Graduate Macro: Lecture 4

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This Lecture

- Consumption Smoothing.
- Hand to Mouth.
- Incomplete markets.
- Krusell-Smith.
- Rich Hand to Mouth.
- HANK.

Consumption Smoothing

Consider impact of temporary transfers on consumption:

- Deterministic problem just to simplify.
- Partial equilibrium.
- Exogenous constant interest rate, $\beta = 1/\left(1+r\right)$.

$$
\begin{aligned}
& \max \sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \\
c_t + b_{t+1} &= y_t + \left(1+r\right) b_t - \tau_t \\
& b_0 \geq 0 \text{ given}
\end{aligned}
$$

subject to a no-Ponzi game restriction to the natural borrowing constraint:

$$
\begin{aligned}
\lim_{T\to\infty} \left(1+r\right)^{-T} b_{t+T} &= 0 \\
b_t &\geq \underline{b}_t^{nat}
\end{aligned}
$$

Ricardian Equivalence - Households

Suppose we impose Inada condition on consumption:

$$\lim_{c \to 0_+} u(c) = \infty$$

- In this case, household will never choose to be in a situation where consumption can go negative.
- Operating the budget constraint forwards then implies:

The natural borrowing limit:

$$b_t \geq \overline{b}_t^{nat} = -\sum_{j=0}^{\infty} (1+r)^{-j} \left(y_{t+j} - \tau_{t+j} \right)$$

- (Aiyagari, 1994).

Consumption smoothing

First-order necessary conditions:

$$
\begin{aligned}
u'(c_t) &\geq \beta(1+r)\,u'(c_{t+1}) \;\; \forall t \geq 0 \\
&\text{and} \\
u'(c_t) &> \beta(1+r)\,u'(c_{t+1}) \;\; \text{implies } b_{t+1} = \underline{b}_t
\end{aligned}
$$

- Since $\beta(1+r) = 1$, and with the natural borrowing limit (which consumers will choose never to be constrained by):

$$c_t = \overline{c}$$

where $\overline{c}$ is given as:

$$\overline{c} = \frac{r}{1+r}\sum_{j=0}^{\infty}(1+r)^{-j}\left(y_{t+j} - \tau_{t+j}\right) + rb_0$$

Ricardian Equivalence - Households

This is the permanent income hypothesis.

- The MPC out of a temporary transfer is

$$MPC = \frac{r}{1+r}$$

- At the annual level, this would be around 4 percent;
- PIH also predicts that households increase consumption when they receive information about income change, not when it occurs.
- Suppose group A gets $d\tau$ at t_0 and group B get $(1+r)^{t_1 - t_0} d\tau$ at t_1 :

$$\begin{aligned} dc_{t_0}^A &= dc_{t_0}^B = \frac{r}{1+r} d\tau \\ dc_{t_1}^A &= dc_{t_1}^B = 0 \end{aligned}$$

Consumption Smoothing

Johnson, Parker and Souleles (AER, 2006) study the impact of small transfers on US household spending:

- EGTRRA (Economic Growth and Tax Reconciliation Act) of 2001: Cut in taxes from 2002.
- Households sent a tax rebate check (an advance on a 5 percent reduction in the tax rate on the first $6,000 earned per household member) in 2001, most received $300 tax rebate checks.
- Timing of checks was randomized according to last two digits of social security number.
- PIH would predict that timing difference should have no impact on consumption.
- Johnson, Parker and Souleles used the timing differences for identification.

MPCs

JPS regressions:

$$\triangle c_{it} = \beta_1' X_{it-1} + \beta_2 R_{it} + \text{month fixed effects} + \varepsilon_{it}$$

- $\triangle c_{it}$: household consumption.
- X_{it-1} : individual characteristics (age of head, size, etc.).
- R_{it}: dollar value of the tax rebate received at date t.

What does β_2 measure?

- Suppose we have early 2001:Q2 (A) and late 2001:Q3 (B) recipients of the check

$$\beta_2 = \frac{\left(\triangle c^A_{Q2} - \triangle c^B_{Q2}\right) + \left(\triangle c^B_{Q3} - \triangle c^A_{Q3}\right)}{2}$$

- If everyone anticipated the check coming and no one liquidity constrained: $\beta_2 = 0$

Results from Consumer Expenditure Survey (CEX) data:

TABLE I

ESTIMATES OF THE 2001 REBATE COEFFICIENT ($\hat{\beta}_2$)[a]

	Nondurables
JPS 2006, 2SLS ($N = 13{,}066$)	0.375 (0.136)
Trim top & bottom 0.5%, 2SLS ($N = 12{,}935$)	0.237 (0.093)
Trim top & bottom 1.5%, 2SLS ($N = 12{,}679$)	0.219 (0.079)
MS 2011, IVQR ($N = 13{,}066$)	0.244 (0.057)

- Much larger MPCs than anything consistent with PIH.
- Lower bound on MPC around 20 percent.

Complete Markets

Representative agent models

- Now suppose stochastic income, complete markets, general equilibrium
- N types of agents with endowments ω_i.
- Households face idiosyncratic endowment risk.
- Can solve for complete markets allocation from planner's problem.

In an N-agent model, the efficient solution solves

$$\begin{aligned} U &= \max \sum_{i=1}^{N} \phi_i u_i \left(c_i \right) \\ \sum_i c_i &\leq \sum_i \omega_i \end{aligned}$$

- $\phi_i \geq 0$: the planner's welfare weight associated to agent of type i

Complete Markets

First-order condition:

$$\phi_i u_i' \left(c_i \right) = \lambda$$

- where λ is the multiplier on the resource constraint.

Since λ is the multiplier on the resource constraint, it is independent of i:

$$\underbrace{\phi_i u_i' \left(c_i \right)}_{\substack{\text{agent i's marg. utility} \\ \text{times constant}}} = \underbrace{\phi_j u_j' \left(c_j \right)}_{\substack{\text{agent j's marg. utility} \\ \text{times constant}}}$$

- **Optimal solution implies risk sharing**: Weighted marginal utilities are equalized across agents in any state of nature.
- Idiosyncratic risk has no consequence for the allocation neither at the aggregate level nor at the individual level.
- transfers as such should have no effects.

Hand to Mouth

One possible resolution to this would be to assume that there are two types of households:

- Ricardian households: Hold assets, can insure
- Hand to Mouth: hold no assets, lack insurance
- Transfers to the latter will impact on their consumption even if transitory

Hand to Mouth

Ricardian households - share γ of aggregate

$$\begin{aligned} V_0^R &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t^R - \frac{\phi^R}{1+\kappa} \left(n_t^R \right)^{1+\kappa} \right) \\ c_t^R + i_t^R &= w_t n_t^R + r_t^R k_t^R + T_t^R \\ k_{t+1}^R &= (1-\delta) k_t^R + i_t^R \end{aligned}$$

Hand to Mouth - share $1-\gamma$ of aggregate

$$\begin{aligned} V_0^H &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t^H - \frac{\phi^H}{1+\kappa} \left(n_t^H \right)^{1+\kappa} \right) \\ c_t^H &= w_t n_t^H + T_t^H \end{aligned}$$

- differences in ϕ^R allowed for to control for wealth effects

Hand to Mouth

Production - competitive firms:

$$y_t = k_t^{1-\alpha} n_t^{\alpha}$$

Government - balances budget:

$$\gamma T_t^R = -\left(1-\gamma\right) T_t^H$$

Goods and labor **market clearing**:

$$\begin{aligned} y_t &= \gamma\left(c_t^R + i_t^R\right) + \left(1-\gamma\right) c_t^H \\ n_t &= \gamma n_t^A + \left(1-\gamma\right) n_t^B \\ k_t &= \gamma k_t^R \end{aligned}$$

Hand to Mouth

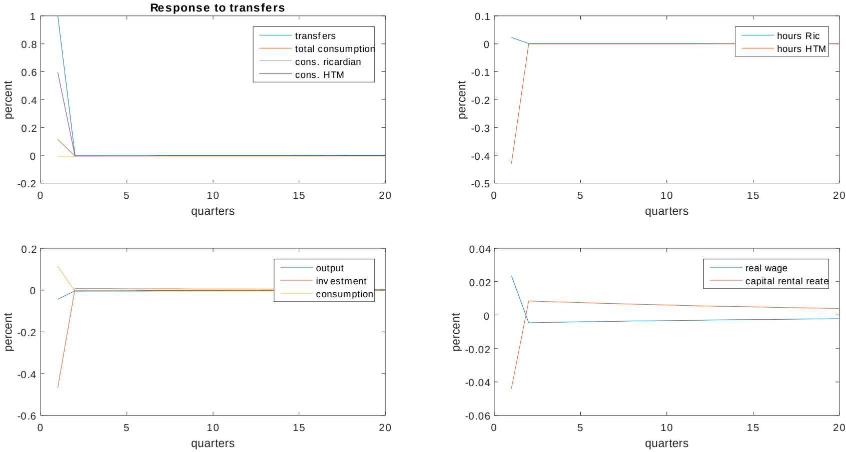
Calibration:

- $\beta = 1/1.01$
- $\alpha = 2/3$
- $n^H = n^R = 0.3$
- $\delta = 0.025$
- $1/\kappa = 3/4$
- $\gamma = 0.2$

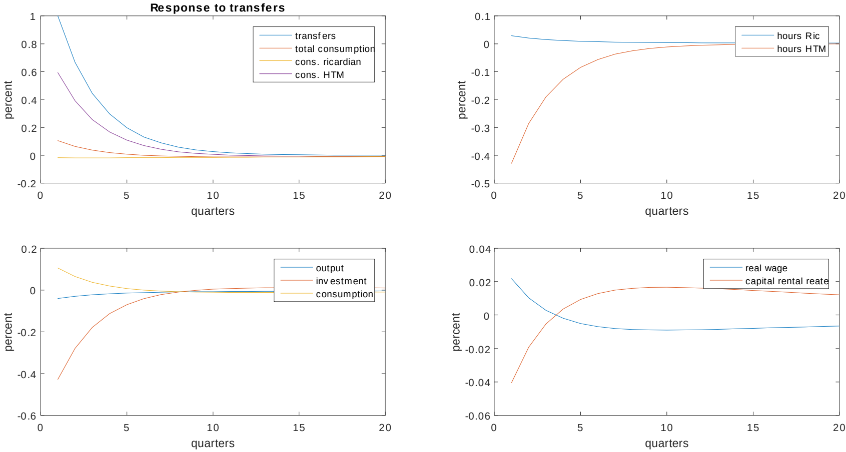
$$\frac{dT_t^H}{c^H} = \rho_T \frac{dT_{t-1}^H}{c^H} + \varepsilon_t$$

- implies that $c^R = 1.19c^{HTM}$

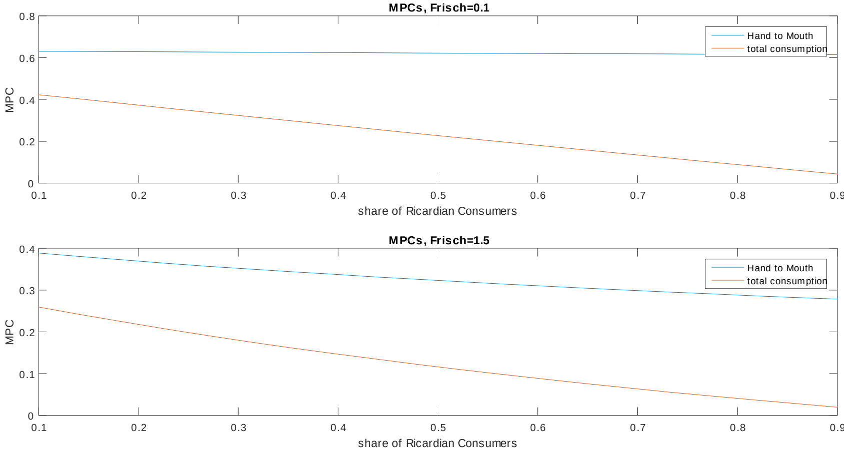
Hand to Mouth - purely transitory



Hand to Mouth - around 3 months persistence



Hand to Mouth - MPCs



Hand to Mouth

Hard to get large aggregate MPCs

- share of HTM need to be at least 50-60 percent to get large HTMs
- this also assumes that HTM are quite rich - not poor, just unable to smooth

Idiosyncratic Risk

Idiosyncratic risk plays no in the models we have examined so far because we have either assumed complete markets or risk sharing within large families

- But idiosyncratic risk may be very important for aggregate fluctuations
- Shocks can impact on the demand side beyond intertemporal and intratemporal motives
- Idiosyncratic risk and aggregate fluctuations may be related

Incomplete Markets

Incomplete markets: Lack of perfect insurance against idiosyncratic risk
Main implications:

1. Marginal rates of substitution may differ across agents - imperfect risk sharing
2. Agents may differ in wealth, consumption and utility - model can say something about inequality
3. Equilibrium allocations will often be history dependent
4. Aggregate equilibrium may depend on distribution

A Savings Problem

Households face **idiosyncratic** labor income risk but **no** aggregate uncertainty

- s_t: the individual employment level, or alternatively as the efficiency level of an individual agent,
- Cross-sectional average of s_t: aggregate employment (in efficiency units) which is assumed constant

Labor income risk **cannot** fully be insured - Partial self-insurance through savings in a single asset a_t with state non–contingent payoffs

- s_t is discrete and evolves according to an n-state homogeneous Markov chain with transition probability matrix P
- Discretize assets: $a_t \in A = [a_1 < .. < a_n]$
- Discretization implies the existence of borrowing constraints through the assumption on the minimum level of assets

A Savings Problem

The households problem:

$$\begin{aligned} & \max E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \\ st \quad : \quad & c_t + a_{t+1} = \left(1+r\right) a_t + ws_t, \\ a_{t+1} \quad \in \quad & A \end{aligned}$$

Borrowing constraints required for a well defined solution

A Savings Problem

Bellman's equation for the household's problem:

$$v(a, s) = \max_{a' \in A} \left\{ u\left(ws + (1+r)a - a'\right) + \beta E\left(v\left(a', s'\right)\right) \right\}$$

Using the Markov structure:

$$v(a_l, s_j) = \max_{a' \in A} \left\{ u\left(ws_j + (1+r)a_l - a'\right) + \beta \sum_{h=1}^{n} P(j, h)\, v\left(a', s_h\right) \right\}$$

- $P(j, h) = P(s' = s_h | s = s_j)$

A Savings Problem

Solution has **borrowing limits** built into it through the grid imposed on a

- Borrowing limits are a necessary condition for the existence of an equilibrium - otherwise agents could keep on accumulating debt

Which debt limit to impose?

- It must be a debt limit that we know that the agent can actually observe: The agent must always be able to pay her debt
- But, no unique choice of borrowing limit without imposing more structure

A Savings Problem

Aiyagari: Impose that consumption must be positive:

$$c_t \geq 0$$

Budget constraint implies:

$$\begin{aligned} c_t + a_{t+1} &= ws_t + (1+r)\, a_t \Rightarrow \\ a_t &= \frac{1}{1+r} \sum_{i=0}^{\infty} (1+r)^{-i} \left(c_{t+i} - ws_{t+i} \right) \end{aligned}$$

so $c_t \geq 0$ implies that:

$$a_t \geq -\frac{1}{1+r} \sum_{i=0}^{\infty} (1+r)^{-i}\, ws_{t+i}$$

- RHS is a **stochastic variable** that depends on future idiosyncratic earnings shocks: It cannot be imposed only in expected value - in that case there will be states where it does not hold!

A Savings Problem

Impose the worst case scenario:

$$s_1 = \min s$$

The natural debt limit is:

$$\phi = -\frac{1}{1+r}\sum_{i=0}^{\infty}(1+r)^{-i}\, ws_1 = \frac{s_1 w}{r}$$

- This level of debt can be paid with probability 1 with non-negative levels of consumption

We **can impose debt limits stronger** than this but **not weaker** unless we allow for default

Distributions

The stationary distribution of agents over assets and over employment states (a, s), $\pi_\infty(a, s)$, found from iterating on:

$$\pi_{t+1}(a', s') = \sum_s \sum_a \pi_t(a, s) P(s', s)$$

- π_∞ can also be interpreted as the fraction of agents that at each point in time are characterized by (a_l, s_j)

The wealth distribution:

$$H_\infty(a) = \pi_\infty(a, s) G(s)$$

- where $G(s)$ is the ergodic probability of state s :

$$\begin{aligned} G(s) &= \left(A'A\right)^{-1} A' e_{n+1} \\ A &= \begin{bmatrix} I_n - P \\ \mathbf{1}' \end{bmatrix}, \quad e_{n+1} = \text{col n+1 of } I_{n+1} \end{aligned}$$

The equilibrium interest rate

In the non-stochastic economy, the equilibrium real interest rate will be $(1+r) = 1/\beta$

- This **cannot** be the case in the stochastic economy
- Let the borrowing limit be ϕ and assume $u'(c_t) \geq 0$

The Euler equation implies that:

$$\begin{aligned} u'(c_t) &\geq E_t \beta (1+r) u'(c_{t+1}) \\ \text{with equality if } a_{t+1} &> \phi \end{aligned}$$

- Define:

$$K_t = \beta^t (1+r)^t u'(c_t) \geq 0$$

The equilibrium interest rate

The Euler equation can then be expressed as:

$$E_t \left(K_{t+1} - K_t \right) \leq 0$$

- K_t is a **supermartingale**: A stochastic variable for which $E_t K_{t+1} \leq K_t$
- Since K_t is non-negative, the **supermartingale convergence theorem** holds: K_t converges almost surely to a non-negative random variable K

The equilibrium interest rate

Case 1: $(1+r) > 1/\beta$: **Ruled out**

- If K_t converges to a constant, then $u'(c_t)$ has to converge to zero since $\beta^t (1+r)^t$ is diverging to infinity
- If $u'(c_t)$ converges to zero, c_t diverges to infinity (as do assets).

Case 2: $(1+r) = 1/\beta$: **Ruled out**

- $\beta^t (1+r)^t = 1$ so consumption diverges as in the $(1+r) > 1/\beta$ case as do assets

Case 3: $(1+r) < 1/\beta$: **Possible**.

- convergence of K_t allows for consumption to be finite (and stochastic)
- Equilibrium real interest rates under idiosyncratic risk will be less than $1/\beta$

Aiyagari QJE 1994

Model with physical capital and production

- Evolution of household's capital stock:

$$k_{t+1} = (1 - \delta)\, k_t + x_t$$

- and budget constraint:

$$c_t + x_t = Rk_t + ws_t$$

- Combining them:

$$\begin{aligned} c_t + k_{t+1} &= (1 + r)\, k_t + ws_t \\ r &= R - \delta \end{aligned}$$

Aiyagari QJE 1994

Production by competitive firms with CD technologies:

$$
\begin{aligned}
Y_t &= AK^{\alpha}N^{1-\alpha} \\
(1-\alpha)\,AK^{\alpha}N^{-\alpha} &= w \\
\alpha AK^{\alpha-1}N^{1-\alpha} &= r+\delta
\end{aligned}
$$

- Employment is determined exogenously through s : ie. no aggregate shocks
- The model will converge to a stationary distribution

Aiyagari QJE 1994

Equilibrium asset demand and asset supply schedules

- **Asset demand schedule**: From firms' profit maximization: $F_K = r + \delta$. The higher is the interest rate, the lower is the demand
- **Asset supply schedule**: From households' savings decision:

$$K = \sum_{k,s} \pi_j (k, s) \, g (k, s)$$

Aiyagari QJE 1994

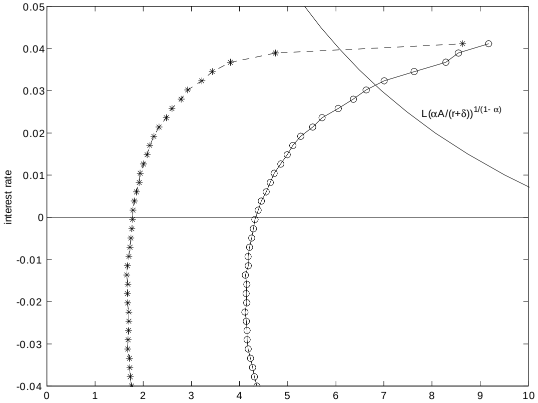


Figure: The stars and cicles curves are the asset supplies for two different productivity processes, the circled one with higher variance

Aiyagari QJE 1994

Bisection:

1. Start with a guess on $r_0 = 1/\beta - 1 - \epsilon$ where ϵ is very small
2. Compute capital labor ratio and wages given guess on r_j
3. Solve the households problem
4. Compute stationary distribution $\pi_j (k, s)$
5. Compute the average capital stock K_j^*
6. Compute the return on capital implied by $K_j^* : r = R - \delta$ where R is the marginal product of capital
7. Update the interest rate by bisection and repeat until convergence

- Bisection exploits that the equilibrium interest rate is below $1/\beta - 1$. The first guess will almost surely be too high. It will imply a low interest rate in the next step. We then have a r_{low} (the implied interest rate) and a r_{high} (r_0). Now take $r_1 = (r_{low} + r_{high})/2$. If the implied interest rate is above r_1 then replace r_{high} with the implied interest rate. If the implied interest rate is below r_1 replace r_{low} with the implied interest rate.

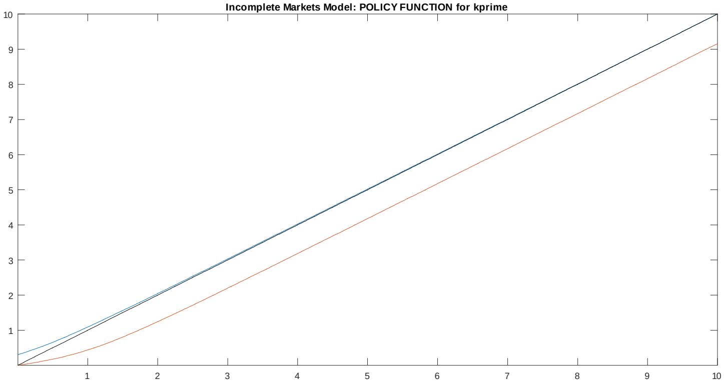
Aiyagari QJE 1994

Example:

- $s_t = [1, 0]$
- $P = \begin{bmatrix} 0.95 & 0.9 \\ 0.05 & 0.1 \end{bmatrix}$
- $\alpha = 0.4$
- $\delta = 0.025$
- $u(c) = \left(c^{1-\sigma} - 1\right) / (1 - \sigma)$, $\sigma = 2$
- $\beta = 0.99$
- natural borrowing limit

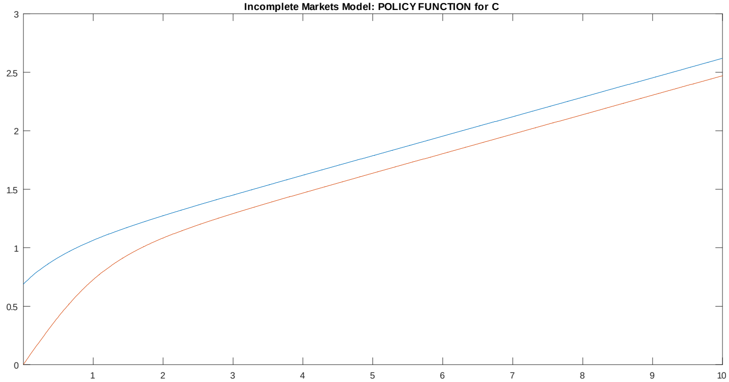
Implies long run unemployment rate of 5.3 percent, and expected unemployment duration of 1.11 months

Aiyagari QJE 1994



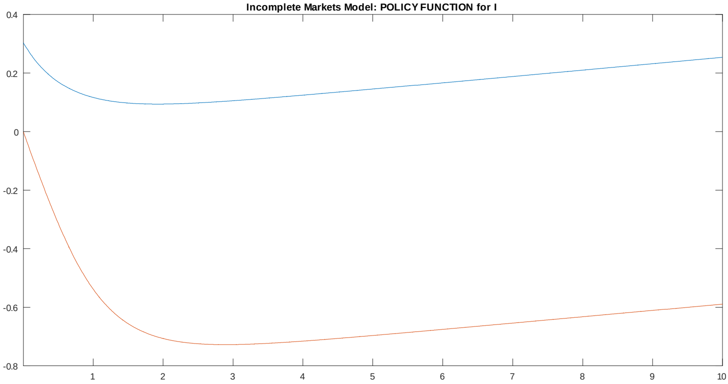
- save if you are poor and employed, dis-save when you become unemployed

Aiyagari QJE 1994



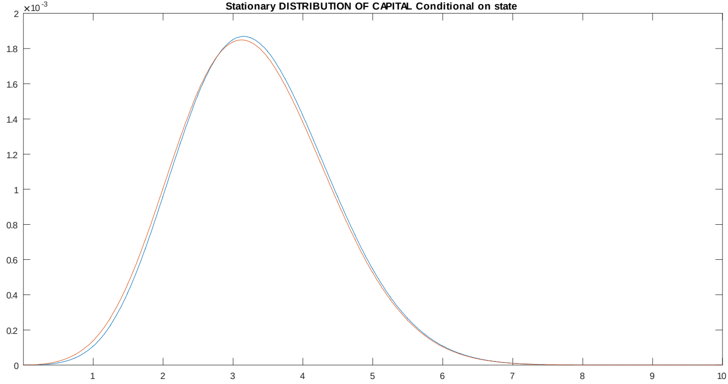
- MPC low apart from the very poor (equilibrium wage is around 1)

Aiyagari QJE 1994



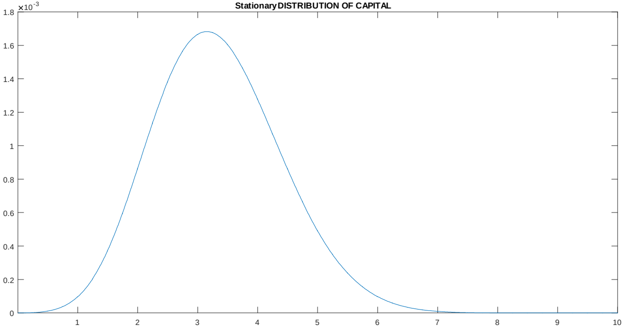
- savings policies

Aiyagari QJE 1994



- Unemployment duration is short so conditional wealth distributions are similar

Aiyagari QJE 1994



- very few (0.46 percent) hold less wealth than one period wage
- probability of being unemployed two periods or more conditional upon job loss is $1/0.9 = 11.1$ percent only

Krusell-Smith

Above: No aggregate risk

- Krusell and Smith, 1998 - two sources of risk
- **Idiosyncratic employment risk** - averages out
- **Aggregate productivity shocks**: z

Fundamental consequences: Every household's wealth will depend on the history of aggregate shocks and the sequence of individual shocks

- Across households the individual and aggregate shock histories will then be correlated
- Implication: Cross-sectional distribution of (k, s) will vary over time and so will wages and interest rates

Krusell-Smith

How can this be handled?

- Let $\lambda(k, s)$ denote the joint distribution of k and s
- The model:

$$
\begin{aligned}
v(k, s, \lambda, z) &= \max\left(u(c) + \beta E v\left(k', s', \lambda', z'\right) \mid (s, \lambda, z)\right) \\
c + k' &= r(K, N, z)\, k + w(K, N, z)\, s + (1 - \delta)\, k \\
r(K, N, z) &= z\alpha K^{\alpha-1} N^{1-\alpha} \\
w(K, N, z) &= z(1-\alpha) K^{\alpha} N^{-\alpha} \\
\lambda' &= H(\lambda, z) \\
K &= \int\int k\lambda(k, s)\, dkds
\end{aligned}
$$

- (employment is still constant)

Krusell-Smith

Definition

A recursive competitive equilibrium is a pair of price functions $r(K, N, z)$ and $w(K, N, z)$, a value function and decision rule $k'(k, s, \lambda, z)$, and a law of motion $H(\lambda, z)$ such that (i) given the price functions and the law of motion H, the value function solves the Bellman equation and the decision rule is $k'(k, s, \lambda, z)$, (ii) the decision rule $k'(k, s, \lambda, z)$ and the stochastic processes for s and z imply that $\lambda(k, s)$ is mapped into $\lambda'(k, s)$ by H.

Krusell-Smith

The households problem includes the first order condition:

$$u'\left(c\left(k,s,\lambda,z\right)\right)=\beta\mathbb{E}u'\left(c\left(k',s',\lambda',z'\right)\left(1-\delta\right)+R\left(K',N'\right)\right)$$

Future prices affect current decisions
$\Rightarrow$ need to forecast aggregate capital stock
$\Rightarrow$ need for forecast of cross-sectional distribution of capital

- Involves mapping distributions into distributions and then find fixed points of this mapping

Krusell-Smith

Finding fixed points of mappings of distributions into distributions is very difficult

- Krusell and Smith: Approximate the equilibrium ("bounded rationality")
- Assume that

$$\lambda(k, s) \approx \widehat{\lambda}(k, s) = F_I(k)$$

- where F_I denotes a linear function of the first I moments of of the distribution of k
- In practice, they used only first moment

Krusell-Smith

Let the vector of the relevant moments be denoted by m

- Given a functional form, H then induces m' given m

Algorithm:

1. Assume initial parameters for H_j
2. Given this solve:

$$v(k, s, m, z) = \max\left(u(c) + \beta E v\left(k', s', m', z'\right) \mid (s, \lambda, z)\right)$$

3. Simulate the economy for a large number of agents N for a very long period T
4. Compute the empirical distribution of λ
5. Compute a time series for m
6. Estimate new transition function $\widehat{H}_j$
7. If $\widehat{H}_j = H_j$ stop. Else update H_j and return to step 2.

Krusell-Smith

Aggregate Time Series

Model	Mean(k_t)	Corr(c_t, y_t)	Standard Deviation (i_t)	Corr(y_t, y_{t-4})
Benchmark:				
Complete markets	11.54	.691	.031	.486
Incomplete markets	11.61	.701	.030	.481
$\sigma = 5$:				
Complete markets	11.55	.725	.034	.551
Incomplete markets	12.32	.741	.033	.524
Real business cycle:				
Complete markets	11.56	.639	.027	.342
Incomplete markets	11.58	.669	.027	.339
Stochastic-β:				
Incomplete markets	11.78	.825	.027	.459

- Approximate aggregation

Krusell-Smith

Why approximate aggregation?

- Key difference between this incomplete markets model and RBC: Agents close to the borrowing constraint cannot insure against negative shock
- This induces ceteris paribus closer correlation between consumption and income than complete markets model

But agents find it costly to be at the constraint

- strong incentive to accumulate sufficient wealth that the risk of hitting the borrowing constraint is small
- few agents will therefore end up being constrained - minor aggregate effects
- moreover, those who end up at the constraint are poor - very minor aggregate effects
- In earlier Aiyagari example, 0.5 percent of households have less wealth than the wage and mean MPC is around 0.15

Hand to Mouth

Under incomplete markets, not many agents end up at the borrowing constraint

- those not close to the constraint do intertemporal substitution over time - will not be affected by timing of income
- those at the constraint will consume extra income with high MPC
- But being constrained is costly and they are poor and therefore account for little of total consumption

Hand to Mouth

Kaplan and Violante (Econometrica, 2014) argued that the estimated MPC may be consistent with a model with two types of liquidity constrained agents:

- **Poor hand to mouth:** No wealth like in the standard Aiyagari model
- **Rich hand to mouth:** Have wealth but held in illiquid assets
- both types off their Euler equations and therefore may have high MPCs

Analyze the impact of this in a two asset life-cycle model:

- Liquid asset (cash) - low return, allow for debt (unsecured consumer credit)
- Illiquid asset (housing) - higher return, fixed cost of withdrawal, must be non-negative

Two asset economy

Demographics: Continuum of households live for J periods, i is individual, j is age

- Work until age J^w, retire for J^r periods

Preferences: Epstein-Zin preferences:

$$V_{ij} = \left[(1-\beta) \left(c_{ij}^{\phi} s_{ij}^{1-\phi} \right)^{1-\sigma} + \beta \left(\mathbb{E} V_{i,j+1}^{1-\gamma} \right)^{(1-\sigma)/(1-\gamma)} \right]^{1/(1-\gamma)}$$

- $1/\sigma$ is the intertemporal elasticity of substitution, γ is risk aversion
- c is non-durables, s is service flow from housing

Two asset economy

Earnings: Labor log earnings given as

$$\log y_{ij} = \chi_j + \alpha_i + z_{ij}$$

- χ_j deterministic age profile common across households, α_i fixed effect
- z_{ij} idiosyncratic component, follows Markov process

Assets: two assets:

- **illiquid** a: Return $1/q^a$, withdrawal cost κ, $a \geq 0$, provides housing service

  $$s_{ij} = \zeta a_{it} + h_{ij}$$

  where h_{ij} is rental housing
- **liquid** m: Return $1/q^m$, free withdrawal, $m \geq \underline{m}(y_{ij})$
- Interest on debt higher than on assets, $\overline{q}^m < q^m$

Two asset economy

Government: Balances budget period by period

- spends G on purchasing goods, makes social security transfers to retirees
- taxes consumption (τ^c), asset income (τ^a, τ^m), payroll $(\tau^{ss}(y_{ij}))$ and labor income $(\tau^y(y_{ij}))$, $T(y_{ij}, a_{ij}, m_{ij})$ is combined income tax liability

Two asset economy

Household problem: Due to fixed withdrawal cost, chooses every period with to not to withdraw (0) or to withdraw (1) if it holds illiquid assets:

$$V_j(s_j) = \max\left[V_j^0(s_j), V_j^1(s_j)\right]$$

- if non-adjust:

$$\begin{aligned}(1+\tau^c)(c_j+h_j) + q^m(m_{j+1})\, m_{j+1} &= y_j + m_j - T_j(y_j, a_j, m_j) \\ q^a a_{j+1} &= a_j, m_j \geq \underline{m}(y_{ij}), h_j \geq -\zeta a_j\end{aligned}$$

- if adjust:

$$\begin{aligned}&(1+\tau^c)(c_j+h_j) + q^m(m_{j+1})\, m_{j+1} + q^a a_{j+1} \\ &= y_j + m_j - T_j(y_j, a_j, m_j) + a_j - \kappa \\ m_j &\geq \underline{m}(y_{ij}), h_j \geq -\zeta a_j,\ a_{j+1} \geq 0\end{aligned}$$

Consumption / Portfolio / Savings decision: four scenarios

- **Scenario 1**: Household off Euler equations when $m_{j+1} = -m_{j+1}(y_j)$ - poor HTM - up against borrowing constraint
- **Scenario 2**: Household off Euler equations when $m_{j+1} = 0$ because kink in budget set. Rich HTM - have assets but no liquidity.
- **Scenario 3**: Those with liquid assets or not up against borrowing constraint will be on short run Euler equation

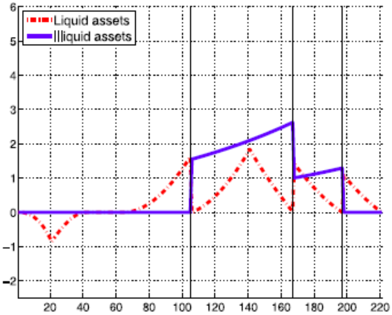
$$u'(c_j) = \frac{\beta}{q^m(m_{j+1})} u'(c_{j+1})$$

- **Scenario 4**: Households adjusting illiquid asset will do it so that:

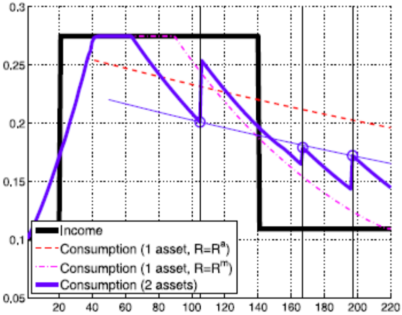
$$u'(c_j) = \left(\frac{\beta}{q^a}\right)^N u'(c_{j+N})$$

- where we imagine the household expects to adjust every N periods
- illiquid assets will be adjusted occasionally and when it happens, liquid assets will go to zero so will have no liquid assets and be at a kink

Two asset economy



(a) Life-cycle asset accumulation



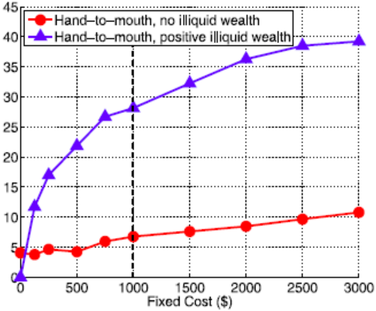
(b) Life-cycle income and consumption path

Two asset economy

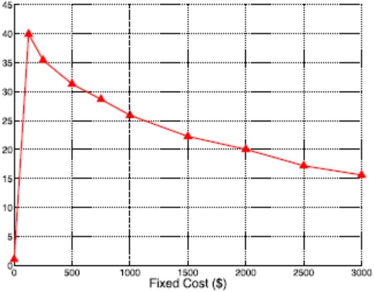
	Median ($2001)	Mean ($2001)	Fraction Positive	Return (%)
Earnings plus benefits (age 22–59)	41,000	52,745	–	–
Net worth	62,442	150,411	0.90	1.7
Net liquid wealth	2,629	31,001	0.77	−1.5
Cash, checking, saving, MM accounts	2,858	12,642	0.92	−2.2
Directly held MF, stocks, bonds, T-Bills	0	19,920	0.29	1.7
Revolving credit card debt	0	1,575	0.41	–
Net illiquid wealth	54,600	119,409	0.93	2.3
Housing net of mortgages	31,000	72,592	0.68	2.0
Retirement accounts	950	34,455	0.53	3.5
Life insurance	0	7,740	0.27	0.1
Certificates of deposit	0	3,807	0.14	0.9
Saving bonds	0	815	0.17	0.1

- HtM defined as those who have less liquid wealth than half earnings per period, the poor ones are those that also have negative liquid wealth adjusted for reported credit limit
- Kaplan and Violante argue that 17.5-35% of population are HTM, 40-80 percent of these are rich

Two asset economy



(c) Percentage of HtM households

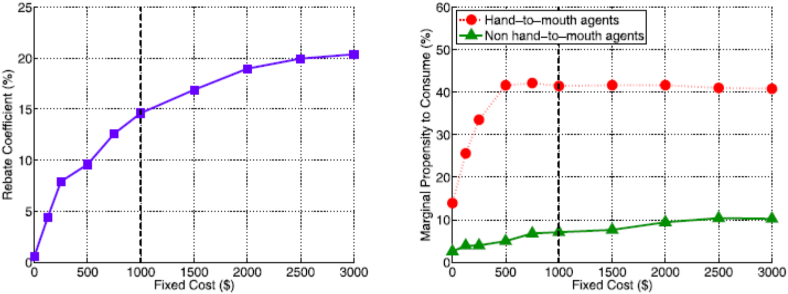


(d) Percentage of borrowers

Rebate Experiment

Kaplan - Violante implement the rebate experiment as:

- economy initially in a steady-state
- next period a rebate check (\$500) is sent to half the eligible population
- the period after the other half get the check
- financed by government issuing debt, repaid after 10 years by increasing the payroll tax



(a) Rebate coefficient

(b) Average marginal prop. to consume

Two asset economy

TABLE IV

BREAKDOWN OF THE MODEL'S REBATE COEFFICIENT INTO DIFFERENT COMPONENTS FOR THE THREE DIFFERENT INFORMATIONAL ASSUMPTIONS[a]

	Δc^A_{Q2}	Δc^B_{Q2}	Δc^A_{Q3}	Δc^B_{Q3}	β_2
Baseline	0.20	0.06	−0.09	0.07	0.15
Anticipated by all	0.07	0.00	−0.08	0.07	0.11
Surprise for all	0.20	0.00	−0.09	0.20	0.25

HtM: Data

Kaplan, Violante and Weidner (Brookings, 2014) look at micro data to estimate HtM

- Look at survey evidence on household portfolios in US, Canada, Australia, UK, Germany, France and Italy

Sample selection and measurements:

- household heads 22-79 years old, drop self employed, those with negative income
- **income** is sum of labor income and government transfers, correction for taxes for some countries, capital and self-employment income for some
- **liquid wealth**: checking, saving, money market and call accounts, bonds and stocks, mutual funds but cash holdings missing from data, imputed for US data, liquid debt is credit card balances
- net liquid wealth used to infer about HtM
- **Illiquid wealth**: value of housing net of mortgages plus retirement accounts

HtM: Data

Table 2. Household Income, Liquid and Illiquid Wealth Holdings, and Portfolio Composition, Sample Countries[a]

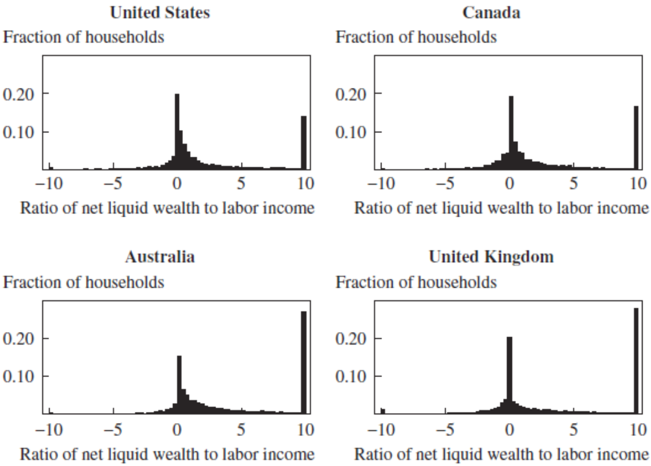
	United States[b]		*Canada*[c]		*Australia*		*United Kingdom*	
	Median	*Fraction positive*	*Median*	*Fraction positive*	*Median*	*Fraction positive*	*Median*	*Fraction positive*
Income (age 22–59)	47,040	0.984	49,905	1.000	79,555	0.993	29,340	0.979
Net worth	56,721	0.883	112,418	0.877	380,889	0.984	187,157	0.880
Net liquid wealth	1,714	0.750	2,643	0.716	12,139	0.880	2,111	0.632
Cash, checking, saving, MM accounts	2,640	0.923	2,873	0.864	8,709	0.978	2,639	0.766
Directly held stocks	0	0.142	0	0.109	0	0.351	0	0.160
Directly held bonds	0	0.014	0	0.106	0	0.015	0	0.154
Revolving credit card debt	0	0.382	0	0.412	0	0.296	0	0.405
Net illiquid wealth	52,000	0.761	100,713	0.752	347,500	0.939	17,4999	0.843
Housing net of mortgages	29,000	0.629	64,238	0.648	250,000	0.714	81,400	0.677
Retirement accounts	1,508	0.526	871	0.518	61,000	0.863	58,560	0.766
Life insurance	0	0.186	0	0.033	0	0.064	0	0.110

HtM: Data

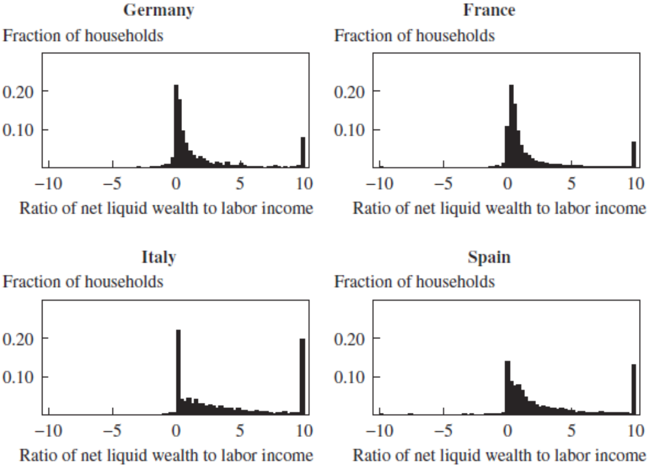
	Germany		France		Italy		Spain	
	Median	*Fraction positive*	*Median*	*Fraction positive*	*Median*	*Fraction positive*	*Median*	*Fraction positive*
Income (age 22–59)	35,444	0.994	31,518	0.999	26,116	0.987	26,961	0.991
Net worth	46,798	0.949	108,976	0.966	165,420	0.919	178,925	0.967
Net liquid wealth	1,319	0.853	1,453	0.925	5,226	0.769	2,685	0.890
Cash, checking, saving, MM accounts	1,154	0.876	1,255	0.953	4,181	0.769	2,261	0.908
Directly held stocks	0	0.110	0	0.151	0	0.043	0	0.106
Directly held bonds	0	0.050	0	0.015	0	0.146	0	0.014
Revolving credit card debt	0	0.225	0	0.076	0	0.049	0	0.086
Net illiquid wealth	39,306	0.876	104,214	0.922	148,524	0.803	171,161	0.885
Housing net of mortgages	0	0.476	86,372	0.607	148,524	0.716	162,491	0.847
Retirement accounts	0	0.245	0	0.039	0	0.088	0	0.037
Life insurance	0	0.493	0	0.378	0	0.193	0	0.245

HtM: Data

Figure 2. Distribution of Liquid Wealth to Monthly Income Ratios, Sample Countries[a]



HtM: Data

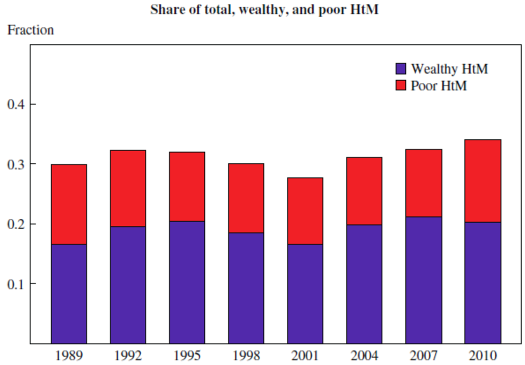


Germany
Fraction of households
0.20
0.10
−10
−5
0
5
10
Ratio of net liquid wealth to labor income
France
Fraction of households
0.20
0.10
−10
−5
0
5
10
Ratio of net liquid wealth to labor income
Italy
Fraction of households
0.20
0.10
−10
−5
0
5
10
Ratio of net liquid wealth to labor income
Spain
Fraction of households
0.20
0.10
−10
−5
0
5
10
Ratio of net liquid wealth to labor income

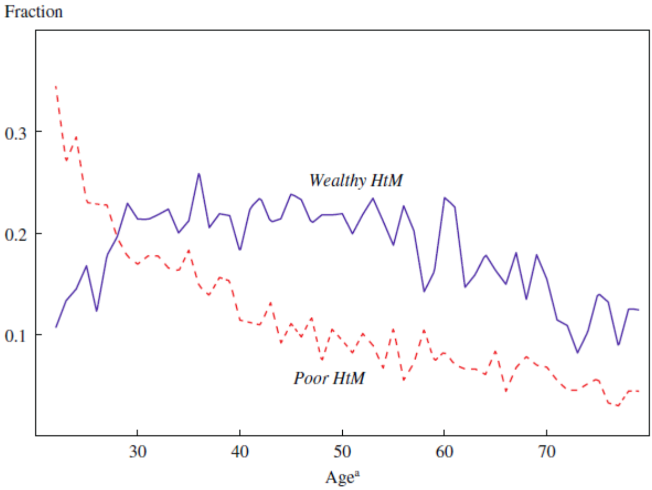
HtM: Data

Similar to Kaplan and Violante HtM is defined as:

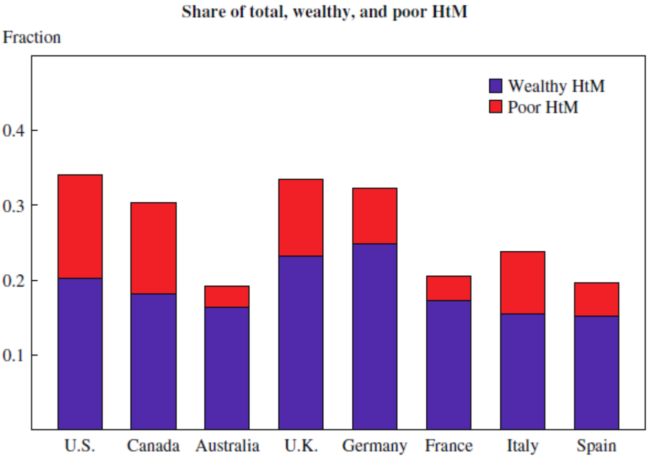
- **Poor HtM**: No illiquid assets, liquid assets less than half period income adjusted for possible credit limit
- **Rich HtM**: Positive illiquid assets, liquid asset less than half period income adjusted for possible credit limit



HtM: Data



HtM: Data



Share of total, wealthy, and poor HtM
Fraction
Wealthy HtM
Poor HtM
0.4
0.3
0.2
0.1
U.S.
Canada
Australia
U.K.
Germany
France
Italy
Spain

Aggregate Consequences

Does this matter for macro? New literature on **HANK** (Heterogeneous Agents New Keynesian)

- incomplete markets, aggregate shocks
- frictions like in representative agent models
- sticky prices (HANK)
- labor market frictions (HANK&SAM) - next time

Kaplan, Moll and Violante

Monetary Policy According to HANK

- HANK model with "rich hand-to-mouth consumers"
- Compare HANK with Representative Agent NK (RANK)
- Application to monetary policy

Argue that response of monetary policy can be decomposed

$$\begin{array}{lcccc} dC|_{dm} & = & \underbrace{\text{direct response to } r} & + & \underbrace{\text{indirect GE response due to } dY} \\ RANK & & > 95\% & & < 5\% \\ HANK & & < 25\% & & > 75\% \end{array}$$

RANK

Assume for simplicity that

- CRRA preferences with risk aversion γ and discount rate ρ
- technology linear in labor, $Y_t = N_t$
- prices perfectly rigid, output is demand determined

$$\frac{du'\left(c_t\right)/dt}{u'\left(c_t\right)} = \rho - r_t$$

- where monetary authority sets $r_t = \rho + e^{-\eta t}\left(r_0 - \rho\right)$
- jump in r at date 0, then reverts to ρ at constant speed η
- assume also that economy returns to initial steady-state in the long run

RANK

Equilibrium consumption response is then

$$
\begin{aligned}
C_t &= \overline{C} \exp\left(-\frac{1}{\gamma}\int_t^{\infty}(r_s-\rho)\,ds\right) \Rightarrow \\
\frac{d\log C_0}{dr_0} &= \frac{d\left(-\frac{1}{\gamma}\left(\int_0^{\infty}(r_s-\rho)\,ds\right)\right)}{dr_0} \\
&= \frac{d\left(-\frac{1}{\gamma}\left(\int_0^{\infty}\exp(-\eta t)(r_0-\rho)\,ds\right)\right)}{dr_0} \\
&= \frac{d\left(\frac{1}{\gamma\eta}(r_0-\rho)\left[\exp(-\eta t)\,ds\right]_{t=0}^{\infty}\right)}{dr_0} = \frac{d\left(-\frac{1}{\gamma\eta}(r_0-\rho)\right)}{dr_0} \\
&= -\frac{1}{\eta\gamma}
\end{aligned}
$$

- intertemporal substitution will account for a lot.

RANK

Decompose consumption change from interest rate change into

- Direct effect: Change in consumption for constant income path
- Indirect effect: Change in consumption because labor income changes too

$$
\begin{aligned}
dC_0 &= \int_0^\infty \frac{\partial C_0}{\partial r_t} dr_t dt + \int_0^\infty \frac{\partial C_0}{\partial Y_t} dY_t dt && (1) \\
d \log Y_t &= -\frac{1}{\gamma} \int_t^\infty dr_s ds && (2)
\end{aligned}
$$

- The response due to intertemporal substitution plus indirect effect through income.
- Because of constant prices, output is demand determined whcih gives the last term.

RANK

KMV show that for small change dr_t, the total change in consumption $d \log C_0 = -\frac{1}{\gamma} \int_0^\infty dr_t dt$

$$d \log C_0 = -\frac{1}{\gamma} \int_0^\infty e^{-\rho t} dr_t dt - \frac{\rho}{\gamma} \int_0^\infty e^{-\rho t} \int_t^\infty dr_s ds dt$$

- For the interest rate path above

$$\frac{d \log C_0}{dr_0} = \frac{1}{\gamma\eta} \quad [\underbrace{\frac{\eta}{\rho + \eta}}_{\text{direct}} \quad + \quad \underbrace{\frac{\rho}{\rho + \eta}}_{\text{indirect}}]$$

- $\gamma = 2$, $\rho = 0.005$, $\eta = 0.5 \Rightarrow$ direct effect $= 99\%$

RANK

Now assume that the government has debt, $B_0 > 0$

- Monetary policy must be accompanied by fiscal policy because of impact of interest rate change on government budget constraint - lower/higher interest rate, means less/more interest paid on government debt
- Because of Ricardian equivalence, total effect is unchanged but split into direct and indirect effects alters
- For the interest rate path above

$$\frac{d \log C_0}{dr_0} = \frac{1}{\gamma\eta} \quad [\underbrace{\frac{\eta}{\rho+\eta}\left(1-\rho\gamma\frac{B_0}{Y}\right)}_{\text{direct}} + \underbrace{\frac{\rho}{\rho+\eta}}_{\text{indirect Y}} + \underbrace{\frac{\eta}{\rho+\eta}\rho\gamma\frac{B_0}{Y}}_{\text{Indirect T}}]$$

- Unless government debt very large, little effect from the last term

TANK

Now introduce HTM households (Two Agent NK)

- share λ of spenders who consume all income

$$\frac{d \log C_0}{dr_0} = \frac{1}{\gamma\eta} \quad [\underbrace{\frac{\eta}{\rho+\eta}(1-\lambda)}_{\text{direct}} \quad + \quad \underbrace{(1-\lambda)\frac{\rho}{\rho+\eta}+\lambda}_{\text{indirect Y}}]$$

- λ around 0.3, direct effect still around 70 percent

	RANK				TANK		
	$B = 0$	$B > 0$	S-W	$B, K > 0$	$B = 0$	$B > 0$	$B, K > 0$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Elasticity of C	-2.00	-2.00	-0.74	-2.07	-2.00	-2.43	-2.77
P.E. elast. of C	-1.98	-1.96	-0.73	-1.95	-1.38	-1.39	-1.39
Direct effects	99%	98%	99%	94%	69%	57%	50%

Table 1: Elasticity of aggregate consumption and share of direct effects in several versions of the RANK and TANK models.

Model

Households: Idiosyncratic earnings risk, die with prob. λ

- portfolio of liquid and illiquid assets

Firms: Monopolistically competitive

- face Rotemberg type quadratic adjustment costs

Investment funds: Financial intermediaries
Government: Issues liquid debt

- Taxes, transfers, public spending

Central Bank: Sets nominal interest rate (on liquid asset) with Taylor rule

Households

Preferences:

$$\max_{c_t, l_t} \mathbb{E}_0 \int_0^\infty e^{-(\rho+\lambda)t} u\left(c_t, l_t\right)$$

st.

$$\begin{aligned}
\overset{\bullet}{b}_t &= r\left(b_t\right) b_t + \left(1-\tau_t\right) w_t z_t l_t - \mathcal{T}_t - d_t - \chi\left(d_t, a_t\right) - c_t \\
\overset{\bullet}{a}_t &= r_t^a a_t + d_t \\
\chi\left(d_t, a_t\right) &= \chi_0 \left|d\right| + \chi_1 \left(\frac{d}{a}\right)^{\chi_2} a \\
z_t &: \quad \text{Markovian} \\
b_t &\geq \underline{b},\ a_t \geq 0
\end{aligned}$$

- d_t: illiquid deposits
- $\chi\left(d_t, a_t\right)$: adjustment cost function

Firms

Competitive final goods producer (representative firm)

$$Y = \left(\int_j y_j^{1-1/\varepsilon} dj \right)^{1/(1-1/\varepsilon)}$$

Monopolistically competitive intermediate goods producers

$$y_j = k_j^{\alpha} n_j^{1-\alpha}$$

- Rotemberg quadratic price adjustment costs

$$\Theta\left(\frac{\overset{\bullet}{p}}{p}\right) = \theta \left(\frac{\overset{\bullet}{p}}{p}\right)^2 Y$$

- Implies NK Phillips curve in symmetric equilibrium:

$$\left(r^a - \frac{\overset{\bullet}{Y}}{Y} \right) \pi = \frac{\varepsilon}{\theta} \left(mc - mc^* \right) + \overset{\bullet}{\pi}$$

Competitive Investment Funds

Owners of the intermediate goods sector firms

$$a_t = k_t + q_t s_t$$

- Issues illiquid asset with return r^a
- Two sources of income

1. Rents capital to firms

$$\left(r^k - \delta\right) k$$

2. Receives dividends and equity shares

$$\dot{k}_t + q_t \dot{s}_t = \left(r_t^k - \delta\right) k_t + \Pi_t s_t + d_t$$

- lack of arbitrage

$$r^a = \left(r^k - \delta\right) = \frac{\Pi + \dot{q}}{q}$$

Policy

Monetary authority sets interest rate on liquid asset

$$\begin{aligned} r^b &= i - \pi \\ i &= \bar{r}^b + \phi\pi + \epsilon \end{aligned}$$

Fiscal authority must observe government budget constraint:

$$\overset{\bullet}{B}{}_t^g + G_t + T_t = \tau_t \int w_t z l_t d\mu_t + r_t^b B_t^g$$

- Ricardian equivalence is failing here due to market incompleteness
- Lump sum taxes assumed to adjust to keep government budget constraint

Equilibrium

Definitions

Equilibrium is a path for prices $\left(w_t, r_t^k, r_t^a, r_t^b\right)_{\forall t}$, government policies, and quantities such that (i) households, firms, and investment funds are making optimal choices given their constraints taking equilibrium prices for given, (ii) government budget constraint holds, (iii) all markets clear

- liquid asset market

$$B^G + \int_\mu b d\mu = 0$$

- illiquid asset market

$$K + q = A$$

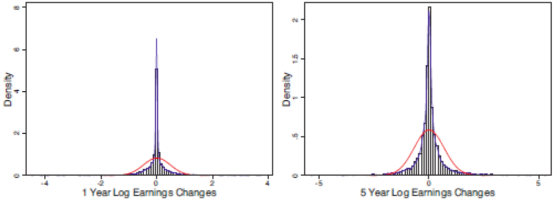
- labor and goods markets

$$\begin{aligned} N &= \int_\mu zl d\mu \\ Y &= C + C^h + I + G + \Theta + \chi + \kappa \int_\mu \max\left(-b, 0\right) d\mu \end{aligned}$$

Calibration: Earnings Process

$$
\begin{aligned}
\log z_{it} &= z_{1,it} + z_{2,it} \\
dz_{j,it} &= -\beta_j z_{j,it} dt + \varepsilon_{j,it} dN_{j,it} \\
\varepsilon_{j,it} &\sim N\left(0, \sigma_j^2\right) \\
dN_{j,it} &\sim \text{Poisson w. rate } \lambda_j
\end{aligned}
$$

Parameter		Component $j = 1$	Component $j = 2$
Arrival rate	λ_j	0.080	0.007
Mean reversion	β_j	0.761	0.009
St. Deviation of innovations	σ_j	1.74	1.53



Calibration: Adjustment costs

- adjustment costs account for 2 percent of GDP on average

Assets

	Liquid (B^h)		Illiquid (A)		Total
Non-productive $B^h + \omega A$	Revolving cons. debt Deposits Corporate bonds Government bonds	-0.03 0.23 0.04 0.02	$0.60\times$ Net housing $0.60\times$ Net durables	0.60×1.09 0.60×0.22	$B^h = 0.26$ $\omega A = 0.79$
Productive $(1-\omega) A = K$	$\times$		Corporate equity Private Equity $0.40\times$ Net housing $0.40\times$ Net durables	1.02 0.59 0.40×1.09 0.40×0.22	$K = 2.13$
Total		0.26		2.92	3.18

- One issue here: Housing only illiquid if there is no equity in the house

Model implication for wealth distribution

	Data	Model
Mean illiquid assets	2.92	2.92
Mean liquid assets	0.26	0.23
Frac. with $b = 0$ and $a = 0$	0.10	0.10
Frac. with $b = 0$ and $a > 0$	0.20	0.19
Frac. with $b < 0$	0.15	0.15

	Liquid Wealth		Illiquid Wealth	
Moment	Data	Model	Data	Model
Top 0.1% share	17%	2.3%	12%	7%
Top 1% share	47%	18%	33%	40%
Top 10% share	86%	75%	70%	88%
Bottom 50% share	-4%	-3%	3%	0.1%
Bottom 25% share	-5%	-3%	0%	0%
Gini coefficient	0.98	0.86	0.81	0.82

Other parameters

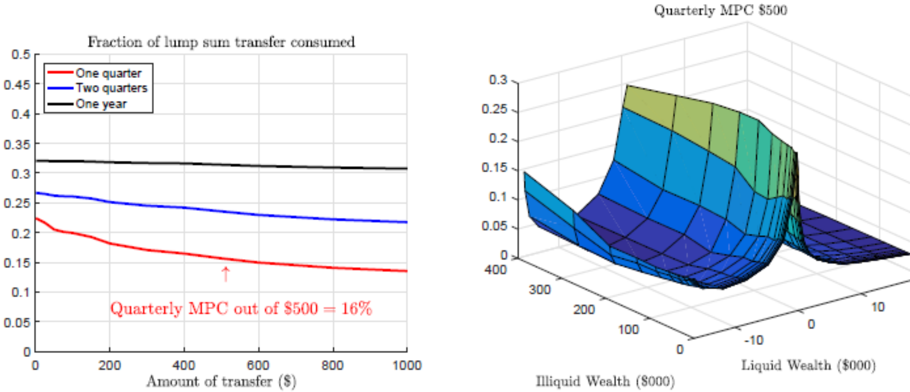
Treatment of profits: Profits are countercyclical in sticky price models - this implies a fall in share prices after a monetary expansion because equity is assumed to be an illiquid asset

- This would generate a decline in investment
- Assume that a share ω of profits are paid into illiquid accounts, the other share is paid out lump-sum to households proportionally to their productivity
- seems very arbitrary
- preferences:

$$u = \frac{c^{1-\gamma}}{1-\gamma} - \varphi \frac{n^{1+\nu}}{1+\nu}, \gamma = 1, \ 1/\nu = 1$$

- $\varepsilon = 10$, $\theta = 100$ (prices fixed for around 3 quarters)
- Taylor rule coefficient, $\phi = 1.25$

MPC



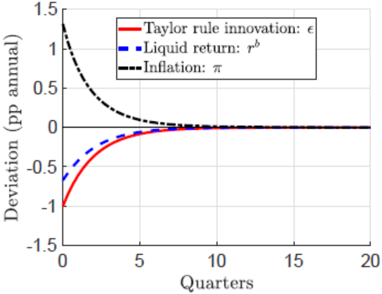
(a) $\int \mathrm{MPC}_\tau^x(a,b,z)d\mu$ by τ, x

(b) $\mathrm{MPC}_1^{\$500}(a,b,z)$

- MPC from small transfers high for the poor and for the rich but liquid asset poor

Impact of Monetary Policy Shock

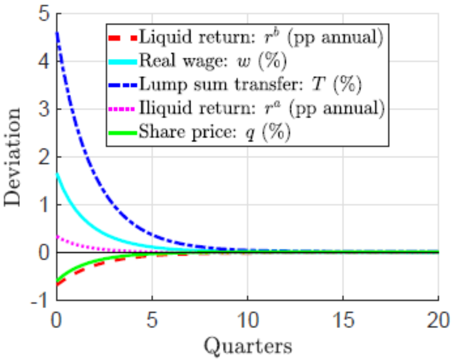
- investigate impact of 25 basis point (at the annual rate) cut in the nominal interest rate
- lump-sum transfers adjusted to keep government budget constraint



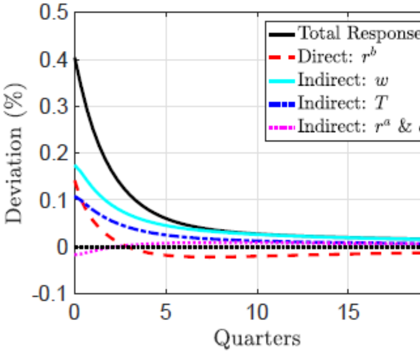
(a) Monetary Shock, Interest Rate, Inflation (b) Aggregate Quantities

- Consumption elasticity is -2.9, 50% higher than RANK (but similar to TANK)

Direct vs. Indirect Effects



(a) Prices



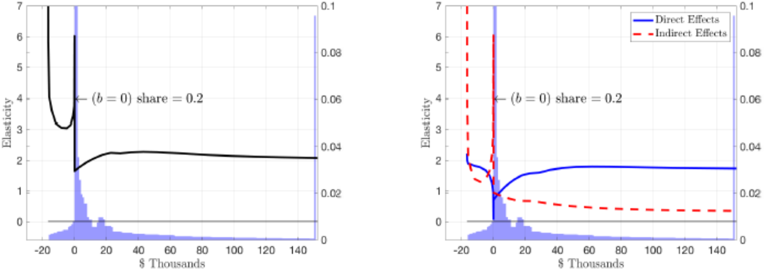
(b) Consumption Decomposition

Direct vs. Indirect Effects

	Baseline (1)	$\omega = 1$ (2)	$\omega = 0.1$ (3)	$\frac{\varepsilon}{\theta} = 0.2$ (4)	$\phi = 2.0$ (5)	$\frac{1}{\nu} = 0.5$ (6)
Change in r^b (pp)	-0.28%	-0.34%	-0.16%	-0.21%	-0.14%	-0.25%
Elasticity of Y	-3.96	-0.13	-24.9	-4.11	-3.94	-4.30
Elasticity of I	-9.43	7.83	-105	-9.47	-9.72	-9.79
Elasticity of C	-2.93	-2.06	-6.50	-2.96	-3.00	-2.87
Partial Eq. Elast. of C	-0.55	-0.45	-0.99	-0.57	-0.59	-0.62
Component of change in C due to:						
Direct effect: r^b	19%	22%	15%	19%	20%	22%
Indirect effect: w	51%	56%	51%	51%	51%	38%
Indirect effect: T	32%	38%	19%	31%	31%	45%
Indirect effect: r^a & q	-2%	-16%	15%	-2%	-2%	-4%

- ω - which is arbitrary - seems to matter enormously

Distributional Effects



(a) Elasticity with respect to r^b

(b) Consumption Change: Indirect and direct

Figure 5: Consumption Responses by Liquid Wealth Position

Importance of Fiscal Policy

	T adjusts (1)	G adjusts (2)	τ adjusts (3)	B^g adjusts (4)
Change in r^b (pp)	-0.28%	-0.23%	-0.33%	-0.34%
Elasticity of Y	-3.96	-7.74	-3.55	-2.17
Elasticity of I	-9.43	-14.44	-8.80	-5.07
Elasticity of C	-2.93	-2.80	-2.75	-1.68
Partial Eq. Elast. of C	-0.55	-0.60	-0.56	-0.71
Component of Change in C due to:				
Direct effect: r^b	19%	21%	20%	42%
Indirect effect: w	51%	81%	62%	49%
Indirect effect: T	32%	-	-	9%
Indirect effect: τ	-	-	18%	-
Indirect effect: r^a & q	-2%	-2%	0%	0%

- interest rate declines $\Rightarrow$ tax revenues up and interest payments down
- stimulus comes from using this fiscal effect on transfers or on direct spending

Readings

Kaplan, Greg, and Gianluca Violante, 2014, "A Model of the Consumption Response to Fiscal Stimulus Payments," Econometrica 82(4), 1199-1239.

Kaplan, Greg, Gianluca Violante, and Justin Weidner, 2018, 2014, "The Wealthy Hand-to-Mouth," Brookings Papers on Economic Activity, spring.

Kaplan, Greg, Benjamin Moll and Gianluca Violante, 2018, "Monetary Policy According to HANK," forthcoming, American Economic Review.

Ljungqvist and Sargent, chapter 14.

Ricardian households - share γ of aggregate

$$
\begin{aligned}
V_0^R &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t^R - \frac{\phi}{1+\kappa} \left(n_t^R \right)^{1+\kappa} \right) \\
c_t^R + i_t^R &= w_t n_t^R + r_t^R k_t^R + T_t^R \\
k_{t+1}^R &= (1-\delta) k_t^R + i_t^R
\end{aligned}
$$

Hand to Mouth - share $1-\gamma$ of aggregate

$$
\begin{aligned}
V_0^H &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t^H - \frac{\phi}{1+\kappa} \left(n_t^H \right)^{1+\kappa} \right) \\
c_t^H &= w_t n_t^H + T_t^H
\end{aligned}
$$

Production - competitive firms:

$$
y_t = k_t^{1-\alpha} n_t^{\alpha}
$$

Government - balances budget:

$$
\gamma T_t^R = -(1-\gamma) T_t^H
$$

Goods and labor **market clearing**:

$$
\begin{aligned}
y_t &= \gamma\left(c_t^R + i_t^R\right) + (1-\gamma)\, c_t^H \\
n_t &= \gamma n_t^A + (1-\gamma)\, n_t^B \\
k_t &= \gamma k_t^R
\end{aligned}
$$

first-order necessary conditions:

$$
\begin{aligned}
k_{t+1}^{R} &= (1-\delta)\,k_t^R + i_t^R \\
\phi^R \left(n_t^R\right)^{\kappa} &= \frac{1}{c_t^R} w_t \\
\frac{1}{c_t^R} &= \beta \mathbb{E}_t \frac{1}{c_{t+1}^R}\left(r_{t+1} + (1-\delta)\right) \\
\phi^H \left(n_t^H\right)^{\kappa} &= \frac{1}{c_t^H} w_t \\
c_t^H &= w_t n_t^H + T_t^H \\
w_t &= \alpha y_t / n_t \\
r_t &= (1-\alpha)\, y_t / \left(\gamma k_t^R\right) \\
y_t &= \gamma \left(c_t^R + i_t^R\right) + (1-\gamma)\, c_t^H \\
y_t &= \left(\gamma k_t^R\right)^{1-\alpha} n_t^{\alpha} \\
n_t &= \gamma n_t^R + (1-\gamma)\, n_t^H
\end{aligned}
$$

The steady-state:

$$
\begin{aligned}
y &= \left(\frac{k}{y}\right)^{(1-\alpha)/\alpha} n \\
w &= \alpha \left(\frac{k}{y}\right)^{(1-\alpha)/\alpha} \\
r &= 1/\beta - (1-\delta) \\
\frac{k}{y} &= \frac{(1-\alpha)}{r} \\
k^R &= \frac{1}{\gamma} k = \frac{1}{\gamma} \frac{(1-\alpha)}{r} y \\
i^R &= \delta k^R \\
\phi^H &= \frac{1}{\left(n^H\right)^{1+\kappa}} \\
\phi^R \left(n^R\right)^{1+\kappa} &= \alpha \frac{n^R}{n} \frac{y}{c^R} \\
\phi^R &= \frac{\alpha \frac{n^R}{n} \frac{y}{c^R}}{\left(n^R\right)^{1+\kappa}}
\end{aligned}
$$

Log linearization:

$$
\begin{aligned}
k_{t+1}^{R} &= (1-\delta)\,k_t^{R} + \frac{i^{R}}{k^{R}} i_t^{R} \\
\kappa n_t^{R} &= w_t - c_t^{R} \\
-c_t^{R} &= -\mathbb{E}_t c_{t+1}^{R} + \frac{r}{r+(1-\delta)} \mathbb{E}_t r_{t+1} \\
\kappa n_t^{H} &= w_t - c_t^{H} \\
c_t^{H} &= w_t + n_t^{H} + T_t^{H}, \;\; T_t^{H} \equiv \frac{dT_t^{H}}{c^{H}} \\
w_t &= y_t - n_t \\
r_t &= y_t - k_t^{R} \\
y_t &= \gamma \frac{1}{y}\left(c^{R} c_t^{R} + i^{R} i_t^{R}\right) + (1-\gamma)\,\frac{c^{H}}{y} c_t^{H} \\
y_t &= (1-\alpha)\,k_t^{R} + \alpha n_t \\
n_t &= \gamma n_t^{R} + (1-\gamma)\,n_t^{H} \\
c_t &= \gamma c_t^{R} + (1-\gamma)\,c_t^{H}
\end{aligned}
$$

HANK

$$
\begin{aligned}
\frac{dC_t}{dt}\frac{1}{C_t} &= \frac{1}{\gamma}\left(r_t-\rho\right) \\
C_t &= \overline{C}\exp\left(-\frac{1}{\gamma}\left(\int_t \left(r_s-\rho\right)ds\right)\right) \\
r_t &= \rho+\exp\left(-\eta t\right)\left(r_0-\rho\right) \\
\frac{d\log C_0}{dr_0} &= \frac{d\left(-\frac{1}{\gamma}\left(\int_0^\infty \left(r_s-\rho\right)ds\right)\right)}{dr_0} \\
-\frac{1}{\gamma}\left(\int_0^\infty \left(r_s-\rho\right)ds\right) &= -\frac{1}{\gamma}\left(\int_0^\infty \exp\left(-\eta t\right)\left(r_0-\rho\right)ds\right) \\
&= -\frac{1}{\gamma}\left(r_0-\rho\right)\left(\int_0^\infty \exp\left(-\eta t\right)ds\right) \\
&= \frac{1}{\gamma\eta}\left(r_0-\rho\right)\left[\exp\left(-\eta t\right)ds\right]_{t=0}^\infty \\
&= -\frac{1}{\gamma\eta}\left(r_0-\rho\right) \\
\frac{d\log C_0}{} &= -\frac{1}{}
\end{aligned}
$$

$$
\begin{aligned}
\log C_0 &= \log \overline{C} - \frac{1}{\gamma}\left(\int_0^\infty (r_s - \rho)\, ds\right) \\
&= \log \overline{C} - \frac{1}{\gamma}\int_0^\infty \exp(-\eta t)(r_0 - \rho)\, dt \\
d\log C_0 &=
\end{aligned}
$$

$$
\begin{aligned}
C_t &= \overline{C}\exp\left(-\frac{1}{\gamma}\left(\int_t (r_s - \rho)\, ds\right)\right) = Y_t \\
\frac{dC_0}{dr_0} &= \int_0^\infty \frac{\partial C_0}{\partial r_t} dr_t dt + \int_0^\infty \frac{\partial C_0}{\partial Y_t} dY_t dt \\
r_t &= \rho + \exp(-\eta t)(r_0 - \rho)
\end{aligned}
$$

$$
\begin{aligned}
\sum (1+r)^{-t} dc_t &= dx_t \\
\frac{1}{1 - \frac{1}{1+r}} &= \frac{1+r}{r} \\
dc_t &= \frac{r}{1+r} dx_t
\end{aligned}
$$

Ricardian Equivalence - Households

We will restrict borrowing further using **one** of the two constraints:

1. **A no-debt constraint:**

$$b_t \geq 0 \; \forall t \geq 0$$

2. **A natural borrowing limit:**

$$b_t \geq \overline{b}_t = -\sum_{j=0}^{\infty} (1+r)^{-j} \left(y_{t+j} - \tau_{t+j} \right)$$

- The first of these rules out household debt at any point in time - households can only save.
- The second of these permits borrowing but limits debt to the maximum amount that the household can repay with zero consumption. Aiyagari (1994).