metoda Strassen'a

$$\underbrace{\begin{pmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{pmatrix}}_{\text{wspołczynniki stałe !}} \begin{pmatrix} x_{00} & x_{01} \\ x_{10} & x_{11} \end{pmatrix} = \begin{pmatrix} a_{00}x_{00} + a_{01}x_{10} & a_{00}x_{01} + a_{01}x_{11} \\ a_{10} + x_{00} + a_{11}x_{10} & a_{10}x_{01} + a_{11}x_{11} \end{pmatrix}$$

$\Downarrow$

8 operacji mnożenia

Strassen: Tylko 7 mnożeń, 18 dodawań

$\Downarrow$

11 dodawań (zmodifikowana przez Winograda)

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przy n=100 ⇒ metoda zmod. ~~...~~ szybsza

Strassen: macierzy ~~...~~ rząd potęgi dwójki

jeżeli rząd się nie zgadza dopisane są zero ~~...~~ w liniach i wierszach

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dana macierz:

$$A = \left[\begin{array}{c|c} A_1 & A_2 \\ \hline A_3 & A_4 \end{array}\right]$$

$S_1 = a_{10} + a_{11}$

$S_2 = S_1 - a_{00}$

$S_3 = a_{00} - a_{10}$

$S_4 = a_{01} - S_2$

$S_5 = x_{01} - x_{00}$

$S_6 = x_{11} - S_5$

$S_7 = x_{11} - x_{01}$

$S_8 = x_{10} - S_6$

$m_1 = S_2 S_6$

$m_2 = a_{00} x_{00}$

$m_3 = a_{01} x_{10}$

$m_4 = S_3 S_7$

$m_5 = S_1 S_5$

$m_6 = S_4 x_{11}$

$m_7 = a_{11} S_8$

$t_1 = m_1 + m_2$ $\qquad y_{00} = m_2 + m_3$

$t_2 = t_1 + m_4$ $\qquad y_{01} = t_1 + m_5 + m_6$

$y_{10} = t_2 + m_7$

$y_{11} = t_2 + m_8$

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wektory kolumnowe

⇓

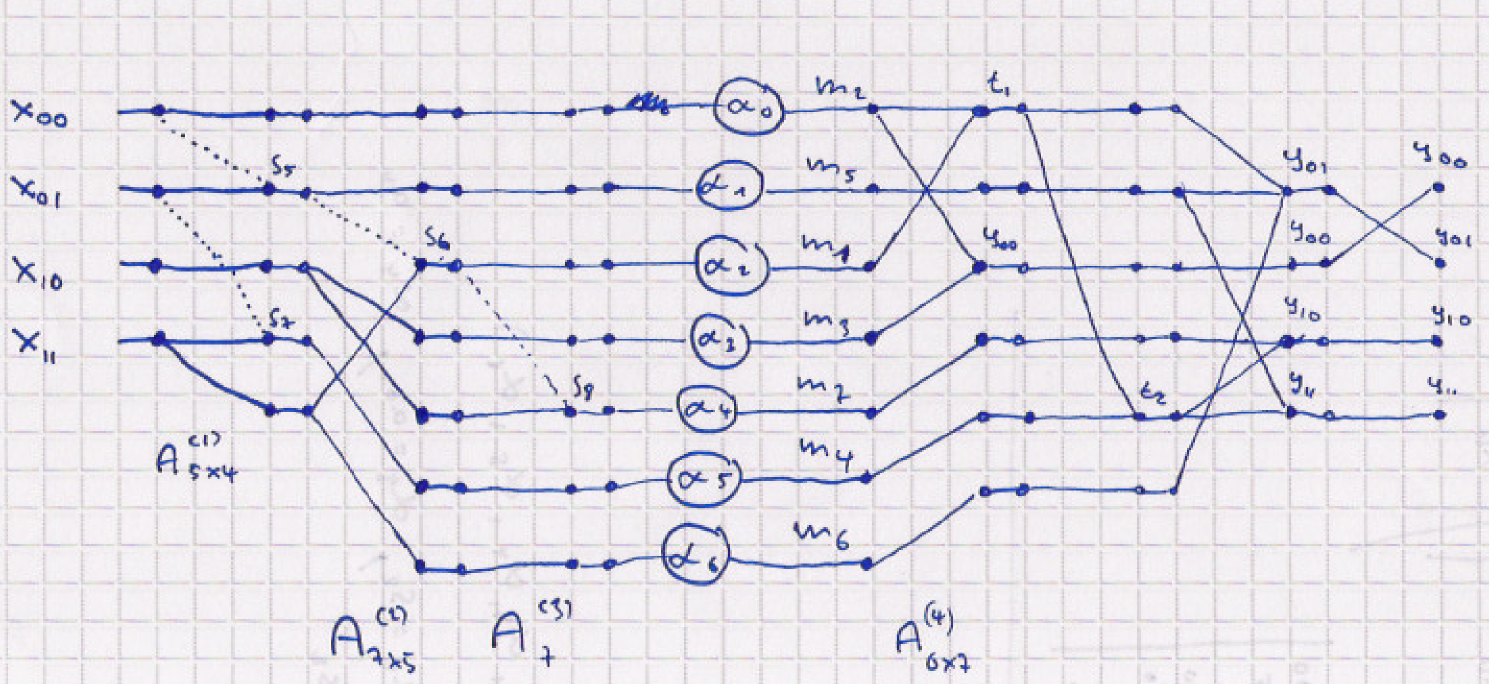
$$Y_{4\times1} = A_4^{(7)} A_{4\times6}^{(6)} A_6^{(5)} A_{6\times7}^{(4)} D_7 A_7^{(3)} A_{7\times5}^{(2)} A_{5\times4}^{(1)} X_{4\times1} \; ;$$

$$X_{4\times1} = [x_{00}, x_{01}, x_{10}, x_{11}]^T$$

$$X_{4\times1} \cdot A_{5\times4}^{(1)} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} x_{00} \\ x_{01} \\ x_{10} \\ x_{11} \end{vmatrix} = \cancel{\begin{vmatrix} x_{00} & -x_{00} & 0 \\ 0 & x_{01} & 0 \\ 0 & 0 & x_{10} \\ 0 & 0 & 0 \end{vmatrix}}$$

$$= \begin{vmatrix} x_{00} + 0 \\ -x_{00} + x_{01} + 0 \quad 0 \\ x_{10} \\ -x_{01} + x_{11} \\ x_{11} \end{vmatrix}$$

$$= \begin{vmatrix} x_{00} \\ -x_{00} + x_{01} \\ x_{10} \\ -x_{01} + x_{11} \\ x_{11} \end{vmatrix}$$



x_{00}
x_{01}
x_{10}
x_{11}
s_5
s_6
s_7
s_8
$A^{(1)}_{5\times4}$
$A^{(2)}_{7\times5}$
$A^{(3)}_{7}$
α_0
α_1
α_2
α_3
α_4
α_5
α_6
m_2
m_5
m_1
m_3
m_7
m_4
m_6
$A^{(4)}_{6\times7}$
t_1
t_2
y_{00}
y_{01}
y_{00}
y_{10}
y_{11}
y_{00}
y_{01}
y_{10}
y_{11}

$$A^{(2)}_{7\times 5} = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} \cdot \left(\begin{vmatrix} x_{00} \\ -x_{00} + x_{01} \\ x_{10} \\ -x_{01} + x_{11} \\ x_{11} \end{vmatrix} = \begin{vmatrix} x_{00} \\ s_5 \\ x_{10} \\ s_7 \\ x_{11} \end{vmatrix} \right)$$

$$= \begin{vmatrix} x_{00} \\ s_5 \\ -s_5 + x_{11} \\ x_{10} \\ x_{10} \\ s_7 \\ x_{11} \end{vmatrix} = \begin{vmatrix} x_{00} \\ s_5 \\ s_6 \\ x_{10} \\ x_{10} \\ s_7 \\ x_{11} \end{vmatrix}$$

$$A^{(3)}_{7} = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

$$D^{(4)}_{7} = \operatorname{diag}(\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6)$$

$\alpha_0 = a_{00}$, $\alpha_1 = s_1$, $\alpha_2 = s_2$, $\alpha_3 = a_{01}$, $\alpha_4 = a_{11}$

$\alpha_5 = s_3$, $\alpha_6 = a_{01} - s_2$

$$A^{(4)}_{6\times 7} = \begin{vmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} \qquad A^{(5)}_{6} = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

$$A^{(6)}_{4\times 6} = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{vmatrix}$$

$$A^{(7)}_{4} = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Zadanie dla $n = 4$ i narysować graf strukturalny

↳ za miesiąc

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iloczyn skalarny

$$A_{4\times1} = [a_{00}, a_{01}, a_{10}, a_{11}]^T$$

$$X_{4\times1} = \begin{bmatrix} x_{00} \\ x_{01} \\ x_{10} \\ x_{11} \end{bmatrix}$$

$$(A, X) \overset{def}{=} A^T_{4\times1} \cdot X_{4\times1} = a_{00}x_{00} + a_{01}x_{01} + a_{10}x_{10} + a_{11}x_{11}$$

Winograd:

$$= (a_{00} + x_{01})(a_{01} + x_{00}) + (a_{10} + x_{11})(a_{11} + x_{10}) - a_{00}a_{01} - a_{10}a_{11} - x_{00}x_{01} - x_{10}x_{11}$$

$$\text{stałe elementy!} \left\{ \begin{matrix} a_{00}a_{01} - \\ a_{10}a_{11} - \end{matrix} \right.$$

$$\text{jednakowa wartość dla danej macierzy!} \leftarrow \left\{ \begin{matrix} x_{00}x_{01} - \\ x_{10}x_{11} \end{matrix} \right.$$

~~macierz przez wektor mnożenie:~~

procedura obliczeniowa:

$$Y_{M\times1} = A_{M\times2M}\left[A_{2M\times1} + P_{2M\times(M+1)}\, A_{(M+1)\times\frac{N(M-1)}{2}}\, B_{\frac{N(M+1)}{2}\times N(M+1)} \circ \left(A_{N(M+1)\times1} + P_{N(M+1)\times N}\, X_{N\times1}\right)\right]$$

macierz wektoryzacji danych:

1) $P_{N(M+1)\times N} = 1_{(M+1)\times 1} \otimes I_N$

2) $A_{N(M+1)\times 1} = \left(I_{\frac{N(M+1)}{2}} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right) \left(C_{NM\times 1} \frac{\square}{\square} 0_{N\times 1} \right)$

3) $B_{\frac{N(M+1)}{2}\times N(M+1)} = I_{\frac{N(M+1)}{2}} \otimes 1_{1\times 2}$

4) $A_{(M+1)\times \frac{N(M+1)}{2}} = I_{M+1} \otimes 1_{1\times \frac{N}{2}}$

5) $P_{2M\times(M+1)} = I_M \oplus 1_{M\times 1}$

6) $A_{2M\times 1} = 0_{M\times 1} \frac{\square}{\square} B_{M\times 1}$

7) $A_{M\times 2M} = I_M \; \square|\square \; (-I_M)$

przykład : macierz 3x4

$$C_{3\times 4} = \begin{vmatrix} c_{00} & c_{01} & c_{02} & c_{03} \\ c_{10} & c_{11} & c_{12} & c_{13} \\ c_{20} & c_{21} & c_{22} & c_{23} \end{vmatrix} \Rightarrow \begin{array}{l} M = 3 \\ N = 4 \end{array}$$

$$\underline{C_{NM\times 1}} = C_{3\cdot 4\times 1} = C_{12\times 1} =$$

$$\left[((c_{00}, c_{01}, c_{02}, c_{03}), (c_{10}, c_{11}, c_{12}, c_{13}), (c_{20}, c_{21}, c_{22}, c_{23}) \right]^T$$

$$B_{3\times 1} = [b_0, b_1, b_2],$$

$$b_0 = c_{00}\, c_{01} + c_{02}\, c_{03}$$

$$b_1 = c_{10}\, c_{11} + c_{12}\, c_{13}$$

$$b_2 = c_{20}\, c_{21} + c_{22}\, c_{33}$$

$$= Y_{3\times 1} = A_{3\times 6}\left[A_{6\times 1} + P_{6\times 4}\, A_{4\times 8}\, B_{8\times 16} \circ (A_{16\times 1} + P_{16\times 4}\, X_{4\times 1}) \right]$$

1) $P_{16\times 4} = I_{4\times 1} \oplus I_4 = \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \end{vmatrix} \otimes \begin{vmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{vmatrix}$

$$= \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \\ \hline 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \\ \hline 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \\ \hline 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

$$P_{16\times 4} \cdot \begin{vmatrix} x_{00} \\ x_{01} \\ x_{10} \\ x_{11} \end{vmatrix} = \left.\begin{bmatrix} x_{00} \\ x_{01} \\ x_{10} \\ x_{11} \\ \hline x_{00} \\ x_{01} \\ x_{10} \\ x_{11} \\ \hline x_{00} \\ x_{01} \\ x_{10} \\ x_{11} \\ \hline x_{00} \\ x_{01} \\ x_{10} \\ x_{11} \end{bmatrix}\right\} \boxed{\text{„wektoryzacja" danych}}$$

zadanie domowe: wyznaczyć wszystkie macierze
i narysować graf strukturalny

↳ ~~na tydzień przynieść~~ za dwa tygodnie