Robotics, Geometry and Control - A Preview

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HYCON-EECI Graduate School - Spring 2008

Broad areas

- ► Types of manipulators - articulated mechanisms, rolling mechanisms and underwater vehicles
- ► Constraints in manipulators - kinematic level and dynamic level
- ► Motivation for geometric framework and differential geometry
- ► Equations of motion
- ► Constraints, path planning, feedback and stabilization

Articulated mechanisms

- ▶ Serial link robots, parallel link robots
- ▶ Objectives - pick and place objects, trajectory following (welding)
- ▶ Realization - compute joint angles for a given position of the end-effector, compute joint angles, velocities and accelerations for a given trajectory of the end-effector
- ▶ Constraints - range of joint angles, length of the links, rate and acceleration constraints on the joint movement
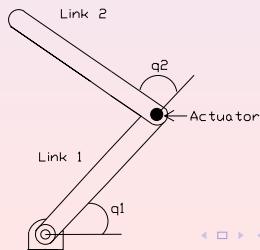
Rolling mechanisms

- ▶ Wheeled mobile robots, fingers handling an object (rolling contact)
- ▶ Objective - move from one configuration to another
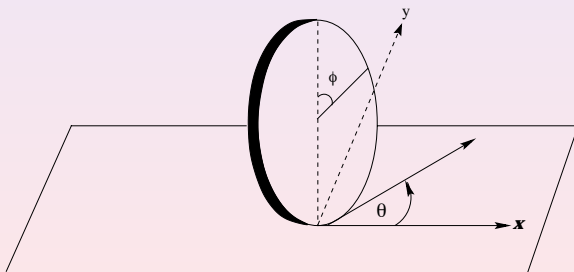- ▶ Constraints - pure rolling (no sliding or slipping)

Figure: Vertical coin on a plane

Underwater vehicles

- *Kinematics*

$$\begin{aligned}
\dot{x} &= v_x \cos\theta - v_y \sin\theta \\
\dot{y} &= v_x \sin\theta + v_y \cos\theta \\
\dot{\theta} &= \omega_z
\end{aligned}$$

- *Dynamics*

$$\begin{aligned}
m_{11}\dot{v}_x - m_{22}v_y\omega_z + d_{11}v_x &= F_x \\
m_{22}\dot{v}_y + m_{11}v_x\omega_z + d_{22}v_y &= 0 \\
m_{33}\dot{\omega}_z + (m_{22} - m_{11})v_x v_y + d_{33}\omega_z &= \tau_z
\end{aligned}$$
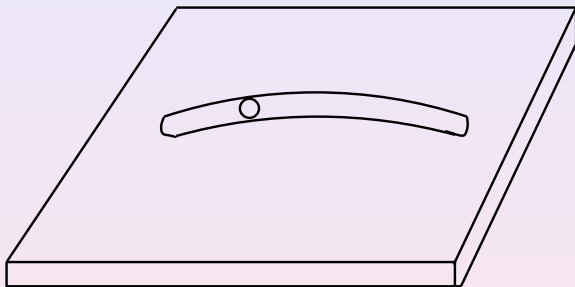
A bead moving in a slot

Figure: A bead in a slot

- ▶ Configuration variables (x, y)
- ▶ Constraints - $g(x, y) = 0$
- ▶ The bead is restricted in the configurations that it can assume (holonomic constraint)

Classification of constraints in mechanical systems

- ▶ **Holonomic constraints** - restrict the allowable configurations of the system
- ▶ **Nonintegrable constraints** - do not restrict the allowable configurations of the system but restrict instantaneous velocities/accelerations
  - ▶ Velocity level constraints - parking of a car, wheeled mobile robots, rolling contacts in robotic applications
  - ▶ Acceleration level constraints - fuel slosh in spacecrafts/launch vehicles, underwater vehicles, underactuated mechanisms (on purpose or loss of actuator) systems - serial link manipulators **Underactuated systems**

Mechanical systems

- ▶ To fix up the configuration we use a set of variables - called the configuration variables
- ▶ Algebraic constraints between these configuration variables and their derivatives
- ▶ Holonomic constraints and nonholonomic constraints

Constraints

- Consider a mechanical system described by

$$\ddot{\boldsymbol{q}} = f(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{u}) \tag{1}$$

  $\boldsymbol{q}$ - generalized coordinates that evolve on a n-dimensional smooth manifold $\mathcal{M}$ called the configuration manifold $f(.)$ - the dynamics and $\boldsymbol{u}$ - m-dimensional vector of generalized inputs.

- Suppose the system is subject to a constraint of the form

$$s(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}) = 0 \quad \text{Say} \quad s \in C^1 \tag{2}$$

- If the following equivalence holds

$$s(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}) = 0 \quad \equiv \quad h(\boldsymbol{q}) = 0 \tag{3}$$

  then the constraint satisfies the complete integrability property

- Completete integrability implies *holonomic constraint* and the system is called a *holonomic system* . Else *nonholonomic system*

Velocity level constraints

▶ *Rolling coin* : Configuration variables

$$\boldsymbol{q} = (x, y, \theta, \phi) \in \mathcal{M} = R^1 \times R^1 \times S^1 \times S^1$$
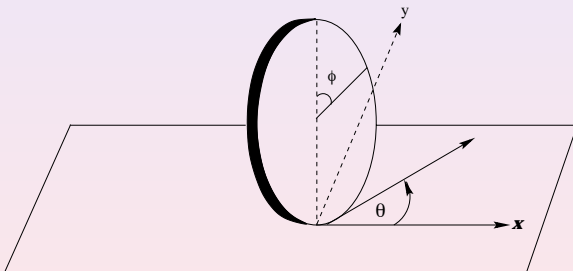

Figure: Rolling coin

▶ Constraints - No slip and no sliding

Velocity level constraints

▶ Constraints of motion are expressed as

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0 \quad \text{No lateral motion}$$

$$\dot{x}\cos\theta + \dot{y}\sin\theta = r\dot{\phi} \quad \text{Pure rolling}$$

▶ Constraints in a matrix form

$$\left[\begin{array}{cccc} \sin(\theta) & -\cos(\theta) & 0 & 0 \\ \cos\theta & \sin\theta & 0 & -r \end{array} \right] \left[\begin{array}{c} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{array} \right] = 0$$
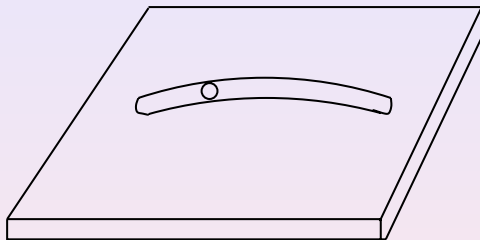
Bead in a slot

Figure: A bead in a slot

- Configuration variables of the bead are (x, y)
- Restriction to stay in the slot gives rise to an algebraic constraint

$$g(x, y) = 0$$

- g is the equation of the curve describing the slot
- Slot restricts the possible configurations that the bead can

Underactuated mechanical systems

Definition

Consider a mechanical system described by (1) The system is said to be underactuated if $m < n$.
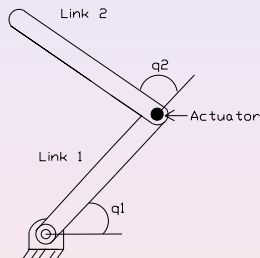
Acceleration level constraints

Figure: A two-link manipulator

► Two link manipulator moving in a vertical plane with one actuator at its second joint (acrobot)

Acrobot

▶

$$d_{11}(q)\ddot{q}_1 + d_{12}(q)\ddot{q}_2 + h_1(q,\dot{q}) + \psi_1(q) = 0$$
$$d_{21}(q)\ddot{q}_1 + d_{22}(q)\ddot{q}_2 + h_2(q,\dot{q}) + \psi_2(q) = \tau_2 \qquad (4)$$

- ▶ First equation (with the right hand side being zero) denotes the lack of actuation at the first joint

- ▶ The acrobot can assume any configuration but cannot assume arbitrary accelerations.

Fuel slosh in a launch vehicle

- ▶ A launch vehicle with liquid fuel in its tank.
- ▶ Unactuated pivoted pendulum model. The motion of the pendulum is solely affected by the motion of the outer rigid body
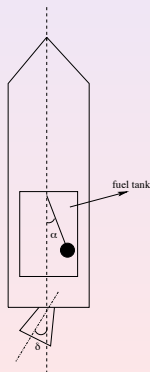

Figure: Fuel slosh phenomenon

► The equations of motion are of the form

$$\ddot{q}_u + f_1(q, \dot{q}, \ddot{q}) = 0$$
$$\ddot{q}_a + f_2(q, \dot{q}, \ddot{q}) = F \tag{5}$$

where $q \stackrel{\triangle}{=} (q_u, q_a)$, q_u corresponds to the configuration variable of the pendulum, q_a corresponds to the configuration variable of the outer rigid body and F is the external force.

► While the acceleration level constraint in the acrobot arises due to purpose of design or loss of actuation, in the case of the launch vehicle it is the inability to directly actuate the fluid dynamics.

Objectives

- **Motion planning** - Given $x(0) = x_0$ (initial state) and x_f (desired final state) and a time interval $[0, t_f]$, find a control history $u(\cdot)$ (from an admissible class of functions) such that $x(t_f) = x_f$.

- **Feedback stabilization** - Find a state (or output) feedback control law to stabilize the system about an equilibrium. Allied questions - domain of convergence/stability

Challenges/Solutions

Challenges

- ▶ Incorporating non-integrable constraints
- ▶ Loss of linear controllability
- ▶ Loss of full state-feedback linearization
- ▶ Lack of existence of a continuous feedback control law that can locally stabilize the system (Brockett's result)

Solutions

- ▶ Discontinuous or time-varying control laws
- ▶ New notions of stability (not the conventional Lyapunov one)
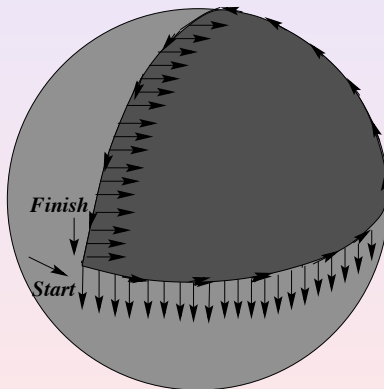
Mechanics, geometry and control
## **The thumb experiment**

Figure: Motion on a sphere

Mechanics, geometry and control

▶ Phenomenon similar to the thumb experiment is also common to a large class of systems which are termed **nonholonomic systems**

▶ Variables describing the motion of these systems can be classified into two sets called the **shape** variables and the **group** variables.

▶ Cyclic motion in the shape variables produces motion in the group (fiber) variables. Biological systems (fishes, snakes, paramecia) and robotic mechanisms, the falling cat, the steering of a car, the motion of underactuated systems of linkages.

▶ The net changes in position due to changes in the shape variables is explicable either due to an interaction with the environment or some conservation law.

Geometry

- Provides better insight, better framework for solution (control law) and an elegant setting for these problems
- Essential tools - Lie groups and differentiable manifolds

Dynamical systems and geometry
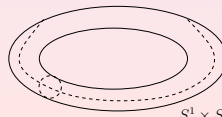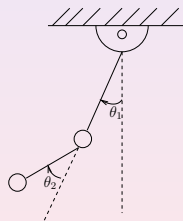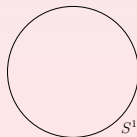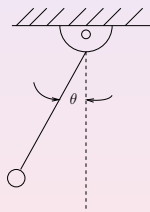
A glimpse of differential geometry

- ▶ Familiar with calculus (differentiation, integration) on the real line ($\mathbb{R}$) or $\mathbb{R}^n$. Extend this calculus to surfaces - say on a circle, a sphere, a torus and more complicated surfaces

- ▶ But why ? From a dynamical systems and control theory point of view, systems need not always evolve on the real line ($\mathbb{R}$) or multiple real lines ($\mathbb{R}^n$). They may evolve on *non-Euclidean* surfaces. On such surfaces, we wish to talk of "rate of change (velocity) $\Rightarrow$ differentiation, to talk of "rate of rate of change (acceleration), to talk of accumulation $\Rightarrow$ integration

- ▶ Can these surfaces which are *not Euclidean* be *locally represented* as Euclidean ? This would allow us to employ the calculus that we are familiar with for these surfaces

- ▶ Just as a cartographer maps the surface of the earth on a plane sheet, differential geometry begins with such a map - called charts and atlases of the surface - words borrowed from the cartographer's lexicon

Stability, controllability - from the linear to the nonlinear

Linear systems

State-space linear model

$$\dot{x} = Ax + Bu \qquad\qquad x(0) = x_0 \qquad\qquad x(t) \in R^n$$

- ▶ Properties are global
- ▶ Stability - Eigen values in the open left-half of the complex plane (alternatively *poles* of a transfer function)
- ▶ Controllability - The ability to transfer the system from any given initial state to a prescribed final state.
- ▶ Control algorithms - state feedback, estimated state feedback, robust and optimal algorithms (LQG, H_2, H_∞)

Nonlinear systems

$$\dot{x} = f(x, u) \qquad\qquad x(0) = x_0 \qquad\qquad x(t) \in R^n$$

- ▶ Properties are local - valid in open subsets of the state-space
- ▶ Various notions of controllability - local accessibility, strong local accessibility, small-time local controllability
- ▶ Lyapunov stability, asymptotic stability, stability in a set
- ▶ Control algorithms - based on Lyapunov functions, energy methods, passivity, strategies may be discontinuous, time-varying, sliding surfaces