Non-linear Optimization

Introduction

Niclas Börlin Klas Markström

[1]Department of Computing Science
Umeå University
niclas.borlin@cs.umu.se

[2]Department of Mathematics and Mathematical Statistics
Umeå University
klas.markstrom@math.umu.se

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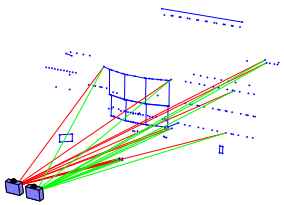
Travelling Salesman

- Given n cities, find the shortest route that visits each city exactly once.
- A robot moves its arm over a circuit board in order to place components at given locations. We want to mimize the total distance, and thererby time, the arm has to move in order to place all components on the board.

Optical Photogrammetry

- Given a number of measurements in multiple images, calculate the positions of the object points and cameras in 3D such that the points projected into the cameras are as close as possible to the measured points.





Dåvamyran

- The Dåvamyran power plant has four heaters with different maximum energy output H_i.
- Each heater has a fixed startup cost S_i and linear energy cost E_i.
- Given an expected energy demand D, find the cheapest way to meet the demand, i.e. which heaters should be operating and at what level?

Shortest path

- Find the shortest distance between two nodes *a* and *b* in a graph.
- A data package is to be sent from a webserver to a surfer on the internet. Find the fastet route for the data package through the web.

Max cut

- Given a graph *G* find a partition of its vertex set into two sets *A* and *B* such that as many edges as possible has one endpoint in *A* and one endpoint in *B*
- An antiferromagnet is a material where neighbouring atoms minimize their interaction energy if they have spins in opposite direction. Find a way of setting the spins to values *A* and *B* so that the total energy of the crystal is minimized. This is called a ground state of the the crystal.

X-ray Photogrammetry

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Max Flow/Min Cut

- Given a graph and two nodes in the graph, and capacities for the edges, find the largest possible flow from one vertex to the other.
- An oil rig is connected to one, or several, refinieries by system of piplines. How much oil can be transported from the rig through the current pipeline system?

Airline Scheduling

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Structure Optimization

- Given a shape of a structure, minimize its weight while maintaining its strength.

GPS

- Given a number of measurements of the time delay of signals from several satellites, determine the position of the receiver.

Graph Coloring

- Given a graph G, find an assignment of colours to its vertices so that neighbouring vertices have different colours, and as few colours a possible is used.
- A collection of radio masts are placed throughout a country. Masts which are close to each other must broadcast at different frequencies in order to avoid interference. How many different frequencies are reuquired?

Protein Folding

- A string of protein is constructed in a stretched out state. How can the protein be folded up in order to minimize its energy?

Flight Path Optimization

- Given a wind forecast and a take-off weight, what is the optimal flight path and altitude between Umeå and Stockholm?

Optimal Log Cutting

- Given a tree log, a what lengths should it be cut to produce maximum revenue?

Optimization Problem Features

- We must have something to minimize (maximize) — *The objective function*.
- The objective function may be continous or discrete.
- We have some *variables* we wish to determine.
- The variables may be continous or discrete, e.g. integers or binary.
- We may have relationships or limitations on some variables (*constraints*).
- Common assumptions
  - A solution exists.
  - The objective function is differentiable (for continous problems).

Optimization Problem Requirements

- What kind of solution do we need?
  - A globally optimal solution.
  - A locally optimal solution.
  - Upper/lower bounds on the optimal solution.
  - A “good enough” solution.
- How much time do we have?
- Do we have access to derivatives (for continuous problems)?

Optimization Algorithm Types

- Complete vs. incomplete algorithms.
- Deterministic vs. stochastic algorithms.
- Direct vs. iterative algorithms.

Solving an Optimization Problem

- Transform a described problem into a mathematical problem.
  - Formulate an objective function.
  - Determine the metric (max error, avg error, etc.)
  - Formulate constraints.
  - Determine what variables to estimate.
- Classify the optimization problem.
- Determine a starting approximation.
- Solve the optimization problem.
- Analyze the result.