

Summer School on SLAM
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Advanced EKF-SLAM: Building Large Maps

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Building Large Maps

1. The curse of correlations

2. Independent local maps

3. Map joining

4. Multivehicle SLAM

5. Hierarchical SLAM

6. Conclusion

EKF-SLAM

Algorithm 1 SLAM:

$\mathbf{x}_0^B = \mathbf{0}; \mathbf{P}_0^B = \mathbf{0} \{Map\ initialization\}$

$[\mathbf{z}_0, \mathbf{R}_0] = \text{get_measurements}$

$[\mathbf{x}_0^B, \mathbf{P}_0^B] = \text{add_new_features}(\mathbf{x}_0^B, \mathbf{P}_0^B, \mathbf{z}_0, \mathbf{R}_0)$

for $k = 1$ to steps **do**

$[\mathbf{x}_{R_k}^{R_{k-1}}, \mathbf{Q}_k] = \text{get_odometry}$

$[\mathbf{x}_{k|k-1}^B, \mathbf{P}_{k|k-1}^B] = \text{EKF_prediction}(\mathbf{x}_{k-1}^B, \mathbf{P}_{k-1}^B, \mathbf{x}_{R_k}^{R_{k-1}}, \mathbf{Q}_k)$

$[\mathbf{z}_k, \mathbf{R}_k] = \text{get_measurements}$

$\mathcal{H}_k = \text{data_association}(\mathbf{x}_{k|k-1}^B, \mathbf{P}_{k|k-1}^B, \mathbf{z}_k, \mathbf{R}_k)$

$[\mathbf{x}_k^B, \mathbf{P}_k^B] = \text{EKF_update}(\mathbf{x}_{k|k-1}^B, \mathbf{P}_{k|k-1}^B, \mathbf{z}_k, \mathbf{R}_k, \mathcal{H}_k)$

$[\mathbf{x}_k^B, \mathbf{P}_k^B] = \text{add_new_features}(\mathbf{x}_k^B, \mathbf{P}_k^B, \mathbf{z}_k, \mathbf{R}_k, \mathcal{H}_k)$


end for

The curse of correlations

- Map:

$$\hat{\mathbf{x}}_{\mathcal{F}}^F = \begin{bmatrix} \hat{\mathbf{x}}_{F_0}^F \\ \vdots \\ \hat{\mathbf{x}}_{F_n}^F \end{bmatrix}; \mathbf{P}_{\mathcal{F}}^F = \begin{bmatrix} \mathbf{P}_{F_0 F_0}^F & \cdots & \mathbf{P}_{F_0 F_n}^F \\ \vdots & \ddots & \vdots \\ \mathbf{P}_{F_n F_0}^F & \cdots & \mathbf{P}_{F_n F_n}^F \end{bmatrix}$$

$$\hat{\mathbf{x}}_{\mathcal{F}_k}^B = \hat{\mathbf{x}}_{\mathcal{F}_{k|k-1}}^B + \mathbf{K}_k(\mathbf{z}_k - \mathbf{h}_k(\hat{\mathbf{x}}_{\mathcal{F}_{k|k-1}}^B))$$

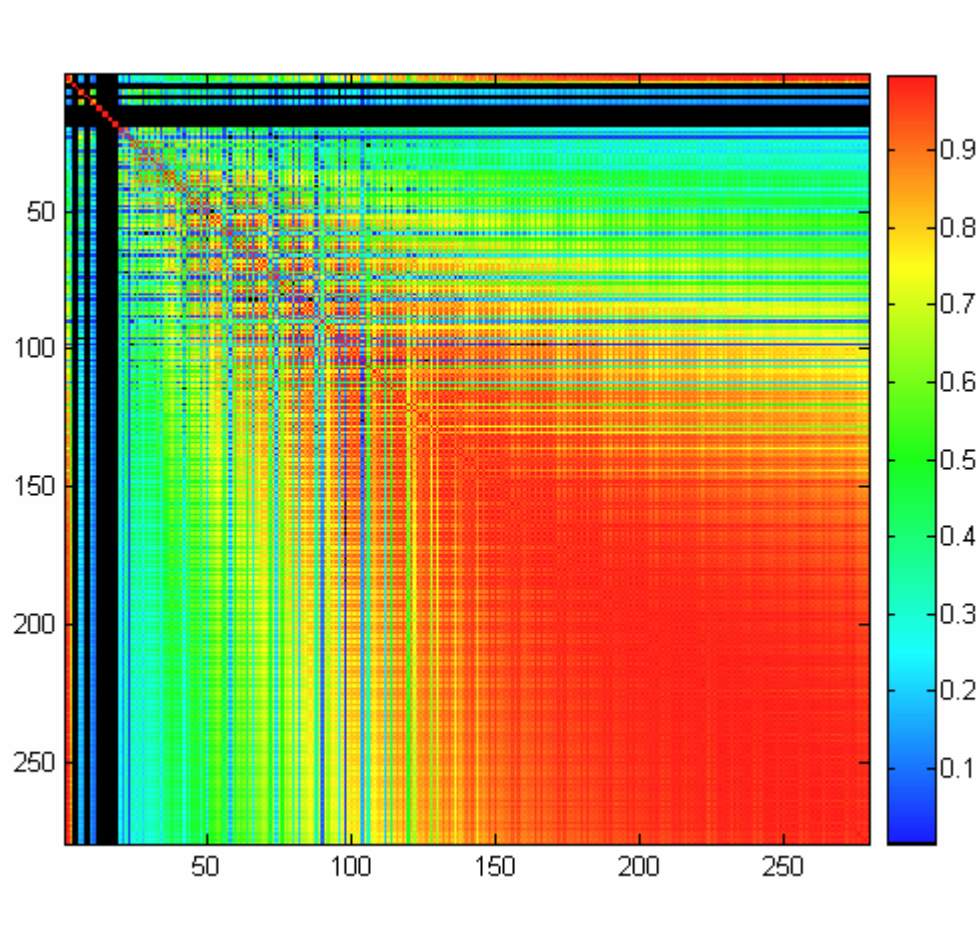


$$\mathbf{P}_{\mathcal{F}_k}^B = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{\mathcal{F}_{k|k-1}}^B$$

$$\mathbf{K}_k = \mathbf{P}_{\mathcal{F}_{k|k-1}}^B \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_{\mathcal{F}_{k|k-1}}^B \mathbf{H}_k^T + \mathbf{R}_k)^{-1}$$

- Updating a full stochastic map of n features is well known to be $\mathbf{O}(n^2)$ (Castellanos et al. 1999, Guivant and Nebot 2001)

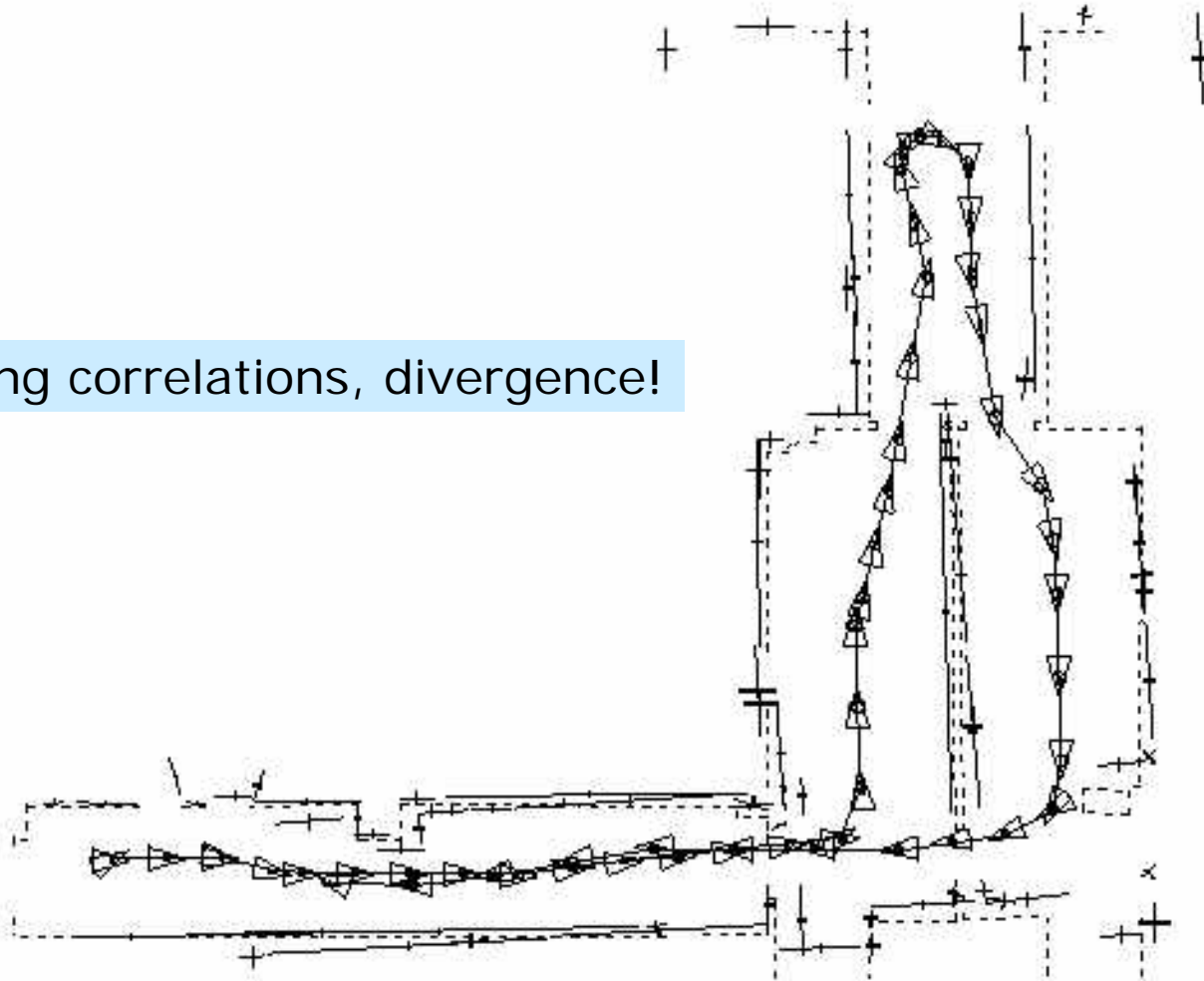
Are correlations necessary?



```
sigmas = sqrt(diag(Cov))';  
Corr=diag(1./sigmas)*Cov*diag(1./sigmas) ;
```

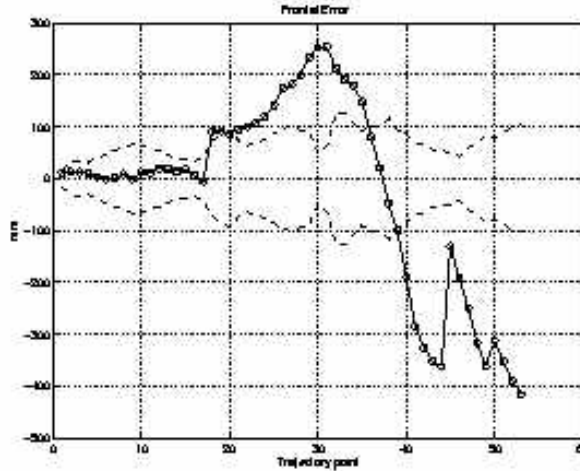
The importance of Correlations

Ignoring correlations, divergence!

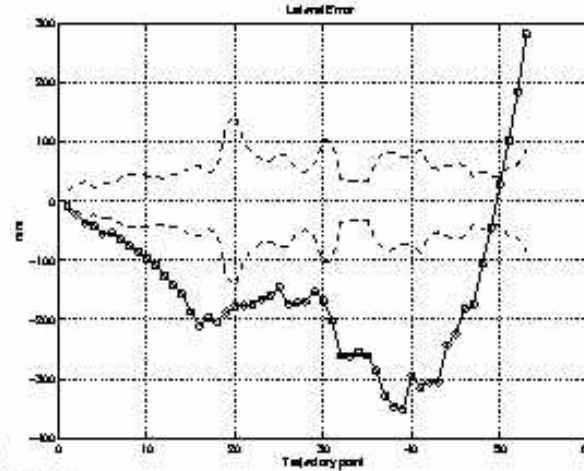


The importance of Correlations

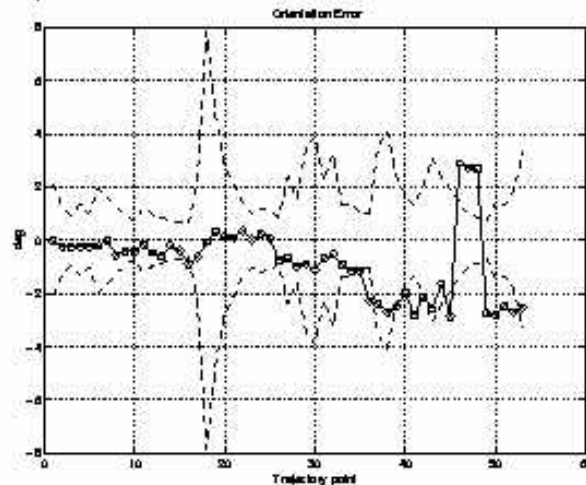
Frontal



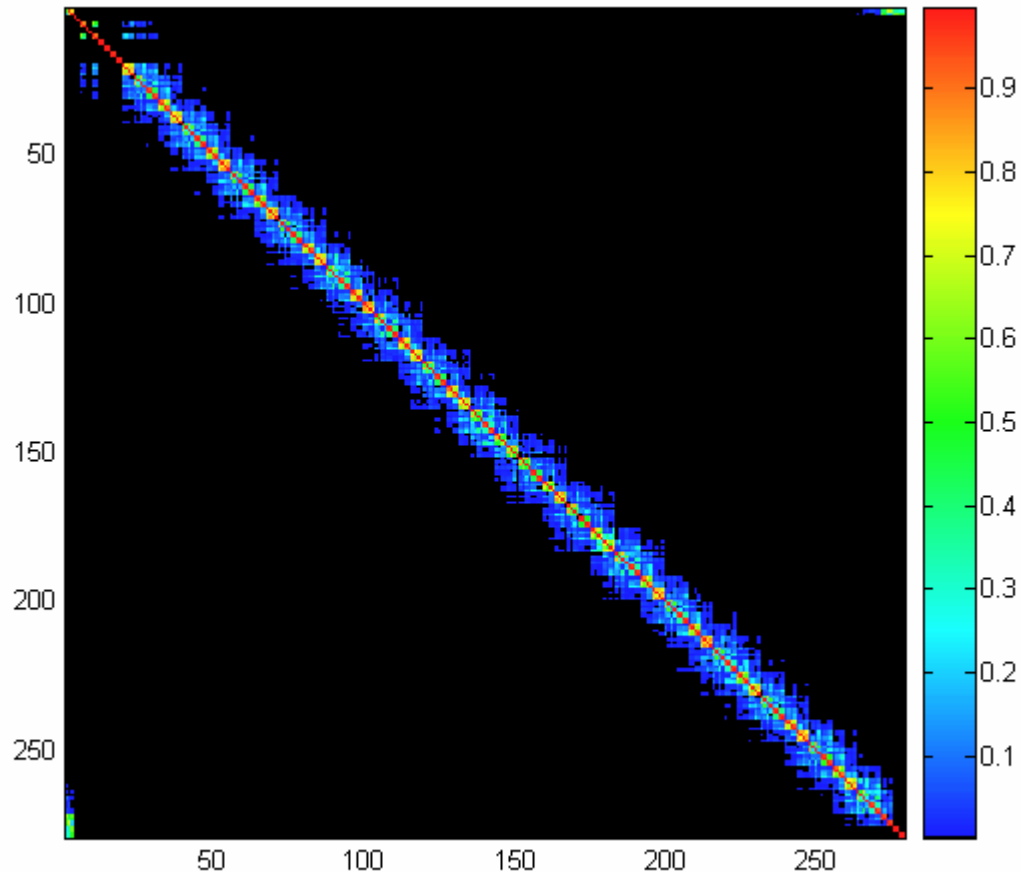
Lateral



Angular

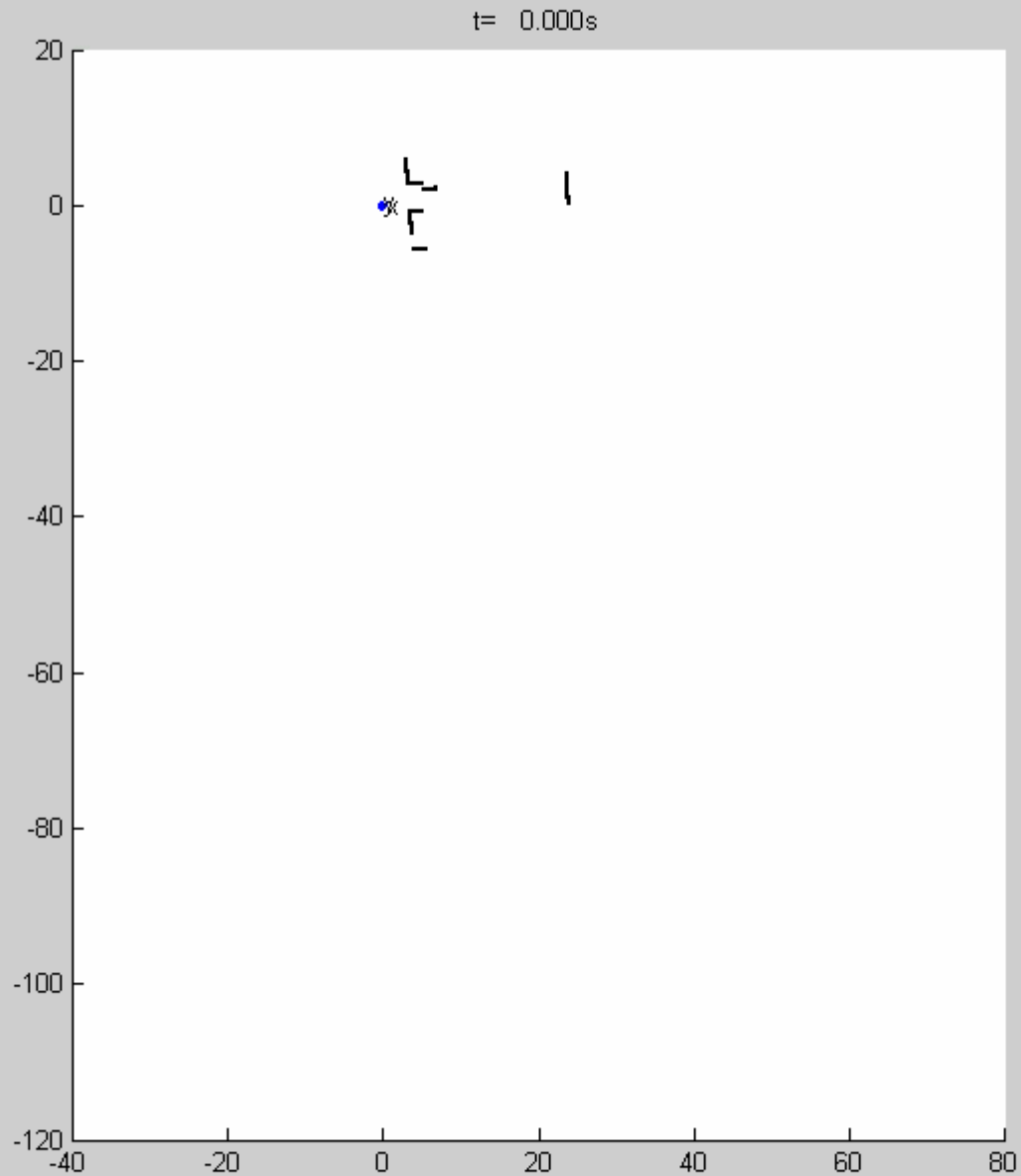


Inverse correlations



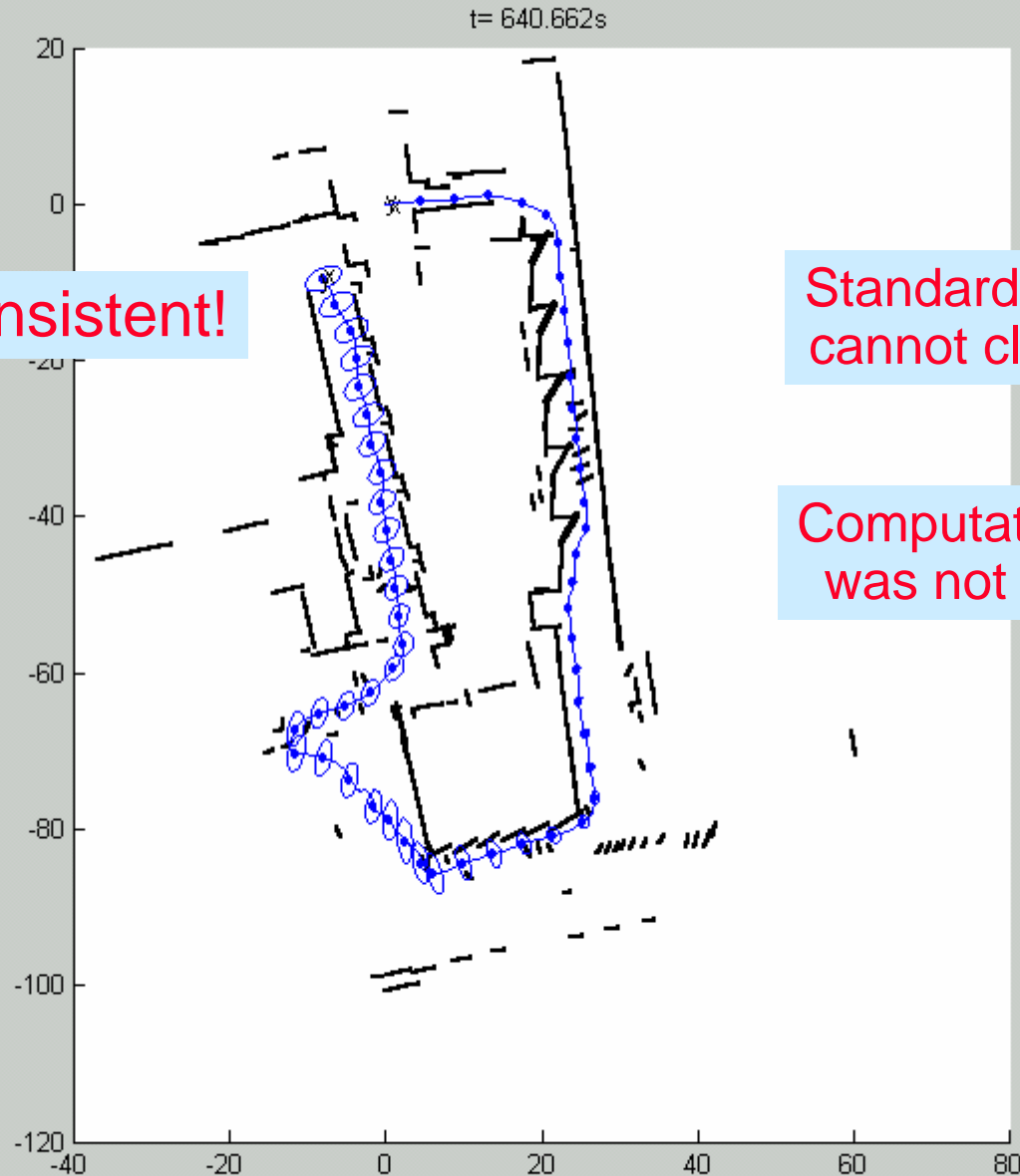
This observation is the basis of SEIFs

EKF-SLAM: Real Example



EKF-SLAM: Real Example

Inconsistent!



Standard data association cannot close a 250m loop

Computational complexity was not a problem here!

Building Large Maps

1. The curse of correlations

2. Independent local maps

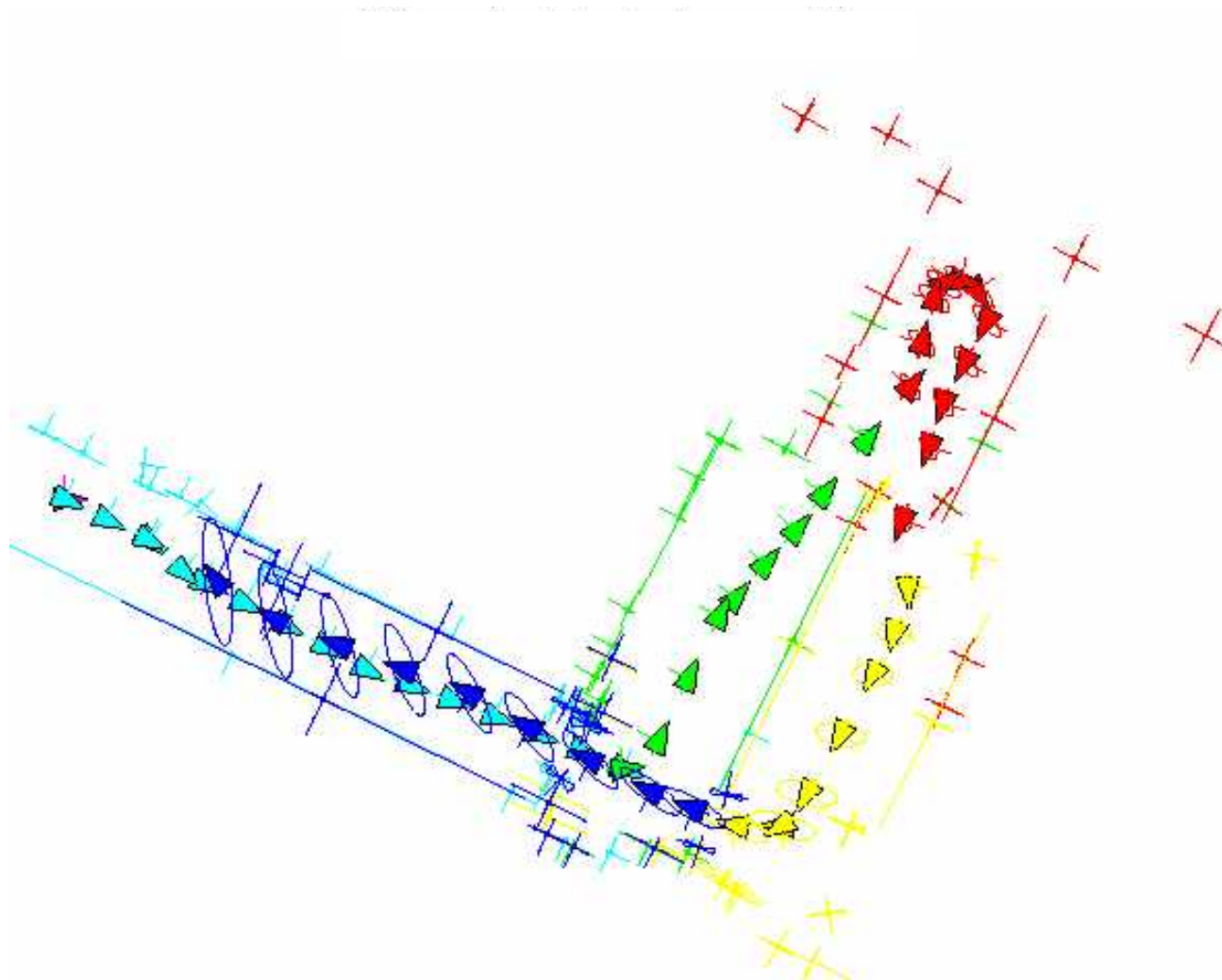
3. Map joining

4. Multivehicle SLAM

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Independent Local Maps



Independent Local Maps

J.D. Tardós, J. Neira, P.M. Newman and J.J. Leonard, Robust Mapping and Localization in Indoor Environments using Sonar Data, The Int. Journal of Robotics Research, Vol. 21, No. 4, April, 2002, pp 311 –330

- Build independent local maps (Tardós 02)
 - Constant time complexity
- Map joining (Tardós 02)
 - Improves consistency and precision
- Map matching (Neira 03)
 - Correct environment topology
- Hierarchical SLAM (Estrada 05)
 - Scalable map representation
 - Good precision after loop closing
 - Fast convergence
- Similar approaches: CLSF (Williams 02), NCFM (Bailey 02), ATLAS (Bosse 03), CTS (Newman 03)



Local map building

- Periodically, the robot starts a new map, relative to its current location:

$$\hat{\mathbf{x}}_{R_0}^B = \mathbf{0}$$

$$\mathbf{P}_{R_0}^B = \mathbf{0}$$

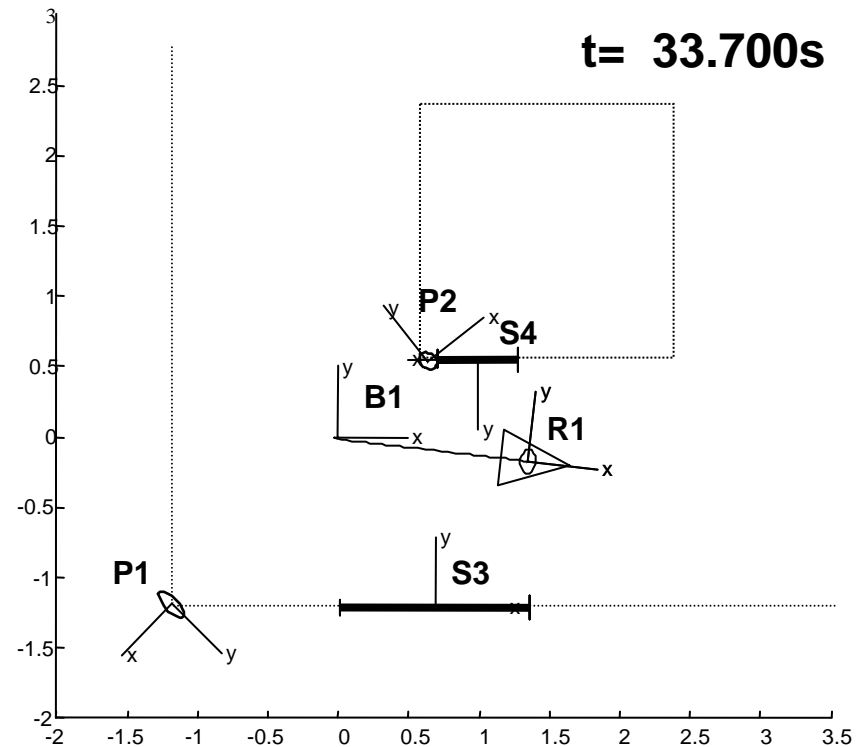
- Given measurements:

$$D^{1\dots k_1} = \left\{ \mathbf{u}_1 \mathbf{z}_1 \dots \mathbf{u}_{k_1} \mathbf{z}_{k_1} \right\}$$

$$\mathbf{u}_k = \hat{\mathbf{x}}_{R_k}^{R_{k-1}}$$

- EKF approximates the conditional mean:

$$\hat{\mathbf{x}}_{\mathcal{F}_1}^{B_1} \simeq E \left[\mathbf{x}_{\mathcal{F}_1}^{B_1} \mid D^{1\dots k_1}, \mathcal{H}^{1\dots k_1} \right]$$



Local map building

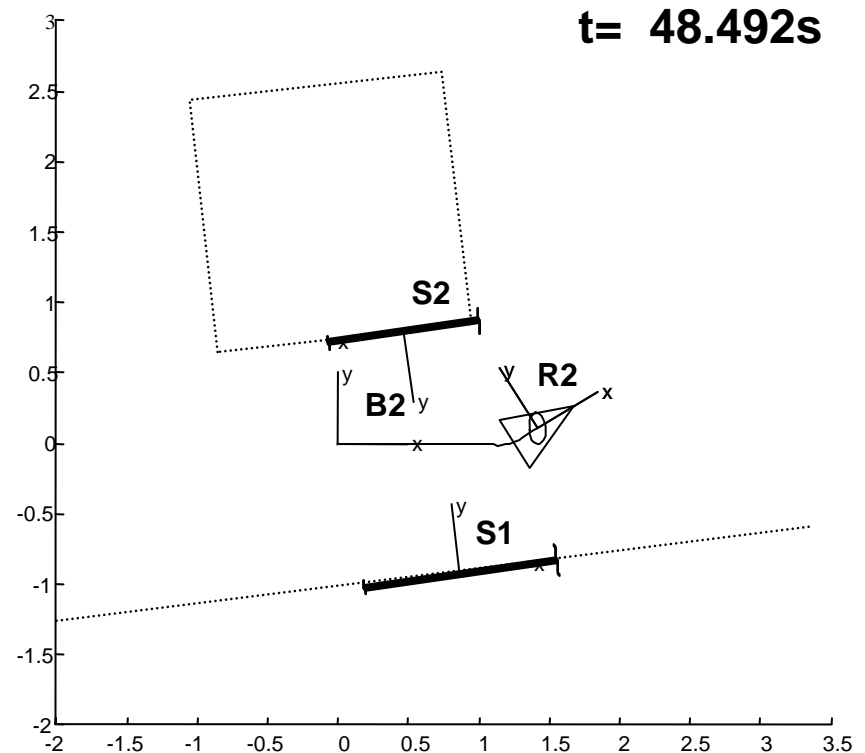
- Second map: $D^{k_1+1 \dots k_2} = \left\{ \mathbf{u}_{k_1+1} \mathbf{z}_{k_1+1} \dots \mathbf{u}_{k_2} \mathbf{z}_{k_2} \right\}$

$$\hat{\mathbf{x}}_{\mathcal{F}_2}^{B_2} \simeq E \left[\mathbf{x}_{\mathcal{F}_2}^{B_2} \mid D^{k_1+1 \dots k_2}, \mathcal{H}^{k_1+1 \dots k_2} \right]$$

- No information is shared:
 $D^{1 \dots k_1} \cap D^{k_1+1 \dots k_2} = \emptyset$

Maps are uncorrelated

- Common reference:
 $B_2 = R_1$



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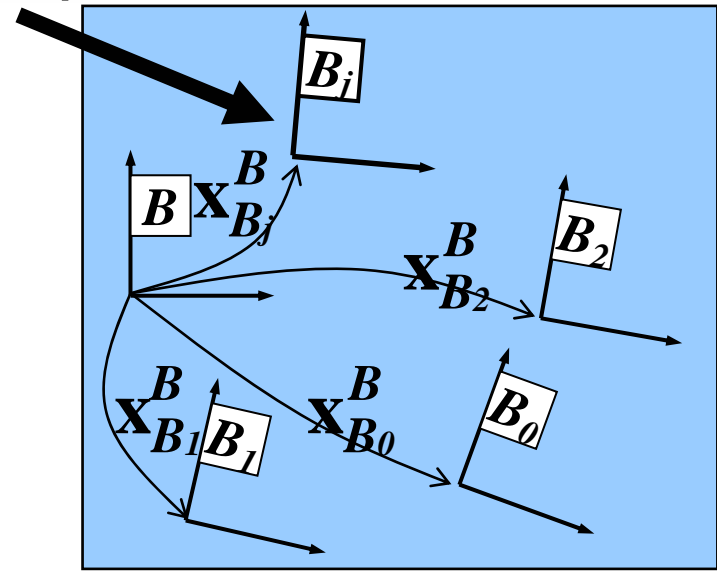
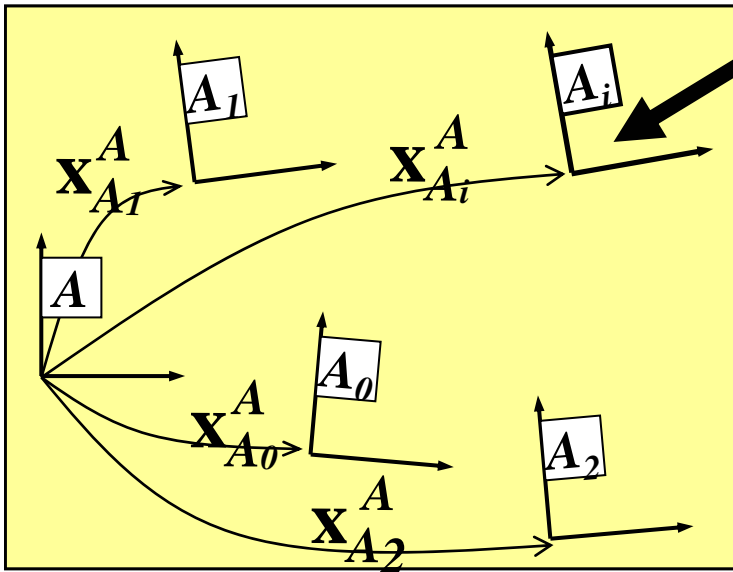
Map Joining

- Given:
 - Two statistically independent stochastic maps
 - A common reference

$$\mathcal{M}_A^A = (\hat{\mathbf{x}}_A^A, \mathbf{P}_A^A)$$

$$A_i = B_j$$

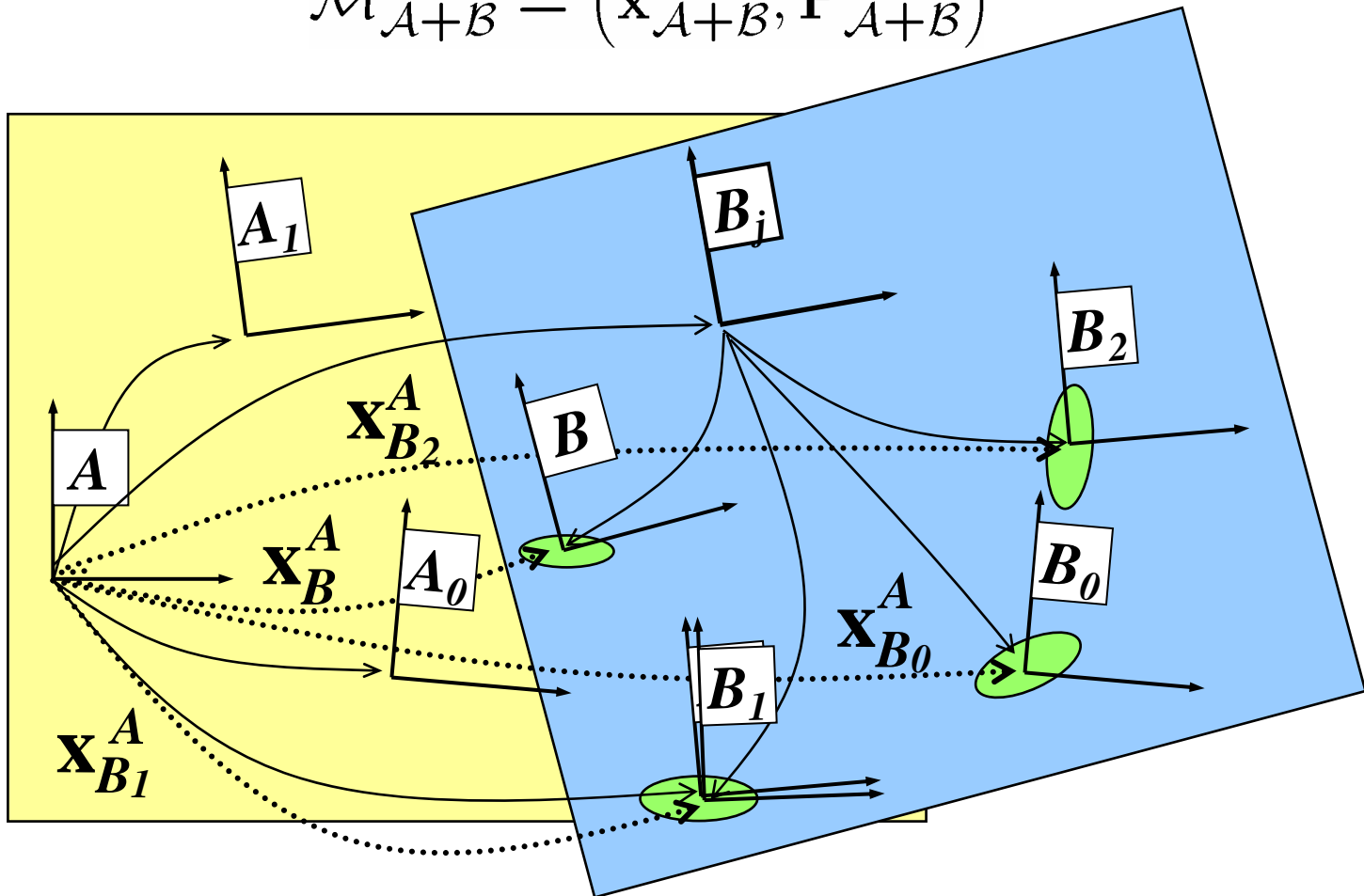
$$\mathcal{M}_B^B = (\hat{\mathbf{x}}_B^B, \mathbf{P}_B^B)$$



Map Joining

- Conveys the information of the two maps into a single **fully consistent** stochastic map:

$$\mathcal{M}_{A+B}^A = (\hat{\mathbf{x}}_{A+B}^A, \mathbf{P}_{A+B}^A)$$



Change the base of map B to B_j

- New state vector:
$$\hat{\mathbf{x}}_{\mathcal{B}}^{B_j} = \begin{bmatrix} \hat{\mathbf{x}}_{B_0}^{B_j} \\ \vdots \\ \hat{\mathbf{x}}_B^{B_j} \\ \vdots \\ \hat{\mathbf{x}}_{B_m}^{B_j} \end{bmatrix} = \begin{bmatrix} \ominus \hat{\mathbf{x}}_{B_j}^B \oplus \hat{\mathbf{x}}_{B_0}^B \\ \vdots \\ \ominus \hat{\mathbf{x}}_{B_j}^B \\ \vdots \\ \ominus \hat{\mathbf{x}}_{B_j}^B \oplus \hat{\mathbf{x}}_{B_m}^B \end{bmatrix}$$

- New covariance matrix:
$$\mathbf{P}_{\mathcal{B}}^{B_j} = \mathbf{J}_B^{B_j} \mathbf{P}_{\mathcal{B}}^B \mathbf{J}_B^{B_j T}$$

$$\mathbf{J}_B^{B_j} = \frac{\partial \hat{\mathbf{x}}_{\mathcal{B}}^{B_j}}{\partial \hat{\mathbf{x}}_{\mathcal{B}}^B} = \begin{bmatrix} \mathbf{J}_{00} & \cdots & \mathbf{J}_{0j} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots & & \vdots \\ \mathbf{0} & \cdots & \mathbf{J}_{jj} & \cdots & \mathbf{0} \\ \vdots & & \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{J}_{mj} & \cdots & \mathbf{J}_{mm} \end{bmatrix}$$

$$\mathbf{J}_{jj} = \mathbf{J}_{\ominus} \left\{ \hat{\mathbf{x}}_{B_j}^B \right\}$$

$$\mathbf{J}_{ij} = \mathbf{J}_{1\oplus} \left\{ \ominus \hat{\mathbf{x}}_{B_j}^B, \hat{\mathbf{x}}_{B_i}^B \right\} \mathbf{J}_{\ominus} \left\{ \hat{\mathbf{x}}_{B_j}^B \right\} \quad i = 0..m, i \neq j$$

$$\mathbf{J}_{ii} = \mathbf{J}_{2\oplus} \left\{ \ominus \hat{\mathbf{x}}_{B_j}^B, \hat{\mathbf{x}}_{B_i}^B \right\} \quad i = 0..m, i \neq j$$



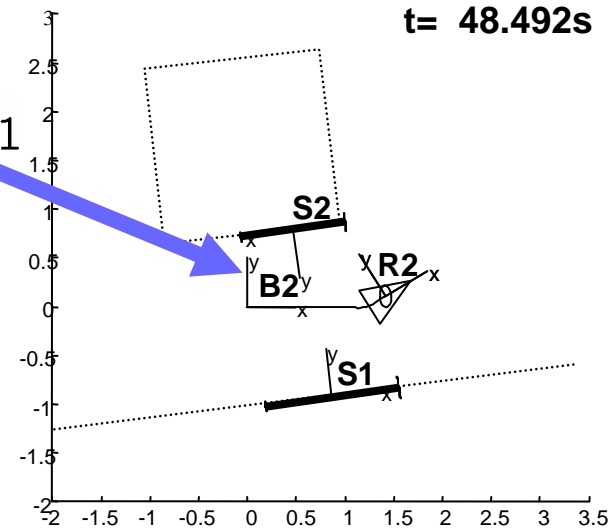
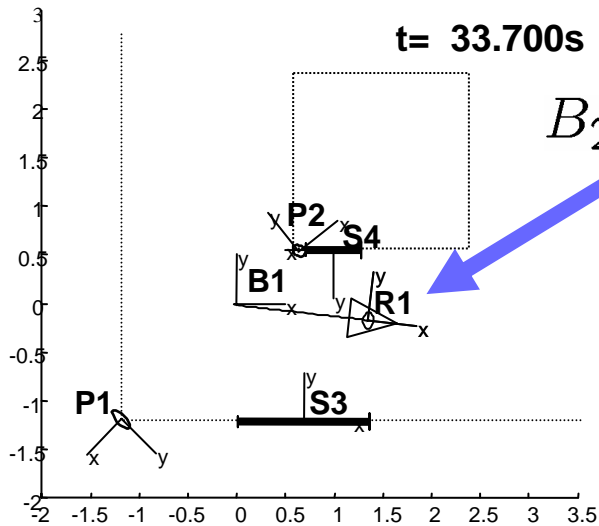
Map Joining

- New state vector: $\hat{\mathbf{x}}_{\mathcal{A}+\mathcal{B}}^A = \begin{bmatrix} \hat{\mathbf{x}}_{\mathcal{A}}^A \\ \hat{\mathbf{x}}_{\mathcal{B}}^A \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{x}}_{\mathcal{A}_i}^A \oplus \hat{\mathbf{x}}_{B_0}^{B_j} \\ \vdots \\ \hat{\mathbf{x}}_{\mathcal{A}_i}^A \oplus \hat{\mathbf{x}}_{B_m}^{B_j} \end{bmatrix}$
- New covariance matrix:

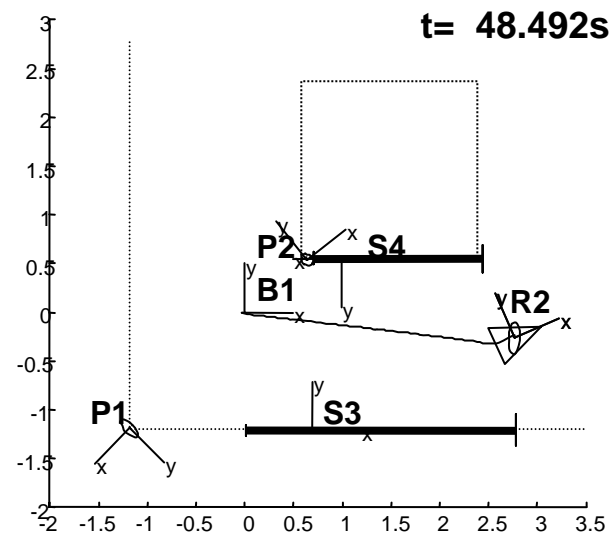
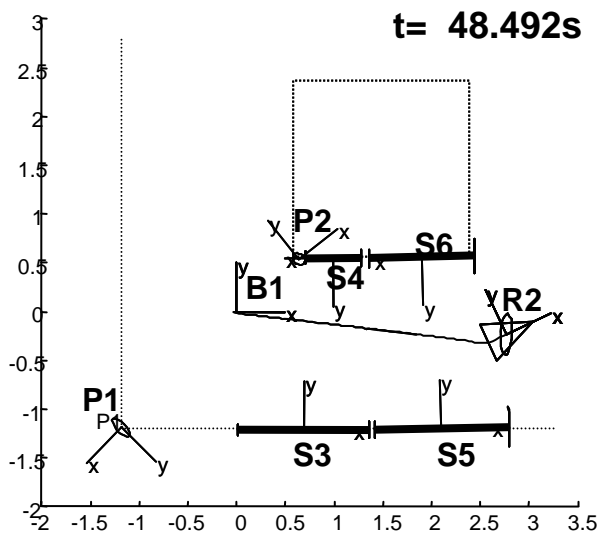
$$\begin{aligned} \mathbf{P}_{\mathcal{A}+\mathcal{B}}^A &= \mathbf{J}_{\mathcal{A}}^{A+\mathcal{B}} \mathbf{P}_{\mathcal{A}}^A \mathbf{J}_{\mathcal{A}}^{A+\mathcal{B}T} + \mathbf{J}_{\mathcal{B}}^{A+\mathcal{B}} \mathbf{P}_{\mathcal{B}}^{B_j} \mathbf{J}_{\mathcal{B}}^{A+\mathcal{B}T} \\ &= \begin{bmatrix} \mathbf{P}_{\mathcal{A}}^A & \mathbf{P}_{\mathcal{A}}^A \mathbf{J}_1^T \\ \mathbf{J}_1 \mathbf{P}_{\mathcal{A}}^A & \mathbf{J}_1 \mathbf{P}_{\mathcal{A}}^A \mathbf{J}_1^T \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_2 \mathbf{P}_{\mathcal{B}}^{B_j} \mathbf{J}_2^T \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \mathbf{J}_{\mathcal{A}}^{A+\mathcal{B}} &= \frac{\partial \hat{\mathbf{x}}_{\mathcal{A}+\mathcal{B}}^A}{\partial \hat{\mathbf{x}}_{\mathcal{A}}^A} = \begin{bmatrix} \mathbf{I} \\ \mathbf{J}_1 \end{bmatrix} & \mathbf{J}_1 &= \begin{bmatrix} \mathbf{0} & \cdots & \mathbf{J}_{1 \oplus \left\{ \hat{\mathbf{x}}_{\mathcal{A}_i}^A, \hat{\mathbf{x}}_{B_0}^{B_j} \right\}} & \cdots & \mathbf{0} \\ \vdots & & \vdots & & \vdots \\ \mathbf{0} & \cdots & \mathbf{J}_{1 \oplus \left\{ \hat{\mathbf{x}}_{\mathcal{A}_i}^A, \hat{\mathbf{x}}_{B_m}^{B_j} \right\}} & \cdots & \mathbf{0} \end{bmatrix} \\ \mathbf{J}_{\mathcal{B}}^{A+\mathcal{B}} &= \frac{\partial \hat{\mathbf{x}}_{\mathcal{A}+\mathcal{B}}^A}{\partial \hat{\mathbf{x}}_{\mathcal{B}}^{B_j}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{J}_2 \end{bmatrix} & \mathbf{J}_2 &= \begin{bmatrix} \mathbf{J}_{2 \oplus \left\{ \hat{\mathbf{x}}_{\mathcal{A}_i}^A, \hat{\mathbf{x}}_{B_0}^{B_j} \right\}} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{J}_{2 \oplus \left\{ \hat{\mathbf{x}}_{\mathcal{A}_i}^A, \hat{\mathbf{x}}_{B_m}^{B_j} \right\}} \end{bmatrix} \end{aligned}$$

Map Joining: Example



$$B_2 = R_1$$

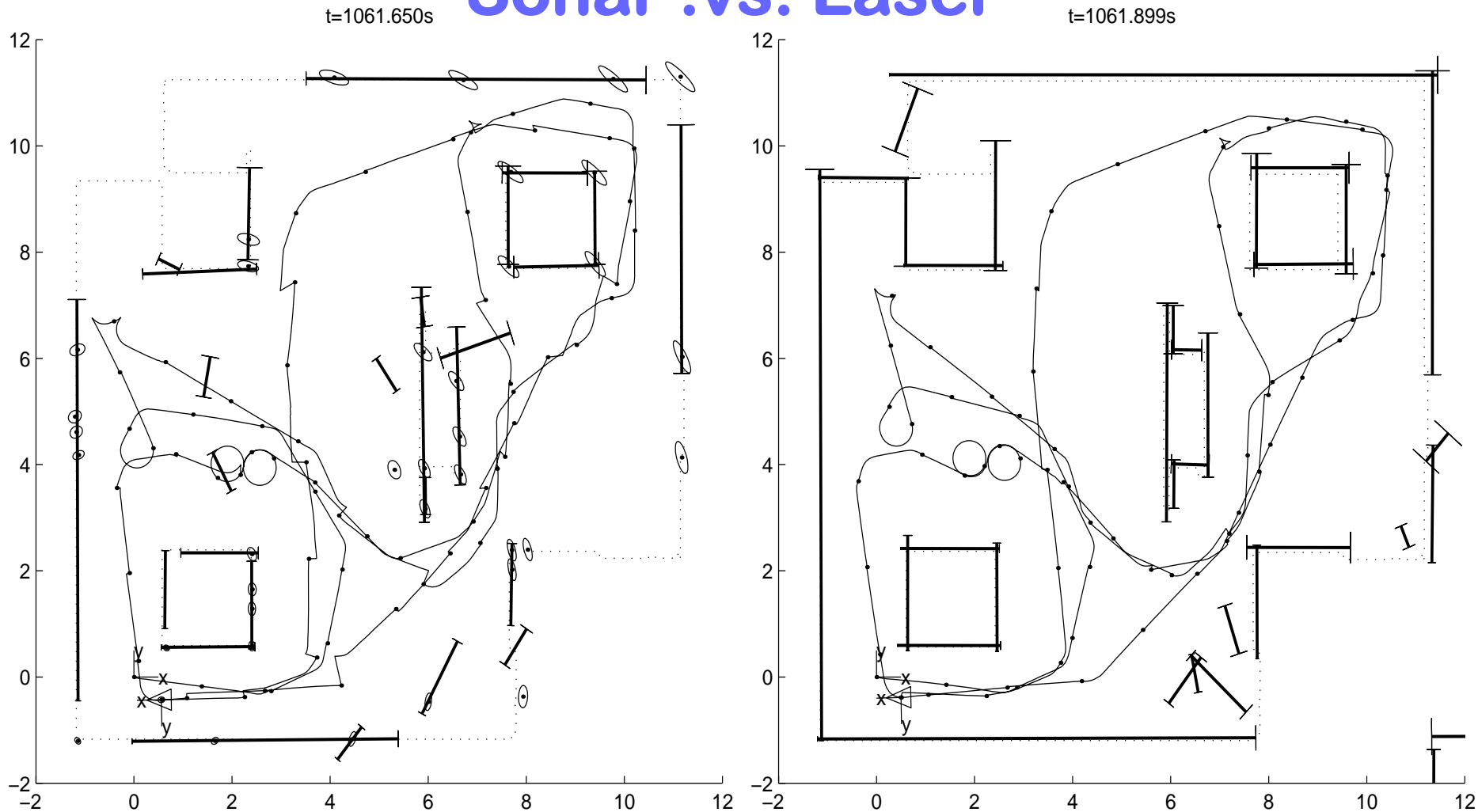


Joined map

After matching and fusion

J.D. Tardós, J. Neira, P.M. Newman and J.J. Leonard, Robust Mapping and Localization in Indoor Environments using Sonar Data, The Int. Journal of Robotics Research, Vol. 21, No. 4, April, 2002, pp 311 –330

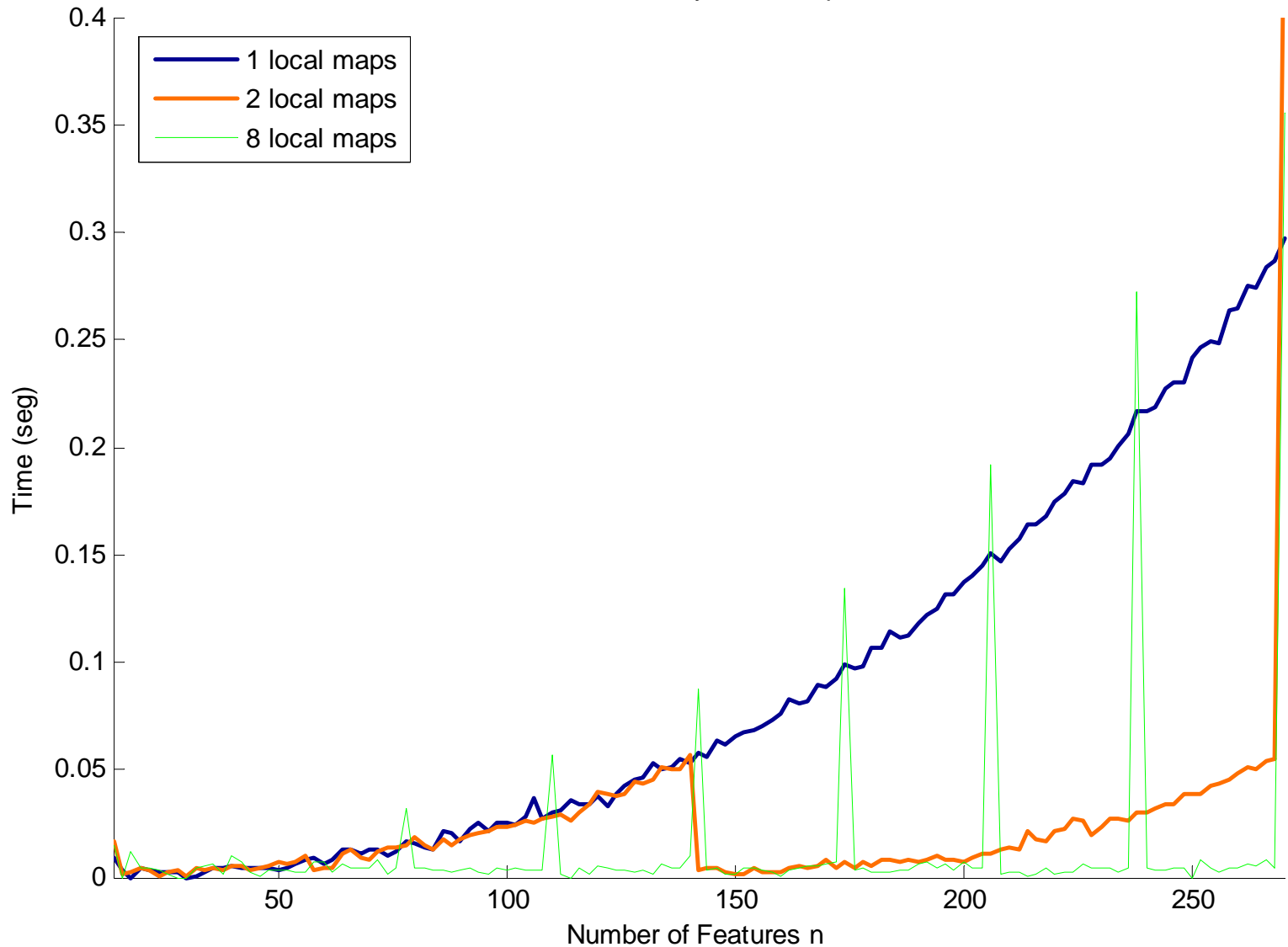
Sonar .vs. Laser



- Very robust map building with sonar
- Local maps: 60x faster than standard SLAM

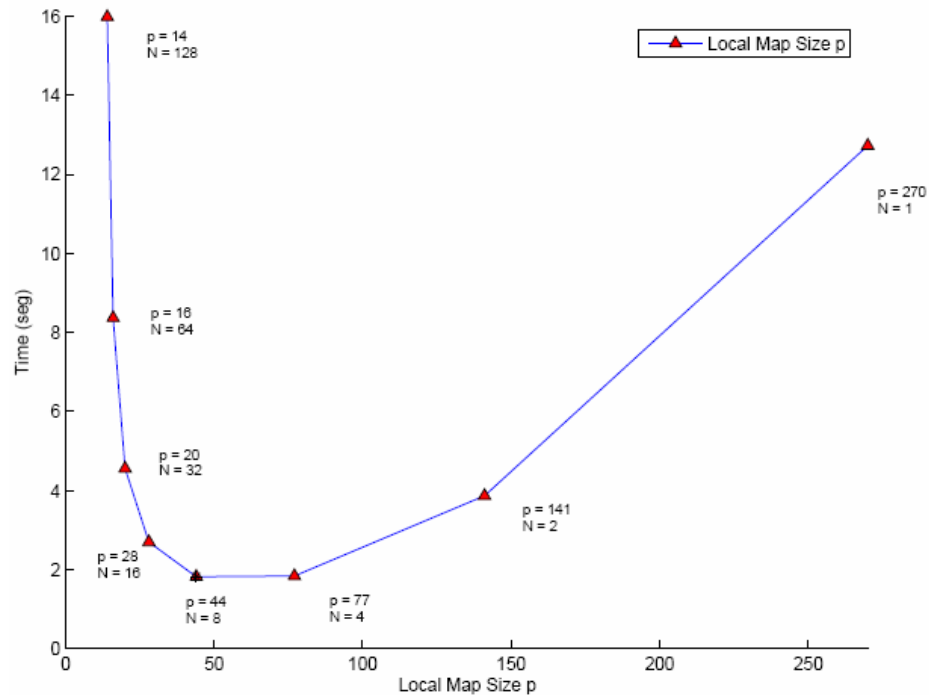
Map Joining

Mean Time by Local Maps



Local map size

- Size matters!



Computational cost for
different map sizes

Local map size

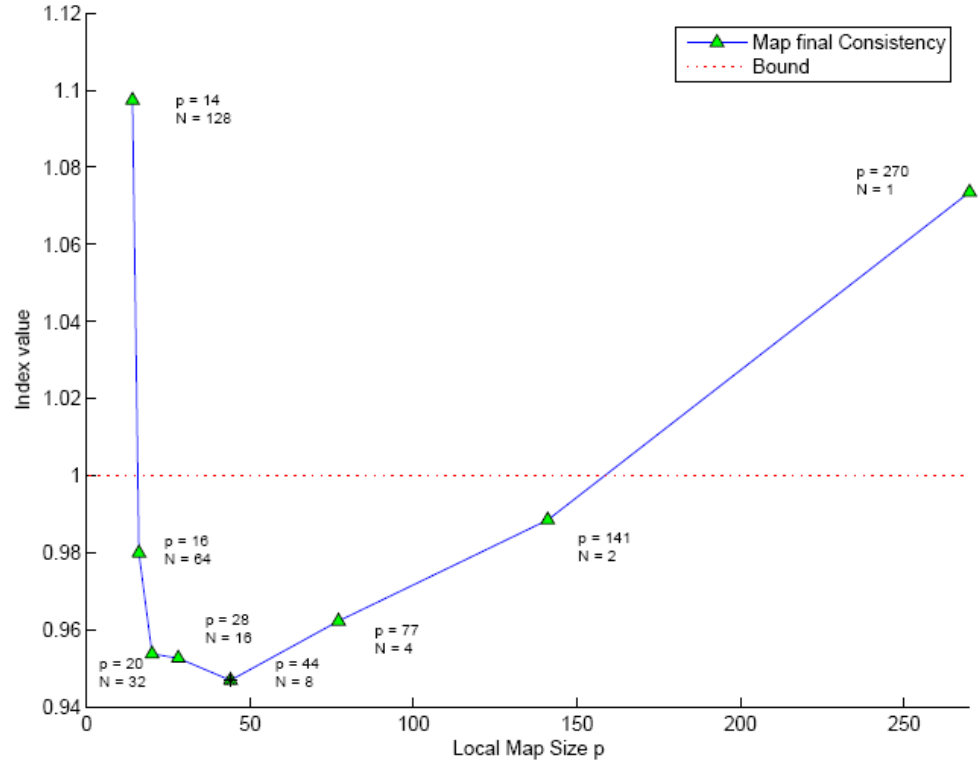
- Monte Carlo runs

$$D^2 = (\mathbf{x} - \hat{\mathbf{x}})^T \mathbf{P}^{-1} (\mathbf{x} - \hat{\mathbf{x}})$$

$$D^2 \leq \chi_{r,1-\alpha}^2$$

- Consistency index

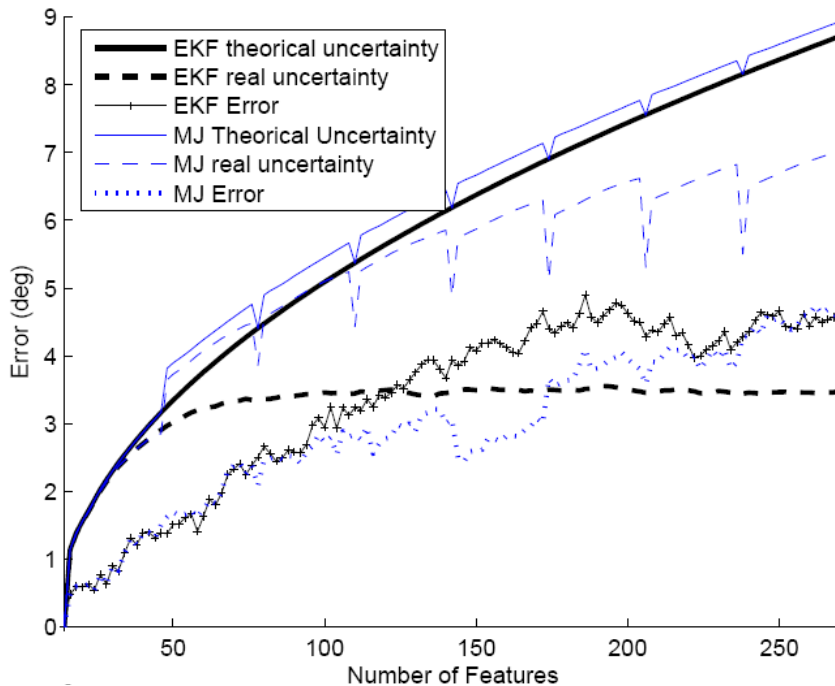
$$CI = \frac{D^2}{\chi_{r,1-\alpha}^2}$$



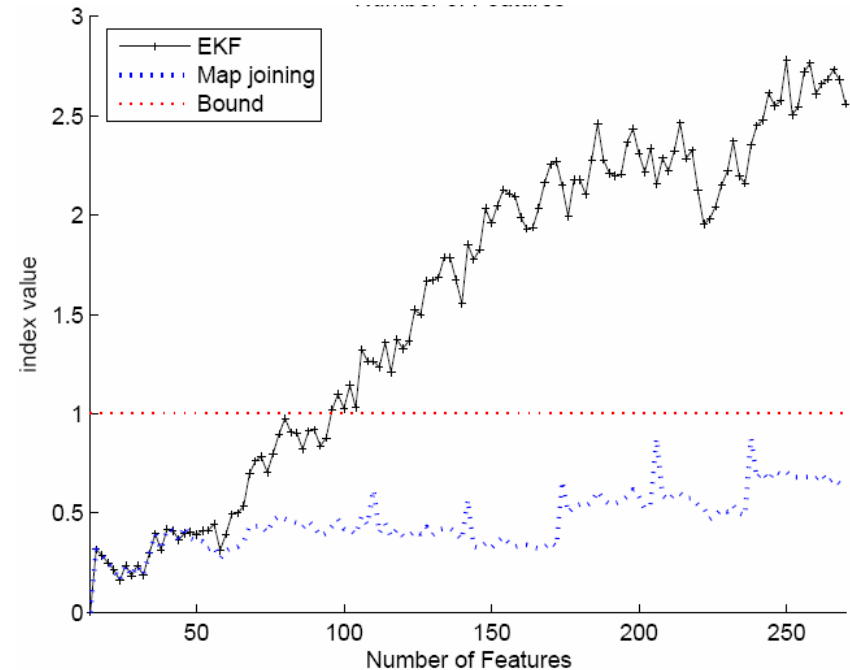
Consistency index for different map sizes

Local map size

- Mean vehicle orientation error for full EKF and Map Joining 8 local maps

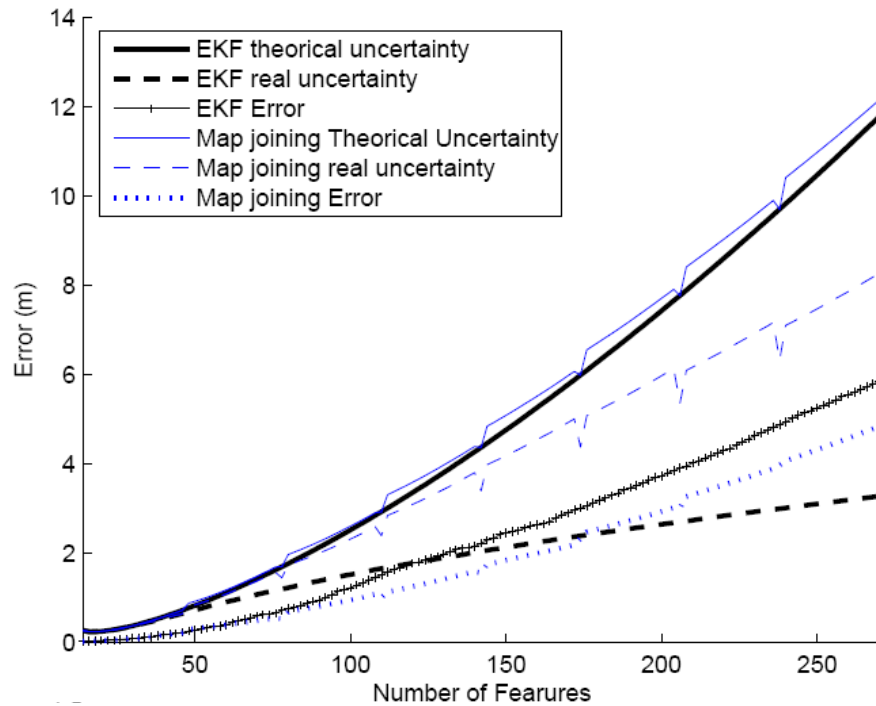


Vehicle orientation
error

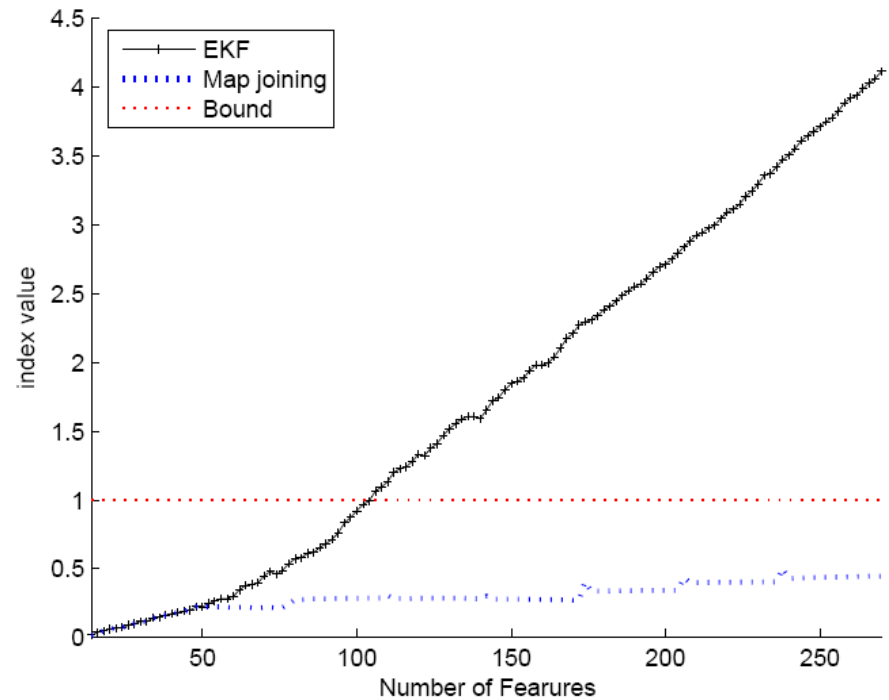


Vehicle orientation
consistency index

Local map size



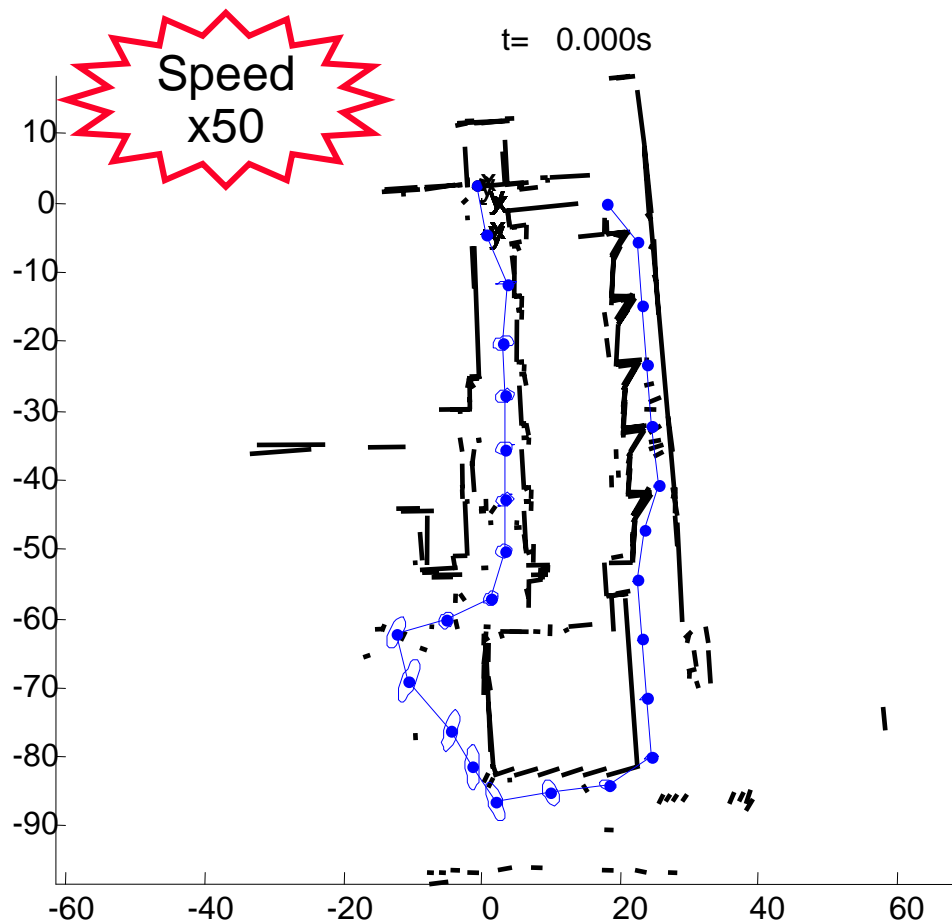
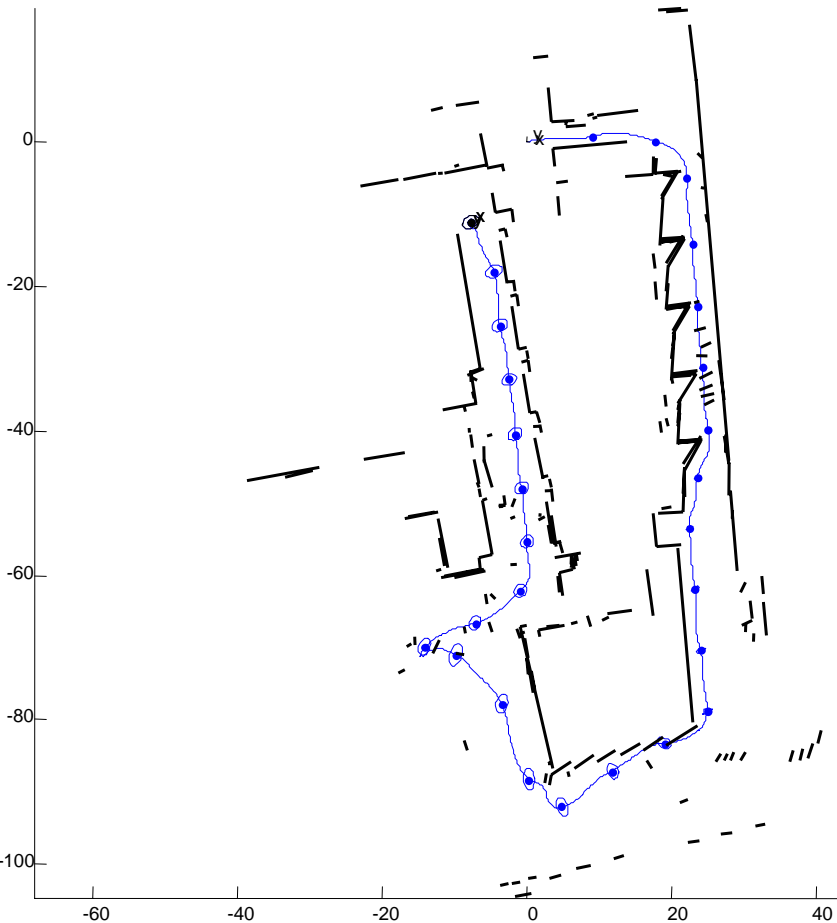
Mean feature
position error



Mean feature
consistency index

Map Joining closes the loop!

- One full SLAM run
- Map joining of 28 local maps

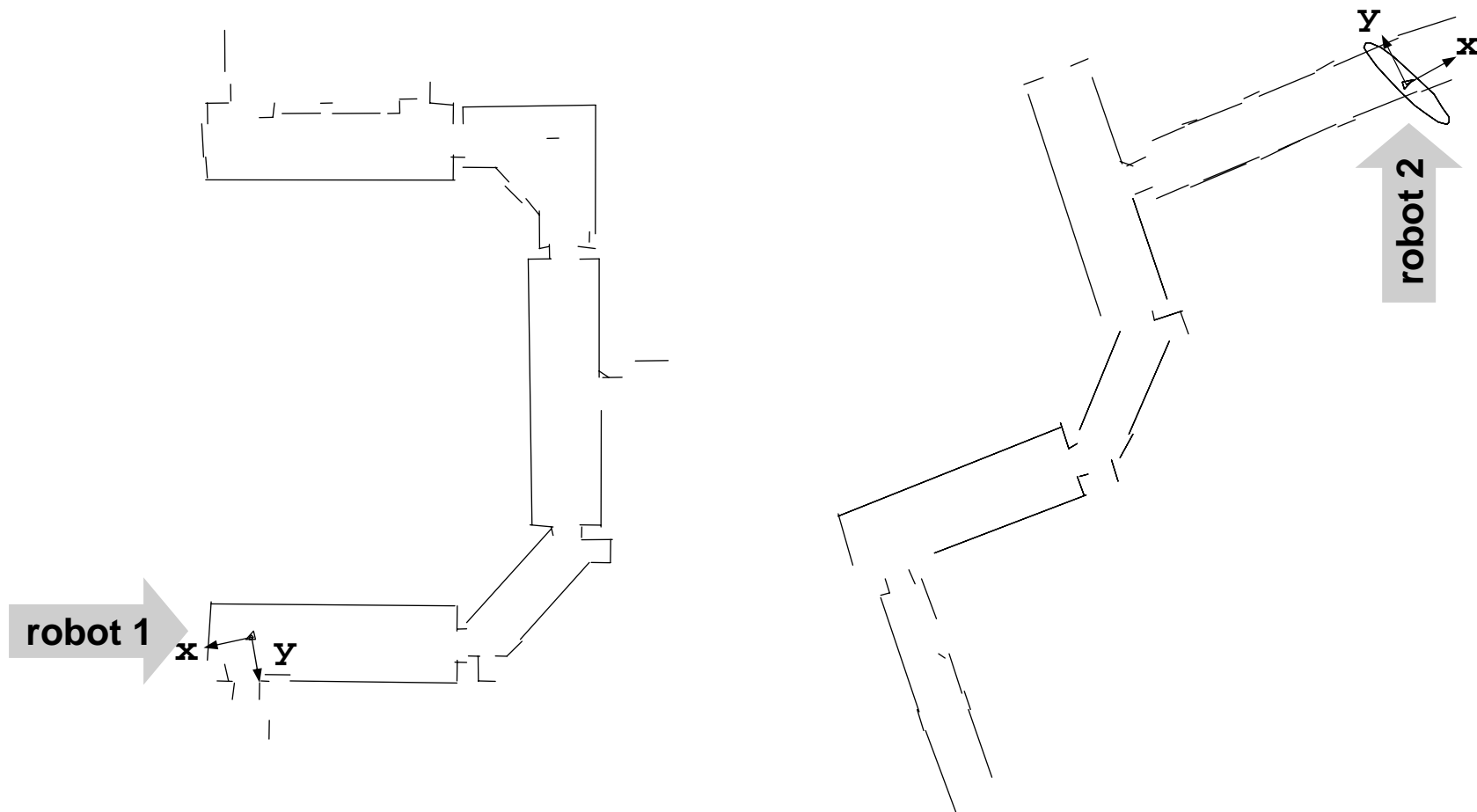


Local maps bound
linearization error effects

Building Large Maps

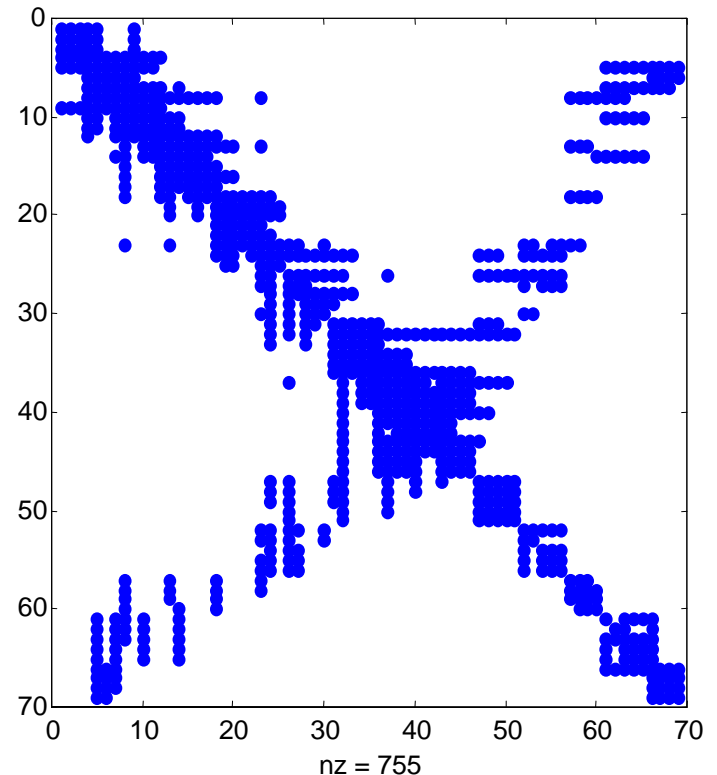
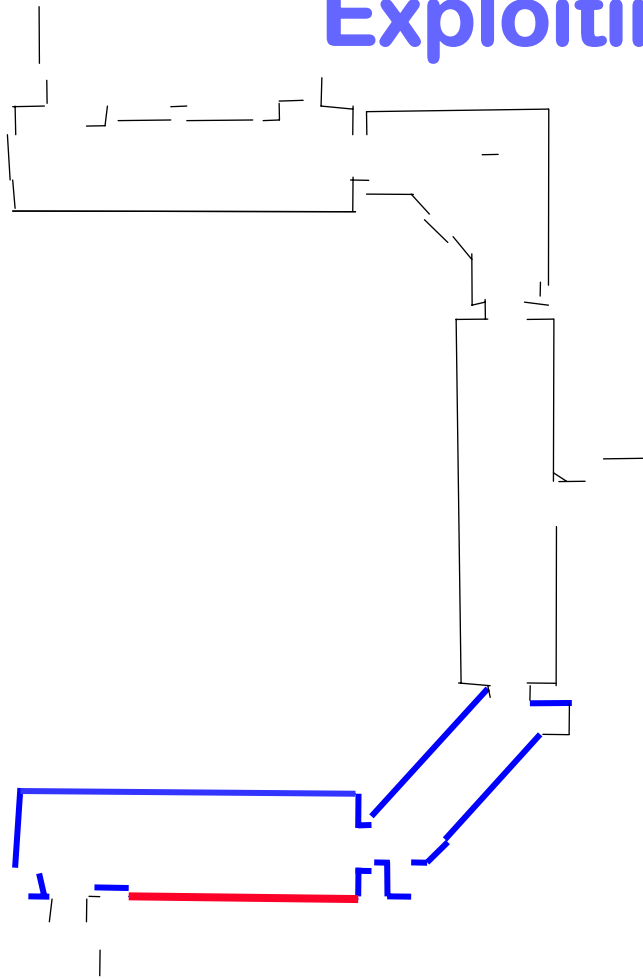
1. The curse of correlations
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Multirobot map building



Match, join and fuse to one full stochastic map

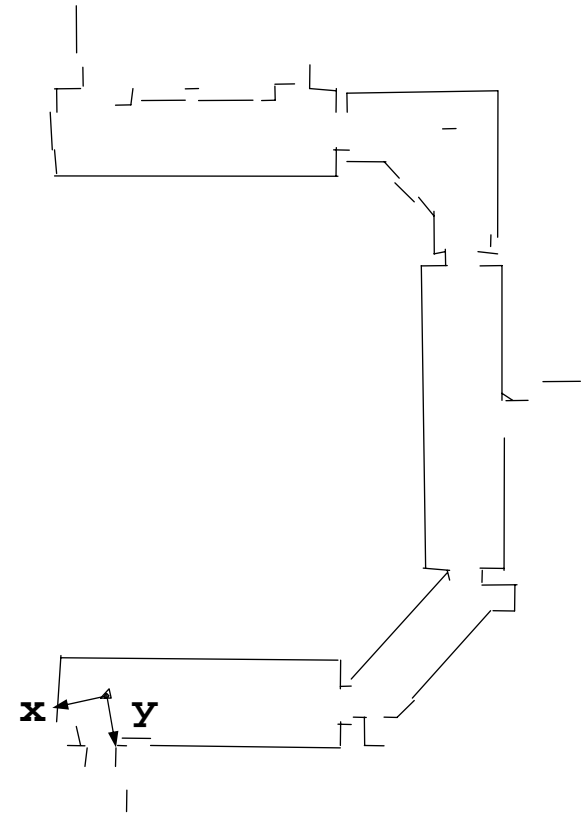
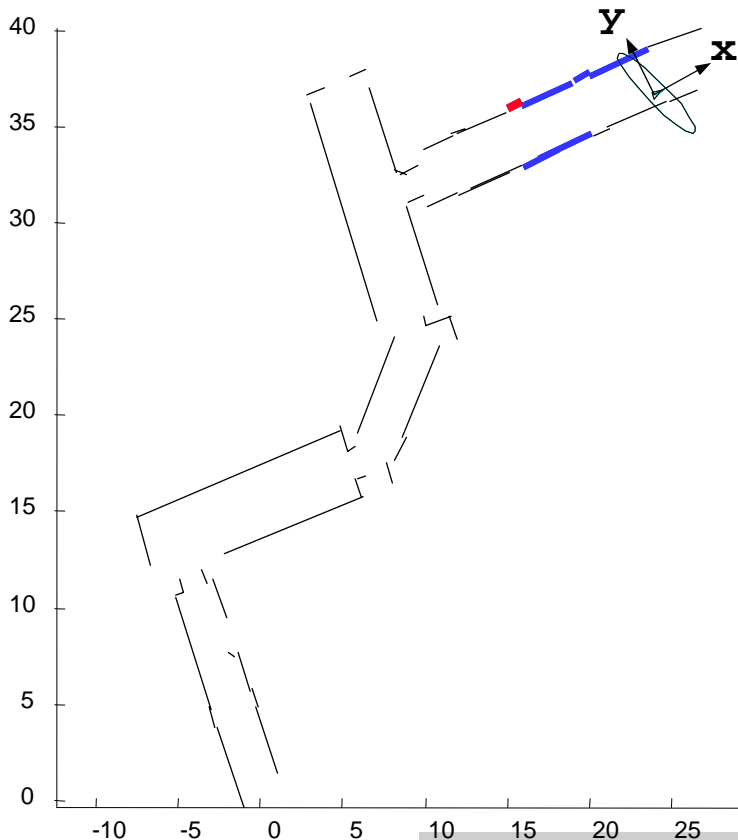
Exploiting locality



- Limit search in the global map to subsets of covisible features
- Locality makes relocation **linear** with the global map size

Multirobot map building

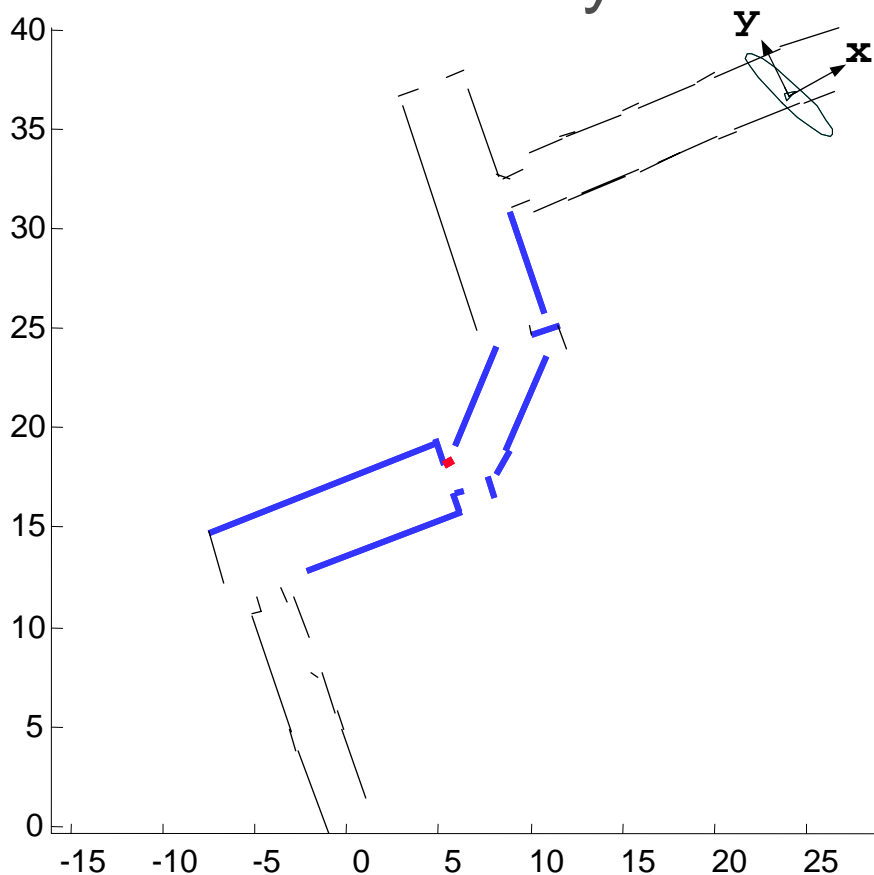
- Randomly select a **feature** in one map
- Try to associate its **covisible features** in the other map



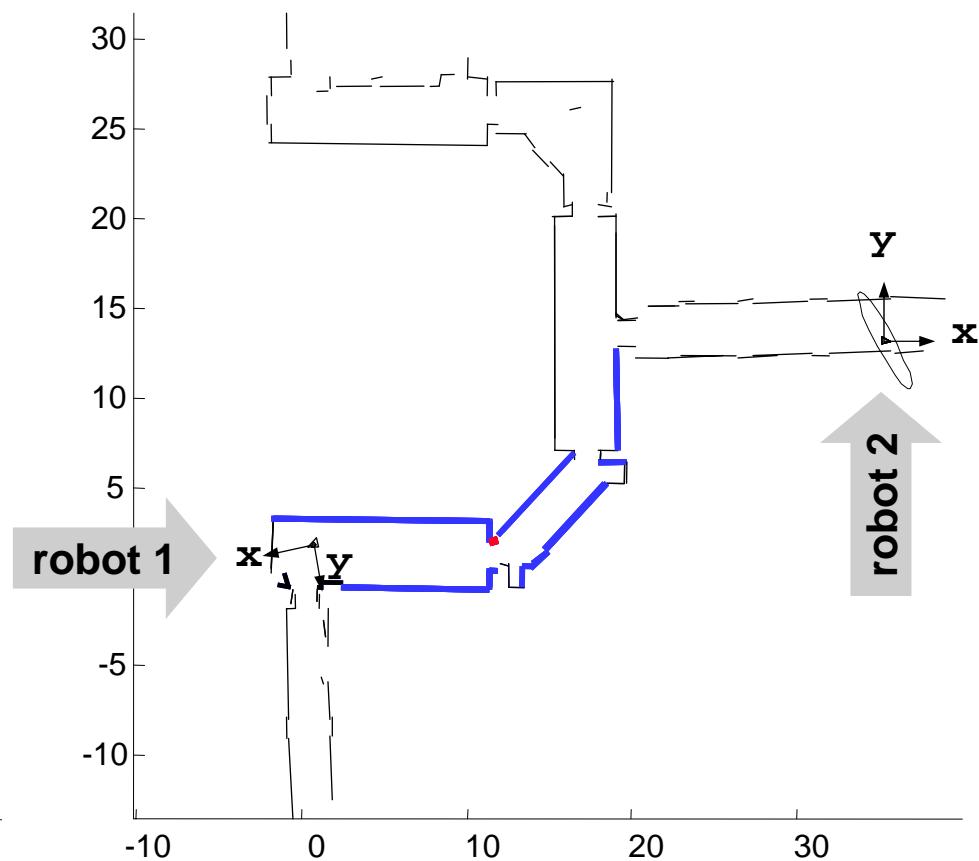
Unsuccessful try

Multirobot map building

- Successful try



- Final map



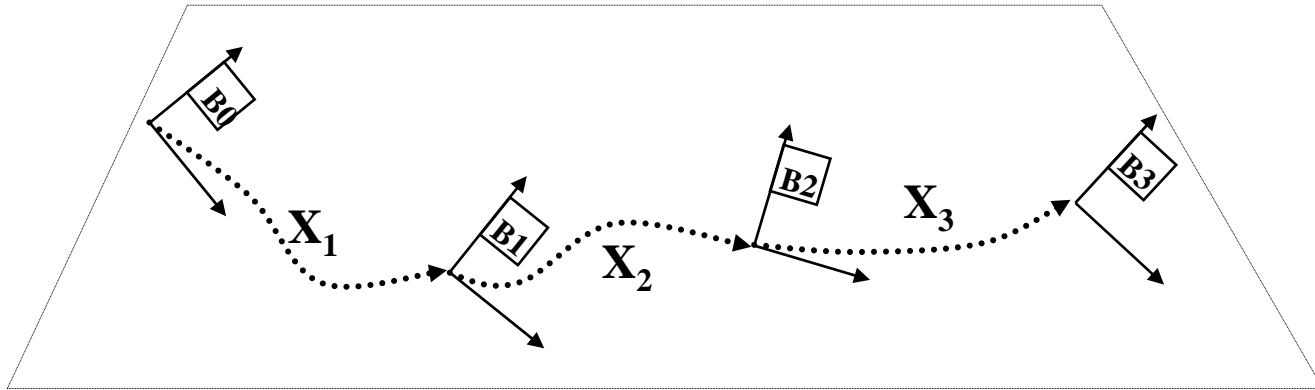
[illegible]

Building Large Maps

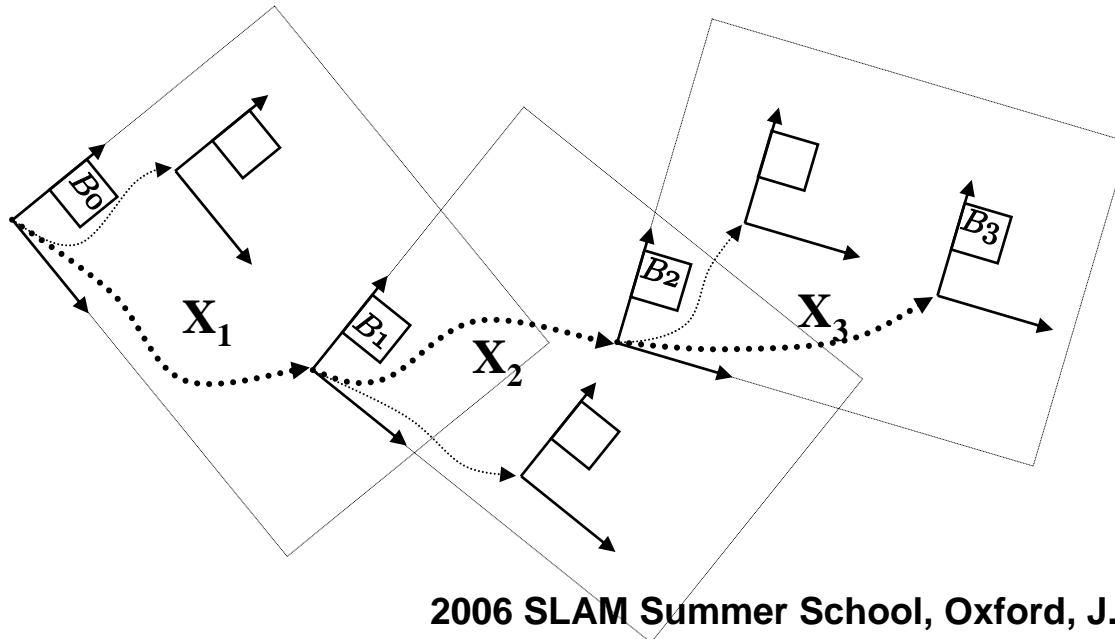
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Hierarchical SLAM

- Global level: adjacency graph and relative stochastic map



- Local level: statistically independent local maps



Hierarchical SLAM

- Local maps:

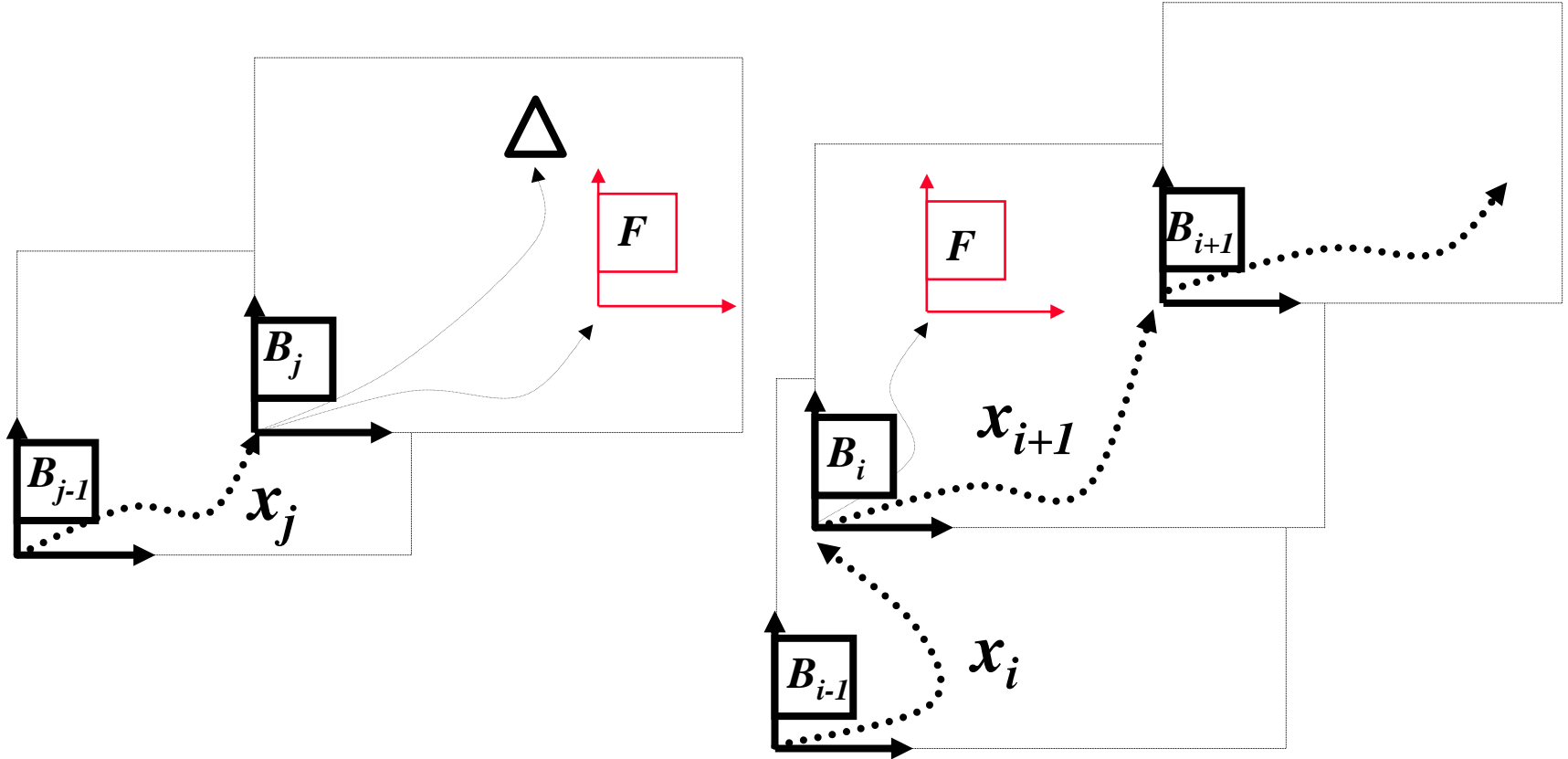
$$\hat{\mathbf{x}}_{\mathcal{F}}^B = \begin{bmatrix} \hat{\mathbf{x}}_R^B \\ \vdots \\ \hat{\mathbf{x}}_{F_n}^B \end{bmatrix}; \mathbf{P}_{\mathcal{F}} = \begin{bmatrix} \mathbf{P}_R^B & \cdots & \mathbf{P}_{RF_n}^B \\ \vdots & \ddots & \vdots \\ \mathbf{P}_{F_n R}^B & \cdots & \mathbf{P}_{F_n F_n}^B \end{bmatrix}$$

- Global relative map:

$$\hat{\mathbf{x}} = \begin{bmatrix} \vdots \\ \hat{\mathbf{x}}_i \\ \vdots \end{bmatrix}; \mathbf{P} = \begin{bmatrix} . & 0 & 0 \\ 0 & \mathbf{P}_i & 0 \\ 0 & 0 & . \end{bmatrix}$$

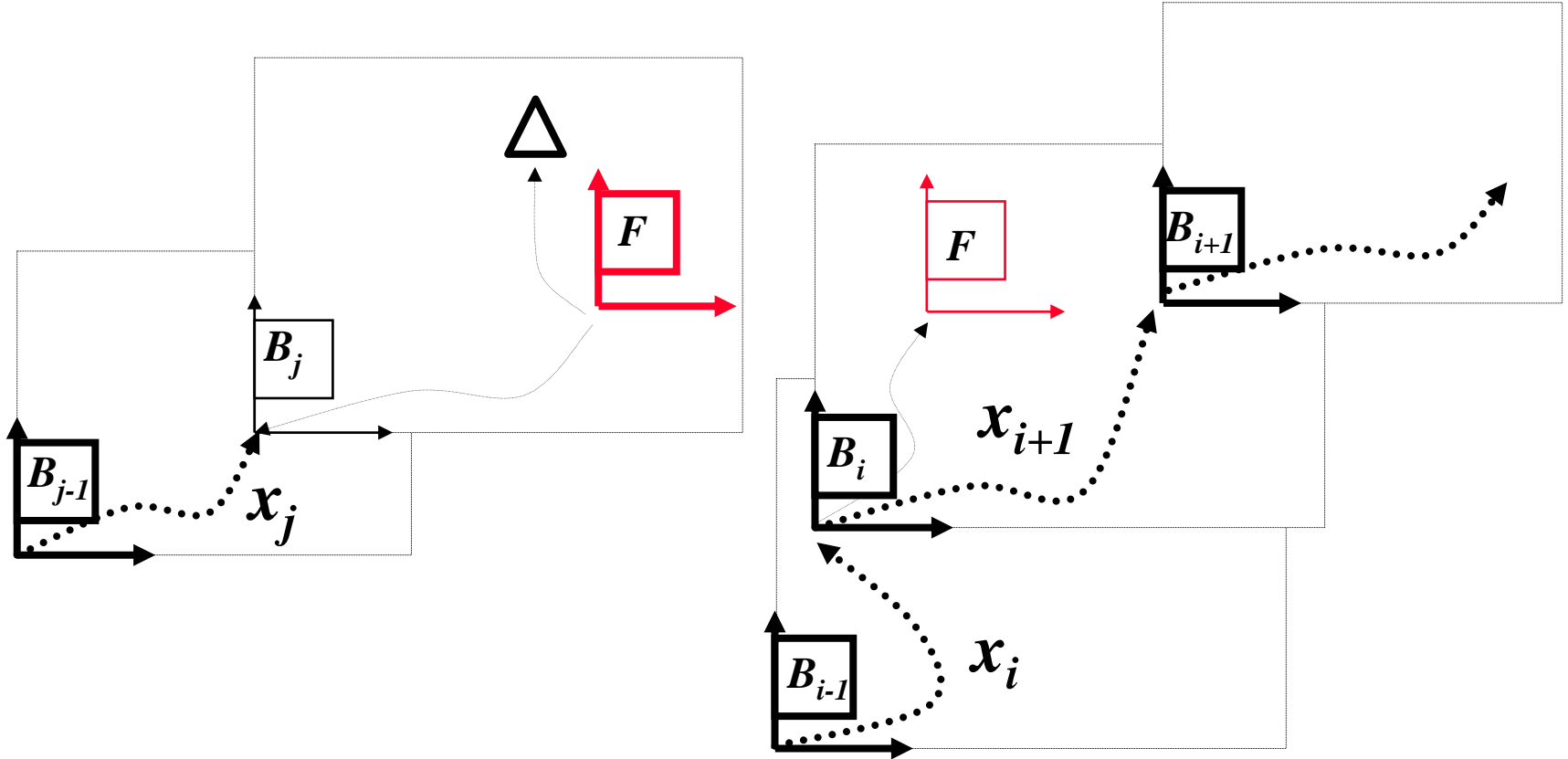
Block diagonal

Loop closing



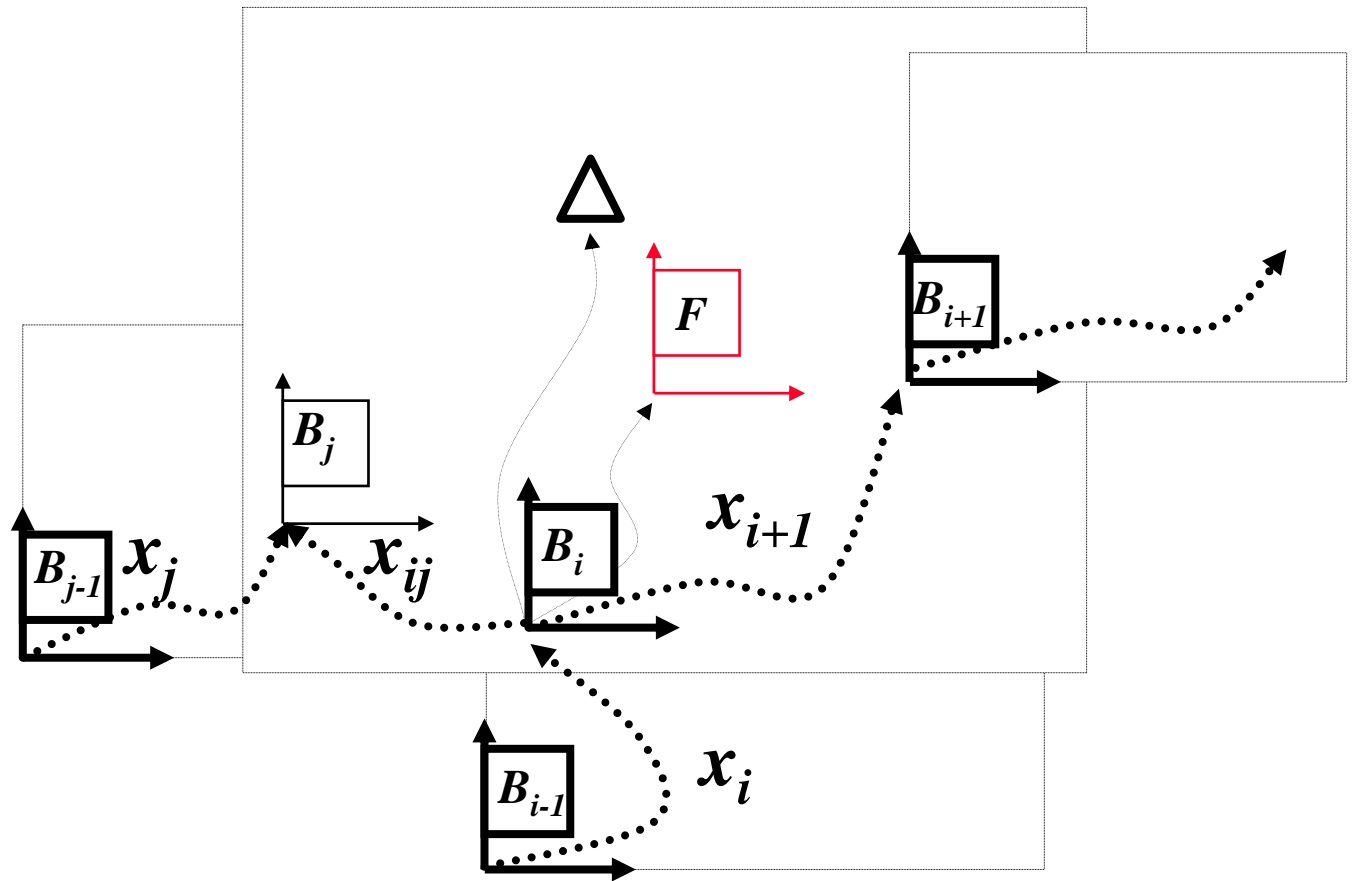
Find a common reference
between maps i and j

Loop closing



Change base reference of
map i to F .

Loop closing



Join maps i and j .

Hierarchical SLAM

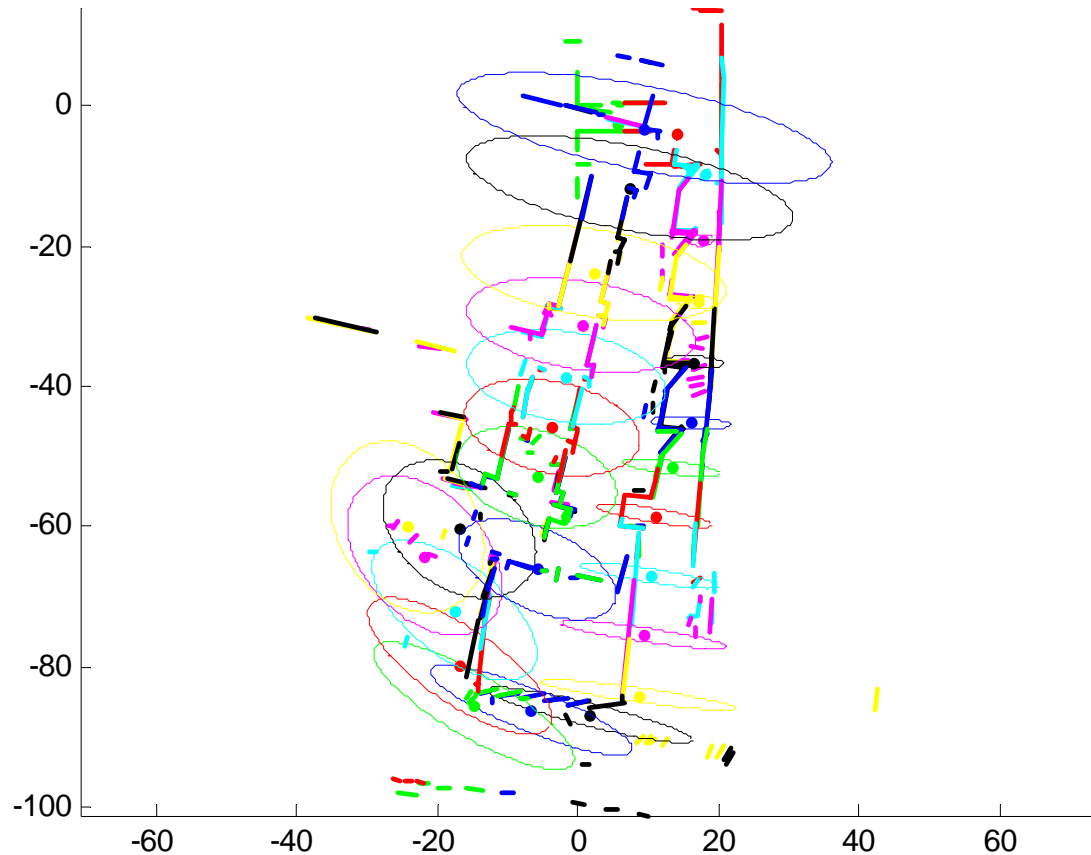
- Global relative map before loop closing:

$$\hat{\mathbf{x}} = \begin{bmatrix} \vdots \\ \hat{\mathbf{x}}_{i+1} \\ \vdots \\ \hat{\mathbf{x}}_j \end{bmatrix}; \mathbf{P} = \begin{bmatrix} . & 0 & 0 & 0 \\ 0 & \mathbf{P}_{i+1} & 0 & 0 \\ 0 & 0 & . & 0 \\ 0 & 0 & 0 & \mathbf{P}_j \end{bmatrix}$$

- After loop closing:

$$\hat{\mathbf{x}} = \begin{bmatrix} \vdots \\ \hat{\mathbf{x}}_{i+1} \\ \hat{\mathbf{x}}_{ij} \\ \vdots \\ \hat{\mathbf{x}}_j \end{bmatrix}; \mathbf{P} = \begin{bmatrix} . & 0 & 0 & 0 & 0 \\ 0 & \mathbf{P}_{i+1} & \mathbf{P}_{i+1,ij} & 0 & 0 \\ 0 & \mathbf{P}_{i+1,ij}^T & \mathbf{P}_{ij} & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & \mathbf{P}_j \end{bmatrix}$$

Imposing loop constraints



$$h(\mathbf{x}) \equiv \mathbf{x}_1 \oplus \mathbf{x}_2 \oplus \cdots \oplus \mathbf{x}_{n-1} \oplus \mathbf{x}_n$$

Nonlinear constrained optimization

- Minimize corrections to the global map, subject to the loop constraint:

$$\min_{\mathbf{x}} \frac{1}{2}(\mathbf{x} - \hat{\mathbf{x}})^T \mathbf{P}^{-1}(\mathbf{x} - \hat{\mathbf{x}})$$
$$\mathbf{h}(\mathbf{x}) = 0$$

- Sequential Quadratic Programming (SQP) :

$$\mathbf{H}_i = \left[\left. \frac{\partial \mathbf{h}}{\partial \mathbf{x}_1} \right|_{\hat{\mathbf{x}}_i} \left. \frac{\partial \mathbf{h}}{\partial \mathbf{x}_2} \right|_{\hat{\mathbf{x}}_i} \cdots \left. \frac{\partial \mathbf{h}}{\partial \mathbf{x}_{n-1}} \right|_{\hat{\mathbf{x}}_i} \left. \frac{\partial \mathbf{h}}{\partial \mathbf{x}_n} \right|_{\hat{\mathbf{x}}_i} \right]$$

$$\mathbf{P}_i = \mathbf{P}_0 - \mathbf{P}_0 \mathbf{H}_i^T \left(\mathbf{H}_i \mathbf{P}_0 \mathbf{H}_i^T \right)^{-1} \mathbf{H}_i \mathbf{P}_0$$

$$\hat{\mathbf{x}}_{i+1} = \hat{\mathbf{x}}_i - \mathbf{P}_i \mathbf{P}_0^{-1} (\hat{\mathbf{x}}_i - \hat{\mathbf{x}}_0) - \mathbf{P}_0 \mathbf{H}_i^T \left(\mathbf{H}_i \mathbf{P}_0 \mathbf{H}_i^T \right)^{-1} \hat{\mathbf{h}}_i$$

» Iterate until convergence

Nonlinear constrained optimization

- A more efficient version:

$$\hat{\mathbf{x}}_{i+1} = \hat{\mathbf{x}}_0 + \mathbf{P}_0 \mathbf{H}_i^T \left(\mathbf{H}_i \mathbf{P}_0 \mathbf{H}_i^T \right)^{-1} \left(\mathbf{H}_i (\hat{\mathbf{x}}_i - \hat{\mathbf{x}}_0) - \hat{\mathbf{h}}_i \right)$$

» Iterate until convergence

- Complexity:
 - \mathbf{P}_0 is block diagonal
 - \mathbf{H}_i is sparse with nonzeros only for the maps in the loop
 - The iteration is linear with the number of maps in the loop
- Convergence:
 - Converges in 2 or 3 iterations (for loops around 300m)
 - For bigger errors ??

Solution 2 (for EKF fans)

- We could impose the loop constraints using an imprecise measurement function:

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{w} = \mathbf{0}$$

- With Covariance:

$$\mathbf{P}_z = Cov(\mathbf{w}) = \begin{bmatrix} \mathbf{P}_{z_1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{z_2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{P}_{z_l} \end{bmatrix}$$

- Estimated errors in closing the loops:

$$\mathbf{h}(\hat{\mathbf{x}}) = \begin{bmatrix} \mathbf{h}_1(\hat{\mathbf{x}}) \\ \mathbf{h}_2(\hat{\mathbf{x}}) \\ \vdots \\ \mathbf{h}_l(\hat{\mathbf{x}}) \end{bmatrix}$$

Iterated Extended Kalman Filter

- Jacobian of the measurement function:

$$\mathbf{H}_i = \left[\left. \frac{\partial \mathbf{h}}{\partial \mathbf{x}_1} \right|_{\hat{\mathbf{x}}_i} \left. \frac{\partial \mathbf{h}}{\partial \mathbf{x}_2} \right|_{\hat{\mathbf{x}}_i} \cdots \left. \frac{\partial \mathbf{h}}{\partial \mathbf{x}_{n-1}} \right|_{\hat{\mathbf{x}}_i} \left. \frac{\partial \mathbf{h}}{\partial \mathbf{x}_n} \right|_{\hat{\mathbf{x}}_i} \right]$$

- Iterated EKF equations:

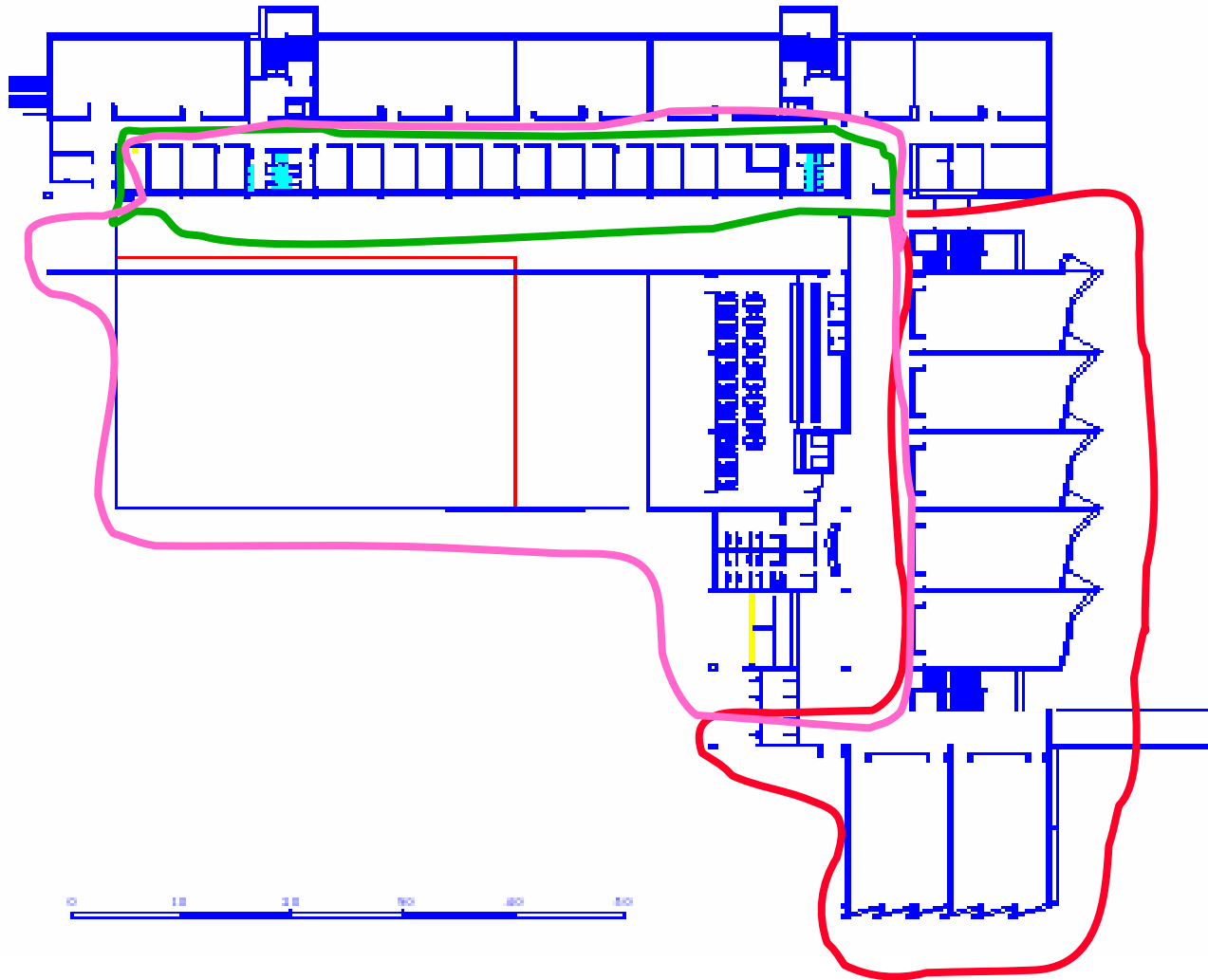
$$\mathbf{P}_i = \mathbf{P}_0 - \mathbf{P}_0 \mathbf{H}_i^T \left[\mathbf{H}_i \mathbf{P}_0 \mathbf{H}_i^T + \mathbf{P}_z \right]^{-1} \mathbf{H}_i \mathbf{P}_0$$

$$\hat{\mathbf{x}}_{i+1} = \hat{\mathbf{x}}_i - \mathbf{P}_i \mathbf{P}_0^{-1} (\hat{\mathbf{x}}_i - \hat{\mathbf{x}}_0) + \mathbf{P}_0 \mathbf{H}_i^T \left(\mathbf{H}_i \mathbf{P}_0 \mathbf{H}_i^T + \mathbf{P}_z \right)^{-1} (\mathbf{z} - \hat{\mathbf{h}}_i)$$

- With exact loop constraint, $\mathbf{z} = 0$ and $\mathbf{P}_z = 0$, IEFK is equivalent to nonlinear optimization with SQP

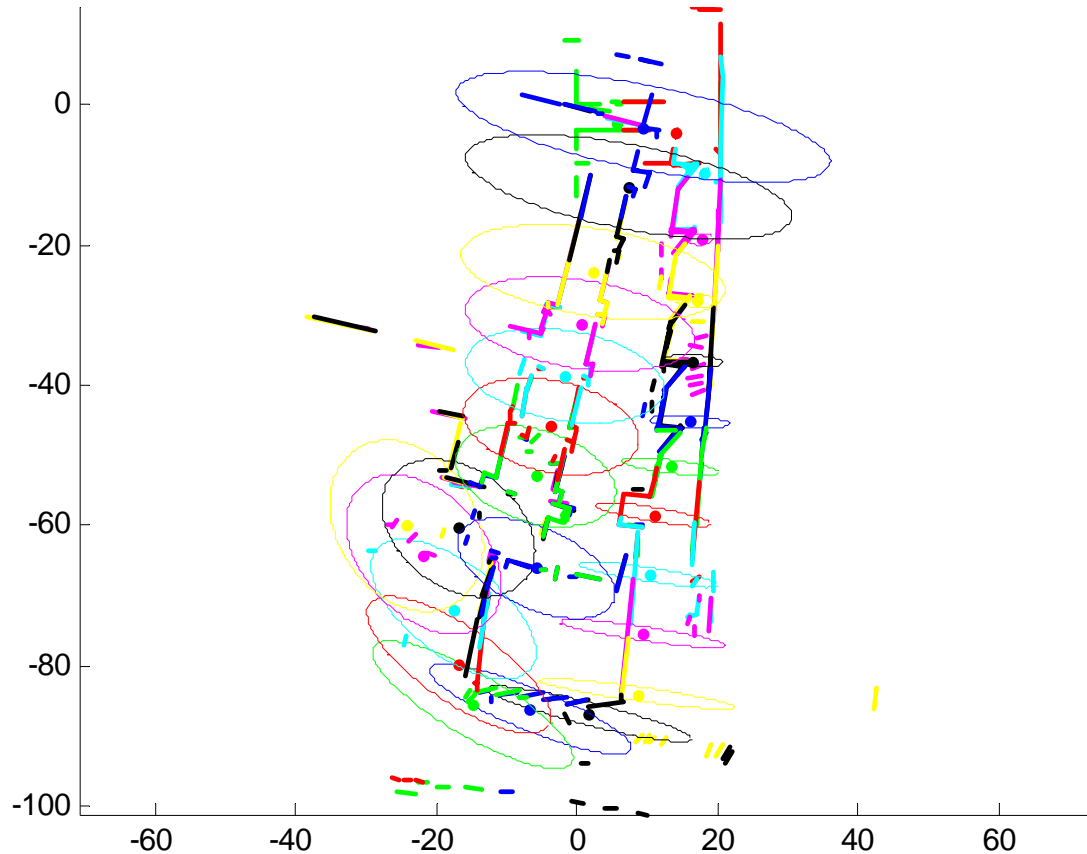
Multivehicle SLAM

- Ada Byron Building, UZ



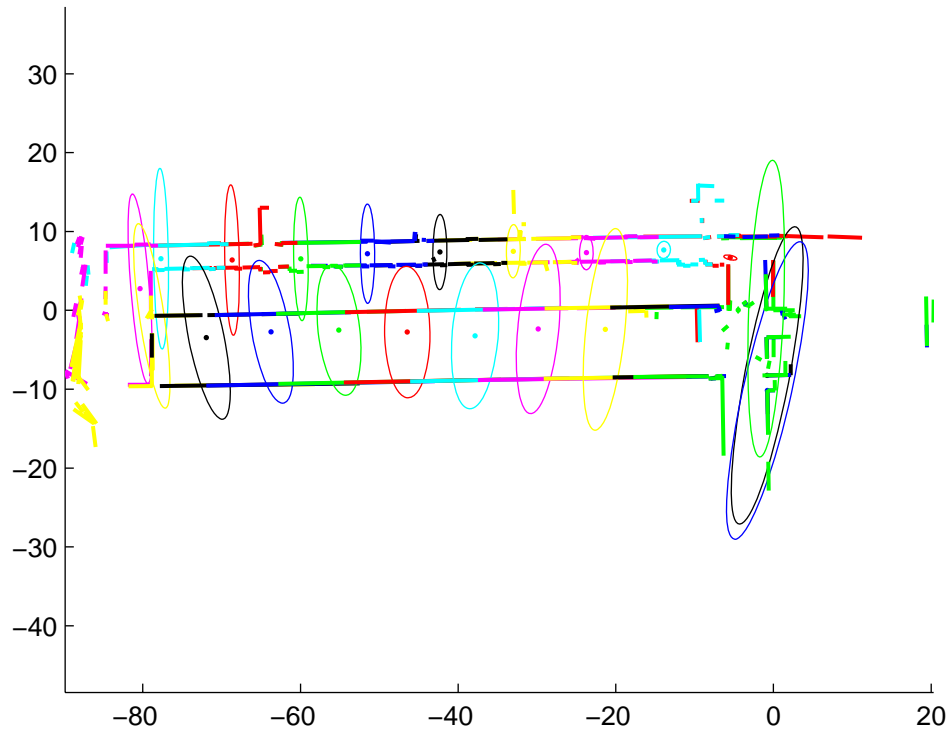
Experiments

- Local maps, first robot



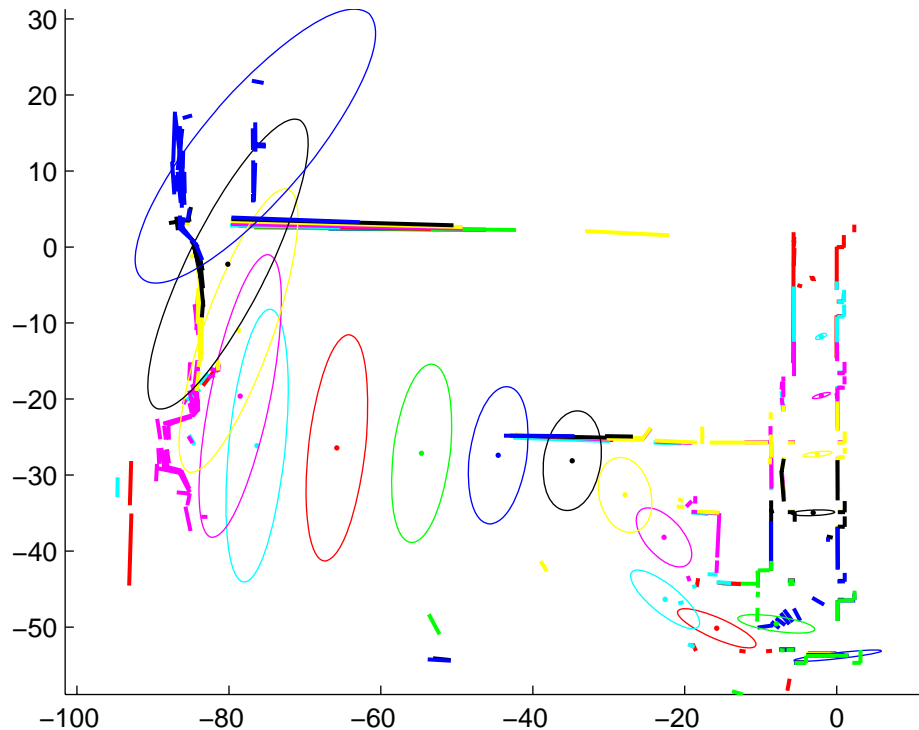
Experiments

- Local maps, second robot



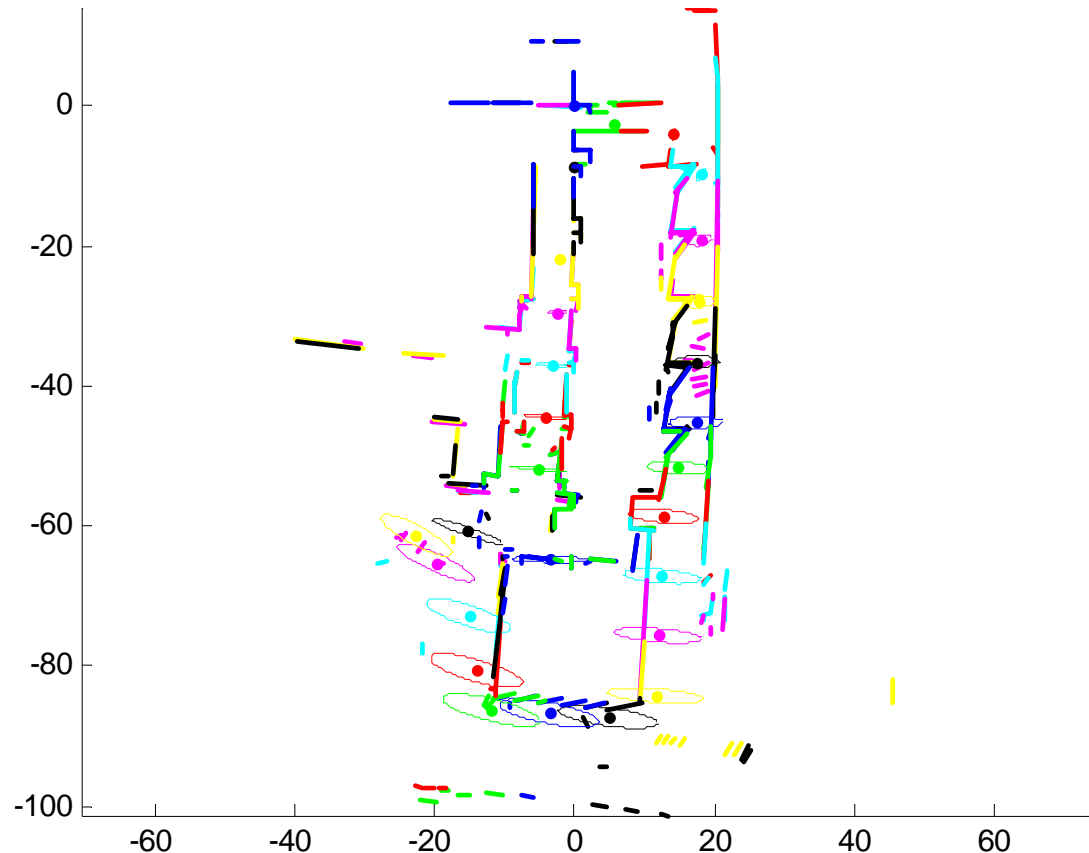
Experiments

- Local maps, third robot



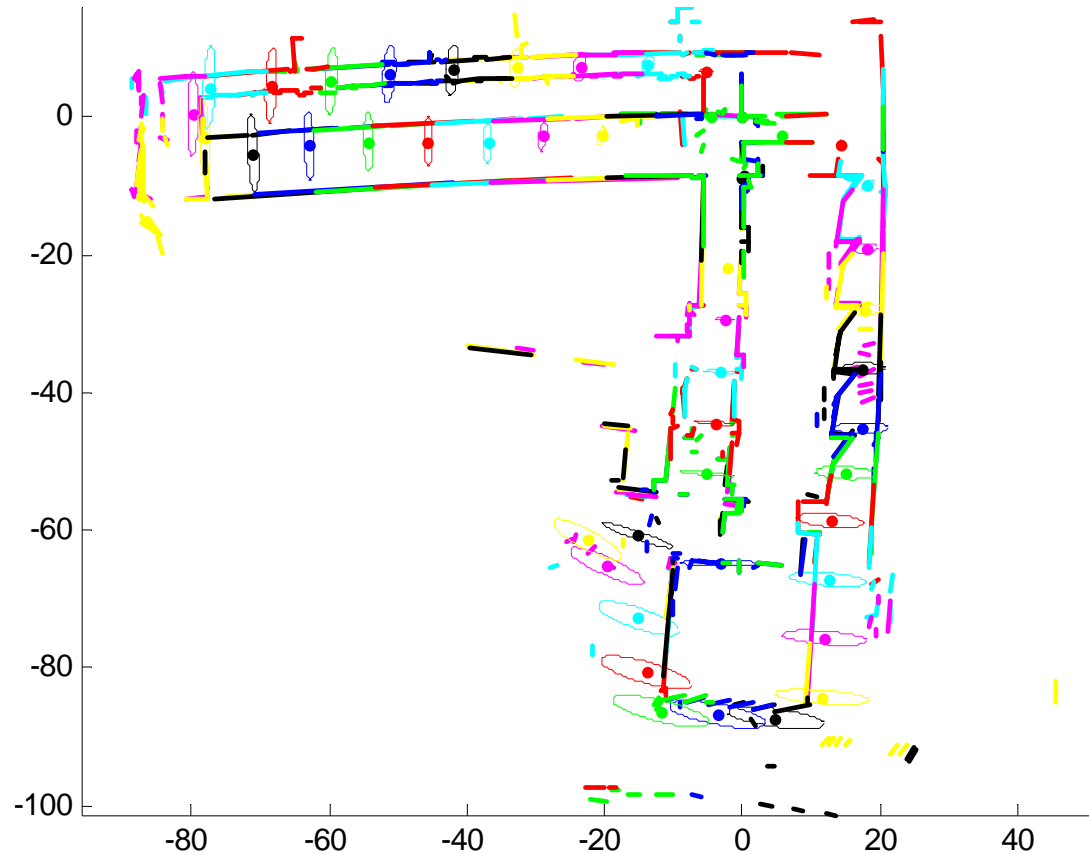
Hierarchical SLAM

- Imposing loop constraint:



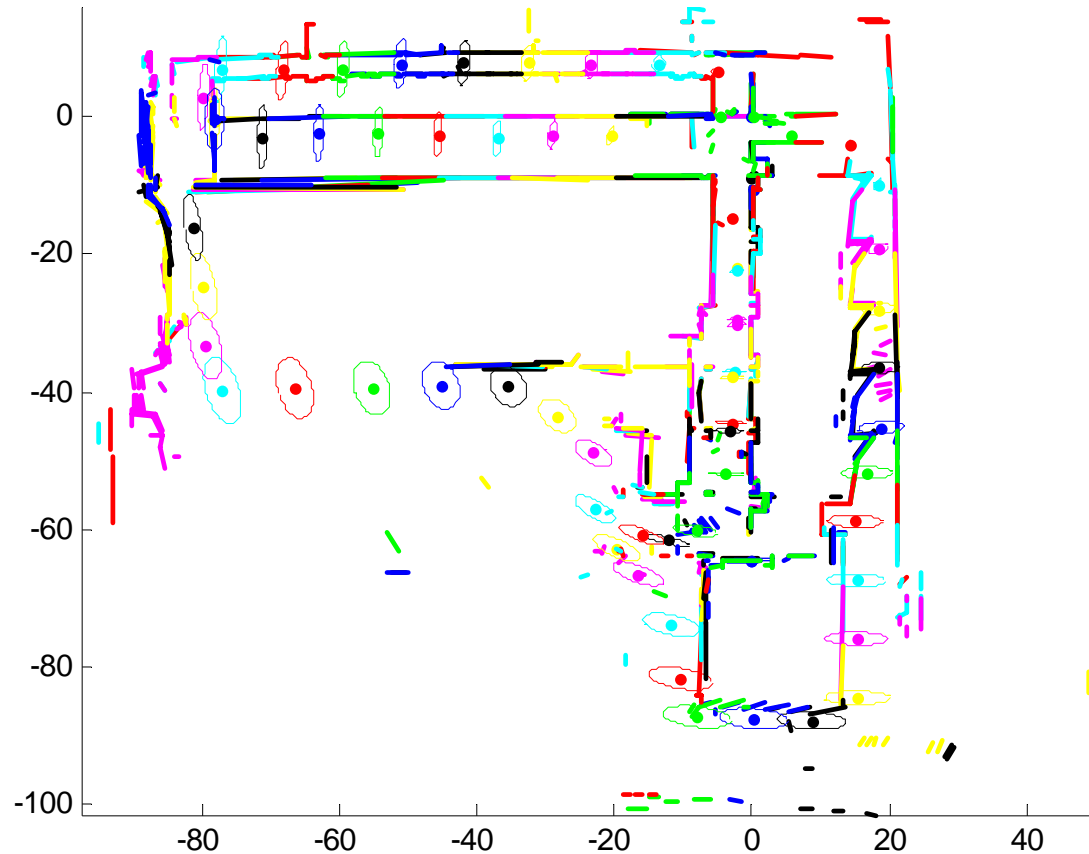
Hierarchical SLAM

- Second robot



Hierarchical SLAM

- Third robot



Results in less than 1s

C. Estrada, J. Neira, J.D. Tardós, Hierarchical SLAM: real-time accurate mapping of large environments. IEEE Transactions on Robotics, 21 (4), pp 588- 596, Aug. 2005

Building Large Maps

1. The curse of correlations
2. Independent local maps
3. Map joining
4. Multivehicle SLAM
5. Hierarchical SLAM
- 6. Conclusion**

Mapping large environments: Summary

- The inconsistency problem appears before the computational complexity problem !
- Local maps improve efficiency and consistency
- Hierarchical SLAM has a **robust, stable and local parametrization** that allows to efficiently maintain loop consistency for large loops

Conclusions

Mapping with Laser segments	Loop 30m	Loop 300m	Longer loop
EKF-SLAM Nearest Neighbor	weak	--- inconsistent	---
EKF-SLAM Joint Compatib.	very good	--- inconsistent	---
Map Joining Joint Compatib.	excellent	good	---
Hierarchical SLAM Relocation	overkill	excellent	future work

Recommended Readings

- J.D. Tardós, J. Neira, P. Newman, and J. Leonard. **Robust Mapping and Localization in Indoor Environments using Sonar Data**, Int. J. Robotics Research, Vol. 21, No. 4, April 2002, pp 311 –330
- C. Estrada, J. Neira, J.D. Tardós, **Hierarchical SLAM: real-time accurate mapping of large environments**. IEEE Transactions on Robotics, 21 (4), pp 588- 596, Aug. 2005
- J.A. Castellanos, J. Neira, J. D. Tardós, **Map Building and SLAM Algorithms**, in Shuzhi Sam Ge, Frank L. Lewis (eds): “Autonomous Mobile Robots: Sensing, Control, Decision-Making and Applications”, CRC Press, May 2006
- J.A. Castellanos, R. Martinez-Cantin, J.D. Tardós and J. Neira, **Robocentric Map Joining: Improving the Consistency of EKF-SLAM**, Robotics and Autonomous Systems, 2006 (to appear)
- L.M. Paz, J. Neira, **Optimal Local Map Size for EKF-based SLAM**, IEEE/RSJ Int.Conf.on Intelligent Robots and Systems, October 9-15, 2006 Beijing, China (to appear)