

1. Give a criterion to automatically estimate  $k$ , the number of dimensions of the eigenspace used in eigenspace learn. That is some criteria of choosing  $k$  automatically.
  
2. Assume that the only source of error in a simple stereo system is the error in estimating the disparity. Assume that this error is fixed and is at least 1 pixel. So if the stereo system says the disparity is 5 pixels it is really between 4 and 6 pixels. And if the stereo system says the disparity is 10 pixels then it is really between 9 and 11 pixels. The change in  $Z$  ( $\Delta Z$ ) due to this one pixel error in estimating the disparity is called the absolute error of the stereo system. Compute the ratio of  $\Delta Z(5 \text{ pixels}) / \Delta Z(10 \text{ pixels})$ . From this answer hypothesize a relationship between  $\Delta Z$  and disparity  $d$ . Prove that your hypothesis is true by computing the partial derivative of  $Z$  with respect to disparity  $d$ .
  
3. There is a simple stereo system with one camera is placed above the other in the  $y$  direction (not the  $x$  direction is as usual) by a distance of  $b$ . In such a case there is no rotation between the cameras, only a translation by a vector  $T = [0, b, 0]$ . First compute the essential matrix  $E$  in this case using the formula provided. Assume that both cameras have the same focal length  $f$ . Prove that in this case, for this given  $E$  that the epipolar lines are vertical. That is for a given point  $(p_b)^T$  in the bottom image prove that the epipolar line defined by the equation  $(p_t)^T E (p_b)^T = 0$  is a horizontal line. Here  $p_t$  is  $(x_t, y_t, f)$  and  $p_b$  is  $(x_b, y_b, f)$  which are the points in the top and bottom image plane. We are given  $p_b$  and  $E$ , and the unknown is  $p_t$ .
  
4. Assume that there is a 3D point  $X$  on a plane that is viewed by two cameras. The projection of this 3D point in camera one is defined by  $x = P X$ , and in camera two by  $x' = P' X$ . Here  $x = [u, v, 1]$  the pixel co-ordinates in image one of  $X$  and  $x' = [u', v', 1]$  the pixel co-ordinates in the other images,  $P$  and  $P'$  are the 3 by 4 projection matrices and  $X$  is a point in 3D space  $= [x, y, z, 1]$ . Prove

that in this case  $x = M x'$ , where  $M$  is a 3 by 3 matrix called a homography. Hint: Define the world co-ordinate frame for the 3D point  $X$  so that the  $x, y$  axis is on the plane. In other words  $X$  is a point on the plane implies that it is defined as  $X = [x, y, 0, 1]$  in homogeneous co-ordinates. Now write down the two projection equations with the elements of  $P$  being simply  $p_{11}, p_{12}, \dots, p_{34}$ , and  $P'$  being defined as  $p'_{11}, p'_{12}, \dots, p'_{34}$ .