"The efficacy of the Gaussian Model"

Discussion moderated by

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LAAS-CNRS
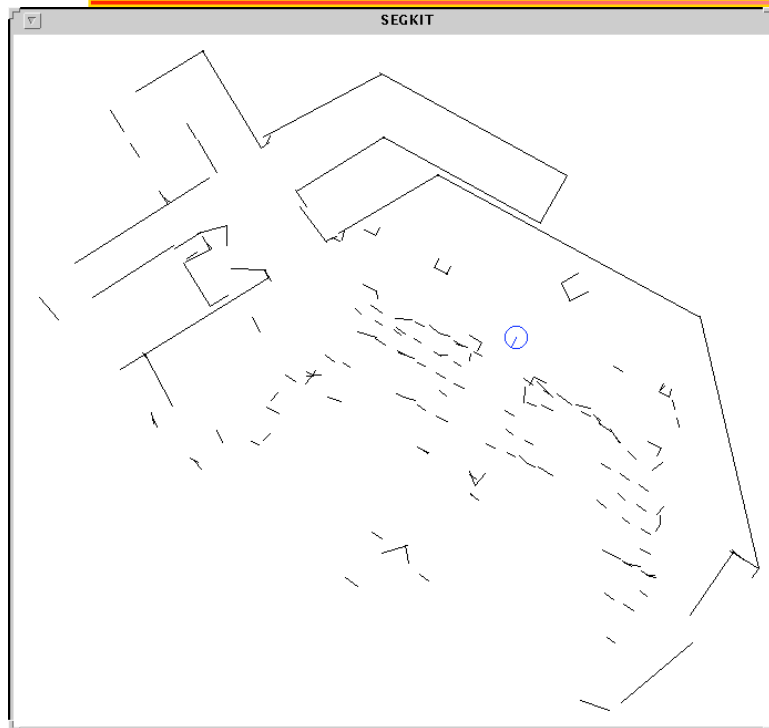
Toulouse, France

SLAM

- **SLAM = Representations + Filtering**

- **Bayesian Filters**
  - General formulation
  - Optimal solutions and Gaussianity
  - Non-optimal solutions and approximate solutions
    - EKF, UKF
    - Monte Carlo - aka Particle filter

- **The Future of SLAM...**

Representations and Maps

SEGKIT

Features
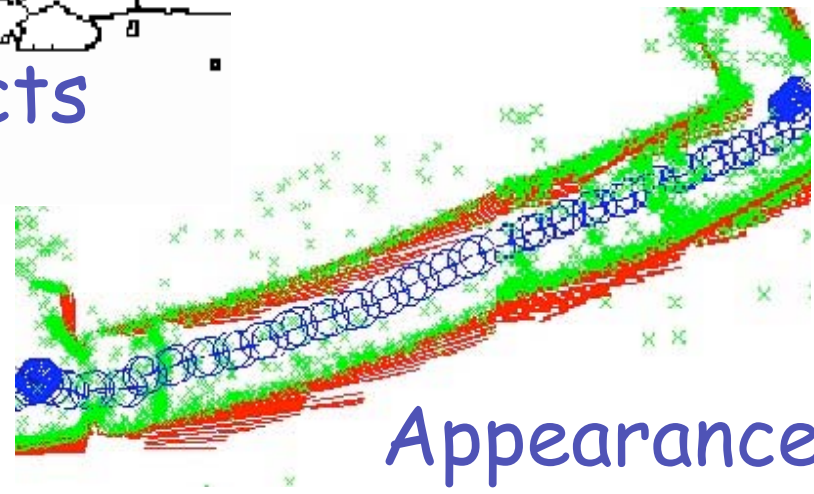
Topology

Objects

Grids

Appearance

Filtering: General Formulation

- State: $\mathbf{x}_k = \mathbf{f}_k(\mathbf{x}_{k-1}, \mathbf{v}_{k-1})$ (u dropped)

- Measure: $\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_k, \mathbf{w}_k)$

- Recursive estimation - Bayes:

$$p(\mathbf{x}_k|\mathbf{z}_{1:k}) = \frac{p(\mathbf{z}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{z}_{1:k-1})}{p(\mathbf{z}_k|\mathbf{z}_{1:k-1})}$$

With:
$$p(\mathbf{x}_k|\mathbf{z}_{1:k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1}, \mathbf{z}_{1:k-1})p(\mathbf{x}_{k-1}|\mathbf{z}_{1:k-1})d\mathbf{x}_{k-1}$$
Prediction

Or (Markov):
$$p(\mathbf{x}_k|\mathbf{z}_{1:k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1})p(\mathbf{x}_{k-1}|\mathbf{z}_{1:k-1})d\mathbf{x}_{k-1}$$

And: $p(\mathbf{z}_k|\mathbf{z}_{1:k-1}) = \int p(\mathbf{z}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{z}_{1:k-1})d\mathbf{x}_k$

Linear Case: Analytical Solution

$$\mathbf{x}_k = F_k \mathbf{x}_{k-1} + \mathbf{v}_{k-1} \qquad \mathbf{z}_k = H_k \mathbf{x}_k + \mathbf{w}_k$$
$$\mathcal{N}(\mathbf{v}_k; 0, Q_k) \qquad\qquad\qquad \mathcal{N}(\mathbf{n}_k; 0, R_k)$$

Prediction
$$m_{k|k-1} = F_k m_{k-1|k-1}$$

$$P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_{k-1}$$

Update:
$$m_{k|k} = m_{k|k-1} + K_k(z_k - H_k m_{k|k-1})$$
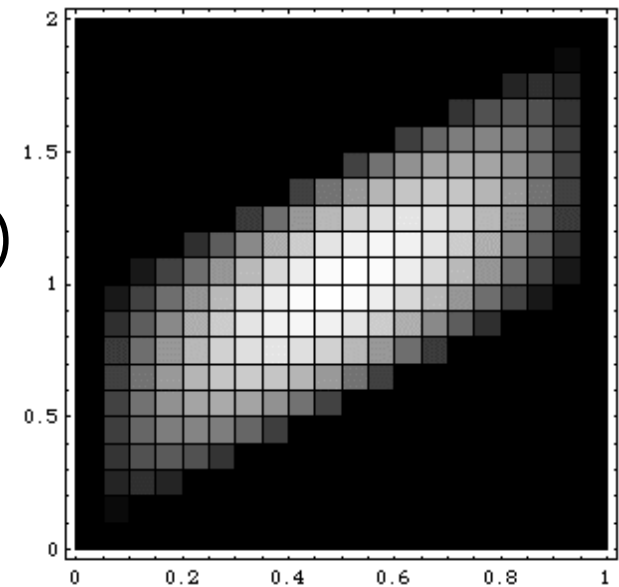
$$P_{k|k} = P_{k|k-1} - K_k H_k P_{k|k-1}$$

Kalman gain
$$K_k = P_{k|k-1} H_k^T S_k^{-1}$$

with:
$$S_k = H_k P_{k|k-1} H_k^T + R_k$$

Optimal Filters

- Kalman optimal (minimizes covariance) if linear equations and Gaussian density.

- Kalman can be derived without the Gaussian restriction (Least squares). Possible loss of optimality.

- Discrete representations (grids)

R. W. R. Darling

Non-linear: Approximate filters - EKF, UKF

$$\mathbf{x}_k = \mathbf{f}_k(\mathbf{x}_{k-1}, \mathbf{v}_{k-1}) \qquad \mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_k, \mathbf{w}_k)$$

Prediction
$$m_{k|k-1} = \mathbf{f}_k(m_{k-1|k-1})$$

$$P_{k|k-1} = \nabla F_k P_{k-1|k-1} \nabla F_k^T + Q_{k-1}$$

$$\nabla F_k = \left.\frac{\partial \mathbf{f}_k}{\partial x}\right|_{m_{k-1|k-1}}$$

Update:
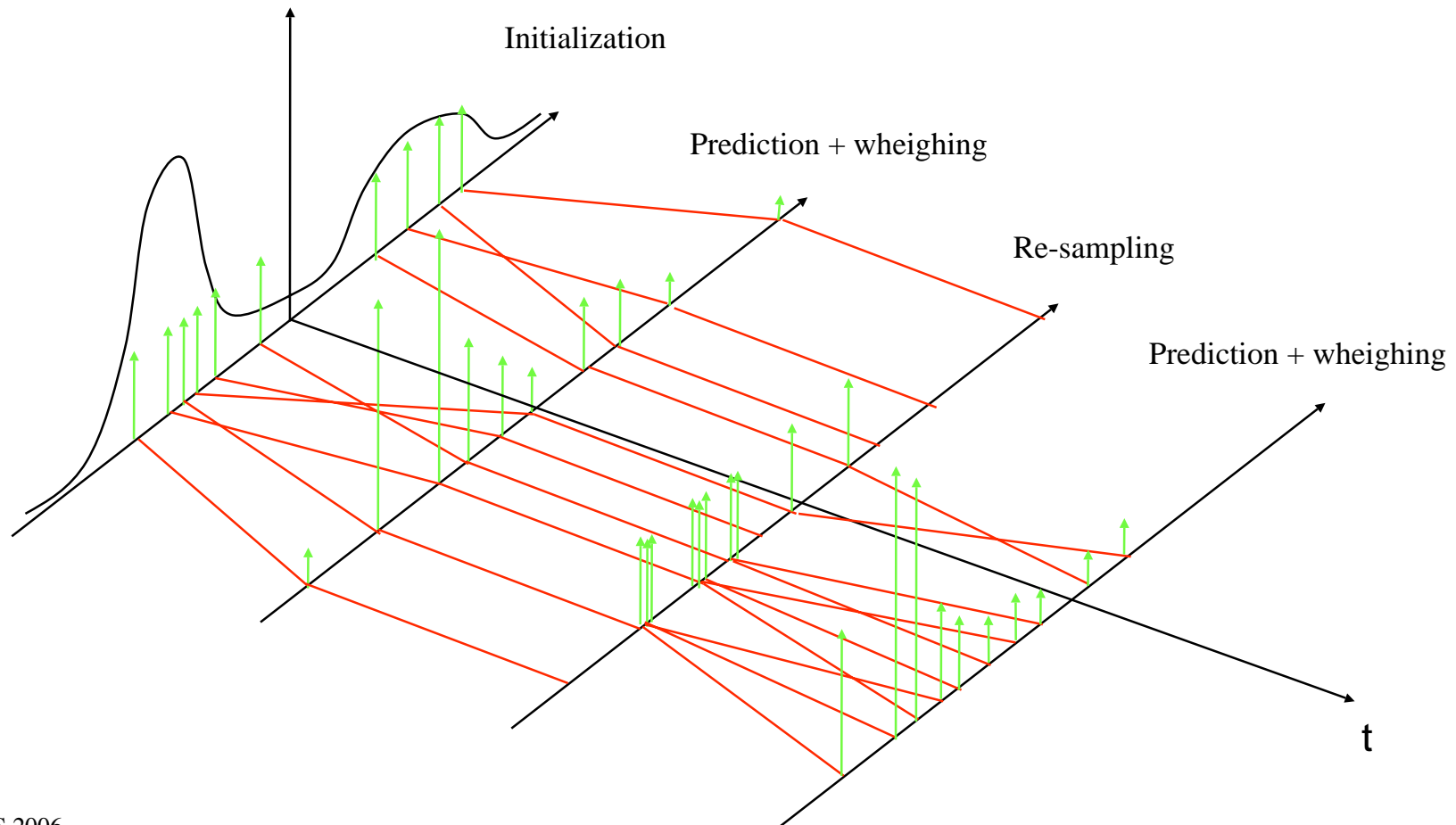$$m_{k|k} = m_{k|k-1} + K_k(\mathbf{z}_k - \mathbf{h}_k(m_{k|k-1}))$$

$$P_{k|k} = P_{k|k-1} - K_k \nabla H_k P_{k|k-1}$$

Kalman gain
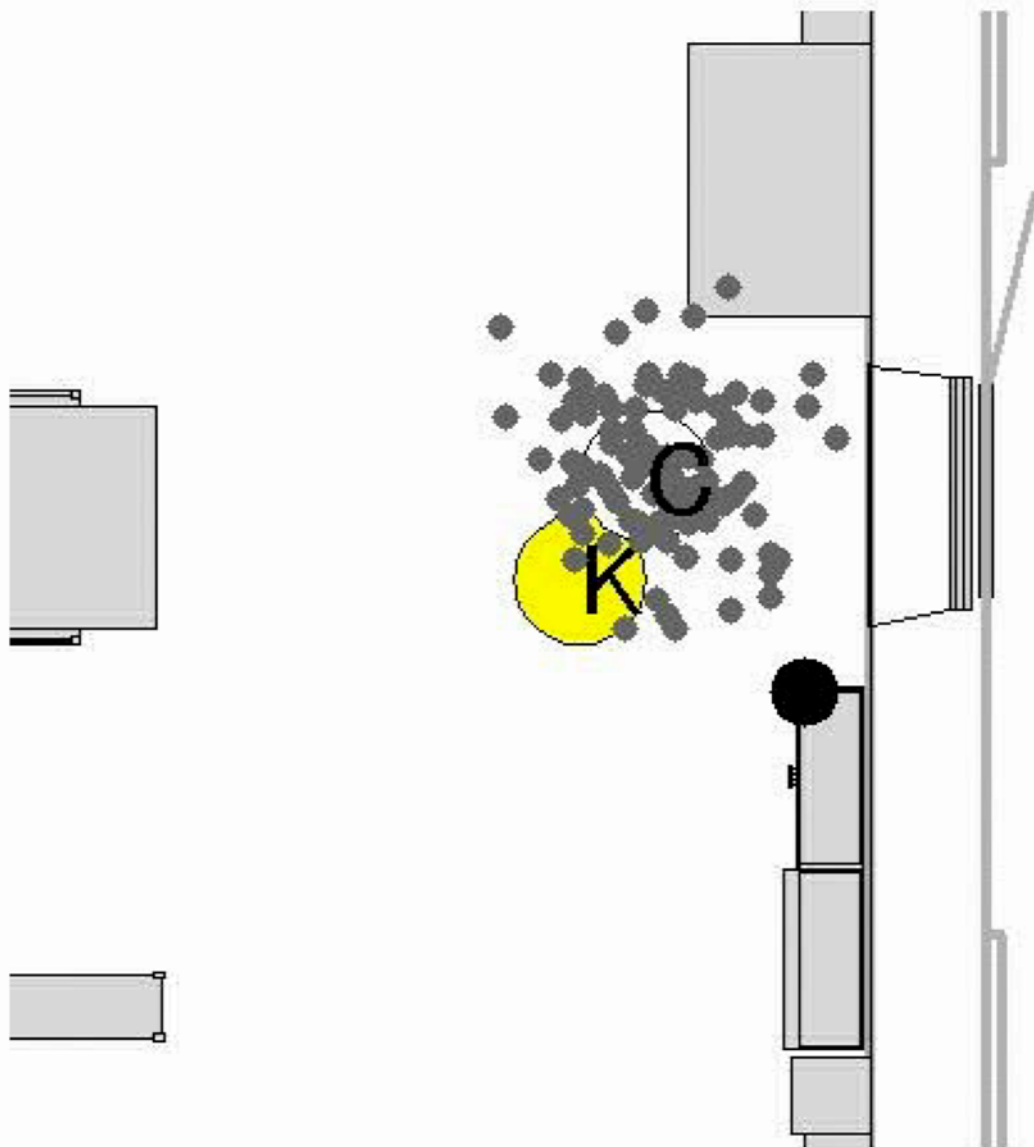$$K_k = P_{k|k-1} \nabla H_k^T S_k^{-1}$$

$$S_k = \nabla H_k P_{k|k-1} \nabla H_k^T + R_k$$

Particle Filter

- Represent PDF by point samples (particles) updated over time $x = \{< x_n, \omega_n >\}_{1 \leq n \leq N}$

Initialization

Prediction + wheighing

Re-sampling
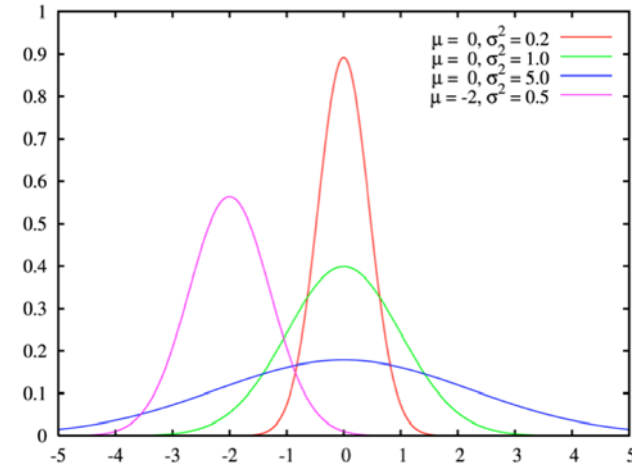
Prediction + wheighing

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K: Kalman est, C: Condensation est
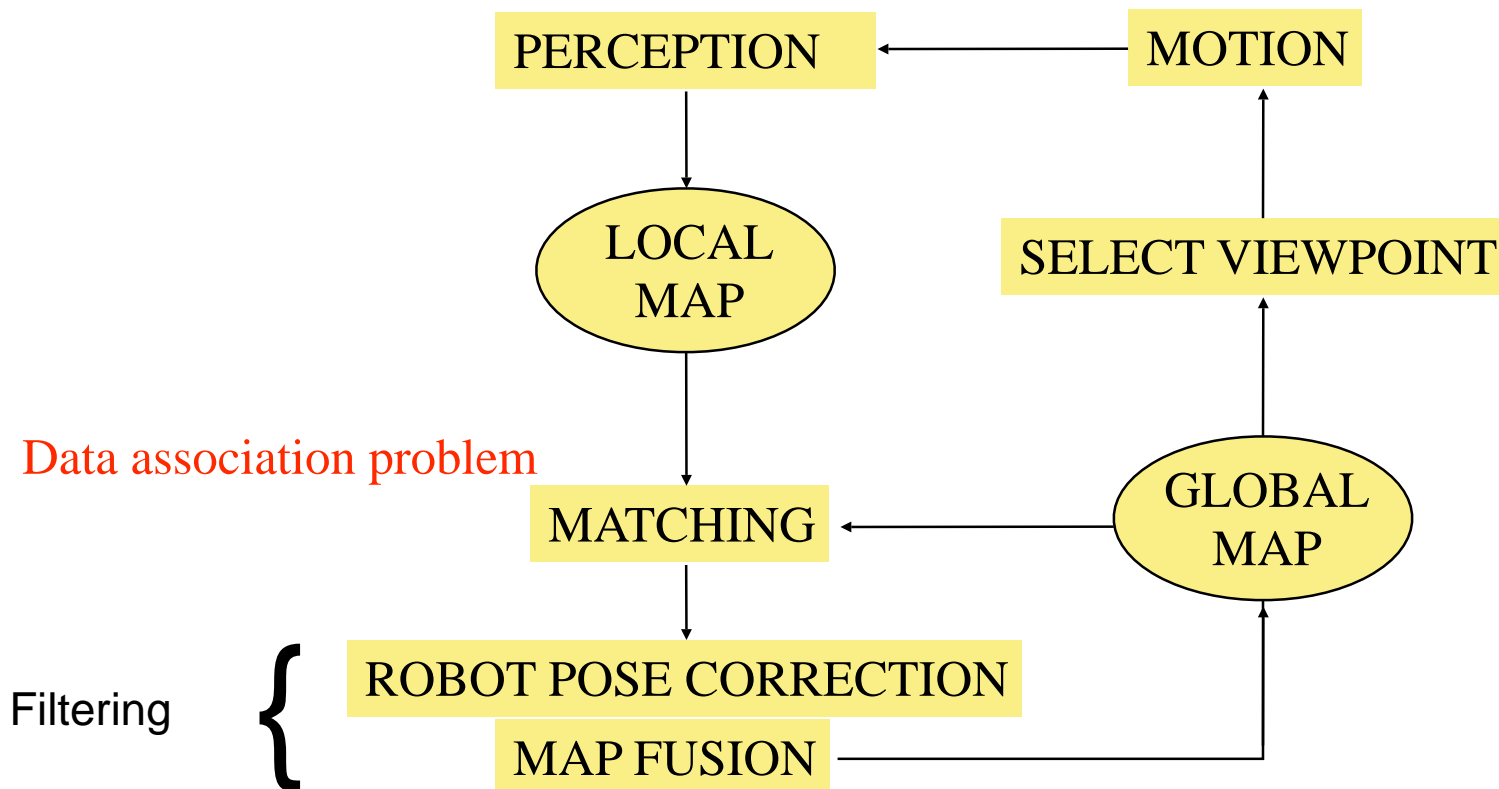
Courtesy Henrik Christensen

Discussion Issues

- Modeling sensor uncertainties
  - laser: gaussian?
  - vision: gaussian?
  - odometry: gaussian?
- Coping with non-gaussianity:
  - Correct position prediction (before filtering):
    Relocate the robot - remove bias.
  - Data Association Problem
- Multiple hypotheses: several EKFs (Gaussian mixture) vs. PF?
- Complexity

S L A M General Process

PERCEPTION ← MOTION

LOCAL MAP

SELECT VIEWPOINT

Data association problem

MATCHING ← GLOBAL MAP

Filtering {

ROBOT POSE CORRECTION

MAP FUSION

A Few Open/Current SLAM Issues

- Environments:
  - Unstructured and natural environments
    - Features, representations?
  - Dynamic environments
    - what is stable, what is used for localization?
  - Large environments
    - complexity, consistency, decorrelation.
- Mapping/modelling: SLAM - What for?
  - Dense 3D
  - Multiple representations
  - Complex fully correlated features ("objects").
  - Not only geometrical. Scene understranding: structure and contents. Semantics

A Few Open/Current SLAM Issues

- Life long SLAM
- Self doubt: question validity
- Persistence and robustness - not only an implementation issue.

- Platforms/sensors:
  - Fast
  - Highly noisy
  - Multisensor.