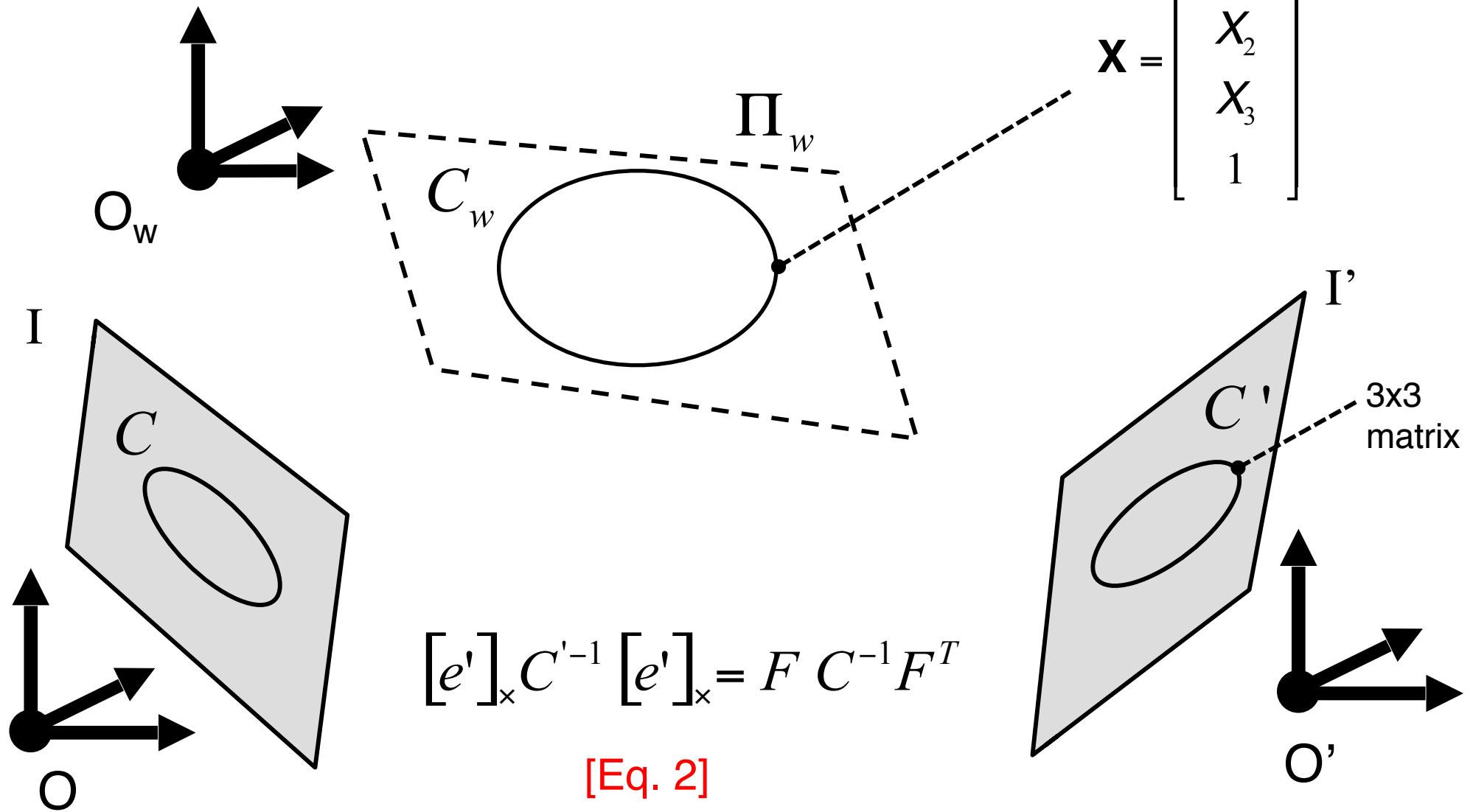
Direct approach

We use the following results:

1. A relationship that maps conics across views
2. Concept of absolute conic and its relationship to K
3. The Kruppa equations

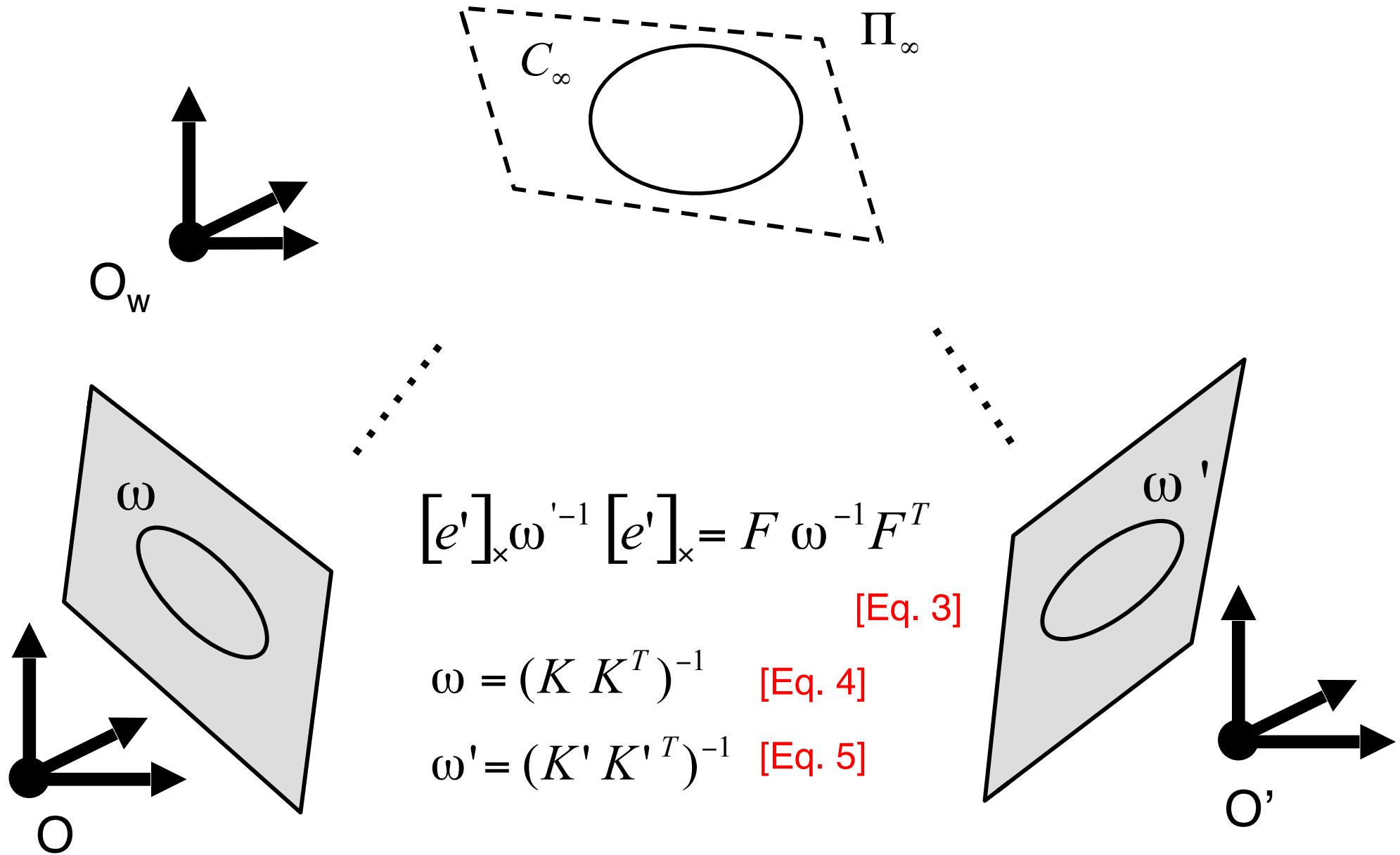
Projections of conics across views

$$X^T C_w X = 0 \quad [\text{Eq. 1}]$$



Projection of absolute conics across views

From lecture 4, [HZ] page 210, sec. 8.5.1



Kruppa equations

[Faugeras et al. 92]

From [HZ] page 471

$$\begin{pmatrix} u_2^T K' K'^T u_2 \\ -u_1^T K' K'^T u_2 \\ u_1^T K' K'^T u_1 \end{pmatrix} \times \begin{pmatrix} \sigma_1^2 v_1^T K K^T v_1 \\ \sigma_1 \sigma_2 v_1^T K K^T v_2 \\ \sigma_2^2 v_2^T K K^T v_2 \end{pmatrix} = 0 \quad [\text{Eq. 6}]$$

where u_i , v_i and σ_i are the columns and singular values of SVD of F

These give us two independent constraints in the elements of K and K'

Kruppa equations

[Faugeras et al. 92]

$$\begin{pmatrix} u_2^T K' K'^T u_2 \\ -u_1^T K' K'^T u_2 \\ u_1^T K' K'^T u_1 \end{pmatrix} \times \begin{pmatrix} \sigma_1^2 v_1^T K K^T v_1 \\ \sigma_1 \sigma_2 v_1^T K K^T v_2 \\ \sigma_2^2 v_2^T K K^T v_2 \end{pmatrix} = 0$$

$$\frac{u_2^T K K^T u_2}{\sigma_1^2 v_1^T K K^T v_1} = \frac{-u_1^T K K^T u_2}{\sigma_1 \sigma_2 v_1^T K K^T v_2} = \frac{u_1^T K K^T u_1}{\sigma_2^2 v_2^T K K^T v_2} \quad [\text{Eq. 7}]$$

- Let's make the following assumption: $K' = K = \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix}$ [Eq. 8]

$$[\text{Eq. 9}] \quad \alpha f^2 + \beta f + \gamma = 0 \longrightarrow f$$

Kruppa equations

[Faugeras et al. 92]

- Powerful if we want to self-calibrate 2 cameras with unknown focal length
- Limitations:
 - Work on a camera pair
 - Don't work if $R=0$

[Eq. 10] $\begin{bmatrix} e' \end{bmatrix}_x \omega^{-1} \begin{bmatrix} e' \end{bmatrix}_x = F \omega^{-1} F^T$ becomes trivial

Since: $F = \begin{bmatrix} e' \end{bmatrix}_x$

Self-calibration

[HZ] Chapters 19 “Auto-calibration”

Several approaches:

- Use single-view metrology constraints (lecture 4)
- Direct approach (Kruppa Eqs) for 2 views
- Algebraic approach
- Stratified approach

Algebraic approach Multi-view approach

Suppose we have a projective reconstruction $\{\tilde{M}_i, \tilde{X}_j\}$

Let H be a homography such that:

$$\begin{cases} \text{First perspective camera is canonical: } \tilde{M}_1 = [\begin{array}{cc} I & 0 \end{array}] \text{ [Eq. 11]} \\ \text{i}^{\text{th}} \text{ perspective reconstruction of the camera (known): } \tilde{M}_i = [\begin{array}{cc} A_i & b_i \end{array}] \text{ [Eq. 12]} \end{cases}$$

$$[\text{Eq. 13}] \quad (A_i - b_i p^T) K_1 K_1^T (A_i - b_i p^T)^T = K_i K_i^T \quad i=2 \dots m$$

$$[\text{Eq. 14}] \quad H = \begin{bmatrix} K_1 & 0 \\ -p^T K_1 & 1 \end{bmatrix} \quad \begin{aligned} p &\text{ is an unknown } 3 \times 1 \text{ vector} \\ K_1 \dots K_m &\text{ are unknown} \end{aligned}$$

Algebraic approach Multi-view approach

Suppose we have a projective reconstruction

Let H be a homography such that:

$$\begin{cases} \text{First perspective camera is canonical: } \tilde{M}_1 = [\begin{array}{cc} I & 0 \end{array}] \text{ [Eq. 11]} \\ \text{i}^{\text{th}} \text{ perspective reconstruction of the camera (known): } \tilde{M}_i = [\begin{array}{cc} A_i & b_i \end{array}] \text{ [Eq. 12]} \end{cases}$$

$$[\text{Eq. 13}] \quad (A_i - b_i p^T) K_1 K_1^T (A_i - b_i p^T)^T = K_i K_i^T \quad i=2 \dots m$$

- How many unknowns?
- 3 from p
 - 5 m from $K_1 \dots K_m$

How many equations? 5 independent equations [per view]

Algebraic approach

Multi-view approach

Suppose we have a projective reconstruction

Let H be a homography such that:

$$\begin{cases} \text{First perspective camera is canonical: } \tilde{M}_1 = [\begin{array}{cc} I & 0 \end{array}] \text{ [Eq. 11]} \\ \text{i}^{\text{th}} \text{ perspective reconstruction of the camera (known): } \tilde{M}_i = [\begin{array}{cc} A_i & b_i \end{array}] \text{ [Eq. 12]} \end{cases}$$

Assume all camera matrices are identical: $K_1 = K_2 \dots = K_m$

$$[Eq. 15] \quad (A_i - b_i p^T) K K^T (A_i - b_i p^T)^T = K K^T \quad i=2 \dots m$$

- How many unknowns?
- 3 from p
 - 5 from K

How many equations? 5 independent equations [per view]

We need at least 3 views to solve the self-calibration problem

Algebraic approach

Art of self-calibration:

Use assumptions on Ks to generate enough equations on the unknowns

<i>Condition</i>	<i>N. Views</i>
• Constant internal parameters	3
• Aspect ratio and skew known • Focal length and offset vary	4
• Skew =0, all other parameters vary	8

Issue: the larger is the number of view,
the harder is the correspondence problem

Bundle adjustment helps!

SFM problem - summary

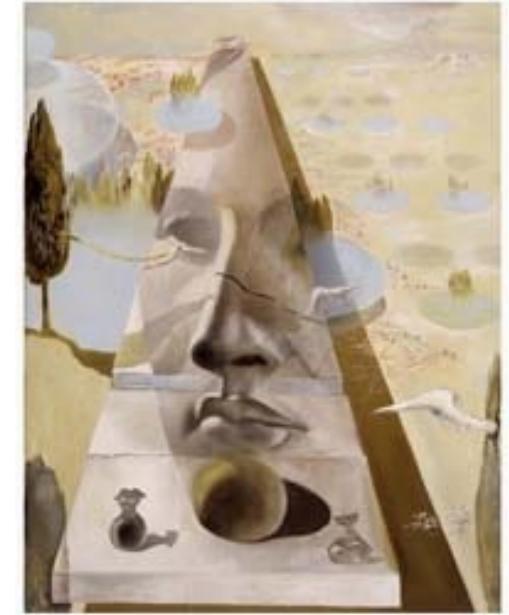
1. Estimate structure and motion up perspective transformation
 1. Algebraic
 2. factorization method
 3. bundle adjustment
2. Convert from perspective to metric (self-calibration)
3. Bundle adjustment

** or **

1. Bundle adjustment with self-calibration constraints

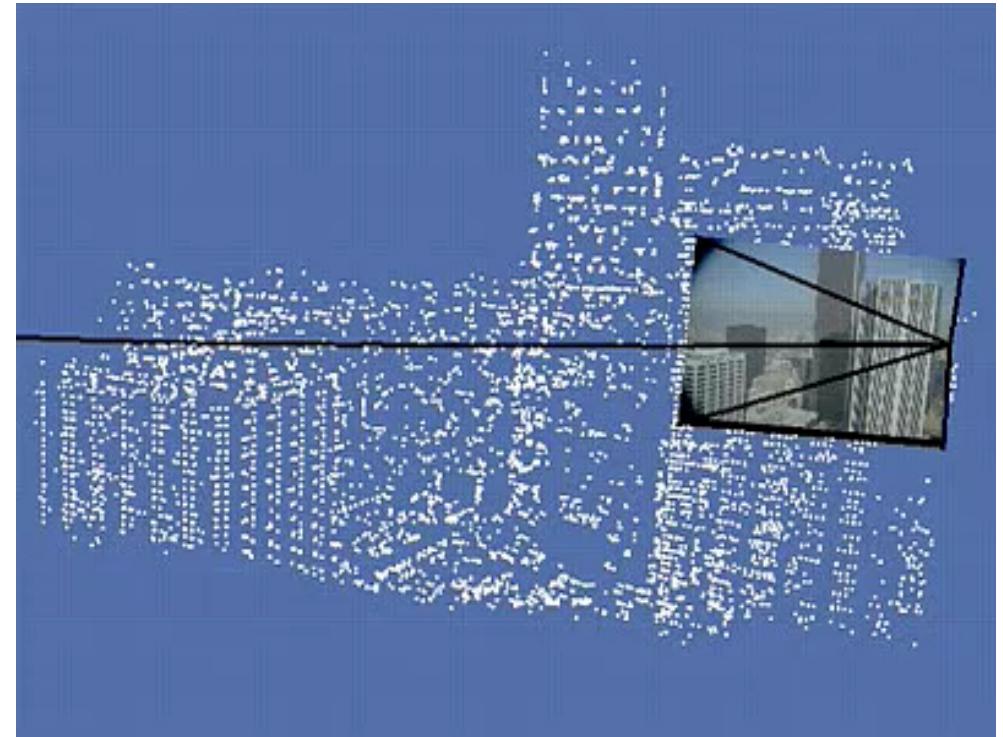
Lecture 7

Multi-view geometry



- The SFM problem
- Affine SFM
- Perspective SFM
- Self-calibration
- Applications

Structure from motion problem



Courtesy of Oxford **Visual Geometry Group**

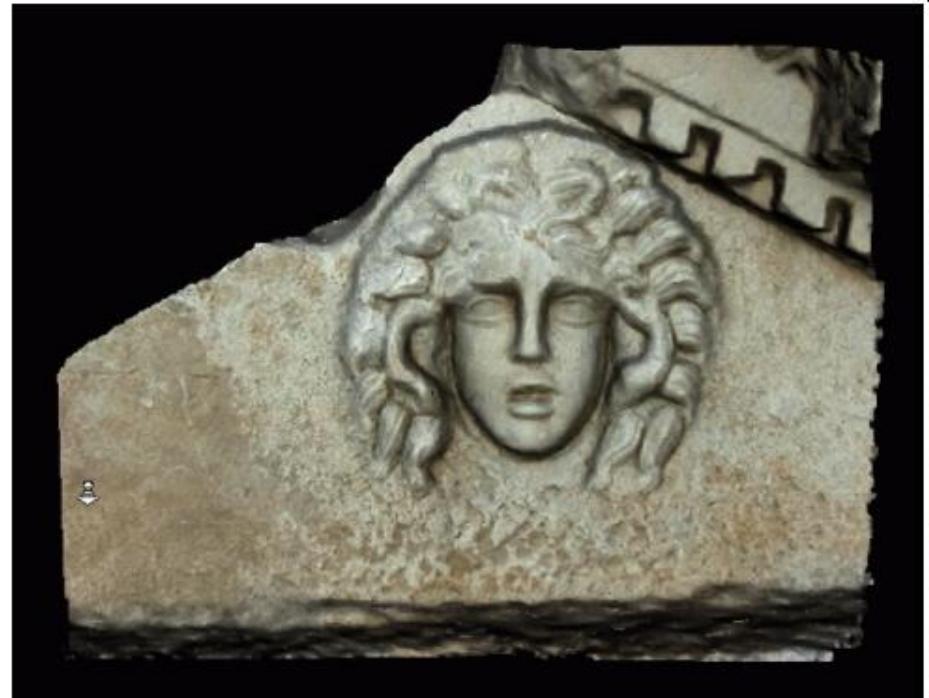
Lucas & Kanade, 81
Chen & Medioni, 92
Debevec et al., 96
Levoy & Hanrahan, 96
Fitzgibbon & Zisserman, 98
Triggs et al., 99
Pollefeys et al., 99
Kutulakos & Seitz, 99

Levoy et al., 00
Hartley & Zisserman, 00
Dellaert et al., 00
Rusinkiewic et al., 02
Nistér, 04
Brown & Lowe, 04
Schindler et al, 04
Lourakis & Argyros, 04
Colombo et al. 05

Golparvar-Fard, et al. JAEI 10
Pandey et al. IFAC , 2010
Pandey et al. ICRA 2011
Microsoft's PhotoSynth
Snavely et al., 06-08
Schindler et al., 08
Agarwal et al., 09
Frahm et al., 10

Reconstruction and texture mapping

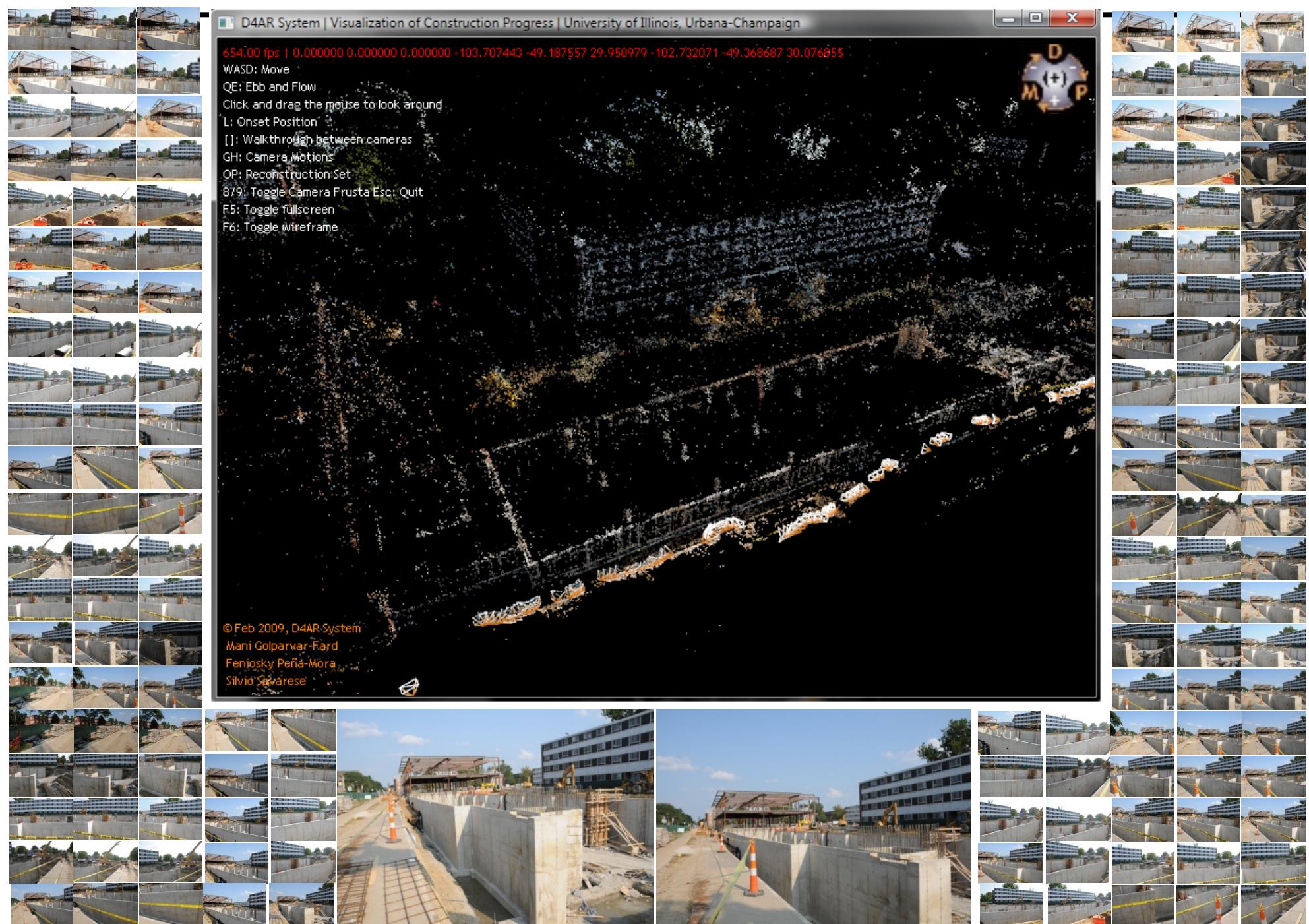
M. Pollefeys et al 98---



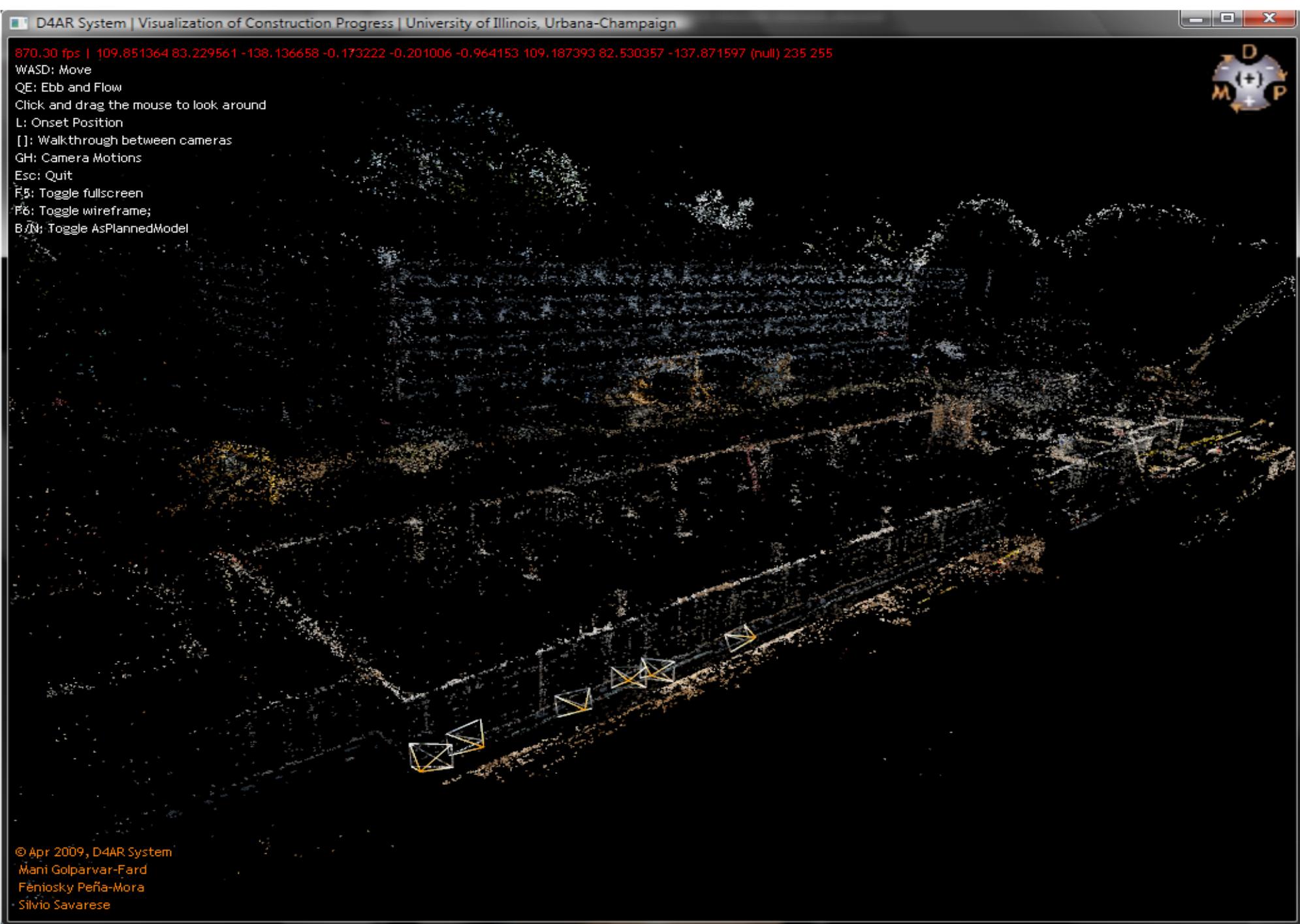
Incremental reconstruction of construction sites

Initial pair – 2168 & Complete Set 62,323 points, 160 images

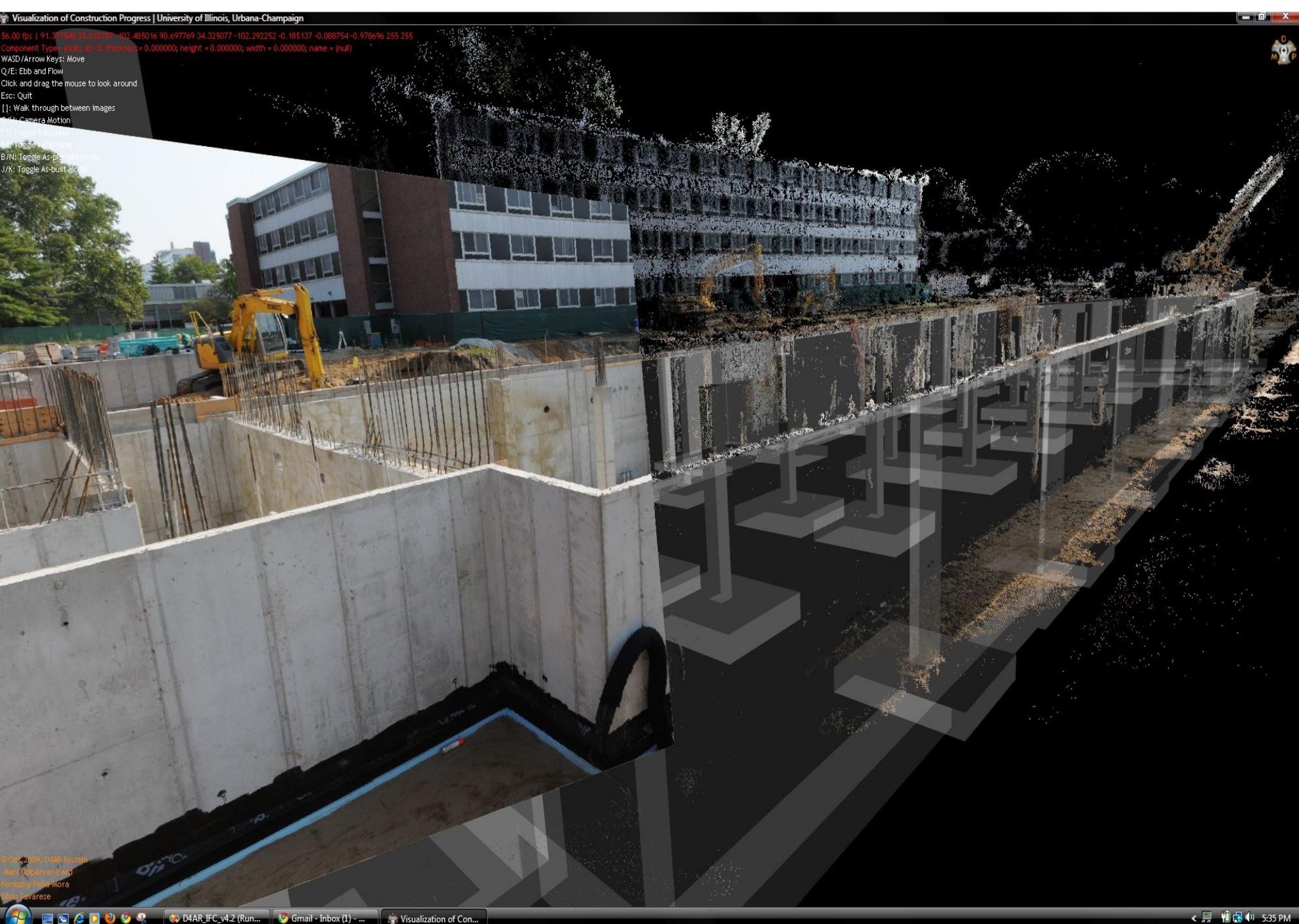
Golparvar-Fard, Pena-Mora, Savarese 2008



Reconstructed scene + Site photos



Reconstructed scene + Site photos



Results and applications

Noah Snavely, Steven M. Seitz, Richard Szeliski, "[Photo tourism: Exploring photo collections in 3D](#)," ACM Transactions on Graphics (SIGGRAPH Proceedings), 2006,

