Comparing the Box-Jenkins Approach with the Exponentially Smoothed Forecasting Model Application to Hawaii Tourists

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Source: *Journal of Marketing Research*, Vol. 12, No. 2 (May, 1975), pp. 182-188
Published by: American Marketing Association
Stable URL: http://www.jstor.org/stable/3150441
Accessed: 01-02-2017 14:32 UTC

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This article compares the forecasting accuracy of two forecasting techniques: (1) Box-Jenkins, and (2) exponential smoothing. The comparison is made using the tourist to Hawaii time series. An analysis of both techniques is included.

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INTRODUCTION

While econometric models have been under scrutiny as to their forecasting accuracy, the Box-Jenkins method has recently been suggested as an effective tool for short-term forecasting. On the highly aggregated level there have been quite a few studies that compare the forecasting performance of this method with those of widely used and highly reputed econometric models such as the FRB-MIT-Penn and the Wharton models [10, 11]. In general, these studies indicated that the Box-Jenkins approach outperforms all other models investigated and suggested that it should be considered seriously as an alternative forecasting technique. Even though the controversy is far from being settled, the argument in favor of the Box and Jenkins method is fairly strong given the fact that it is relatively easy and inexpensive to use. In a similar vein one can argue that various automatic forecasting techniques are easier to understand and more economical to use than the Box-Jenkins, and if they can perform as well, then they should be preferred. Thus the relative place of this method as a forecasting technique could not be determined without comparing it with automatic forecasting tools, since even naive forecasting models seem to fare well when their forecasts are compared with those of large econometric models [13]. Newold and Granger [12] read a paper to the Royal Statistical Society in which they reported on the analysis of approximately 100 time series using the Box-Jenkins and alternative methods. In most, but not all, the Box-Jenkins method outperformed the other methods. However, it is not clear if the improved accuracy was offset by the increased cost of using the Box-Jenkins approach. In addition, it has been recently suggested that the Box-Jenkins method may not be as successful on a less aggregated level and even unsuitable for forecasting series with high multiplicative components or for long lead time forecasting [7].

The purpose of this study is to compare the performance of the Box-Jenkins approach in forecasting Hawaii's tourists with the exponentially smoothed forecasting technique developed by Brown [6]. The IBM IMPACT model and the Honeywell PROFIT model are patterned after the Brown model. This model has proved adequate and successful in a large number of commercial forecasting applications. The comparison intended in the study is especially illuminating in light of the criticism mentioned earlier and the fact that the series investigated is regional with a multiplicative component. However, this article will be presented as a single case study rather than as evidence for or against the Box-Jenkins approach.

APPLICATION

In recent years, the economy of Hawaii has become increasingly dependent on its tourist industry. Forecasts of the number of tourists coming, their length of stay, and their expenditures are used for planning and inventory control. The State's Visitor Bureau, banks, Hawaiian Electric, and several departments at the University of Hawaii are engaged in forecasting tourists coming to Hawaii. These forecasts are used by various tourist-related industries in management

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Journal of Marketing Research
Vol. XII (May 1975), 182-8

planning and decision making.

In February 1973, an unpleasant situation occurred. Thousands of tourists who had confirmed reservations could not find a room in which to stay. Those with rooms were forced to share and cots were placed in their hotel rooms. Several tourists were forced to sleep in lobbies. Some arriving tourists were forced to leave Hawaii without spending more than a few hours at the airport. The state set up an emergency center and asked residents to take tourists into their homes.

As a result of February 1973, the authors decided to see if an improved forecasting model could be developed. Some of Box and Jenkins's original work was done with airline traffic suggesting that for a forecast of Hawaii-bound tourists, the Box-Jenkins approach may perform better than existing models.

THE MODEL

Box and Jenkins postulate that a model which is a mix of autoregressive and moving average processes is sufficient to describe a wide variety of stationary series. The general form of this model can be written as:

$$\phi(B)\nabla^d y_t = \theta(B)\, a_t \tag{1}$$

where ϕ_p, θq are polynomials of degree P, q respectively, B is a backward shift operator, $\nabla^d y_t$ refers to the degree of differencing, and a_t is a white noise process with a mean equal to zero and a variance equal to σ_a^2.

For a better understanding of (1), a more detailed description of the processes it contains may be needed. An autogressive process of order p may be expressed as follows:

$$\text{(1a)}\quad y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + a_t \quad \text{or briefly}$$

$$\phi_p(B)\, y_t = a_t$$

where B is a backward shift operator such that $B^i y_{t-i} = y_t$, $\phi_p(B) = (1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p)$ and a_t is as defined above. A moving average process of order q may be expressed as:

$$\text{(1b)}\quad y_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q} \quad \text{or}$$

$$y_t = \theta_q(B)\, a_t.$$

Combining both processes produces the following model:

$$\phi_p(B)\, y_t = \theta_q(B)\, a_t. \tag{1c}$$

If the series is to be differenced (to gain stationarity), then $\nabla^d y_t$ replaces y_t and (1c) becomes (1). Equation (1) is sometimes referred to as an ARIMA model of order $[p, d, q]$. For example, a model of order [2, 1, 1] is written as follows:

$$(1 - \phi_1 B - \phi_2 B^2)\nabla y_t = (1 - \theta B)\, a_t.$$

Assuming that ϕ_1, ϕ_2, θ are estimated and found to be .8, .5, .6 respectively, the above equation can be written as:

$$(1 - .8B - .5B^2)\nabla y_t = (1 - .6B)\, a_t \quad \text{or}$$

$$\nabla y_t = .8\nabla y_{t-1} + .6\nabla y_{t-2} = a_t - .6\, a_{t-1}$$

$$\text{where}\quad \nabla y_t = y_t - y_{t-1}.$$

The seasonality model can be written as follows:

$$\phi_p(B)\, \Phi_P(B^s)\nabla^d\nabla_s^D y_t = \theta_q(B)\theta_Q(B^s)\, a_t \tag{2}$$

where s is the seasonal order (12 for monthly data), and D is the degree of seasonal differencing. More briefly, the model can be described as being of the form $(p, d, q) \times (P, D, Q)_s$, where p, q, P, Q refer to the numbers of regular autoregressives, of regular moving averages, of seasonal autoregressive, and of seasonal average parameters, and d, D, and s as defined earlier. This is a general model which encompasses a large number of subclasses depending on the numbers and orders of the parameters.

General Aspects of the Box-Jenkins Methods

For an in-depth explanation of the Box-Jenkins method, the reader is referred to [3, 4]. Two main characteristics distinguish this method from other forecasting techniques. The first is that building the model can only be achieved by examining the actual data. This is in contrast to the general econometric approach of fitting the data to a model on the assumption that the structure of the model can be identified a priori. The second is that a systematic process is utilized to build the model which includes identification, estimation, and diagnostic checking.

Identification

To identify the specific model that describes adequately the series, an analysis of its autocorrelation and/or its partial autocorrelation is needed. This is accomplished by estimating the sample autocorrelation function of the differenced series. The function used is:

$$\frac{\gamma_k(x) = C_k(x)}{C_0(x)} \tag{3}$$

$$\text{where}\quad C_k = \frac{1}{n}\sum_{t=1}^{(n-k)} (x_t - \bar{x})(x_{t+k} - \bar{x}) \quad \text{and}$$

$$\bar{x} = \frac{1}{n}\sum_{t=1}^{n} x_t$$

where n is the number of observations available. The original time series is examined to determine how it differs from a "stationary time series." A stationary time series exhibits no trend or seasonality and any segment of the series appears identical to any other segment. By identifying the relationships that cause the original time series to be nonstationary, the forecaster may determine what parameters will give the best fit to the data. In a stationary series each model theoretically has a certain autocorrelation function and it is expected that their estimates behave in a similar fashion.

Stationarity may be achieved by various degrees of regular and seasonal differencing. The speed at which the autocorrelations dampen out determine whether or not the series is stationary. The failure of the autocorrelations to dampen out quickly indicates nonstationarity and differencing of various degrees may be needed. However, even if stationarity is reached, the autocorrelations arrived at are only estimates which may suggest more than one model. Each of the models is considered a candidate for further analysis of parameter estimation and diagnostic checking.

Estimation

The procedure now calls for the use of efficient statistical methods that fit the tentatively identified model to the series by estimating the values of its parameters. This is done by using the maximum likelihood method. If a_t is normally distributed, the conditional maximum likelihood estimates of $(\phi_1, \phi_2, ..., \phi_p)$ and $(\theta_1, \theta_2, ..., \theta_q)$ can be obtained by minimizing the sum of square function $S(\phi, \theta)$ given a large sample size. In addition, minimizing this function will provide a close approximation to the unconditional maximum likelihood estimates. This is achieved by using the Marquant algorithm for nonlinear least squares [3]. Efficient estimation techniques are emphasized so that inadequacy, if indicated, is caused by the use of inappropriate model rather than by inefficient estimation methods.

Diagnostic Checks

When the form of the model is correct and $\hat{\phi}_p$ and $\hat{\theta}_q$ are close to their "true values," the residuals $\hat{a}_t$ will behave like a white noise process. If they show significant autocorrelation, then the model is considered inadequate. Various tests on the autocorrelation of residuals are performed to determine if they are the result of white noise only. Other tests such as the cumulative periodogram check and overfitting may be used [3]. If the model proved to be inadequate, the process of estimation and diagnostic checking is repeated on the models being suggested in the identification stage.

Frequently two models will be identified which produce nearly the same results. In such cases and the absence of a justified criterion, the forecaster is advised to use the model which has the fewest parameters (parsimony).

The Box-Jenkins Approach Vs. the Exponentially Weighted Average Approach

In the exponentially weighted average it is assumed that the current forecast of one period ahead is a function of the forecast at the previous period and the actual current observation [16]. In its simplest form it can be written as:

$$Y_{(t,1)} = \lambda\, Y_t + (1 - \lambda)\, Y_{(t-1,1)}$$

where $Y_{t,1}$ is the forecast of Y at time t for lead time one month, Y_t is the actual observation of time t, and λ is the smoothing constant which determines the weight given past data. Written in the form of past observations (4) can be expressed as follows:

$$Y_{(t,1)} = \lambda \sum_{i=0}^{n} (1 - \lambda)^i\, Y_{t-i} + (1 - \lambda)^{n+1}\, \bar{Y}_0 \tag{5}$$

where $\bar{Y}_0$ is the initial forecast. Equation (5) is based on the assumption that the series can be expressed as a function of past observation and a white noise. The weight assigned to each observation decays with time exponentially. This form can be written as:

$$Y_t = \lambda \sum_{i=1}^{\infty} (1 - \lambda)^{i-1}\, Y_{t-1} + a_t. \tag{6}$$

In the Box-Jenkins language (6) is an infinite autoregressive process and it can easily be shown that this is the same as a model of the form that is zero autoregressive, one regular difference and one moving average. Equation (6) can be written as:

$$\begin{aligned} Y_t &= \lambda\, Y_{t-1} + \lambda(1 - \lambda)\, Y_{t-2} \\ &\quad + \lambda(1 - \lambda)^2\, Y_{t-3} + ... + a_t \end{aligned} \tag{7}$$

and

$$\begin{aligned} (1 - \lambda)\, Y_{t-1} &= \lambda(1 - \lambda)\, Y_{t-2} + \lambda(1 - \lambda)^2\, Y_{t-3} \\ &\quad + ... + (1 - \lambda)\, a_{t-1}. \end{aligned} \tag{8}$$

Subtracting (8) from (7) and rearranging, we get

$$\nabla Y_t = (1 - \theta B)\, a_t \tag{9}$$

where $\nabla Y_t = Y_t - Y_{t-1}$ and $\theta = (1 - \lambda)$.

In our study the exponentially smoothed model uses a smoothing constant of .1 and both a single and double smoothed statistic. Each estimate of sales was also multiplied by a predetermined seasonal factor. Specifically,

$$\hat{y}_{t+T} = S_{t+T}\,[(d_1 S^{[1]}\, y + b_1) + (d_2 S^{[2]}\, y + b_2)] \tag{10}$$

where S_{t+T} is next months seasonal factor, b_1 and b_2 are trend variables, and d_1 and d_2 are lag constants. For a detailed explanation, the interested reader is referred to [6].

A smoothing constant of $\alpha = .1$ was chosen because an alpha analyzer computer program showed this value produced the best forecast for an initialization period of 48 months. This small smoothing constant indicates a well-balanced series in that the pattern of past data is not subject to abrupt changes. An $\alpha = .1$ smoothing

Table 1

AUTOCORRELATIONS OF THE LAGGED SERIES: TOURISTS IN HAWAII IN THE FORMS $\nabla_{12} Y_t$ and $\nabla\nabla_{12} Y_t$

ORIGINAL SERIES
MEAN OF THE SERIES = .17884
ST. ERROR OF MEAN = .008314
NUMBER OF OBSERVATIONS = 228

1-12	0.54	0.45	0.43	0.28	0.29	0.23	0.13	0.14	0.08	0.00	−0.08	−0.26
ST. E.	0.06	0.08	0.09	0.10	0.10	0.11	0.11	0.11	0.11	0.11	0.11	0.11
13-24	−0.09	−0.05	−0.13	−0.08	−0.04	−0.09	−0.10	−0.08	−0.07	−0.07	−0.04	−0.03
ST. E.	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11

MEAN DIVIDED BY ST. ERROR = 21.5

TO TEST WHETHER THIS SERIES IS WHITE NOISE, THE VALUE 245.08 SHOULD BE COMPARED WITH CHI-SQUARE VARIABLE WITH 24 DEGREES OF FREEDOM

DIFFERENCE 1
MEAN OF THE SERIES = .006521
ST. ERROR OF MEAN = .007987
NUMBER OF OBSERVATIONS = 227

1-12	−0.39	−0.08	0.13	−0.17	0.07	0.04	−0.11	0.09	0.01	0.01	0.11	−0.38
ST. E.	0.06	0.07	0.07	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
13-24	0.14	0.13	−0.13	0.01	0.10	−0.06	−0.03	0.02	0.01	−0.03	0.02	0.09
ST. E.	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09

MEAN DIVIDED BY ST. ERROR = .081738

TO TEST WHETHER THIS SERIES IS WHITE NOISE, THE VALUE 106.664 SHOULD BE COMPARED WITH A CHI-SQUARE VARIABLE WITH 24 DEGREES OF FREEDOM

constant allows more time periods to be incorporated into the smoothing statistics [6].

The exponentially weighted average cannot be considered as a subset of the Box-Jenkins approach for two main reasons. The first is that exponentially weighted average is supposed to be a sensible mode of representing the behavior of a simple forecasting problem. This means that the model is identified a priori in contrast to the Box-Jenkins method where a search for adequate model has to be made first. If the Box-Jenkins method would always lead to the model that best describes the series whether it is EWA or not, then the former is obviously superior to the latter. However, sample autocorrelations may be misleading, and thus the Box-Jenkins method may fail to indicate that the model is EWA even if it describes adequately the series investigated. Box and Jenkins pointed out that from their past experiences they have not found this to be troublesome in the context where no irrevocable decisions are being made at the identification stage [2]. For the Box-Jenkins approach to be regarded as a viable alternative to EWA or for that matter to any automatic forecasting techniques, it has to outperform them to a considerable degree since it is more difficult to understand and more costly to use. This is an empirical question and cannot be answered without accumulating more experiments and usages. Later we will introduce the results of our investigations.

BUILDING THE MODEL

The series is a monthly number of tourists visiting Hawaii for the period 1952-1971 inclusive. The original time series was logarithmically transformed because it was observed that these seasonal data fluctuated in a percentage manner. (The amplitude was increasing.) Logarithms are more suitable to get a constant amplitude to the data. At a late stage of this study identifying the model from the original data was attempted and the results confirmed the appropriateness of the transformation.[1]

Our next step was to see whether the series is stationary or not and if not, to transform into a stationary one. As previously stated, stationarity is needed to identify the model from its autocorrelation or partial autocorrelation functions. The procedure is to use differencing of various degrees and orders. When the seasonality is monthly, as it is in this case, at least one seasonal differencing of an order 12 is needed. Higher degrees of seasonal differencing are not recommended unless necessary since it entails the loss of large numbers of degrees of freedom. The autocorrelations of the series in the forms of $\nabla_{12} Y$ and $\nabla\nabla_{12} Y_t$ were computed.

[1] In case the results proved unsuccessful, the Box and Cox transformation may have been needed to obtain more exact transformation (see [5]).

Table 2

PARTIAL AUTOCORRELATIONS OF THE LAGGED SERIES: TOURISTS IN HAWAII IN THE FORM $\nabla\nabla_{12} Y_t$

ORIGINAL SERIES
MEAN OF THE SERIES = .17884
ST. ERROR OF MEAN = .0083184

1-12	0.54	0.22	0.18	−0.07	0.09	−0.01	−0.07	0.03	−0.04	−0.08	−0.14	−0.27
ST. E.	0.06	0.08	0.09	0.10	0.10	0.11	0.11	0.11	0.11	0.11	0.11	0.11
13-24	0.24	0.13	−0.04	−0.04	0.12	−0.06	−0.12	0.06	0.06	−0.12	−0.08	−0.04
ST.E.	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11

From Table 1, it can be seen that a higher degree of regular differencing is not needed since the autocorrelation of the data in each of the above forms becomes insignificant after the 15 lags. We decided in favor of the form $\nabla\nabla_{12} Y$ and against $\nabla_{12} Y_t$ for two reasons. The first is that even though the autocorrelations of $\nabla_{12} Y_t$ become insignificant after a small number of lags, they fade away slowly which suggests a high autoregressive coefficient or nonstationarity. The second is that the series $\nabla_{12} Y_t$ has a significant mean and if it is to be used, the mean must be included in the model. Since the form $\nabla\nabla_{12} Y_t$ has no significant mean, it is thought that a model based on it may be more parsimonious.

The coefficients of the autocorrelation of $\nabla\nabla_{12} T_t$, excluding those at lag 11, 12, and 13, indicate a dampened sine wave starting with the first autocorrelation. This suggests a regular model of the form (1, 1, 1). However, because of the relatively large magnitude of the first autocorrelation and given the sampling errors involved, a cutoff at this point may exist suggesting a model of the form (0, 1, 1). This is partially supported by the partial autocorrelation function of Table 2 which seems to dampen out. Excluding the autocorrelations at lag 11, 12, and 13 from the first consideration enables us to identify the regular model as a first step. These autocorrelations are dependent on the monthly seasonal patterns.

Box and Jenkins reported that one of the two following models gave sufficient representation to most of the seasonal series they worked with: $(0, D, 1)_{12}$ and $(1, D, 1)_{12}$. The series under investigation seems to lend support to this statement since its autocorrelations suggest very strongly the seasonal model $(0, 1, 1)_{12}$. For instance, a special characteristic of the model $(0, 1, 1)\ (0, 1, 1)_s$ is that $P_{s-1} = P_{s+1} = P_1 P_s$ where P is the population autocorrelation coefficient. It is remarkable how close the autocorrelation estimates satisfy the above characteristic since $\gamma_{11} = .11$, $\gamma_{13} = .14$, and $\gamma_1 \gamma_{12} = .148$ (see Table 1).

A different argument, though not as strong, may be made in favor of the seasonal model $(1, 1, 1)_{12}$. The fact that γ_{24} is 0.09 and its standard error is 0.09 may suggest the existence of one seasonal autoregressive. However, it is expected that the coefficient of the autoregressive would be small even if it proved to be significant because of the sharp decline from γ_{12} to γ_{24}. To summarize, the identification stage suggests that the following four models may be considered:

(a) $(0, 1, 1)\ (0, 1, 1)_{12}$
(b) $(1, 1, 1)\ (0, 1, 1)_{12}$
(c) $(0, 1, 1)\ (1, 1, 1)_{12}$
(d) $(1, 1, 1)\ (1, 1, 1)_{12}$

THE EMPIRICAL RESULTS

A summary of the four models considered in the identification stage after being estimated is:

Forms	*Standard error of residual*
(1) $(1 - B)(1 - B^{12}) y_t$ $= (1 - .61B)(1 - .44B^{12}) a_t$	.095
(2) $(1 + .1B^{12})(1 - B)(1 - B)^{12} y_t$ $= (1 - .59B. (1 - .52B^{12}) a_t$	.084
(3) $(1 - .1B)(1 - B)(1 - B^{12}) y_t$ $= (1 - .68B)(1 - .44B^{12}) a_t$	.095
(4) $(1 - .05B)(1 + .12B^{12})(1 - B)(1 - B^{12}) y_t$ $= (1 - .63B)(1 - .5B^{12}) a_t$	.084

It is interesting to note that each of the models presented proved to be adequate after checking the autocorrelation of residuals separately using a *t*-test and combined using the χ^2 test. The hypothesis that these autocorrelations are zeros could not be rejected. Thus on the basis of these tests it was not possible to select one model as being the best that describes the series. The magnitude of the standard error of residuals for each model could not be used to designate the best model either, because these do not seem to differ significantly. In addition, the model that fits best is not necessarily the one that forecasts best. The failure of the Box-Jenkins method to select one model is one of its recent criticisms [7]. However, the authors tend to agree with Box and Jenkins that each of the adequate models is an approximation and any one of them may be used. Actually, this fact seems to us to be a source of strength rather than a source of weakness in this approach. Identification is not exact; it can be confounded by estimation errors and if so, the existence of more than one adequate model would be helpful. Model 2 in Table 3 was our choice because it gave the best forecast for one period

Table 3
COMPARISON OF FORECAST ERRORS

Time period	*Exponential Smooth*[a]		*Box and Jenkins*[b]
	Forecast error	*Actual value*	*Forecast errors*
1	−1883	15592	375
2	14	15169	−336
3	1251	15469	−2093
4	−819	12085	−1539
5	447	13037	140
6	72	11584	644
7	799	12798	628
8	162	11782	−822
9	1902	12258	−946
10	−30	14808	1247
11	1074	12472	275
12	−427	13069	−1454
13	−2629	18024	−692
14	−1697	18806	1263
15	−2545	21707	4269
16	−39	13463	49
17	914	14930	−969
18	1331	12287	−1241
19	−665	16248	416
20	2478	11466	−2179
21	3307	12672	−2381
22	2764	13630	−3861
23	57	14422	−160
24	−781	14441	602

$U_{ES} = .103$ $U_{Bj} = .102$

[a]Double smooth $\alpha = .1$.
[b]$(0, 1, 1)\,(1, 1, 1)_{12}$.

ahead. A description of the criterion used and its justification are given in the next section.

COMPARISON OF THE BOX-JENKINS AND EXPONENTIAL SMOOTHED MODELS

Comparing forecasting models is a very difficult task. The difficulty is in selecting the span of time on which a comparison is made. Another problem is in determining what statistic should be used to compare the forecasting accuracy of the model being considered. We selected the last 24 time periods and the forecasts made one time period $(t + 1)$ ahead as a basis for comparison. This choice was dictated by pragmatics, since for the tourist forecast one month lead time is adequate for planning and inventory management. Also, the two years decision was dictated by user's interest in a currently accurate forecast. It should be noted that none of the 24 periods were included in the modeling process. The statistic selected was the U coefficient suggested by Theil [14, 15]. U is favored by many practitioners [1, 8, 9]:

$$U = \frac{\sum_{i=1}^{24} [p_i - A_i]^{2^{1/2}}}{\sum_{i=1}^{24} A_i^{2^{1/2}}}$$

where p_i is the forecasted value and A_i is the actual value.

Table 3 shows the forecast errors for both techniques and their U's. Using the U coefficient to determine the best model does not enable one to choose any of the two models since the difference is slight enough to be brought about by rounding errors. Given the arbitrary nature of the number of periods and lead times selected, one would be even more hesitant to pick one model as being the best. On the basis of the results in Table 3, it can be said that both models seem to perform well.

Conclusion

On the basis of our results, the exponentially smoothed model patterned on Brown's model and the Box-Jenkins approach seem to perform equally well. However, it should be emphasized that the number of periods or lead times chosen was dictated by practical considerations to a specific situation. Selecting forecasts for different lead time periods may have led to a different conclusion. In addition, it should be noted that computing the U coefficients from the actual forecasts and forecast errors favor the exponentially smoothed model since logarithmic transformation was performed on the series when using the Box-Jenkins approach. The failure of the Box-Jenkins approach to outperform the exponentially smoothed model leads to the conclusion that the latter should be preferred, since it is cheaper to use and may be easier to work with. This conclusion applies to the time series—"Hawaii's tourists"—and cannot be extended to other time series.

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