Introduction to Synthetic Control

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Beyond Parallel Trends

Warning

Difference-in-differences requires some form of parallel trends: similar time trends for treated and untreated units.

Note

Synthetic control can accommodate non-parallel trends without requiring explicit time trend modeling.

Synthetic control methodology as a tool for evaluating population-level health interventions

Janet Bouttell,[1] Peter Craig,[2] James Lewsey,[1] Mark Robinson,[3] Frank Popham[2]

Motivating Example: State Health Policies

Legislative Mandate for Tobacco Control - Proposition 99

CA Prop 99 in 1988:

Raised cigarette taxes

Funding for health ed

OH May 2021:

Lottery for vaccinated people

Followed by other states

Question

What is the effect of these policies on cigarette sales and vaccine uptake, respectively?

Synthetic Control Method

Panel Data/Comparative Case Study

Generally requires:

- One (or few) treated units
- Many untreated units
- Long pre-treatment history of outcomes for all units
- Post-treatment outcomes for time period of interest

Idea

Find a weighted average of the control units that **best approximates** the pre-treatment history of the treated unit(s).

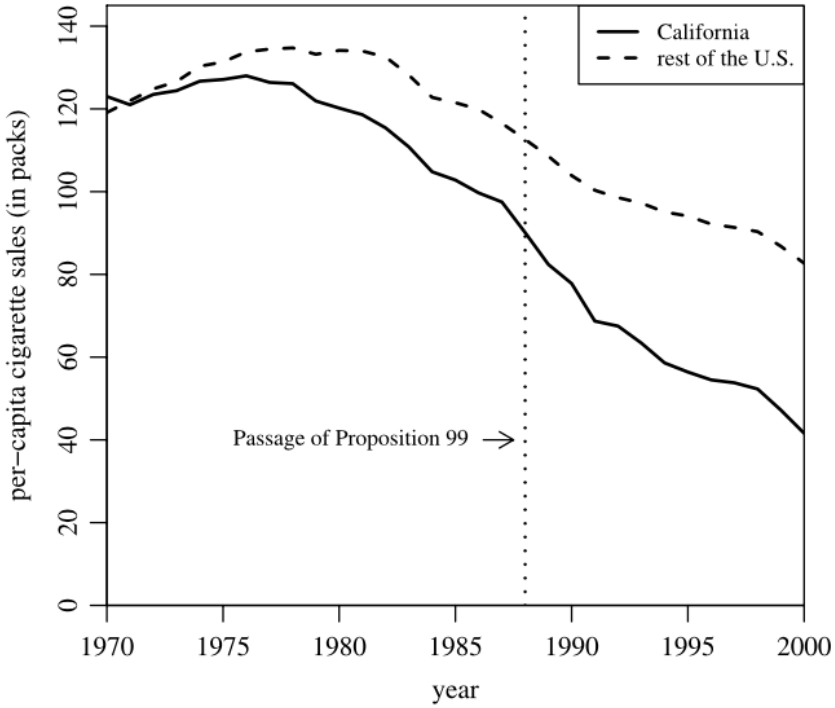
This is the **synthetic control** unit, which is then compared to the treated unit's outcomes in the post-treatment period.

Example of Idea

Synthetic Control Methods for Comparative Case Studies: Estimating the Effect of California's Tobacco Control Program

Alberto ABADIE, Alexis DIAMOND, and Jens HAINMUELLER

Journal of the American Statistical Association
June 2010, Vol. 105, No. 490, Applications and Case Studies
DOI: 10.1198/jasa.2009.ap08746



Abadie et al. (2010), Figure 1

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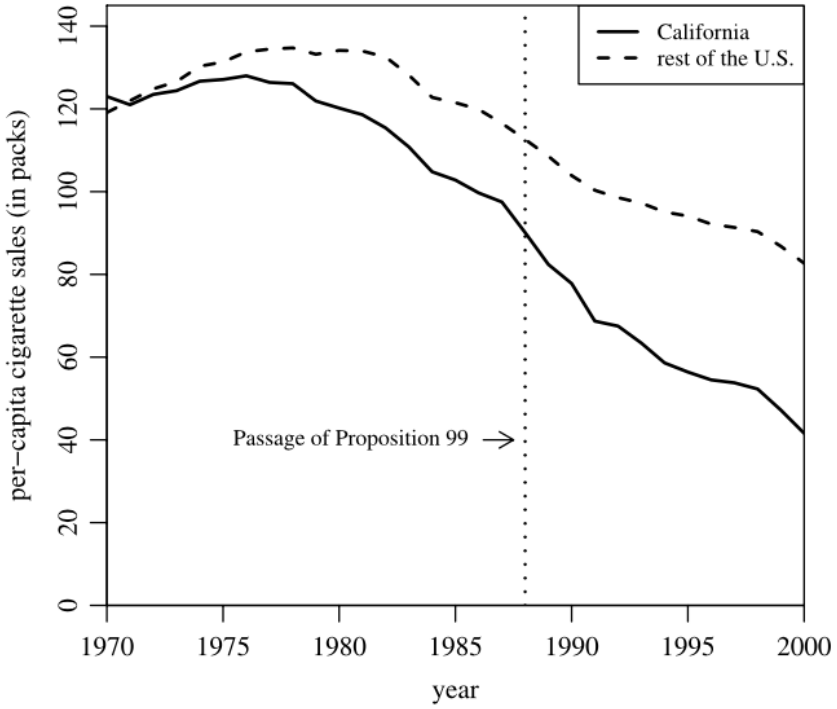

Abadie et al. (2010), Figure 1

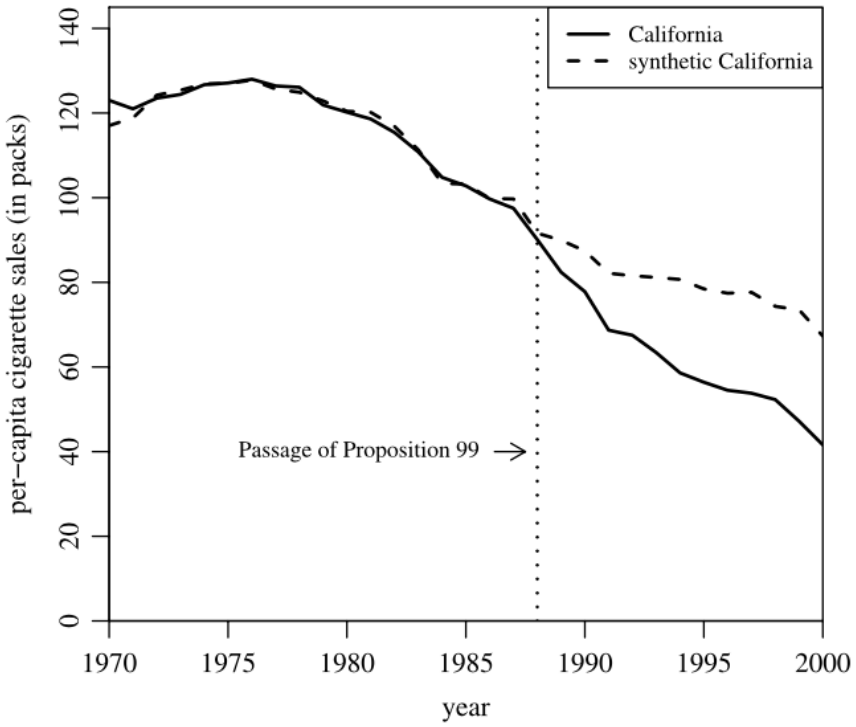

Figure 2. Trends in per-capita cigarette sales: California vs. synthetic California.

Abadie et al. (2010), Figure 2

Key Requirements

Using Synthetic Controls: Feasibility, Data Requirements, and Methodological Aspects†

ALBERTO ABADIE*

Practice of Epidemiology

Synthetic Control Methods for the Evaluation of Single-Unit Interventions in Epidemiology: A Tutorial

Carl Bonander*, David Humphreys, and Michelle Degli Esposti

Key Requirements

- No anticipation
- No spillover
- Suitable control units: “stable weights”
- Convex hull: non-extreme treated unit(s)
- Effect size larger than routine fluctuations
- Appropriate time horizon

Formal Specification

$$\hat{\theta}_t = Y_{1t} - \sum_{i=1}^{n} w_i Y_{0it},$$

where Y_{1t} is the outcome of the treated unit in period t and Y_{0it} is the outcome of the ith untreated unit in period t.

The weights $w_1, \ldots, w_n$ are nonnegative and sum to 1.

Minimizing Pre-Trend Difference

In the simplest form, the weights are chosen to minimize:

$$\sum_{t=1}^{T-1} \left(Y_{1t} - \sum_{i=1}^{n} w_i Y_{0it} \right)^2,$$

where $T-1$ is the last period for which the treated unit is pre-intervention.

Incorporating Covariates

Covariates can be incorporated in the weight minimization. For covariates (including pre-treatment outcomes) labelled $k = 1, \ldots, K$, choose a weight vector w that minimizes:

$$\sum_{k=1}^{K} v_k \left(X_{1k} - \left(\sum_{i=1}^{n} X_{0ik} w_i \right) \right)^2,$$

where v_k are weights on the importance of each covariate, which can themselves be chosen to minimize the pre-intervention difference or by cross-validation on a split sample of pre-intervention times.

Weights

- Do not use post-intervention outcomes
- Generally sparse
- Avoid extrapolation, permit (linear) interpolation
- Transparent and (somewhat) interpretable

Table 2. State weights in the synthetic California

State	Weight	State	Weight
Alabama	0	Montana	0.199
Alaska	–	Nebraska	0
Arizona	–	Nevada	0.234
Arkansas	0	New Hampshire	0
Colorado	0.164	New Jersey	–
Connecticut	0.069	New Mexico	0
Delaware	0	New York	–
District of Columbia	–	North Carolina	0
Florida	–	North Dakota	0
Georgia	0	Ohio	0
Hawaii	–	Oklahoma	0
Idaho	0	Oregon	–
Illinois	0	Pennsylvania	0
Indiana	0	Rhode Island	0
Iowa	0	South Carolina	0
Kansas	0	South Dakota	0
Kentucky	0	Tennessee	0
Louisiana	0	Texas	0
Maine	0	Utah	0.334
Maryland	–	Vermont	0
Massachusetts	–	Virginia	0
Michigan	–	Washington	–
Minnesota	0	West Virginia	0
Mississippi	0	Wisconsin	0
Missouri	0	Wyoming	0

Abadie et al. (2010), Table 2

Estimand

> **Important**
>
> The estimand is again the **average treatment effect on the treated (ATT)**: the effect of the policy on the treated unit compared to if it had not been treated.

The choice of time and scale for the comparison can be made by the investigator based on subject-matter knowledge.

Gap Plot

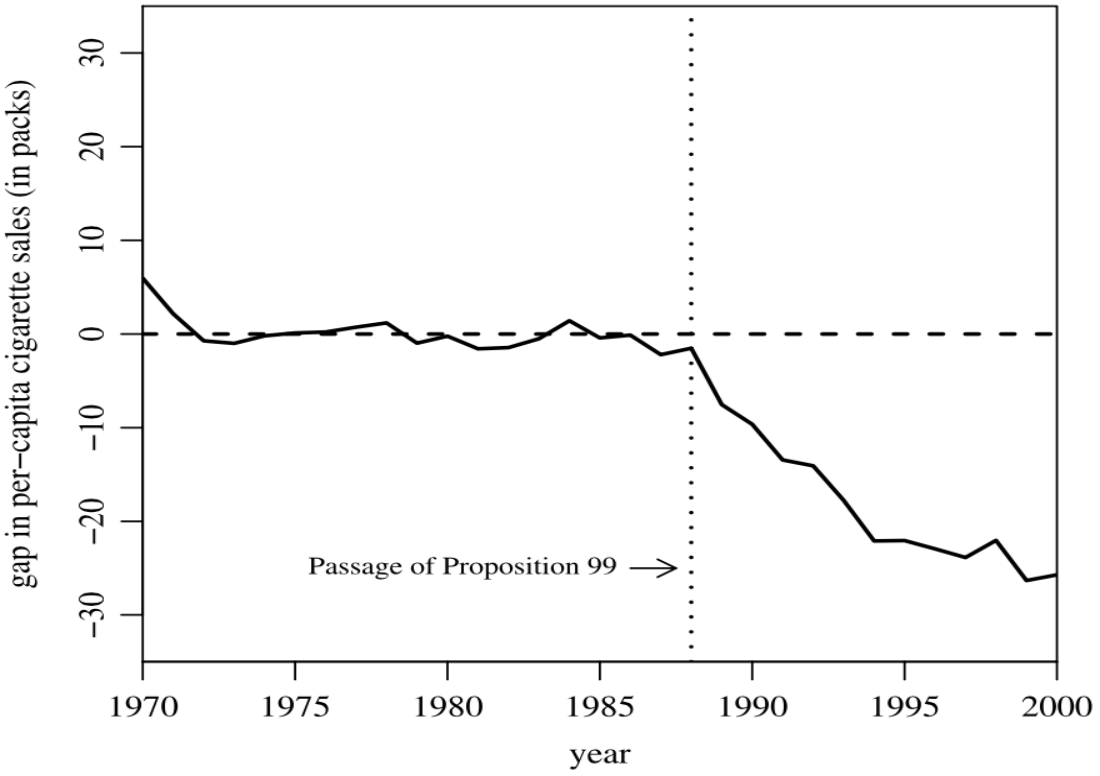

Figure 3. Per-capita cigarette sales gap between California and synthetic California.

Abadie et al. (2010), Figure 3

Robustness and Inference

Assessing Fit

Table 1. Cigarette sales predictor means

Variables	California		Average of 38 control states
	Real	Synthetic	
Ln(GDP per capita)	10.08	9.86	9.86
Percent aged 15–24	17.40	17.40	17.29
Retail price	89.42	89.41	87.27
Beer consumption per capita	24.28	24.20	23.75
Cigarette sales per capita 1988	90.10	91.62	114.20
Cigarette sales per capita 1980	120.20	120.43	136.58
Cigarette sales per capita 1975	127.10	126.99	132.81

NOTE: All variables except lagged cigarette sales are averaged for the 1980–1988 period (beer consumption is averaged 1984–1988). GDP per capita is measured in 1997 dollars, retail prices are measured in cents, beer consumption is measured in gallons, and cigarette sales are measured in packs.

Abadie et al. (2010), Table 1

Specification Tests

The pre-intervention **mean squared prediction error (MSPE)** of the SC fit can be used to assess fit.

To test robustness of results, can change:

- Control units
- Time frame considered
- Covariates used

Placebo Test In-Space

Conduct the SC analysis with the **same specifications** for each control unit, excluding the treated unit. This gives a **null distribution of estimates**.

Can exclude those with much higher pre-intervention MSPEs.

Placebo Test In-Space

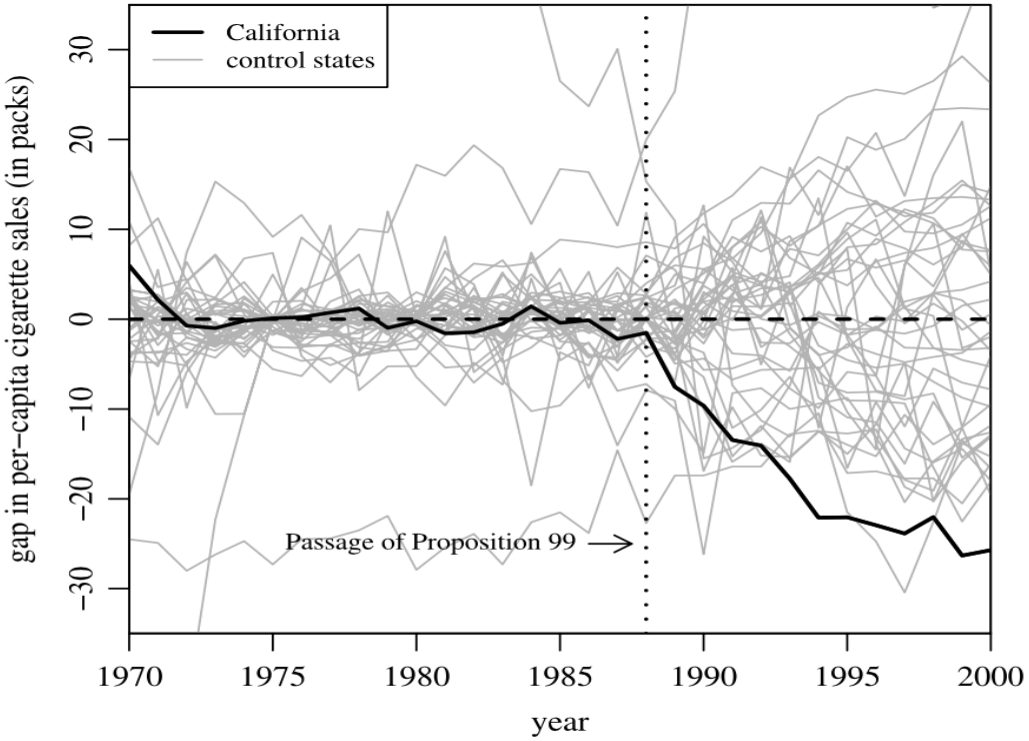

Figure 4. Per-capita cigarette sales gaps in California and placebo gaps in all 38 control states.

Abadie et al. (2010), Figure 4

Placebo Test In-Space

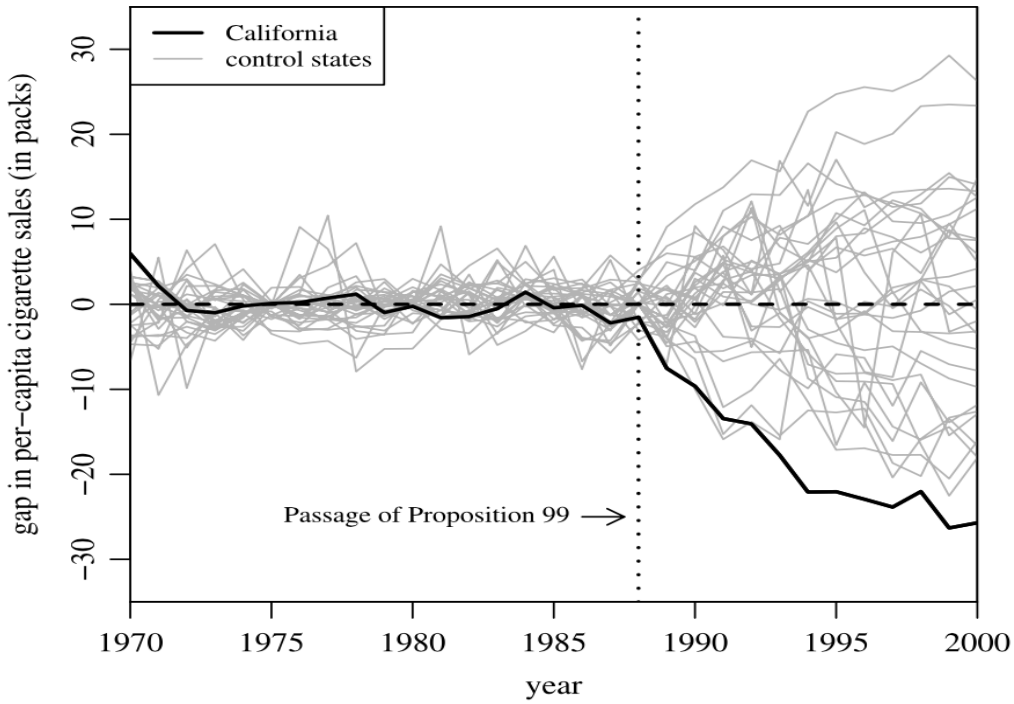

Figure 6. Per-capita cigarette sales gaps in California and placebo gaps in 29 control states (discards states with pre-Proposition 99 MSPE five times higher than California's).

Abadie et al. (2010), Figure 6

Placebo Test Estimators

Visual inspection of the observed result compared to the null distribution can follow. Or a specific estimator can be used to conduct a hypothesis test.

Common choices are:

- First-period effect
- Average effect over all included post-treatment periods
- Post-treatment root mean square prediction error (RMSPE)
- Post-treatment RMSPE/Pre-treatment RMSPE Ratio

Placebo Test In-Time

Can also run the analysis on dummy intervention time points.

> **Note**
>
> This is similar to the cross-validation approach sometimes used to select covariate weights.
>
> If using both, interpret with caution.

Placebo Test In-Outcome/Population

In some cases, non-affected outcomes or populations may be available. These can be used as a null control or distribution.

Estimating the population-level impact of vaccines using synthetic controls

Christian A. W. Bruhn[a], Stephen Hetterich[b], Cynthia Schuck-Paim[b], Esra Kürüm[a,c], Robert J. Taylor[b], Roger Lustig[b], Eugene D. Shapiro[a,d], Joshua L. Warren[a,e], Lone Simonsen[b,f,g], and Daniel M. Weinberger[a,1]

Recommendations

Summary

Advantages:

- Allows non-parallel trends
- Interpretability of weights
- Counterfactual estimate can be used for many ATT estimands

Disadvantages/Limitations:

- Requires linear interpolation of trends
- Lots of researcher degrees of freedom
- Can be highly variable or sensitive to specifications
- Clearest with one or few treated units

Recommendations

- Ensure ATT is appropriate
- Consider trade-offs: more vs. fewer units, more vs. fewer time periods, interpolation vs. extrapolation, etc.
- Pre-specify analyses: control units, covariates, years, placebo tests, MSPE restrictions, etc.
- Run robustness checks wherever possible
- Interpret results in context

Questions?