COMP 4900D: Assignment 1
Due: Tuesday, Feb. 7, 2006

1. (2 points) The rotation matrix in the x-y plane is

$$R(\phi) = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}$$

   (a) Verify $R(\phi_1)R(\phi_2) = R(\phi_1 + \phi_2)$ from the identities for $\cos(\phi_1 + \phi_2)$ and $\sin(\phi_1 + \phi_2)$.

   (b) What is $R(\phi)R(-\phi)$?

2. (4 points) Scale a vector $\begin{bmatrix} x & y \end{bmatrix}^T$ in the plane can be achieved by

   $x' = sx$ and $y' = sy$

   where s is a scalar.

   (a) Write out the matrix form of this transformation.

   (b) Write out the transformation matrix for homogeneous coordinates.

   (c) If the transformation also includes a translation

   $x' = sx + t_x$ and $y' = sy + t_y$

   Write out the transformation matrix for homogeneous coordinates.

   (d) What is the equivalent of the above matrix for three-dimensional vectors?

3. (2 points) Find the least square solution $\bar{x}$ for $Ax = b$ if

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

   Verify that the error vector $b - A\bar{x}$ is orthogonal to the columns of A.

4. (2 points) A pinhole camera has focal length $f = 500$, pixel sizes $s_x = s_y = 1$, and its principal point is at $(o_x, o_y) = (320,240)$. The world coordinate frame and the camera coordinate frame can be related by $X_c = RX_w + T$, where

$$R = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad T = \begin{bmatrix} 70 \\ 95 \\ 120 \end{bmatrix}$$

   (a) Write out the 3x4 projection matrix that project a point in world coordinate frame onto the image plane in pixel coordinate.

   (b) What are the pixel coordinates of the world point $X_w = \begin{bmatrix} 150 & 200 & 400 \end{bmatrix}^T$?