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List of Acronyms

AABB	Axis Aligned Bounding Box
ADF	Adaptively sampled Distance Field
BRep	Boundary Representation
BReps	Boundary Representations
BVH	Bounding Volume Hierarchy
CAD	Computer-Aided Design
CSG	Constructive Solid Geometry
FRep	Function Representation
GPU	Graphics Processing Unit
MVC	Mean Value Coordinates
NURBS	Non-Uniform Rational B-Spline
OBB	Oriented Bounding Box
PCA	Principal Component Analysis
RBF	Radial Basis Function
SAH	Surface Area Heuristic
SDF	Signed Distance Field
STTI	Space Time Transfinite Interpolation
SARDF	Signed Approximate Real Distance Functions
TI	Transfinite Interpolation

1 Physics

In order to simulate granular materials, it is necessary to define what they are and understand how they physically behave. For this reason, this section will look at how granular materials are described in the physics and engineering literature rather than the computer graphics literature. However, the scope of this report clearly does not allow a thorough discussion of the physics of granular matter. It will give an introduction, geared towards the techniques in the computer graphics community, which are the subject of the next sections. It is interesting to add that the physics of granular matter is still a subject of debate DEGENNES.

As a first step towards understanding, we need to define what granular material is. Referring back to the physics literature, de Gennes (REF) defines a granular material as a material made up out of a large collection of grains or particles, where the size d (for diameter) of the individual grains are larger than one micron. At this scale, the Brownian motion and thermal agitation can be ignored. deGennesREF. This means that entropy can be ignored and the behaviour is determined by dynamical effects. JAEGERREF

Hence, to understand the behaviour, it necessary to investigate the dynamics of the material. The first important property is the friction, both static and dynamic, between the different grains. Coulomb's law is a widely accepted and well-known way of modelling (dry) friction between two surfaces in contact:

$$F_f \leq \mu F_n \tag{1}$$

where F_f is the friction force, tangential to the contact surface, F_n the force perpendicular to the contact surface, and μ the coefficient of friction. Notice the relation, \leq , indicating that the friction force will be as big as necessary to avoid sliding in the case of static friction, i.e. the friction force will exactly oppose sliding and keep the objects in the

same position relative to each other. In the case of kinetic sliding, the \leq symbol disappears, making it an equality. Hence, when the objects are sliding relative to each other, kinetic friction results in a constant force opposing the sliding motion. Typically, the friction coefficient μ is different for static μ_s and kinetic friction μ_k .

Another property of granular material is the inelastic collision between the particles JAEGERREF. This, combined with the friction, makes that granular material are dissipative systems, where a lot of kinetic energy is converted in heat rather than bounces JEAGERREFHowever. It is interesting to note that the some grains might be subject to an infinite amount of collisions in a finite amount of time, a phenomenon called inelastic collapse McNamara and Young REF.

All of this results in some quite particular behaviour. For instance, a well-known result of these physics is that the pressure in a granular material does not linearly increase with height as it does in fluid. REFS This phenomenon is what makes hourglasses work. REFHOURLASS. The static friction also makes the sand pile up, without the piles collapsing, along as the slope of the pile is less steep than the angle of repose. JAEGERREF. If the slope is steeper, an avalanche will occur, where only the top layer of the granular material flows. ARANSONREF

These physics, and the particular behaviour of granular matter, lead physicists to describe them as a material with both solid-like and fluid-like behaviour. DEGENNESJAEGERARANSONREF. When approaching the material as a fluid, a continuum model is necessary. This continuum model is usually formulated using a model based on a variation of the Navier-Stokes equations, adapted to model the granular behavior, and complemented with a elasto-plastic formulation of a yield criterion, for example the Mohr-Coulomb yield criterion. DEGENNESREF The Mohr-Coulomb yield criterion is defined as REFZAB: EQUATION Explain. This criterion governs the difference between the solid-like and

fluid-like behaviour of the granular matter. DEGENNESREF.

While discussing the necessary changes to the Navier-Stokes equations to model granular material is interesting in its own rights, this would be out of scope for this report, as the computer graphics techniques usually bring it down to two particular differences from regular fluid simulations REFTOTECH: one needs to account for the friction and a specific form of (in)compressibility: dry granular materials are usually simulated with unilateral incompressibility, while wet granular materials, e.g. snow, are typically compressible. Unilateral incompressibility means that there is no cohesive pressure when the material is diverging REFTECH. The computer graphics techniques based on the continuum approach usually take existing fluid simulators and change these two aspects. The specific formulations for the two differ per technique and will be given while explaining specific techniques in the next section.