algorytmy 28.3.09

2)

$$A_{16\times1} = \left(I_8 \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right) \left(C_{12\times1} \;\vdots\; 0_{4\times1} \right)$$

$$A_{16\times1} = \begin{bmatrix}
0&1&&&&&&&&&&&&&& \\
1&0&&&&&&&&&&&&&& \\
&&0&1&&&&&&&&&&&& \\
&&1&0&&&&&&&&&&&& \\
&&&&0&1&&&&&&&&&& \\
&&&&1&0&&&&&&&&&& \\
&&&&&&0&1&&&&&&&& \\
&&&&&&1&0&&&&&&&& \\
&&&&&&&&0&1&&&&&& \\
&&&&&&&&1&0&&&&&& \\
&&&&&&&&&&0&1&&&& \\
&&&&&&&&&&1&0&&&& \\
&&&&&&&&&&&&0&1&& \\
&&&&&&&&&&&&1&0&& \\
&&&&&&&&&&&&&&0&1 \\
&&&&&&&&&&&&&&1&0
\end{bmatrix} \cdot \begin{bmatrix} C_{00} \\ C_{01} \\ C_{02} \\ C_{03} \\ C_{10} \\ C_{11} \\ C_{12} \\ C_{13} \\ C_{20} \\ C_{21} \\ C_{22} \\ C_{23} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$* \begin{bmatrix} C_{01} \\ C_{00} \\ C_{03} \\ C_{02} \\ C_{11} \\ C_{10} \\ C_{13} \\ C_{12} \\ C_{21} \\ C_{20} \\ C_{23} \\ C_{22} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} C_{01} + x_0 \\ C_{00} + x_1 \\ C_{03} + x_2 \\ C_{02} + x_3 \\ C_{11} + x_0 \\ C_{10} + x_1 \\ C_{13} + x_2 \\ C_{12} + x_3 \\ C_{21} + x_0 \\ C_{20} + x_1 \\ C_{23} + x_2 \\ C_{22} + x_3 \\ x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = M$$

3) macierz mashowania mnożeń

$$B_{8\times 16} = I_8 \otimes 1_{1\times 2}$$

$$B_{8\times 16} = \begin{bmatrix} 1 & 1 & & & & & & & & & & & & & & \\ & & 1 & 1 & & & & & & & & & & & & \\ & & & & 1 & 1 & & & & & & & & & & \\ & & & & & & 1 & 1 & & & & & & & & \\ & & & & & & & & 1 & 1 & & & & & & \\ & & & & & & & & & & 1 & 1 & & & & \\ & & & & & & & & & & & & 1 & 1 & & \\ & & & & & & & & & & & & & & 1 & 1 \end{bmatrix}$$

$$\Rightarrow B_{8\times 16} \circ M$$

$$\begin{bmatrix} (c_{01}+x_0)(c_{00}+x_1) \\ (c_{03}+x_2)(c_{02}+x_3) \\ (c_{11}+x_0)(c_{10}+x_1) \\ (c_{13}+x_2)(c_{12}+x_3) \\ (c_{21}+x_0)(c_{20}+x_1) \\ (c_{23}+x_2)(c_{22}+x_3) \\ x_0 x_1 \\ x_2 x_3 \end{bmatrix}_{8\times 1} = (M^*)$$

4) $A_{4\times 8} = I_4 \otimes 1_{1\times 2}$

$$= \begin{bmatrix} 1 & 1 & & & & & & \\ & & 1 & 1 & & & & \\ & & & & 1 & 1 & & \\ & & & & & & 1 & 1 \end{bmatrix}_{4\times 8}$$

$\Rightarrow \quad A_{4\times 8} \cdot M^{*}$

$$= \begin{bmatrix} (c_{01}+x_0)(c_{00}+x_1) + (c_{03}+x_2)(c_{02}+x_3) \\ (c_{11}+x_0)(c_{10}+x_1) + (c_{13}+x_2)(c_{12}+x_3) \\ (c_{21}+x_0)(c_{20}+x_1) + (c_{23}+x_2)(c_{22}+x_3) \\ x_0 x_1 + x_2 x_3 \end{bmatrix}_{4\times 1} = M^{**}$$

5) $P_{6\times 4} = I_3 \oplus 1_{3\times 1}$

$$= \left[\begin{array}{ccc|c} 1 & & & \\ & 1 & & \\ & & 1 & \\ \hline & & & 1 \\ & & & 1 \\ & & & 1 \end{array}\right]$$

$$P_{6\times 4} \cdot M^{**} = \begin{bmatrix} (c_{01}+x_0 \ldots \\ (c_{11}+x_0 \ldots \\ (c_{21}+x_0 \ldots \\ x_0 x_1 + x_2 x_3 \\ x_0 x_1 + x_2 x_3 \\ x_0 x_1 + x_2 x_3 \end{bmatrix} = M^{***}$$

6) $A_{6\times1} = O_{3\times1} \,\frac{\square}{\square}\, B_{3\times1}$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix}$$

$$\Rightarrow A_{6\times1} + M^{***} = \begin{bmatrix} (c_{01} \dots) + (\quad) \\ (c_{11} \dots) + (\quad) \\ (c_{21} \dots) + (\quad) \\ x_0 x_1 + x_2 x_3 + b_0 \\ x_0 x_1 + x_2 x_3 + b_1 \\ x_0 x_1 + x_2 x_3 + b_2 \end{bmatrix}_{6\times1} = M^{4*}$$

7) $A_{3\times6} = I_3 \,\square\, (-I_3)$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right]$$

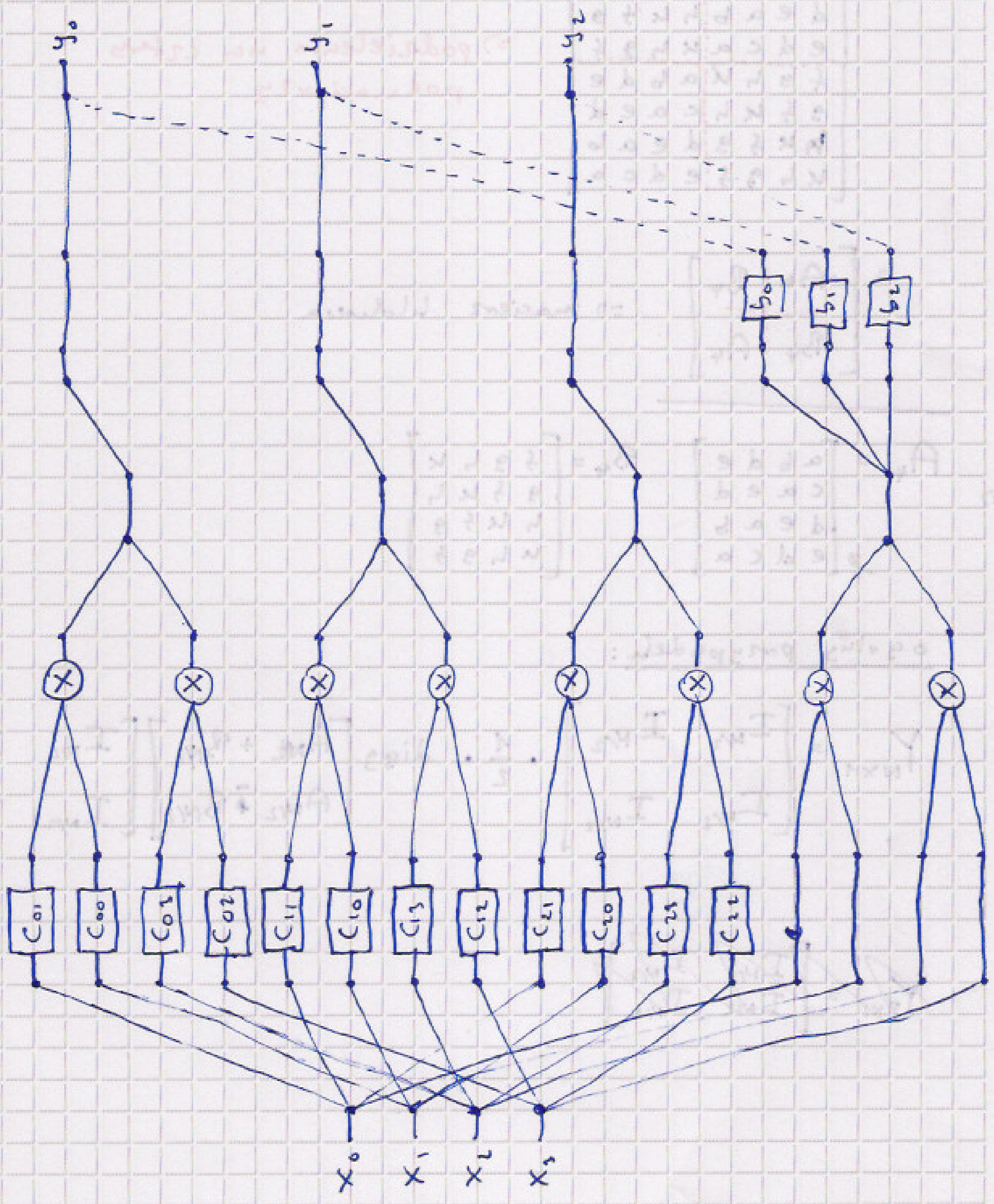
$A_{3\times6} \cdot M^{4*}$

$$\begin{bmatrix} (c_{01}+x_0)(c_{00}+x_1) + (c_{03}+x_2)(c_{02}+x_3) - b_0 - x_0 x_1 - x_{23} \\ (c_{11}+x_0)(c_{10}+x_1) + (c_{13}+x_2)(c_{12}+x_3) - b_1 - x_0 x_1 - x_{23} \\ (c_{21}+x_0)(c_{20}+x_1) + (c_{23}+x_2)(c_{22}+x_3) - b_2 - x_0 x_1 - x_{23} \end{bmatrix}_{3\times1}$$

ckc group

zadanie domowe: procedure i graf dla

$\boxed{M=4 \quad N=6}$



graf strukturalny

Strategia redukcji operacji arytmetycznych przy wyznaczaniu iloczynu macierzowo wektorowego

niech będzie podana macierz 8go rzędu:

$$A_8 = \left[\begin{array}{cccc:cccc} a & b & d & e & f & g & h & k \\ c & a & e & d & g & f & k & h \\ d & e & a & b & h & k & f & g \\ e & d & c & a & k & h & g & f \\ \hdashline f & g & h & k & a & b & d & e \\ g & f & k & h & c & a & e & d \\ h & k & f & g & d & e & a & b \\ k & h & g & f & e & d & c & a \end{array}\right]$$

=> podzielenie na cztery podmacierzy

$$= \left[\begin{array}{c:c} A_4 & B_4 \\ \hdashline B_4 & A_4 \end{array}\right]$$

=> macierz blokowa

=> $A_4 = \begin{bmatrix} a & b & d & e \\ c & a & e & d \\ d & e & a & b \\ e & d & c & a \end{bmatrix}$ $B_4 = \begin{bmatrix} f & g & h & k \\ g & f & k & h \\ h & k & f & g \\ k & h & g & f \end{bmatrix}$

ogólny przypadek:

$$Y_{N\times 1} = \begin{bmatrix} I_{N/2} & I_{N/2} \\ I_{N/2} & I_{N/2} \end{bmatrix} \cdot \frac{1}{2} \cdot \operatorname{diag} \begin{bmatrix} A_{N/2} + B_{N/2} \\ A_{N/2} - B_{N/2} \end{bmatrix} \begin{bmatrix} I_{N/2} & I_{N/2} \\ I_{N/2} & I_{N/2} \end{bmatrix} X_{4\times 1}$$

$$\cancel{Y_{8\times 1} = \begin{bmatrix} I_{N/2} & I_{N/2} \\ I_{N/2} & I_{N/2} \end{bmatrix}}$$

przykład:

$$\Rightarrow Y_{8\times1} = \begin{bmatrix} 1_4 & 1_4 \\ 1_4 & -1_4 \end{bmatrix} \cdot \frac{1}{2}\, \mathrm{diag} \begin{bmatrix} A_4 + B_4 \\ A_4 - B_4 \end{bmatrix} \begin{bmatrix} I_4 & I_4 \\ I_4 & I_4 \end{bmatrix} X_{4\times1}$$

$$\Rightarrow Y_{8\times1} = \left(E_2 \otimes I_4\right) \cdot D_8^{(1)} \cdot$$

I. krok

$\boxed{(1)}$ $$\left(E_2 \otimes I_4\right) X_{4\times1}$$

$$D_8^{(1)} = \frac{1}{2}\left(A_4 + B_4\right) \oplus \frac{1}{2}\left(A_4 - B_4\right)$$

$$A_4 + B_4 = \begin{array}{c} \begin{array}{cccc} & \nearrow A_2 & & \nearrow B_2 \end{array} \\ \left[\begin{array}{cc|cc} a+f & b+g & d+h & e+k \\ c+g & a+f & e+k & d+h \\ \hline d+h & e+k & a+f & b+g \\ e+k & d+h & c+g & a+f \end{array}\right] \end{array}$$

$$A_4 - B_4 = \left[\begin{array}{cc|cc} a-f & b-g & d-h & e-k \\ c-g & a-f & e-k & d-h \\ \hline d-h & e-k & a-f & b-g \\ e-k & d-h & c+g & a+f \end{array}\right]$$

dzielenie na cztery czesci

$$\Rightarrow Y_{4\times1} = \begin{bmatrix} I_2 & I_2 \\ I_2 & I_2 \end{bmatrix} \frac{1}{2}\, \mathrm{diag} \begin{bmatrix} A_2 + B_2 \\ A_2 - B_2 \end{bmatrix} \begin{bmatrix} I_2 & I_2 \\ I_2 & I_2 \end{bmatrix} X_{4\times1}$$

$A_4 + B_4$

$$Y_{4\times1} = \begin{bmatrix} 1_2 & 1_2 \\ 1_2 & 1_2 \end{bmatrix} \frac{1}{2}\, \mathrm{diag} \begin{bmatrix} C_2 + D_2 \\ C_2 - D_2 \end{bmatrix} \begin{bmatrix} I_2 & I_2 \\ I_2 & I_2 \end{bmatrix} X_{4\times1}$$

$A_4 - B_4$

$\boxed{(2)}$ $$D_8^{(2)} = \frac{1}{2}\left(C_2 + D_2\right) \oplus \frac{1}{2}\left(C_2 - D_2\right)$$

II. krok

zadanie domowe: obliczyc $Y_{8\times1}$ dla 2. kroku