Life Tables: 1948-2006

2024-09-05

- M1 MIDS MA7BY020
- Université Paris Cité
- Année 2024-2025
- Course Homepage
- Moodle



Objectives

Loading

Definitions

Check on http://www.mortality.org the meaning of the different columns.

See: Demography: Measuring and Modeling Population Processes by SH Preston, P Heuveline, and M Guillot. Blackwell. Oxford. 2001.

Document Tables de mortalité françaises pour les XIXe et XXe siècles et projections pour le XXIe siècle contains detailed information on the construction of Life Tables for France.

Period tables versus cohort tables

Two kinds of Life Tables can be distinguished: *Period tables (Table du moment)* which contain for each period (here a period is a calendar year), the mortality risks at different age ranges (here, we have one year ranges) for that very period; and *Tables de génération* which contain for a given birthyear, the mortality risks at which an individual born during that year has been exposed.

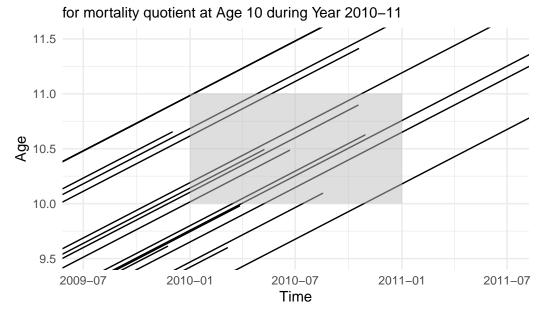
The life tables investigated in this lab are *Table du moment*. According to the document by Vallin and Meslé, building the life tables required ,decisions and doctoring.

Lexis diagrams

Lexis diagrams provide a graphical device that summarizes the construction of mortality quotients (and other rates in demography).

```
birth_dates <- as_date("1999-01-01") + duration(sample(2*365, size=20, replace=T),units="d
death_dates <- as_date("2009-07-01") + duration(sample(3*365, size=20, replace=T), units="d
b_period <- as_date("2010-01-01")</pre>
b_frame <- as_date(b_period - duration(1, units = "year"))</pre>
b_age <- 10L
tb_ld <- tibble(birth=birth_dates, death=death_dates)</pre>
tb_ld |>
  ggplot() +
  geom_segment(aes(x=b_frame,
                   xend=death,
                   y=interval(birth, b_frame)/years(1),
                   yend=interval(birth, death)/years(1))
               ) +
  annotate(geom="rect",
           xmin=b_period,
           xmax=as_date(b_period + duration(1, units = "year")),
           ymin=b_age,
           ymax=b_age + 1L,
           fill="grey",
          alpha=.5) +
  ylab("Age") +
  xlab("Time") +
  coord_cartesian(xlim=c(as_date(b_period - duration(6, units = "months")),
                          as_date(b_period + duration(18, units = "months"))),
                  ylim=c(b_age - .5, b_age+1.5)) +
  labs(
    title="A Lexis diagram",
    subtitle = "for mortality quotient at Age 10 during Year 2010-11"
  )
```

A Lexis diagram



Each line represents the *life line* of an individual born during years 1999 and 2000 and deceased beetween mid 2009 and mid 2012. In order to compute the mortality quotient at age 10 for year 2010, we have to compute the relevant *number of occurrences*, that is the number of segments ending in the grey rectangle, and the *sum of exposure times*, which is proportional to the sum of the lengths of the segments crossing the grey rectangle.

Have a look at Lexis diagram or at Preston et al.

Definitions can be obtained from www.lifeexpectancy.org. We translate it into mathematical (rather than demographic) language.

The mortality quotients define a probability distribution over \mathbb{N} . This probability distribution is a *construction* that reflects the health situation in a population at a given time. This probability distribution does not describe the sequence of sanitary situations experienced by a *cohort* (people born during a specific year).

One works with a period, or current, life table (table du moment). This summarizes the mortality experience of persons across all ages in a short period, typically one year or three years. More precisely, the death probabilities q_x for every age x are computed for that short period, often using census information gathered at regular intervals. These q_x 's are then applied to a hypothetical cohort of 100000 people over their life span to produce a life table.

```
small_tb <- life_table |>
  filter(Country=='France', Year== 2010, Gender=='Female', Age < 10 | between(Age, 80, 89)
  select(Age, qx, mx, lx, dx, Lx, Tx, ex)
small_tb</pre>
```

```
# A tibble: 20 x 8
     Age
                               lx
                                     dx
                                            Lx
                                                    Tx
              qx
                       mx
                                                           ex
   <int>
           <dbl>
                    <dbl>
                           <int> <int>
                                        <int>
                                                 <int> <dbl>
       0 0.00324 0.00325 100000
 1
                                    324 99722 8465207 84.6
2
       1 0.00032 0.00032
                           99676
                                     32 99660 8365484 83.9
3
       2 0.00015 0.00015
                           99645
                                        99637 8265824 83.0
4
       3 0.00011 0.00011
                           99630
                                        99624 8166187 82.0
       4 0.00008 0.00008
                           99619
                                      8 99615 8066563 81.0
5
```

```
5 99608 7966948 80.0
6
       5 0.00005 0.00005
                          99611
7
       6 0.00008 0.00008
                          99606
                                     8 99602 7867339 79.0
8
       7 0.00008 0.00008
                          99598
                                     8 99594 7767737 78.0
9
       8 0.00008 0.00008
                          99590
                                     8 99586 7668143 77
10
       9 0.00007 0.00007
                          99582
                                     7 99578 7568557 76
      80 0.0298
                 0.0302
                          75619
                                  2252 74493
                                              802295 10.6
11
                                              727802
12
      81 0.0346
                 0.0352
                           73367
                                  2535 72099
                                                      9.92
13
      82 0.0398
                 0.0406
                          70832
                                  2818 69423
                                              655702
                                                      9.26
14
      83 0.0464
                 0.0475
                           68014
                                  3158 66435
                                              586280
                                                      8.62
15
      84 0.0539
                 0.0554
                           64856
                                  3493 63109
                                              519845
                                                      8.02
16
      85 0.0610
                 0.0630
                           61362
                                  3745 59490
                                              456736
                                                      7.44
17
      86 0.0699
                 0.0725
                           57617
                                  4029 55603
                                              397246
                                                      6.89
18
      87 0.0793
                 0.0826
                          53588
                                  4249 51464
                                              341643
                                                      6.38
19
      88 0.0922 0.0966
                           49339
                                  4547 47066
                                              290180
                                                      5.88
20
      89 0.105
                 0.111
                           44792
                                  4706 42440
                                              243114
                                                      5.43
```

Question

The table above is not as readable as it should. Use package gt to get a more tunable outpout.

Reorder and filter the columns so that Age comes first (they identify rows), then qx, mx up to ex. You can use select or relocate, or both to do this. Note that Gender, Country, Year are constant in this tibble and need to be reported in the table header, but nowhere else.

Columns qx and mx (for mortality quotient and central death rate) should be dsplayed in scientific notation so that the fact that their range extends over several orders of magnitude shows up.

Columns lx, dx, Lx, Tx contain integer values.

Column ex (residual life expectancy) is a (fictional) decimal number of years

```
small_tb |>
  gt() |>
  tab_header(
    title = "Life table (extract)",
    subtitle = "France, Women, 2010"
) |>
  fmt_integer(columns=c(lx, dx, Lx, Tx)) |>
  fmt_engineering(columns = c(qx,mx), exp_style = "e", drop_trailing_zeros = T ) |>
  tab_source_note(source_note = "From https://mortality.org")
```

Life table (extract) France, Women, 2010

Age	qx	mx	lx	dx	Lx	Tx	ex
0	3.24e - 03	3.25e - 03	100,000	324	99,722	8,465,207	84.65
1	320e - 06	320e - 06	99,676	32	99,660	8,365,484	83.93
2	150e - 06	150e - 06	99,645	15	99,637	8, 265, 824	82.95
3	110e - 06	110e - 06	99,630	11	99,624	8, 166, 187	81.97
4	80e - 06	80e - 06	99,619	8	99,615	8,066,563	80.97
5	50e - 06	50e - 06	99,611	5	99,608	7,966,948	79.98
6	80e - 06	80e - 06	99,606	8	99,602	7,867,339	78.98
7	80e - 06	80e - 06	99,598	8	99,594	7,767,737	77.99
8	80e - 06	80e - 06	99,590	8	99,586	7,668,143	77.00

9	70e - 06	70e - 06	99,582	7	99,578	7,568,557	76.00
80	29.78e - 03	30.24e - 03	75,619	2,252	74,493	802,295	10.61
81	34.55e - 03	35.16e - 03	73,367	2,535	72,099	727,802	9.92
82	39.78e - 03	40.59e - 03	70,832	2,818	69,423	655,702	9.26
83	46.44e - 03	47.54e - 03	68,014	3,158	66,435	586, 280	8.62
84	53.86e - 03	55.36e - 03	64,856	3,493	63,109	519,845	8.02
85	61.03e - 03	62.95e - 03	61,362	3,745	59,490	456,736	7.44
86	69.93e - 03	72.46e - 03	57,617	4,029	55,603	397,246	6.89
87	79.29e - 03	82.56e - 03	53,588	4,249	51,464	341,643	6.38
88	92.15e - 03	96.6e - 03	49,339	4,547	47,066	290, 180	5.88
89	105.05e - 03	110.88e - 03	44,792	4,706	42,440	243, 114	5.43

From https://mortality.org

Understanding the columns of the life table

In the sequel, we denote by F_t the *cumulative distribution function* for year t. We agree on $\overline{F}_t = 1 - F_t$ and $F_t(-1) = 0$. Henceforth, \overline{F} is called the *survival* function.

qx (age-specific) risk of death at age x, or mortality quotient at given age x for given year t.

About the definition of q_{t,x}

Defining and computing $q_{t,x}$ does not boil down to knowing the number of people at age x at the beginning of ear t and knowing how many of them died during year t. If we want to be rigorous, we need to know all life lines in the Lexis diagram, or equivalently, how many people at Age x were alive on each day of Year t

△ Mortality quotients define a probability distribution

For a given year t, the sequence of mortality quotients define a survival function \overline{F}_t using the following recursion:

$$q_{t,x} = \frac{\overline{F}_t(x) - \overline{F}_t(x+1)}{\overline{F}_t(x)}$$

with boundary condition $\overline{F}_t(-1) = 1$.

This recursion can also be read as:

$$\overline{F}_t(x+1) = \overline{F}_t(x) \times (1-q_{t,x+1}) \, .$$

This artificial probability distribution is used to define and compute life expectancies.

 $q_{t,x}$ is the hazard rate of \overline{F}_t at age x.

mx central death rate at age x during year t. This is connected with $q_{t,x}$ by

$$m_{t,x} = -\log(1 - q_{t,x}),$$

or equivalently

$$q_{t,x} = 1 - \exp(-m_{t,x})$$

About central death rate

If we want to define a continuous probability distribution G over $[0,\infty)$ so that G and F coincide over integers and G has piecewise constant hazard rate, we can pick $m_{t,x}$ as the piecewise constant hazard rate.

lx the so-called *survival function*: the scaled proportion of persons alive at age x. These values are computed recursively from the $q_{t,x}$ values using the formula

$$l_t(x+1) = l_t(x) \times (1 - q_{t,x}),$$

with $l_{t,0}$, the radix of the table, (arbitrarily) set to 100000. In the table 1x is rounded to the next integer

Function $l_{t,\cdot}$ and \overline{F}_t are connected by

$$l_{t,x+1} = l_{t,0} \times \overline{F}_t(x)$$
.

- $\mathbf{dx} \ d_{t,x} = q_{t,x} \times l_{t,x}$. The fictitious number of deaths occurring at age x during year t. Again this is a rounded quantity.
- Tx Total number of person-years lived by the cohort from age x to x+1. This is the sum of the years lived by the $l_{t,x+1}$ persons who survive the interval, and the $d_{t,x}$ persons who die during the interval. The former contribute exactly 1 year each, while the latter contribute, on average, approximately half a year, so that $L_{t,x} = l_{t,x+1} + 0.5 \times d_{t,x}$. This approximation assumes that deaths occur, on average, half way in the age interval x to x+1. Such is satisfactory except at age 0 and the oldest age, where other approximations are often used.

Compare with the denominator in the definition of qx and its description using the Lexis diagram.

We will stick to a simplified vision $L_{t,x} = l_{t,x+1}$

ex: Residual Life Expectancy at age x and year t

This is the expectation of X-x for a random variable X distributed according to \overline{F}_t conditionally on the event $\{X \geq x\}$. That is $e_{t,x}$ is the expectation of the probability distribution defined by $\overline{F}_t(\cdot + x - 1)/\overline{F}_t(x - 1)$.

Question

Check dependencies between columns

Life table (relative discrepancies)

Year	Country	Gender	lx	dx	Lx	qx
2015	Spain	Female	17×10^{-6}	10.9×10^{-3}	11.22×10^{-6}	2.16×10^{-3}
2016	Spain	Male	31.74×10^{-6}	33.98×10^{-3}	21.2×10^{-6}	2.14×10^{-3}
2010	Italy	Female	19.78×10^{-6}	49.96×10^{-3}	16.84×10^{-6}	1.5×10^{-3}
2012	Italy	Male	31.08×10^{-6}	95.38×10^{-3}	27.53×10^{-6}	2.31×10^{-3}
2005	France	Female	10.7×10^{-6}	31.67×10^{-3}	12.56×10^{-6}	1.93×10^{-3}
2007	France	Male	34.2×10^{-6}	105.17×10^{-3}	28.96×10^{-6}	2.22×10^{-3}
2013	England & Wales	Female	20.33×10^{-6}	77.95×10^{-3}	10.96×10^{-6}	1.51×10^{-3}
2002	England & Wales	Male	55.97×10^{-6}	41.69×10^{-3}	31.12×10^{-6}	3.15×10^{-3}
2008	Netherlands	Female	21.37×10^{-6}	85.69×10^{-3}	14.31×10^{-6}	2.08×10^{-3}
2009	Netherlands	Male	25.74×10^{-6}	65.96×10^{-3}	27.43×10^{-6}	2.74×10^{-3}
2004	Sweden	Female	19.22×10^{-6}	25.87×10^{-3}	19.81×10^{-6}	1.87×10^{-3}
2005	Sweden	Male	28.5×10^{-6}	32.16×10^{-3}	26.8×10^{-6}	2.88×10^{-3}
2012	USA	Female	25.87×10^{-6}	17.3×10^{-3}	14.73×10^{-6}	1.98×10^{-3}
2010	USA	Male	40.17×10^{-6}	62×10^{-3}	36.57×10^{-6}	2.53×10^{-3}

From https://mortality.org

Western countries in 1948

Several pictures share a common canvas:

i Question

Plot mortality quotients (qx) against age using a logarithmic scale on the y axis. Countries are identified by aesthetics (shape, color, linetype).

- 1. Use facetting to plot qx of all countries at all ages for years 1950, 1960, ..., 2010.
- 2. Use plotly to build an animated plot using Year for the frame aesthetics.

• Abiding to the DRY principle, define a prototype ggplot (alternatively plotly) object.

The prototype will then be fed with different datasets and decorated and arranged for the different figures.

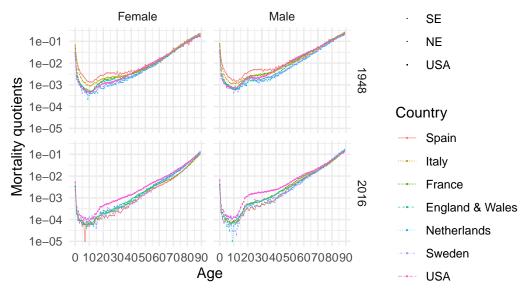
```
dummy_data <- filter(life_table, FALSE)
proto_plot <- dummy_data |>
```

```
ggplot() +
  aes(x=Age,
      y=qx,
      col=Country,
      linetype=Country,
      shape=Area) +
  scale_y_log10() +
  scale_x_continuous(breaks = c(seq(0, 100, 10), 109)) +
  ylab("Mortality quotients") +
  labs(linetype="Country") +
  theme_minimal()
```

p_1

European countries and US, 1948-2016

Sweden and the Netherlands consistently ahead Area



```
proto_plt2 <-
    ggplot() +
    aes(x=Age, y=qx, colour=Area, frame=Year, linetype=Country) +
    geom_point(size=.1) +
    geom_line(size=.1) +
    scale_y_log10() +
    labs(linetype=c("Country")) +
    scale_x_continuous(breaks = c(seq(0, 100, 10), 109)) +
    xlab("Age") +
    ylab("Mortality quotients") +
    facet_grid(cols=vars(Gender)) +
    theme_minimal()</pre>
```

)

In 1948, NE and the USA exhibit comparable mortality quotients at all ages for the two genders, the USA looking like a more dangerous place for young adults. Spain lags behind, Italy and France showing up at intermediate positions.

By year 1962, SE has almost caught up the USA. Italy and Spain still have higher infant mortality while mortality quotients in the USA and France are almost identical at all ages for both genders. Mortality quotients attain a minimum around 10-12 for both genders. In Spain the minimum central death rate has been divided by almost ten between 1948 and 1962.

If we dig further we observe that the shape of the male mortality quotients curve changes over time. In 1962, in the USA and France, mortality quotients exhibit a sharp increase between years 12 and 18, then remain almost constant between 20 and 30 and afterwards increase again. This pattern shows up in other countries but in a less spectacular way.

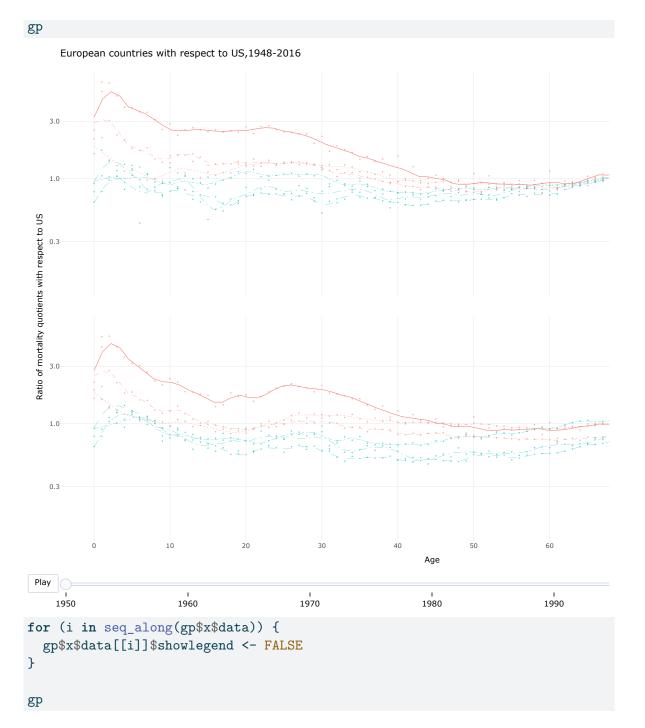
Twenty years afterwards, during years 1980-1985, death rates at age 0 have decreased at around 1% in all countries while it was 7% in Spain in 1948. The male central death curve exhibits a plateau between ages 20 and 30. Mortality quotients at this age look higher in France and the USA.

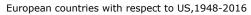
By year 2000, France is back amongst European countries (at least with respect to mortality quotients). Young adult mortality rates are higher in the USA than in Europe. This phenomenon became more pregnant during the last decade.

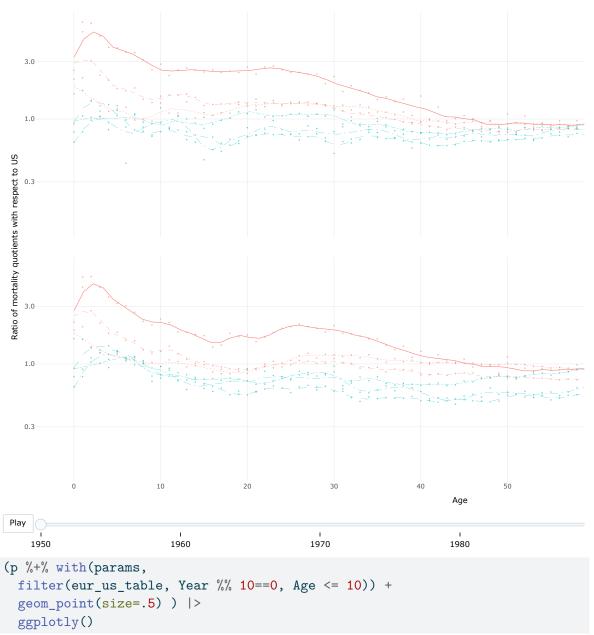
i Question

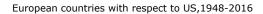
Plot ratios between mortality quotients (qx) in European countries and mortality quotients in the USA in 1948.

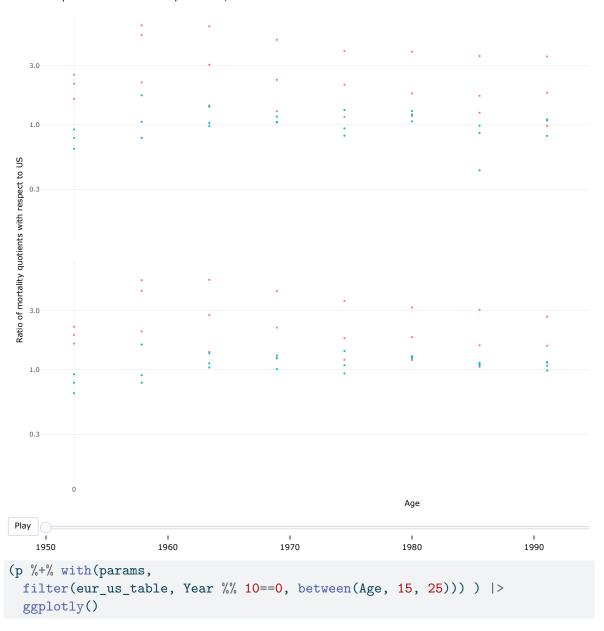
```
p <- with(params,
  (filter(eur us table, Year \\\ 10==0)
    ggplot() +
    aes(x=Age,
        y=Ratio,
        frame=Year) +
    aes(linetype=Country, show.legend = FALSE) +
    aes(color=Area, show.legend = FALSE) +
    scale_y_log10() +
    scale_x = c(seq(0, 100, 10), 109)) +
    geom_point(size=.1) +
    geom_smooth(method="loess",
                formula= 'y~ x',
                se=FALSE,
                span=.1,
                size=.1) +
    ylab("Ratio of mortality quotients with respect to US") +
    ggtitle(label = glue("European countries with respect to US,{year_p}-{year_e}"),
            subtitle = "Sweden consistently ahead") +
   facet_grid(rows = vars(Gender))
  ))
gp <- p |>
 ggplotly()
```





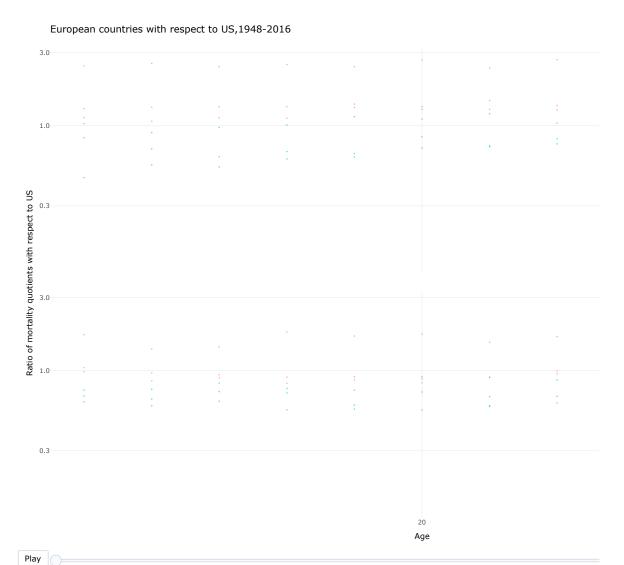






1990

1980



Comment

1950

This animation reveals less than the preceding one since we just have ratios with respect to the USA. But the patterns followed by European societies emerge in a more transparent way. The divide between northern and southern Europe at the onset of the period is even more visible. The ratios are important across the continent: there is a factor of 10 between spanish and swedish infant mortality rates. But the ratios at ages 50 and above tend to be similar. By the early 60s, the gap between southern and northern Europe has shrinked. By now, the ratios between mortality quotients tend to be within a factor of 2 across all ages, and even less at ages 50 and above.

1970

Death rates evolution since WW II

1960

Question

Plot mortality quotients (column qx) for both genders as a function of Age for years 1946, 1956, ... up to 2016. Use aesthetics to distinguish years. You will need to categorize the Year column (forcats:: may be helpful).

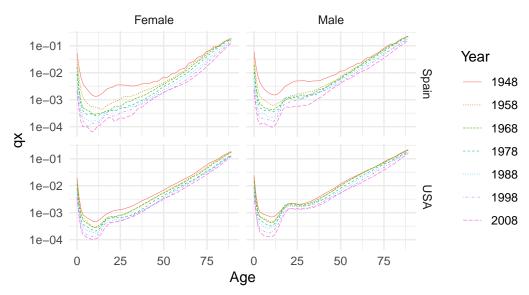
1. Facet by Gender and Country

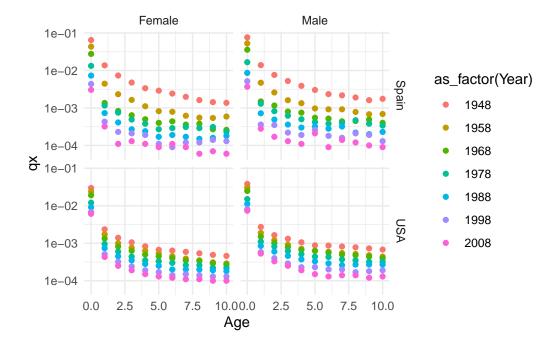
2. Pay attention to axes labels, to legends. Assess logarithmic scales.

```
p_3 <- (p %+%
filter(life_table,
    Year %in% post_ww_II,
    Gender!="Both",
    Age < 90,
    Country %in% c('Spain', 'USA')) +
labs(title="Mortality quotient per Age",
    subtitle = "Post WW II")
)</pre>
```

p_3

Mortality quotient per Age Post WW II





Question

Write a function ratio_mortality_rates with signature function(df, reference_year=1946, target_years=seq(1946, 2016, 10)) that takes as input:

- a dataframe with the same schema as life_table,
- a reference year ref_year and
- a sequence of years target_years

and that returns a dataframe with schema:

Column Name	Column Type
Year	integer
Age	integer
mx	double
$mx.ref_year$	double
Country	factor
Gender	factor

where (Country, Year, Age, Gender) serves as a *primary key*, mx denotes the central death rate at Age for Year and Gender in Country whereas mx_ref_year denotes central death rate at Age for argument reference_year in Country for Gender.

```
select(Age, Area, Gender, Country, qx, Year) |>
inner_join(right_df, by = jbe)
}
```

i Question

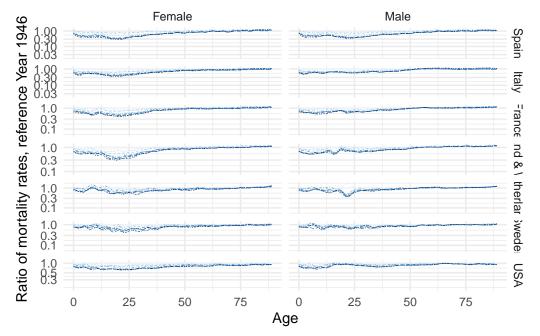
Draw plots displaying the ratio $q_{x,t}/q_{x,1946}$ for ages $x \in 1, ..., 90$ and year t for $t \in 1946, ..., 2013$ where $q_{x,t}$ is the mortality quotient at age x during year t.

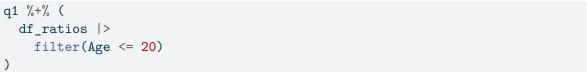
- 1. Handle both genders and all countries
- 2. One properly facetted plot is enough.

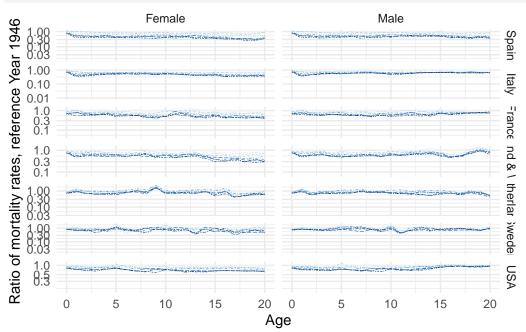
```
geom_smooth_line <- geom_smooth(method="loess",
  formula = y ~ x,
  se= FALSE,
  size = .2,
  span= .1
)

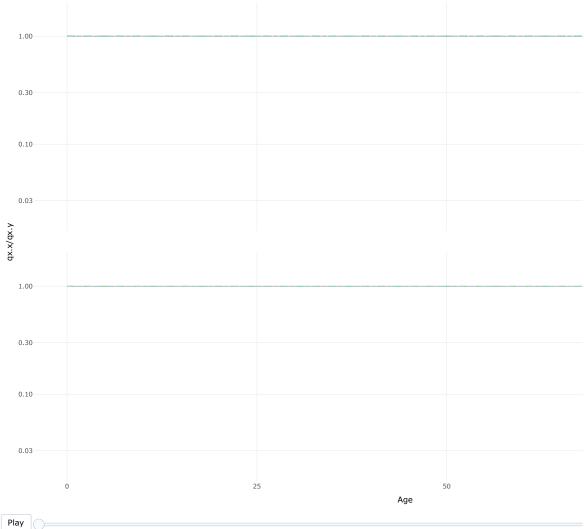
q <- df_ratios |>
```

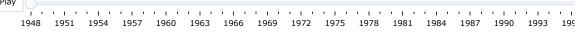
```
ggplot() +
  aes(x=Age,
      y=qx.x/qx.y) +
  geom_smooth_line +
  scale_y_log10()
q1 <- q +
  aes(linetype=as_factor(Year),
      col=as_factor(Year)) +
  ylab("Ratio of mortality rates, reference Year 1946") +
  labs(linetype="Year", col="Year") +
  scale_colour_brewer() +
  theme(legend.position = "none") +
  facet_grid(
    rows = vars(Country),
    cols =vars(Gender),
    scales = "free_y"
  )
q1
```



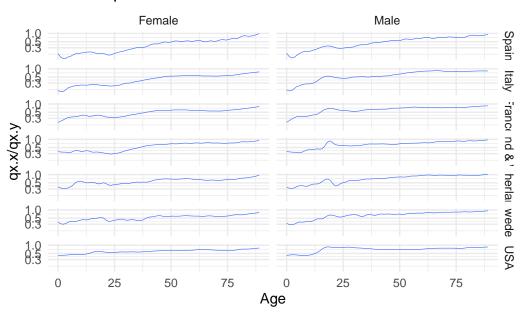












Comment.

During the last seventy years, death rates decreased at all ages in all seven countries. This progress has not been uniform across ages, genders and countries. Across most countries, infant mortality dramatically improved during the first post-war decade while death rates at age 50 and above remained stable until the mid seventies.

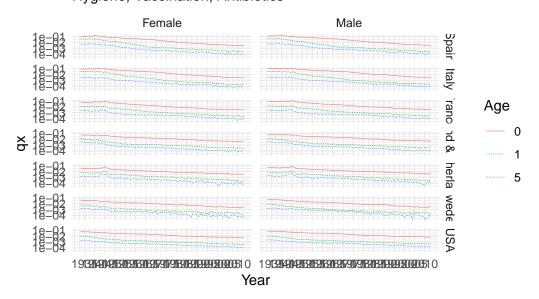
Trends

We noticed that mortality quotients did not evolve in the same way across all ages: first, the decay has been much more significant at low ages; second, the decay of mortality quotients at old ages (above 60) mostly took place during the last four decades. It is worth digging separately at what happened for different parts of life.

Question

Plot mortality quotients at ages 0, 1, 5 as a function of time. Facet by Gender and Country

Infant and child, mortality rate Hygiene, Vaccination, Antibiotics



Comment

All European countries achieved the same infant mortality rates after year 2000. The USA now lag behind.

During years 1940-1945, in the Netherlands and France, gains obtained before 1940 were reversed. Year 1945 was particularly difficult in the Netherlands.

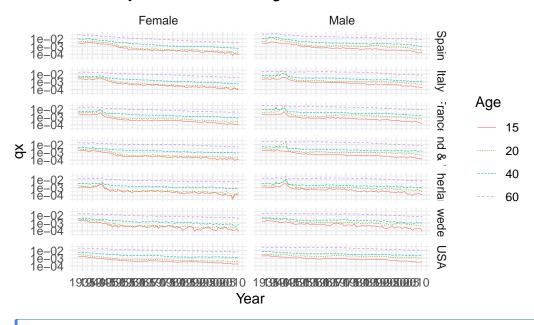
Question

Plot mortality quotients at ages 15, 20, 40, 60 as a function of time. Facet by Gender and Country

ages <-c(15, 20, 40, 60)

```
p_children %+%
filter(life_table,
         Age %in% ages,
         Gender != "Both",
         Year %in% 1933:2013) +
ggtitle("Mortality rate at different ages")
```

Mortality rate at different ages



i Comment.

While death rates at ages 15 and 20 among women are close across all societies, death rates are higher at age 20 than at age 15 among men. In France, at age 20, death rates declined from 1945 until 1960, and then increased back to their initial level until 1980. Male death rates at age 60 started to decline around 1980. Female death rates at age 60 declined steadily throughout the 7 decades. Years 1940-1945 exhibit disruptions with different shapes and intensities in Italy, France, England & Wales, and the Netherlands.