ogolne procedury dla 2. korolu racionalizacji:

$$(2)\quad Y_{8\times1} = (E_2 \otimes I_4)(I_2 \otimes E_2 \otimes I_2)\, D_8^{(2)} (I_2 \otimes E_2 \otimes I_2)\, d$$
$$(E_2 \otimes I_4)\, X_{8\times1}$$

$$D_8^{(2)} = \frac{1}{4}\,\mathrm{diag}\begin{bmatrix} A_2 + B_2 \\ A_2 - B_2 \\ C_2 + D_2 \\ C_2 - D_2 \end{bmatrix}$$

$$A_2 + B_2 = \begin{bmatrix} a+f+d+h & \overbrace{b+g+e+k}^{B_{N/2}} \\ \underbrace{c+g+e+k}_{C_{N/2}} & \underbrace{a+f+d+h}_{A_{N/2}} \end{bmatrix}$$

$$A_2 - B_2 = \begin{bmatrix} a+f-d-h & b+g-e-k \\ c+g-e-k & a+f-d-h \end{bmatrix}$$

$$\left[\begin{array}{c|c} A_N & B_N \\ \hline C_N & A_N \end{array}\right]$$

$$C_2 + D_2 = \begin{bmatrix} a-f+d-h & b-g+e-k \\ c-g+e-k & a-f+d-h \end{bmatrix}$$

$$C_2 - D_2 = \begin{bmatrix} a-f-d+h & b-g-e+k \\ c-g-e+k & a-f-d+h \end{bmatrix}$$

$$Y_{N\times1} = \begin{bmatrix} O_{N/2} & I_{N/2} & I_{N/2} \\ I_{N/2} & O_{N/2} & I_{N/2} \end{bmatrix} \mathrm{diag}\begin{bmatrix} C_{N/2} - A_{N/2} \\ B_{N/2} - A_{N/2} \\ A_{N/2} \end{bmatrix} \begin{bmatrix} I_{N/2} & O_{N/2} \\ O_{N/2} & I_{N/2} \\ I_{N/2} & I_{N/2} \end{bmatrix} X_{N\times1}$$

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przy $N=2$ (3. krok)

$$Y_{2\times1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \operatorname{diag}\begin{bmatrix} S_0 \\ S_1 \\ S_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} X_{2\times1} \Bigg\} A_2 + B_2$$

$$S_0 = c+g+e+k-a-f-d-h$$

$$S_1 = b+g+e+k-a-f-d-h$$

$$S_2 = a+f+d+h$$

$$A_2 - B_2 \Bigg\{ Y_{2\times1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \operatorname{diag}\begin{bmatrix} S_3 \\ S_4 \\ S_5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} X_{2\times1}$$

$$S_3 = c+g-e-k-a-f+d+h$$

$$S_4 = b+g-e-k-a-f+d+h$$

$$S_5 = a+f-d-h$$

$$C_2 + D_2 \Bigg\{ Y_{2\times1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \operatorname{diag}\begin{bmatrix} S_6 \\ S_7 \\ S_8 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} X_{2\times1}$$

$$S_6 = c-g+e-k-a+f-d+h$$

$$S_7 = b-g+e-k-a+f-d+h$$

$$S_8 = a-f+d-h$$

$C_2 - D_2 \left\{ \; Y_{2\times1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \mathrm{diag}\begin{bmatrix} S_9 \\ S_{10} \\ S_{11} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} X_{2\times1} \right.$

$$S_9 = c - g - e + k \quad -a + f + d - h$$

$$S_{10} = b - g - e + k \quad -a + f + d - h$$

$$S_{11} = a - f - d + h$$

$$T_{2\times3} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \qquad \tilde{T}_{3\times2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$(3) \; Y_{8\times1} = \left(E_2 \otimes I_4\right)\left(I_2 \otimes E_2 \otimes I_2\right)\left(I_4 \otimes T_{2\times3}\right) \cdot$$

$$\underbrace{D_{12}^{(3)} \left(I_4 \otimes \tilde{T}_{3\times2}\right)} \cdot \longrightarrow \text{„nowa" część"}$$

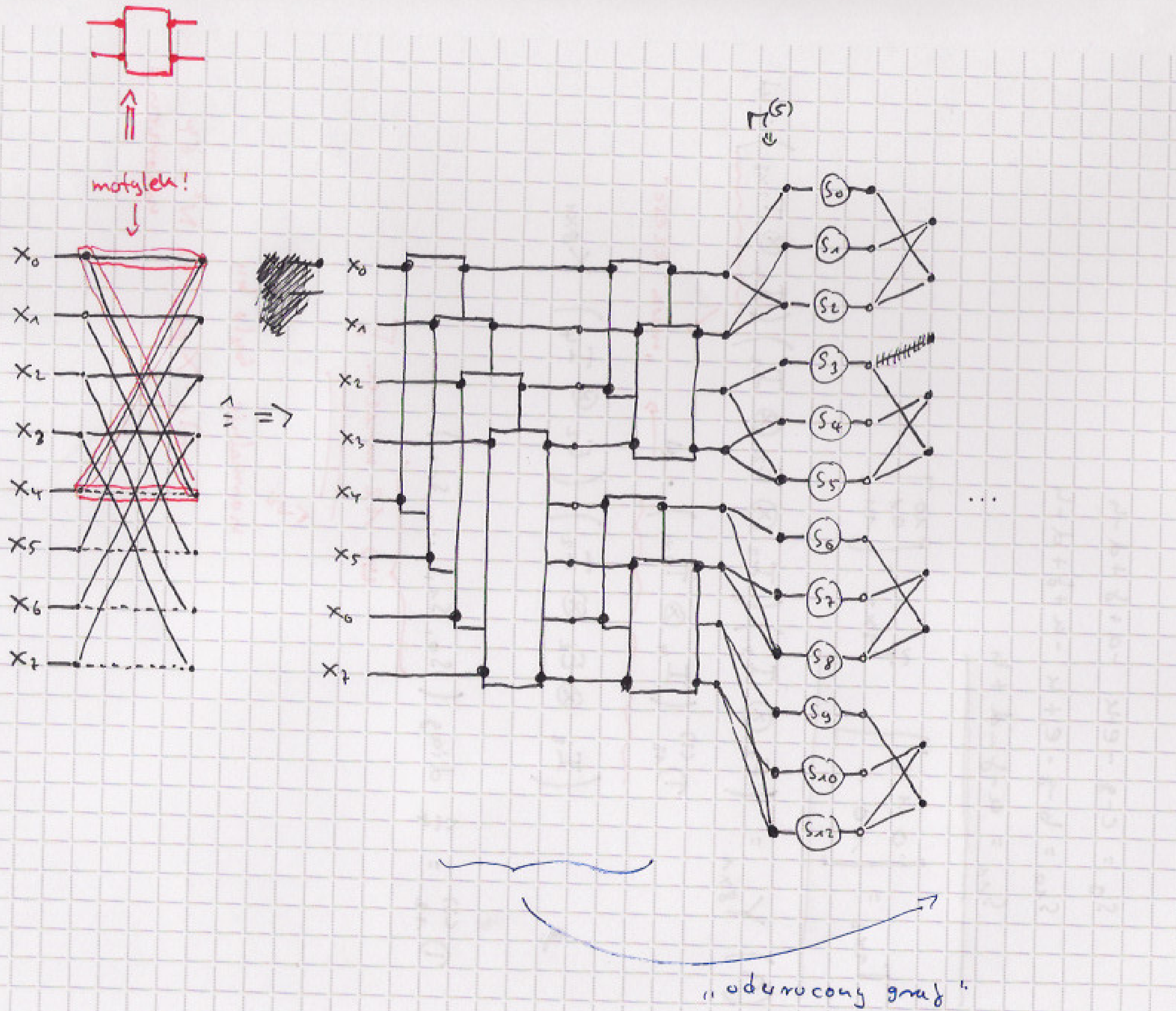
$$\left(I_2 \otimes E_2 \otimes I_2\right)\left(E_2 \otimes I_4\right) X_{8\times1}$$

$$D_{12}^{(3)} = \frac{1}{4}\, \mathrm{diag}\,(\underbrace{S_0, S_1, \ldots, S_{11}})$$

↳ 12 mnożeń !

normalnie było by

$A_8 \cdot X_{8\times1} \Rightarrow N^2 = 64$ mnożeń !


motylek!
x0
x1
x2
x3
x4
x5
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x7
Γ(S)
S0
S1
S2
S3
S4
S5
S6
S7
S8
S9
S10
S12
...
„odwrócony graf"

$$E_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$ macierz Hadamarda

$$I_4 = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \Rightarrow E_2 \otimes I_4 = \left[\begin{array}{cccc|cccc} 1&0&0&0&1&0&0&0 \\ 0&1&0&0&0&1&0&0 \\ 0&0&1&0&0&0&1&0 \\ 0&0&0&1&0&0&0&1 \\ \hline 1&0&0&0&-1&0&0&0 \\ 0&1&0&0&0&-1&0&0 \\ 0&0&1&0&0&0&-1&0 \\ 0&0&0&1&0&0&0&-1 \end{array}\right]$$

$$(E_2 \otimes I_4) X_{8\times1} = \begin{bmatrix} x_0 + x_4 \\ x_1 + x_5 \\ x_2 + x_6 \\ x_3 + x_7 \\ \hdashline x_0 - x_4 \\ x_1 - x_5 \\ x_2 - x_6 \\ x_3 - x_7 \end{bmatrix} = M^{(1)}$$

$\Rightarrow$ motylki ! (patrz na graf!)

$$(I_2 \otimes E_2 \otimes I_2) = \begin{bmatrix} 1&0&1&0&&&& \\ 0&1&0&1&&&& \\ 1&0&-1&0&&&& \\ 0&1&0&-1&&&& \\ &&&&1&0&1&0 \\ &&&&0&1&0&1 \\ &&&&1&0&-1&0 \\ &&&&0&1&0&-1 \end{bmatrix} = M^{(2)}$$

$$M^{(1)} M^{(2)} = \begin{bmatrix} (x_0 + x_4) + (x_2 + x_6) \\ (x_1 + x_5) + (x_3 + x_7) \\ (x_0 + x_4) - (x_2 + x_6) \\ (x_1 + x_5) - (x_3 + x_7) \\ \hdashline (x_0 - x_4) + (x_2 - x_6) \\ (x_1 - x_5) + (x_3 - x_7) \\ (x_0 - x_4) - (x_2 - x_6) \\ (x_1 - x_5) - (x_3 - x_7) \end{bmatrix} = M^{(3)}$$

motylek !

$$I_4 \otimes \tilde{T}_{3\times2} = \begin{bmatrix} 1 &&& \\ & 1 && \\ && 1 & \\ &&& 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 &&&&&& \\ 0 & 1 &&&&&& \\ 1 & 1 &&&&&& \\ && 1 & 0 &&&& \\ && 0 & 1 &&&& \\ && 1 & 1 &&&& \\ &&&& 1 & 0 && \\ &&&& 0 & 1 && \\ &&&& 1 & 1 && \\ &&&&&& 1 & 0 \\ &&&&&& 0 & 1 \\ &&&&&& 1 & 1 \end{bmatrix} = M^{(4)}$$

$$M^{(3)} M^{(4)} = M^{(5)}$$ (G patrz na grafie

$$I_4 \otimes T_{2\times3} = \begin{bmatrix} 1 &&& \\ & 1 && \\ && 1 & \\ &&& 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 &&&&&&&&& \\ 1 & 0 & 1 &&&&&&&&& \\ &&& 0 & 1 & 1 &&&&&& \\ &&& 1 & 0 & 1 &&&&&& \\ &&&&&& 0 & 1 & 1 &&& \\ &&&&&& 1 & 0 & 1 &&& \\ &&&&&&&&& 0 & 1 & 1 \\ &&&&&&&&& 1 & 0 & 1 \end{bmatrix}$$