



第12章 二值数学形态学

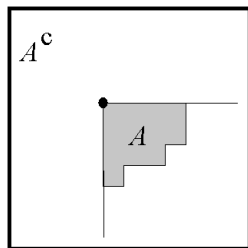
- 12.1 基本集合定义
- 12.2 二值形态学基本运算
- 12.3 二值形态学组合运算
- 12.4 二值形态学实用算法

12.1 基本集合定义

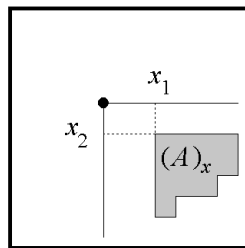
(1) 集合：用大写字母表示，空集记为 \emptyset

(2) 元素：用小写字母表示

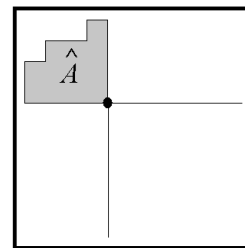
(3) 子集：



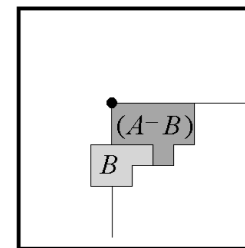
(a)



(b)



(c)



(d)

(4) 并集：

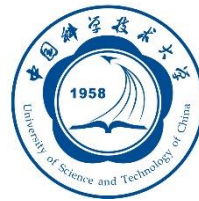
(5) 交集：

(6) 补集： $A^c = \{x \mid x \notin A\}$

(7) 位移： $(A)_x = \{y \mid y = a + x, a \in A\}$

(8) 映像： $\hat{A} = \{x \mid x = -a, a \in A\}$

(9) 差集： $A - B = \{x \mid x \in A, x \notin B\} = A \cap B^c$



12.2 二值形态学基本运算

集合运算:

- A 为图象集合, B 为结构元素 (集合)
- 数学形态学运算是用 B 对 A 进行操作
- 结构元素要指定1个原点 (参考点)

12.2.1 膨胀和腐蚀

12.2.2 开启和闭合

12.2.3 基本运算性质

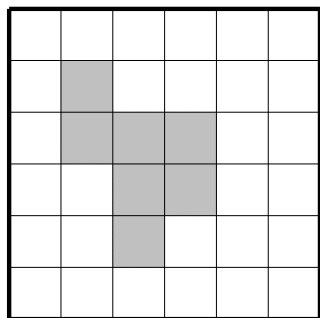
12.2.1 膨胀和腐蚀

1. 膨胀

膨胀的算符为 \oplus

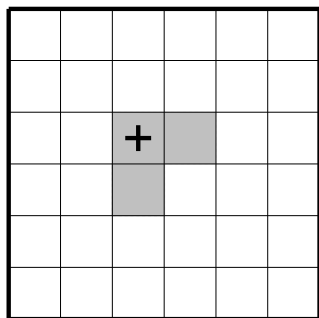
$$A \oplus B = \{x \mid [(\hat{B})_x \cap A] \neq \emptyset\}$$

集合 A



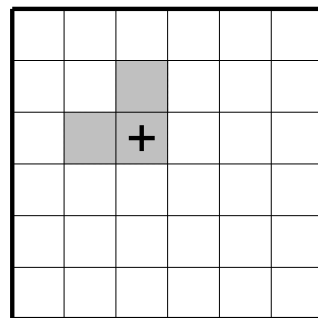
(a)

结构元素 B



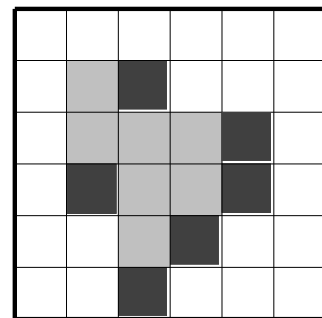
(b)

B 的映象



(c)

集合 $A \oplus B$



(d)

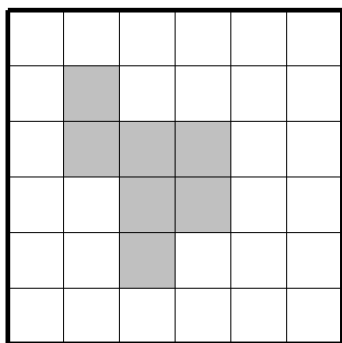
12.2.1 膨胀和腐蚀

2. 腐蚀

腐蚀的算符为

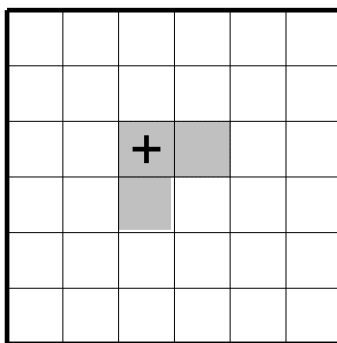
$$A \ominus B = \{x \mid (B)_x \subseteq A\}$$

集合 A



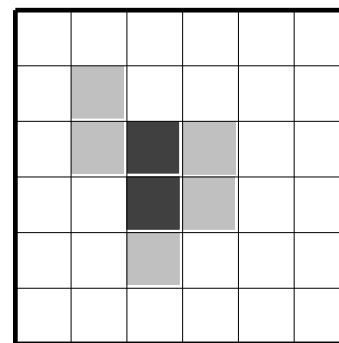
(a)

结构元素 B



(b)

集合 $A \ominus B$



(c)



12.2.1 膨胀和腐蚀

3. 原点不包含在结构元素中的膨胀和腐蚀

原点包含在结构元素中

$$\text{膨胀运算: } A \subseteq A \oplus B$$

$$\text{腐蚀运算: } A \ominus B \subseteq A$$

原点不包含在结构元素中

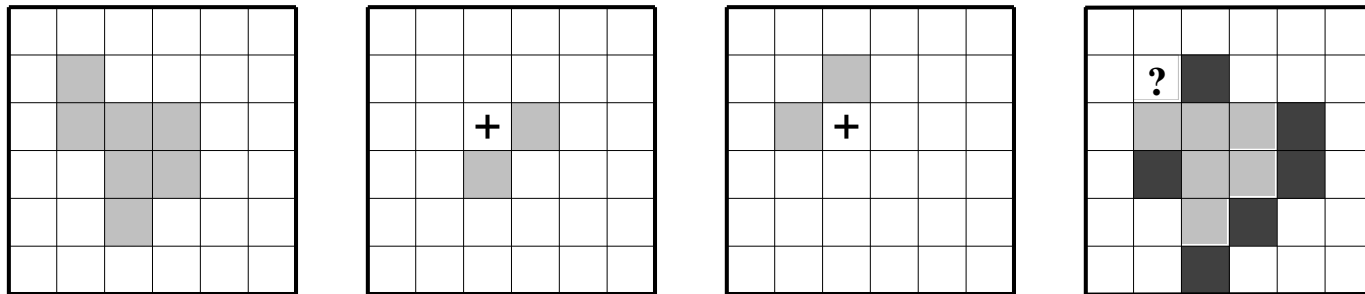
$$\text{膨胀运算: } A \not\subseteq A \oplus B$$

$$\text{腐蚀运算: } A \ominus B \subseteq A, \text{ 或 } A \ominus B \not\subseteq A$$

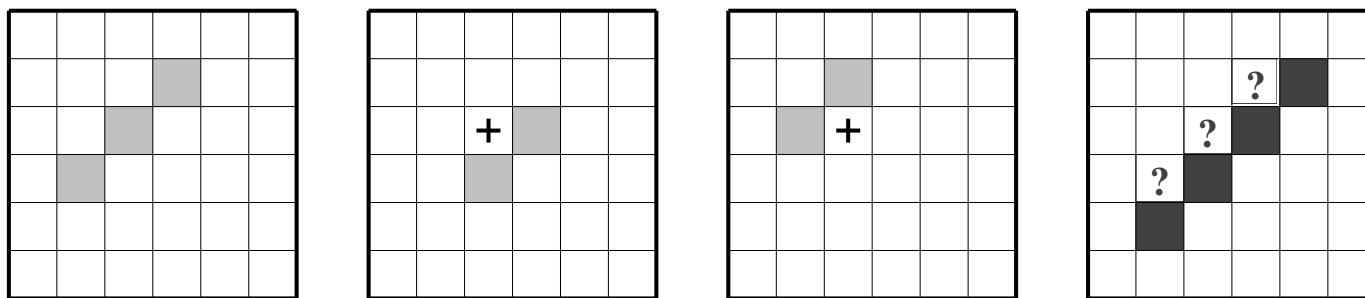
12.2.1 膨胀和腐蚀

原点不包含在结构元素中的膨胀运算

$$A \not\subset A \oplus B$$



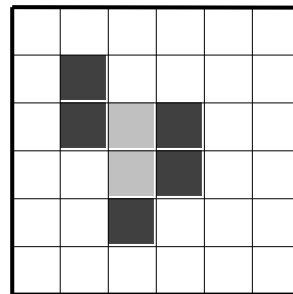
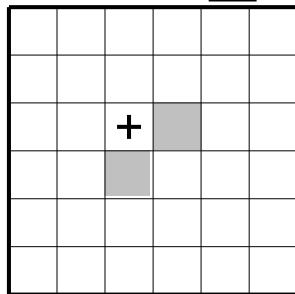
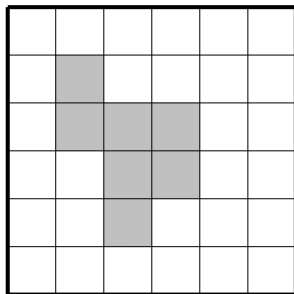
A在膨胀中自身完全消失了



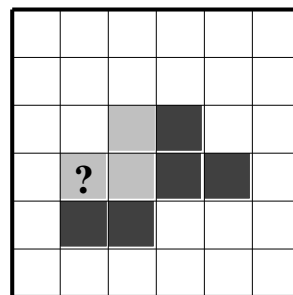
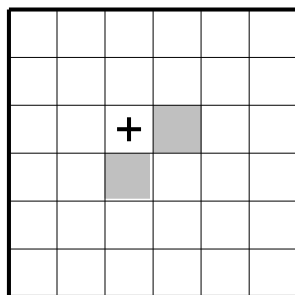
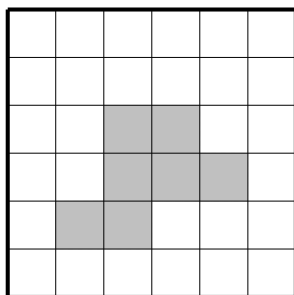
12.2.1 膨胀和腐蚀

原点不包含在结构元素中的腐蚀运算

$$A \ominus B \subseteq A$$



$$A \ominus B \not\subseteq A$$



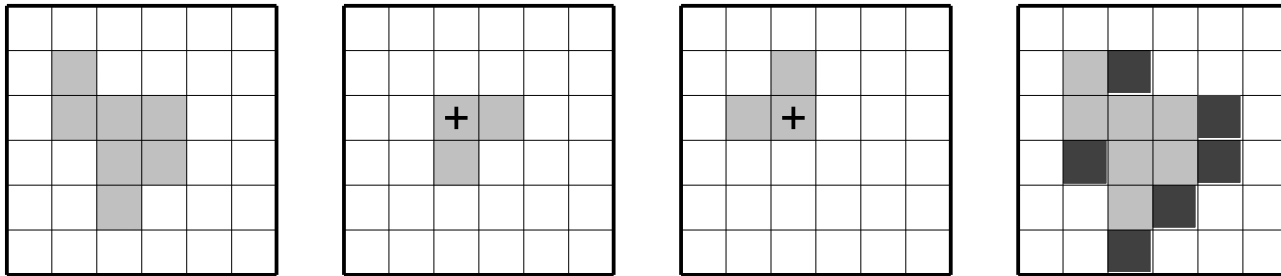
12.2.1 膨胀和腐蚀

4. 用向量运算实现膨胀和腐蚀

$$A \oplus B = \{x \mid x = a + b \quad \text{对某些 } a \in A \text{ 和 } b \in B\}$$

$$A = \{(1, 1), (1, 2), (2, 2), (3, 2), (2, 3), (3, 3), (2, 4)\}$$

$$B = \{(0, 0), (1, 0), (0, 1)\}$$



$$A \oplus B = \{(1, 1), (2, 1), (1, 2), (2, 2), (3, 2), (4, 2), \\ (1, 3), (2, 3), (3, 3), (4, 3), (2, 4), (3, 4), (2, 5)\}$$

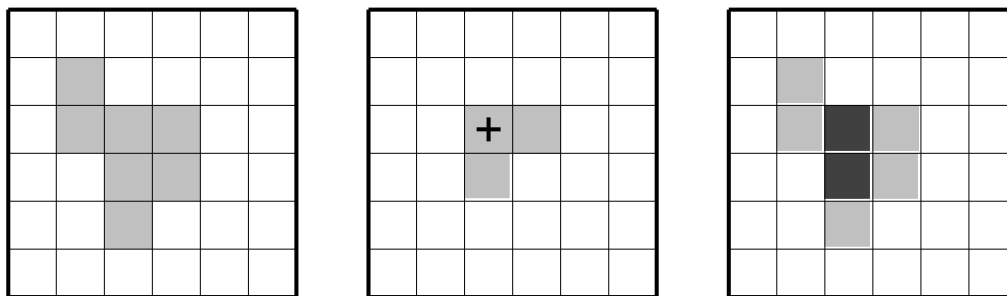
12.2.1 膨胀和腐蚀

4. 用向量运算实现膨胀和腐蚀

$$A \ominus B = \{x \mid (x + b) \in A \text{ 对每一个 } b \in B\}$$

$$A = \{(1, 1), (1, 2), (2, 2), (3, 2), (2, 3), (3, 3), (2, 4)\}$$

$$B = \{(0, 0), (1, 0), (0, 1)\}$$



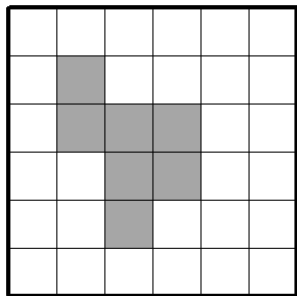
$$A \ominus B = \{(2, 2), (2, 3)\}$$

12.2.1 膨胀和腐蚀

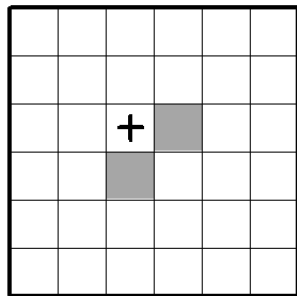
5. 用位移运算实现膨胀和腐蚀

按每个 b 来位移 A 并把结果或（OR）起来

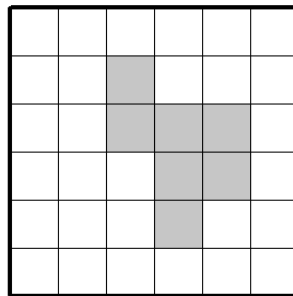
$$A \oplus B = \bigcup_{b \in B} (A)_b$$



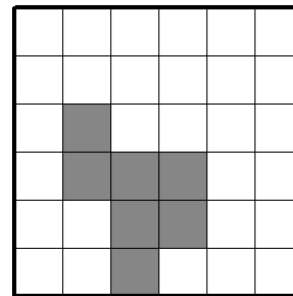
(a)



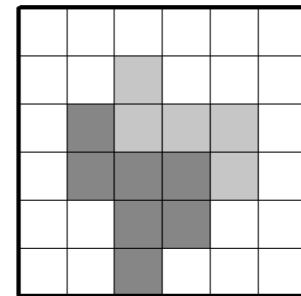
(b)



(c)



(d)



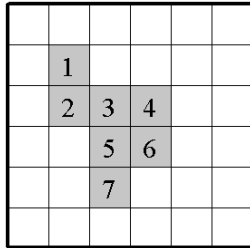
(e)

12.2.1 膨胀和腐蚀

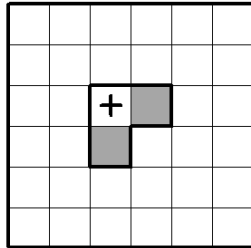
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按每个 a 来位移 B 并把结果或（OR）起来

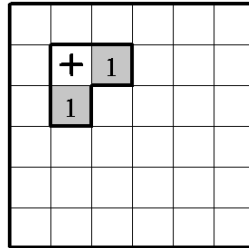
$$A \oplus B = \bigcup_{a \in A} (B)_a$$



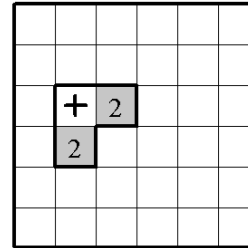
(a)



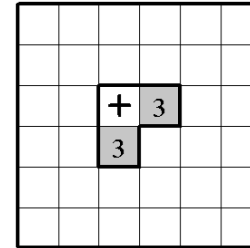
(b)



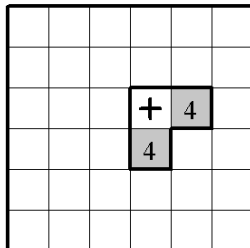
(c)



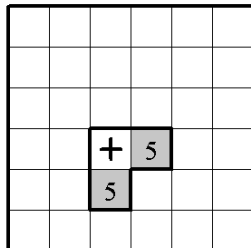
(d)



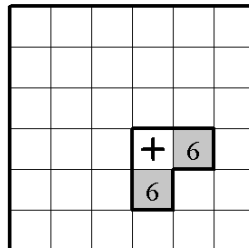
(e)



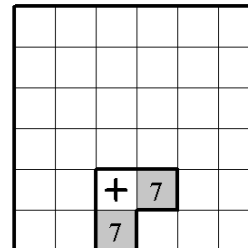
(f)



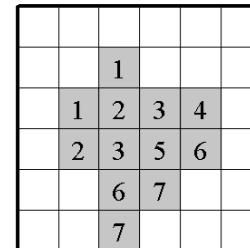
(g)



(h)



(i)



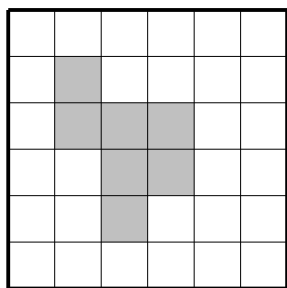
(j)

12.2.1 膨胀和腐蚀

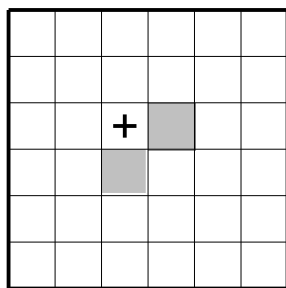
5. 用位移运算实现膨胀和腐蚀

按每个 b 来负位移 A 并把结果交（AND）起来

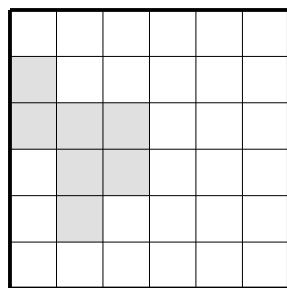
$$A \ominus B = \bigcap_{b \in B} (A)_{-b}$$



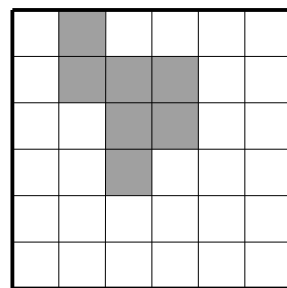
(a)



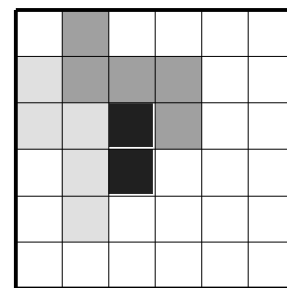
(b)



(c)



(d)



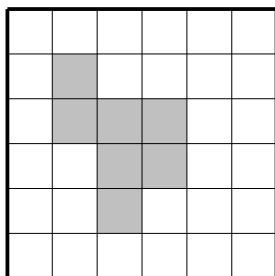
(e)

12.2.1 膨胀和腐蚀

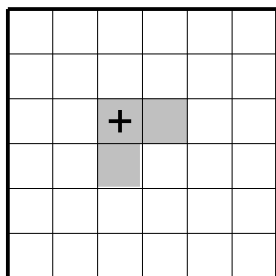
6. 膨胀和腐蚀的对偶性

$$(A \oplus B)^c = A^c \ominus \hat{B}$$

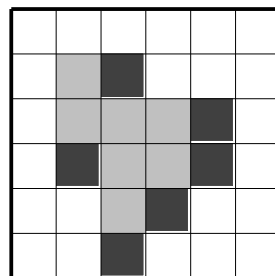
$$(A \ominus B)^c = A^c \oplus \hat{B}$$



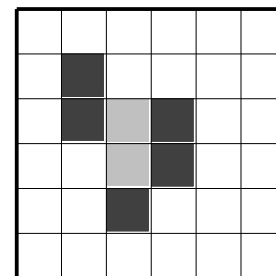
(a)



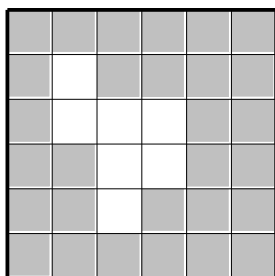
(b)



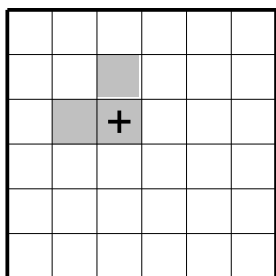
(c)



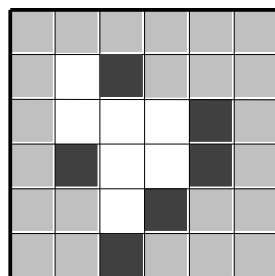
(d)



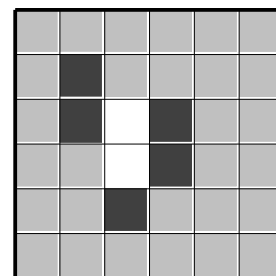
(e)



(f)



(g)



(h)

12.2.1 膨胀和腐蚀

6. 膨胀和腐蚀的对偶性

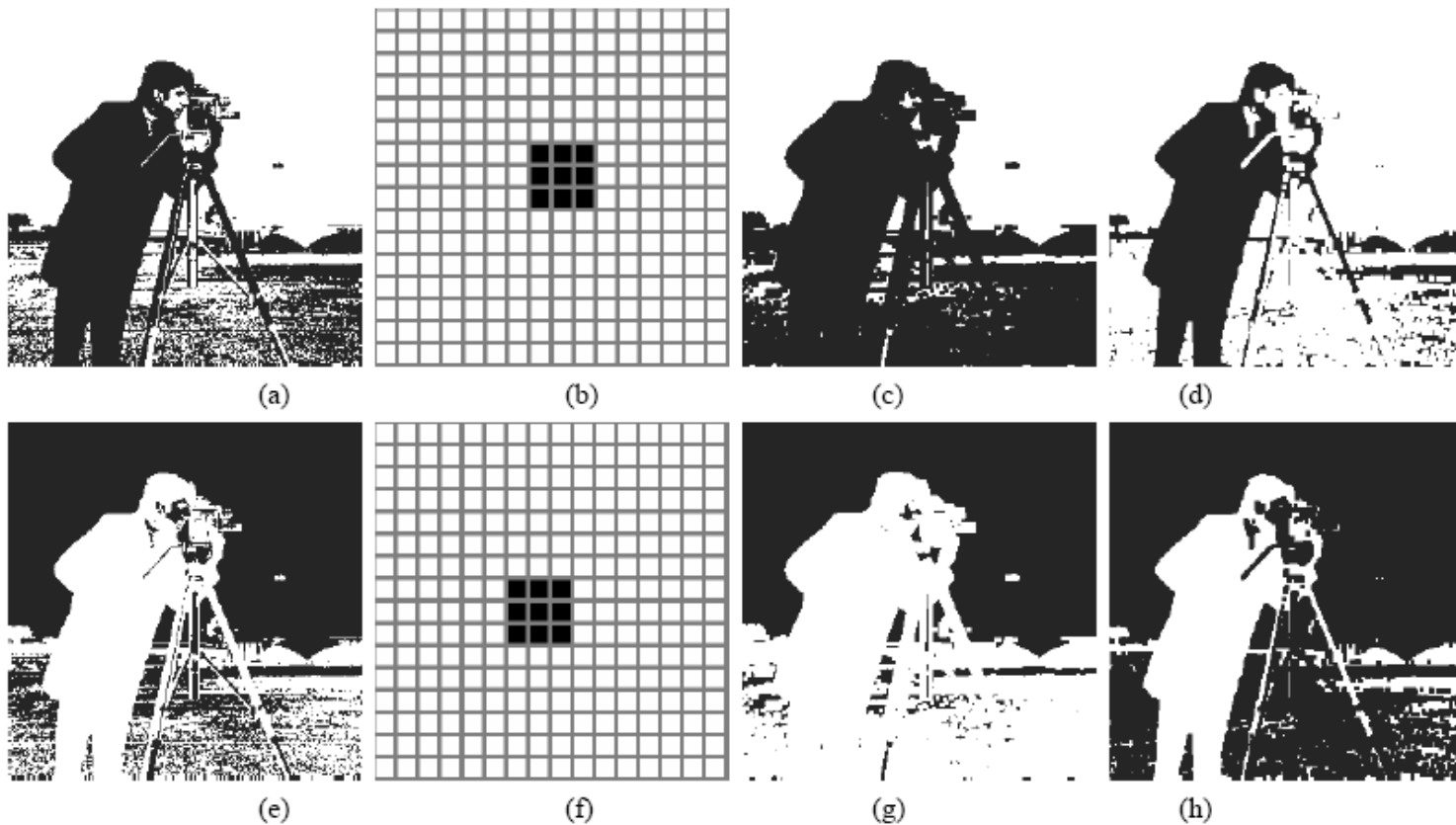
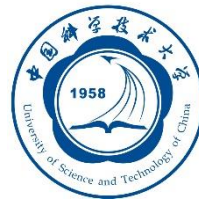


图 14.2.12 膨胀和腐蚀的对偶性验证实例



12.2.2 开启和闭合

1. 开启和闭合定义

膨胀和腐蚀并不互为逆运算

它们可以级连结合使用

开启：先对图象进行腐蚀然后膨胀其结果

$$A \circ B = (A \ominus B) \oplus B$$

闭合：先对图象进行膨胀然后腐蚀其结果

$$A \bullet B = (A \oplus B) \ominus B$$

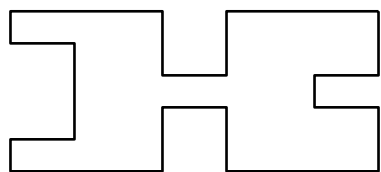
开启和闭合不受原点是否在结构元素之中的影响

12.2.2 开启和闭合

1. 开启和闭合定义

开启运算可以把比结构元素小的突起滤掉

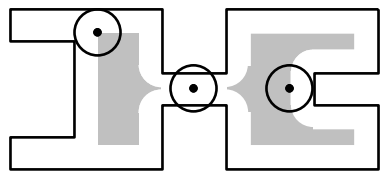
闭合运算可以把比结构元素小的缺口或孔填充上



(a)



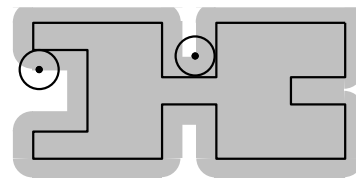
(b)



(c)



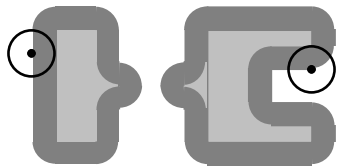
(d)



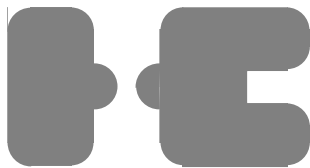
(e)



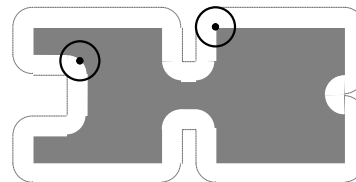
(f)



(g)



(h)



(i)



(j)

12.2.2 开启和闭合

1. 开启和闭合定义



原图



(a)



(b)

图 14.2.14 开启和闭合实例



12.2.2 开启和闭合

2. 开启和闭合的对偶性

开启和闭合也具有对偶性

$$(A \circ B)^c = A^c \bullet \hat{B}$$

$$(A \bullet B)^c = A^c \circ \hat{B}$$

$$(A \circ B)^c = [(A \ominus B) \oplus B]^c = (A \ominus B)^c \ominus \hat{B} = A^c \oplus \hat{B} \ominus \hat{B} = A^c \bullet \hat{B}$$

$$(A \bullet B)^c = [(A \oplus B) \ominus B]^c = (A \oplus B)^c \oplus \hat{B} = A^c \ominus \hat{B} \oplus \hat{B} = A^c \circ \hat{B}$$



12.2.2 开启和闭合

3. 开启和闭合与集合的关系

| 操 作 | 并 集 | 交 集 |
|-----|--|--|
| 开 启 | $\left(\bigcup_{i=1}^n A_i\right) \circ B \supseteq \bigcup_{i=1}^n (A_i \circ B)$ | $\left(\bigcap_{i=1}^n A_i\right) \circ B \subseteq \bigcap_{i=1}^n (A_i \circ B)$ |
| 闭 合 | $\left(\bigcup_{i=1}^n A_i\right) \bullet B \supseteq \bigcup_{i=1}^n (A_i \bullet B)$ | $\left(\bigcap_{i=1}^n A_i\right) \bullet B \subseteq \bigcap_{i=1}^n (A_i \bullet B)$ |

12.2.2 开启和闭合

4. 开启和闭合的几何解释

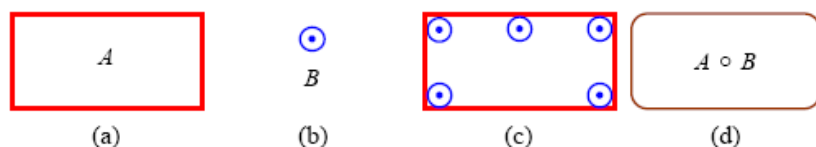


图 14.2.15 开启的填充特性

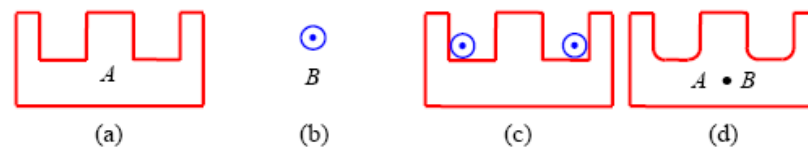
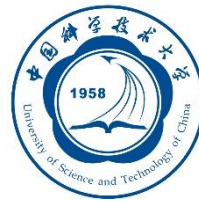


图 14.2.16 闭合的几何解释



12.3 二值形态学组合运算

基本运算:

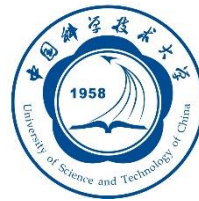
膨胀、腐蚀、开启、闭合

击中-击不中变换 (hit or miss)

组合运算, {基本算法}

12.3.1 击中-击不中变换

12.3.2 组合运算



12.3.1 击中-击不中变换

击中-击不中变换

形状检测的一种基本工具

对应两个操作，所以用到两个结构元素

设 A 为原始图象， E 和 F 为一对不重合的集合

$$A \uparrow (E, F) = (A \ominus E) \cap (A^c \ominus F) = (A \ominus E) \cap (A \oplus F)^c$$

E : 击中结构元素

F : 击不中结构元素

12.3.1 击中-击不中变换

击中-击不中变换

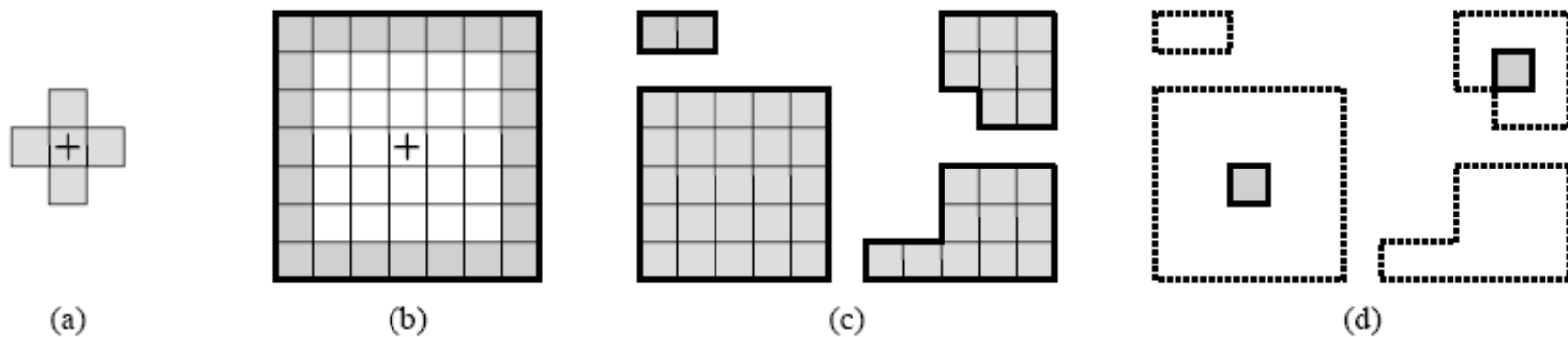


图 14.3.1 击中-击不中变换示例

12.3.1 击中-击不中变换

击中-击不中变换

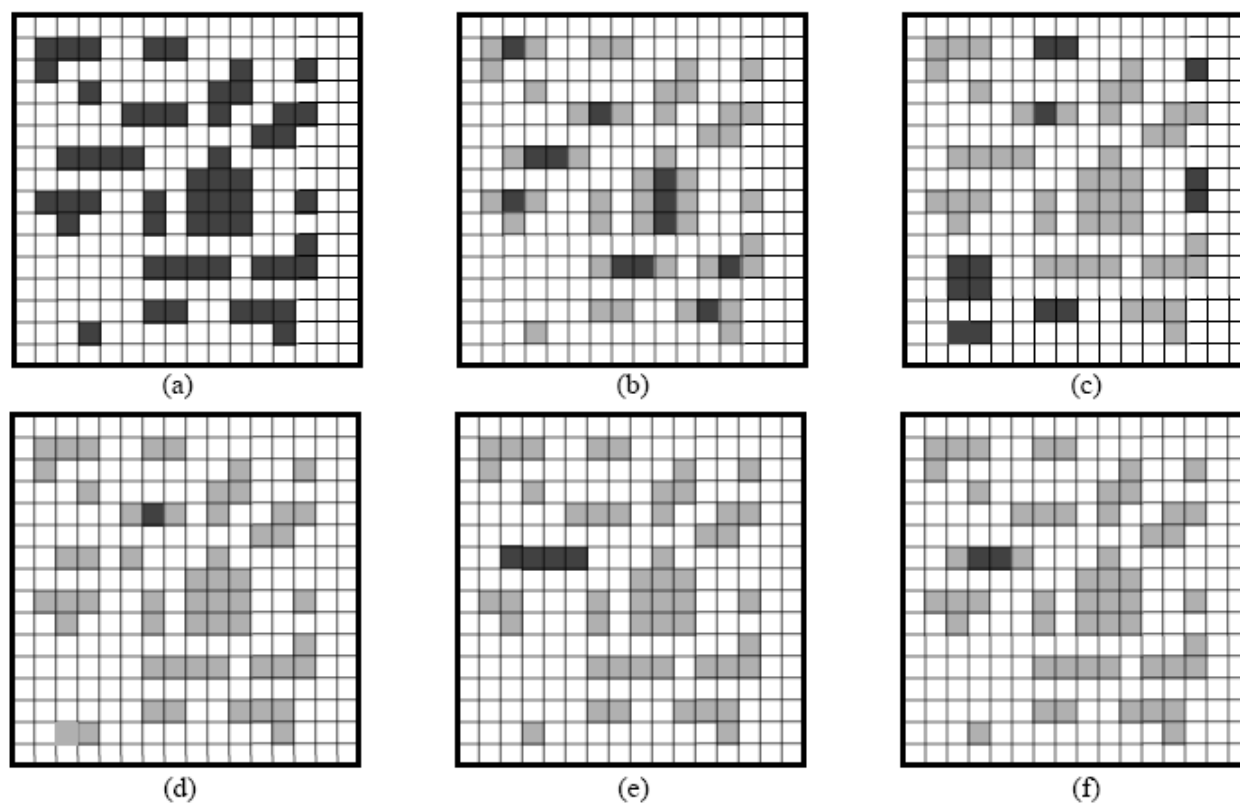


图 14.3.2 利用击中-击不中算子以提取包含水平方向上有连续 3 个像素的线段

12.3.1 击中-击不中变换

击中-击不中变换

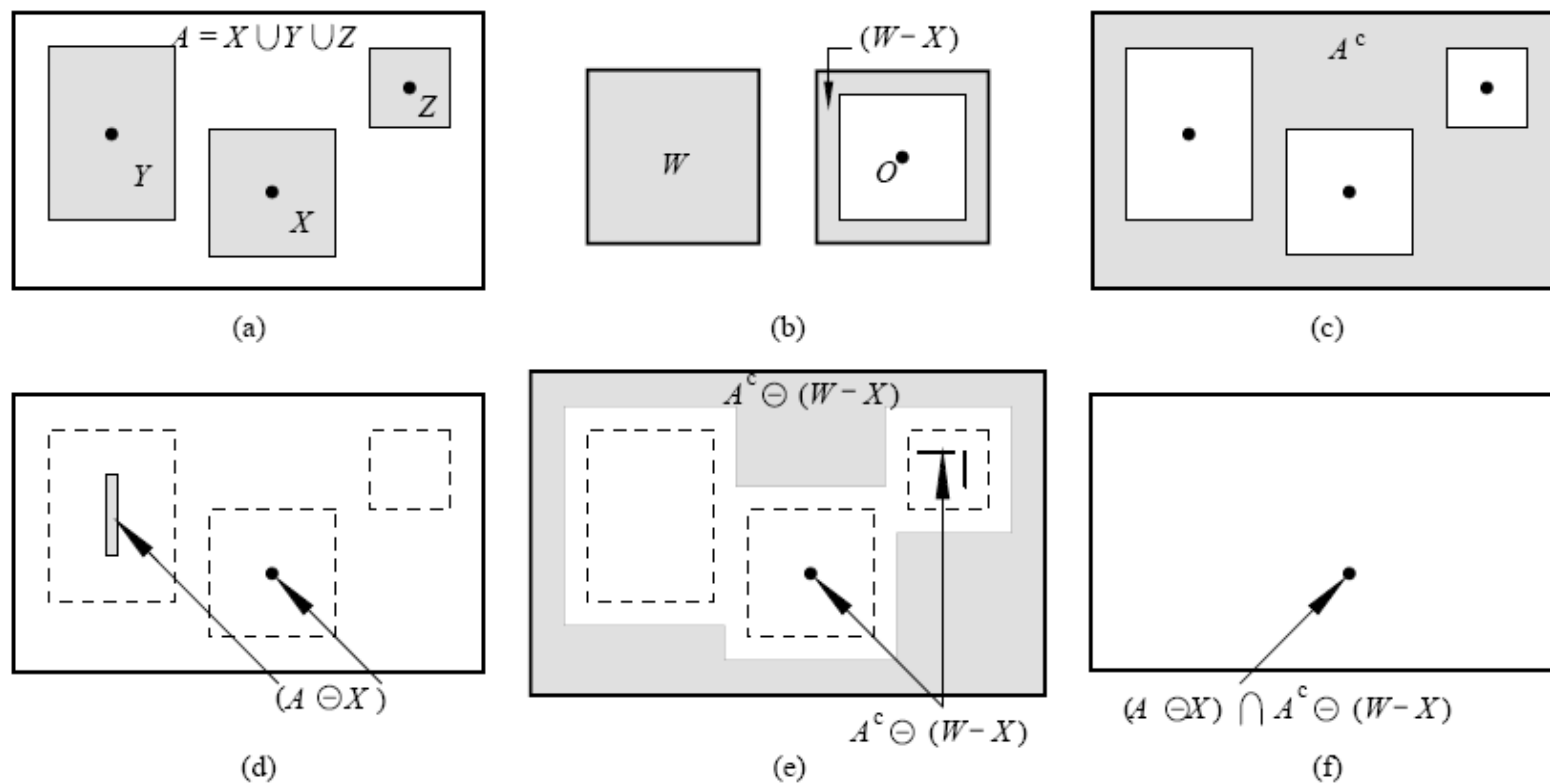
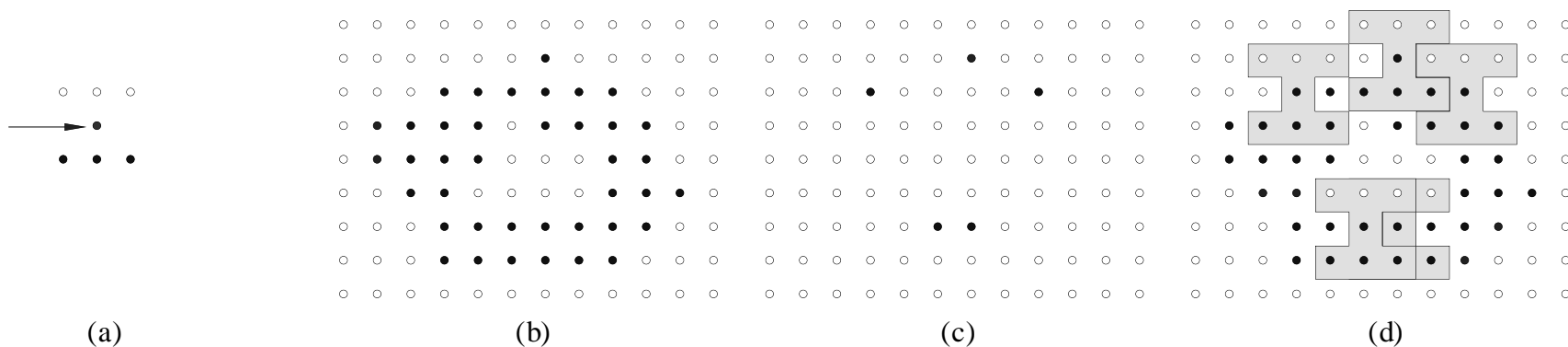


图 14.3.3 击中-击不中变换示例

12.3.1 击中-击不中变换

击中-击不中变换中的结构元素

$A \uparrow B$ 的结果中仍保留的目标像素对应在 A 中其邻域与结构元素 B 对应的像素





12.3.2 组合运算

1. 区域凸包

令 B_i , $i = 1, 2, 3, 4$, 代表4个结构元素, $X_i^0 = A$

构造:

$$X_i^k = (X_i^{k-1} \uparrow B_i) \cup A \quad i = 1, 2, 3, 4 \text{ 和 } k = 1, 2, \dots$$

令 $D_i = X_i^{\text{conv}}$, 上标 “conv”表示在 $X_i^k = X_i^{k-1}$

意义下收敛

A的凸包可表示为: $C(A) = \bigcup_{i=1}^4 D_i$

12.3.2 组合运算

1. 区域凸包

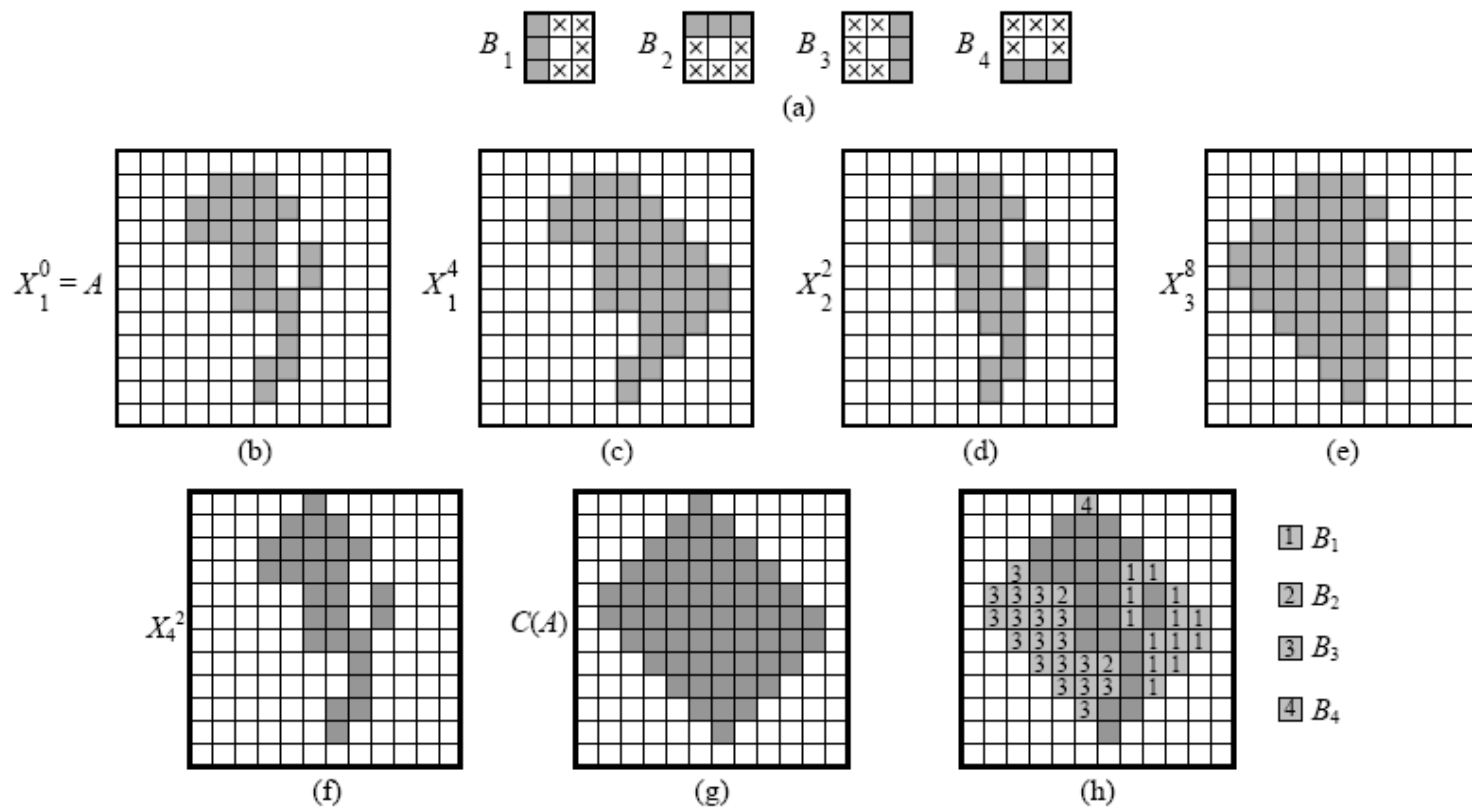


图 14.3.5 构造凸包的示例



12.3.2 组合运算

2. 细化

用结构元素 B 细化集合 A 记作 $A \otimes B$

借助击中-击不中变换定义

$$A \otimes B = A - (A \uparrow B) = A \cap (A \uparrow B)^c$$

定义一个结构元素系列

$$\{B\} = \{B_1, B_2, \dots, B_n\}$$

$$A \otimes \{B\} = A - ((\dots((A \otimes B_1) \otimes B_2) \dots) \otimes B_n)$$

12.3.2 组合运算: 细化

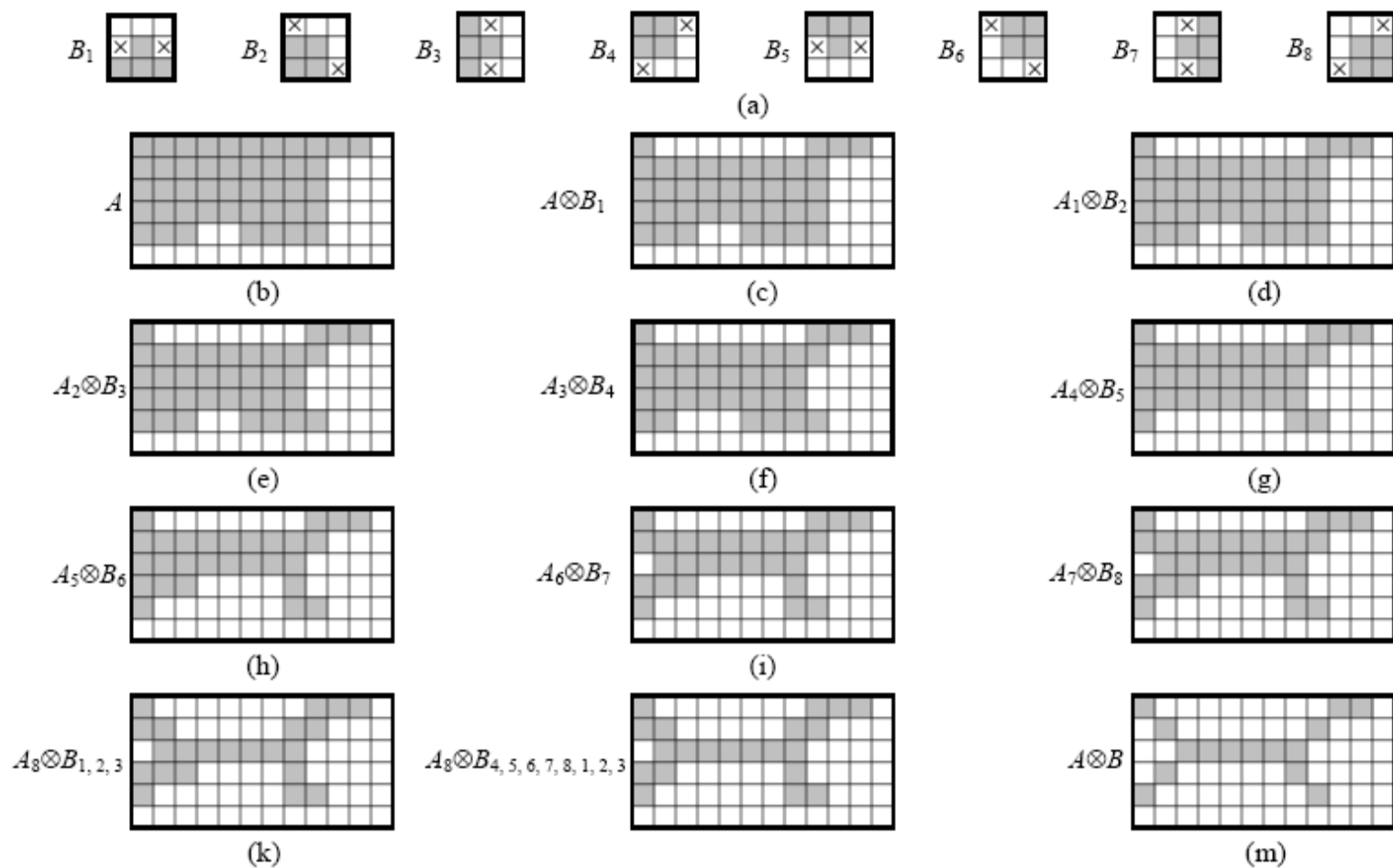


图 14.3.6 细化示例

12.3.2 组合运算

3. 粗化

用结构元素 B 粗化集合 A 记作 $A \oplus B$

$$A \oplus B = A \cup (A \uparrow B)$$

定义为一系列操作

$$A \oplus \{B\} = ((\dots((A \oplus B_1) \oplus B_2) \dots) \oplus B_n)$$

粗化从形态学角度来说与细化是对应的，实际中可先细化背景然后求补以得到粗化的结果。换句话说，如果要粗化集合 A ，可先构造 $C = A^c$ ，然后细化 C ，最后求 C^c 。

12.3.2 组合运算

3. 粗化

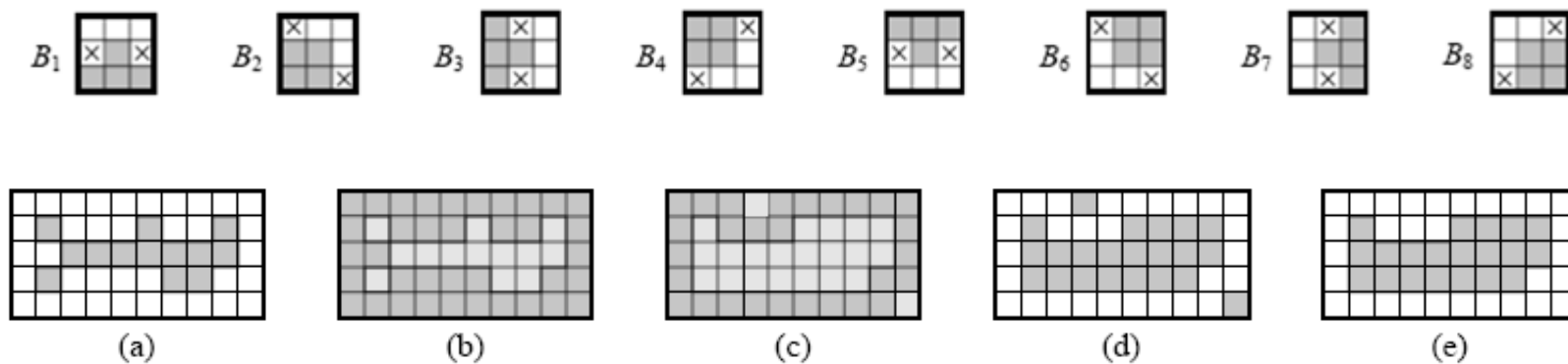


图 14.3.7 利用细化进行粗化

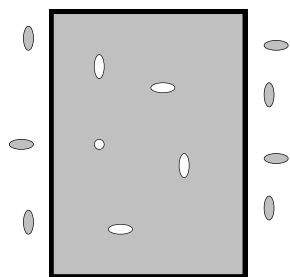
12.4 二值形态学实用算法

1. 噪声滤除

先开启后闭合

$$\{[(A \ominus B) \oplus B] \oplus B\} \ominus B = (A \circ B) \bullet B$$

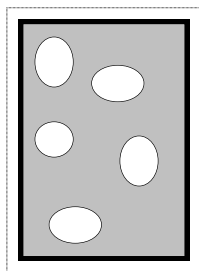
腐蚀 膨胀 膨胀 腐蚀



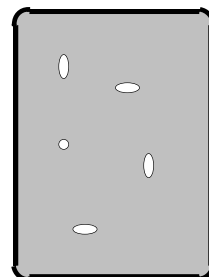
(a)



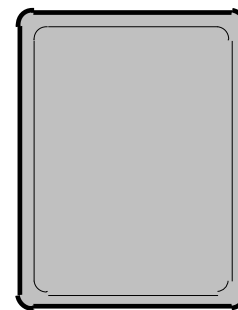
(b)



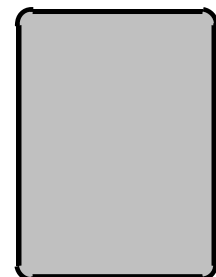
(c)



(d)



(e)



(f)

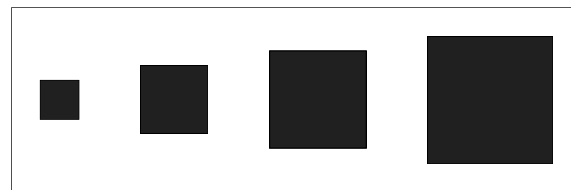
12.4 二值形态学实用算法

2. 目标检测

3×3 , 5×5 , 7×7 和 9×9 的实心正方形

3×3 实心正方形

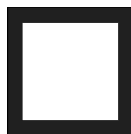
9×9 方框



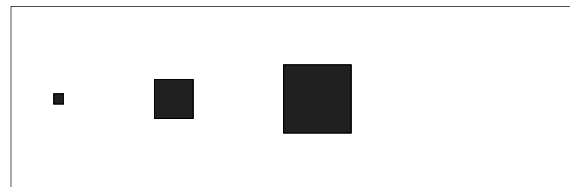
(a)



(b)



(c)



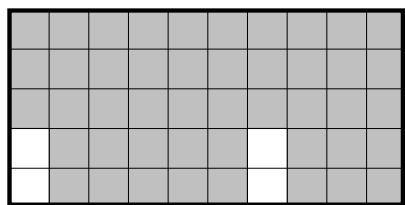
(d)

12.4 二值形态学实用算法

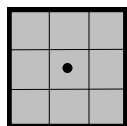
3. 边界提取

先用1个结构元素 B 腐蚀 A ，再求取腐蚀结果和 A 的差集就可得到边界 $\beta(A)$

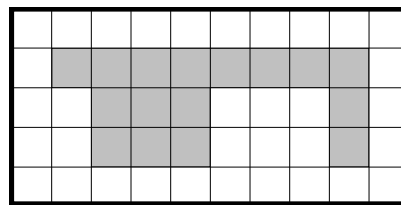
$$\beta(A) = A - (A \ominus B)$$



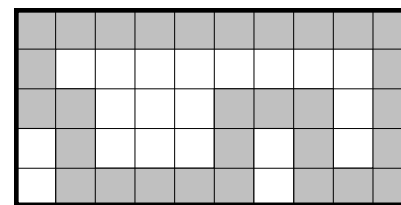
(a)



(b)



(c)



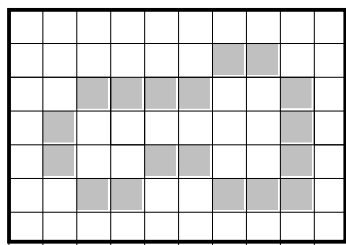
(d)

结构元素是8-连通的，而所得到的边界是4-连通的

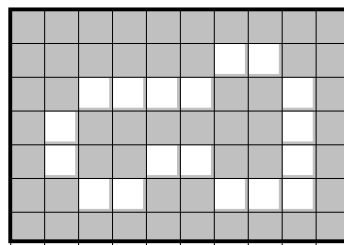
12.4 二值形态学实用算法

4. 区域填充

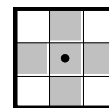
$$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1, 2, 3, \dots$$



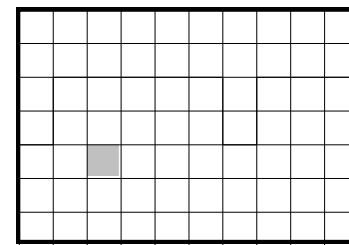
(a)



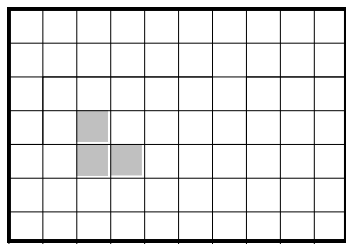
(b)



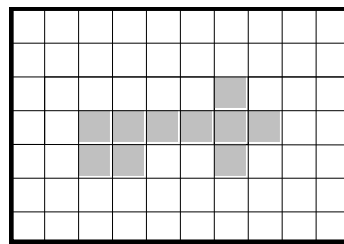
(c)



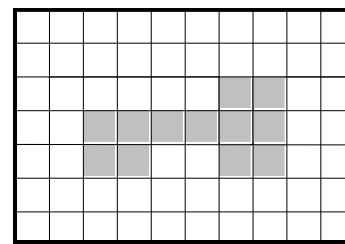
(d)



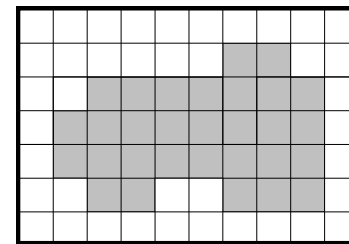
(e)



(f)

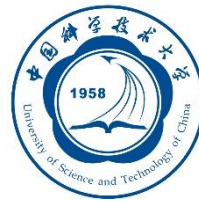


(g)



(h)

结构元素是4-连通的，而原填充的边界是8-连通的



12.4 二值形态学实用算法

5. 区域骨架提取

$$S(A) = \bigcup_{k=0}^K S_k(A) \quad S_k(A) = (A \ominus kB) - [(A \ominus kB) \circ B]$$

$$(A \ominus kB) = ((\dots (A \ominus B) \ominus B) \ominus \dots) \ominus B$$

$$K = \max\{k \mid (A \ominus kB) \neq \phi\}$$

$$A = \bigcup_{k=0}^K (S_k(A) \oplus kB)$$

12.4 二值形态学实用算法

5. 区域骨架提取

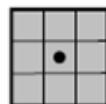
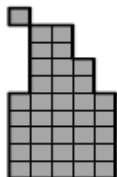
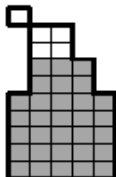
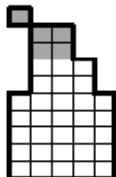
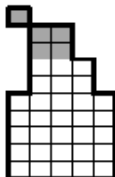
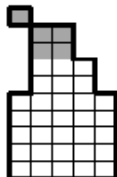
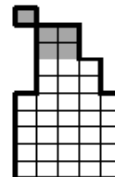
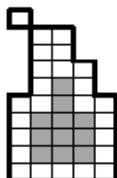
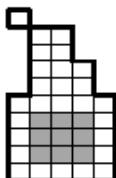
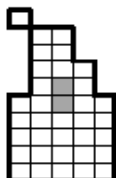
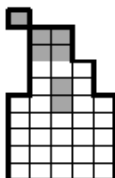
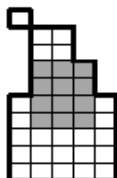
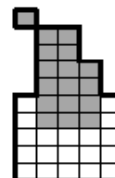
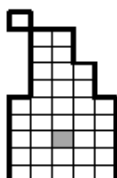
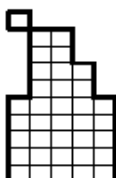
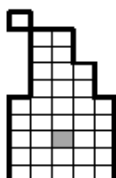
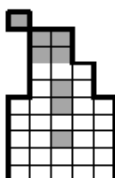
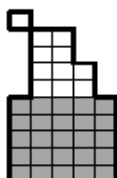
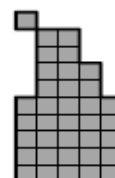


表 14.4.1 区域骨架抽取示例

| 列 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-------|---|---|---|--|---|---|---|
| 运算 | | $A \ominus kB$ | $(A \ominus kB) \circ B$ | $S_k(A)$ | $\bigcup_{k=0}^K S_k(A)$ | $S_k(A) \oplus kB$ | $\bigcup_{k=0}^K [S_k(A) \oplus kB]$ |
| $k=0$ | |  |  |  |  |  |  |
| $k=1$ | |  |  |  |  |  |  |
| $k=2$ | |  |  |  |  |  |  |

12.4 二值形态学实用算法

5. 区域骨架提取



(a)



(b)



(c)



(d)

图 14.4.7 形态学骨架计算实例

(b) 3x3; (c) 5x5; (d) 7x7