Monetary policy transmission, the labour share and HANK models

Jamie Lenney

December 10, 2021

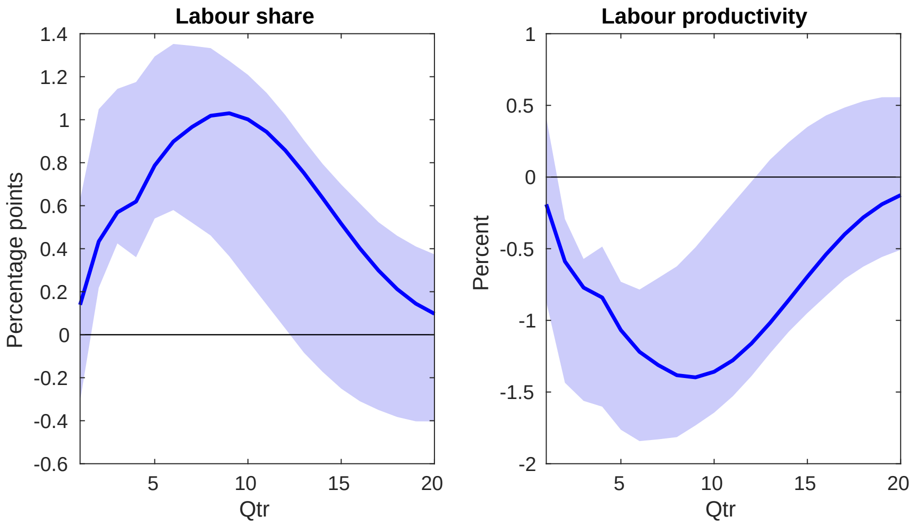
Abstract

I analyse the role of capital income in the transmission of demand shocks, such as monetary policy shocks, in a medium scale DSGE model that produces an empirically consistent counter-cyclical response of the labour share to monetary policy shocks. This is achieved by augmenting the one sector New Keynesian model with an alternate form of labour that seeks to expand the measure of goods available to consumers. I compare and contrast the transmissions of monetary policy shocks in the one sector 'textbook' model relative to the augmented model in both a representative agent (RANK) and heterogeneous agent (HANK) setting that includes a fully endogenous wealth distribution. The comparison highlights the role of capital income in the transmission of monetary policy shocks in these models. When the labour share moves counter-cyclically partial equilibrium decomposition's of monetary policy transmission show a significant contractionary role for capital income.

Code available at https://github.com/s0840389/BWP

My thanks to Ralph Luetticke for his valuable advice and code. I also thank Dario Bonciani, Robin Braun, Cristiano Cantore, Jonathan Haskel, Simon Jurkatis and the students, workshop participants and faculty of the UCL economics department for useful comments, advice and inspiration.

Figure 1: Response to a monetary policy shock.



Note: Figure shows the impulse response of the labour share (LHS) and productivity (RHS) to a 100 basis point increase in the short term interest rate. Following Cantore et al. (2021) the responses are estimated using quarterly US data from 1984-2007 in a seven variable three lag VAR including GDP, the GDP deflator, CPI, real wages, a commodity price index, the labour share and the Federal Funds Rate. The IRF's are identified using the proxy-SVAR external instruments method of Mertens & Ravn (2014) and spliced instruments sourced from Romer & Romer (2004) [pre-1991] and Miranda-Agrippino & Ricco (2021) [from 1991]. Labour productivity is derived from the impulse response of the labour share and wages. The shaded area represents a 68 percent confidence interval based on the the moving block bootstrap routine of Jentsch & Lunsford (2016). The solid blue line is the point estimate. See appendix A1 for more details.

1 Introduction

The textbook sticky price New Keynesian model remains a core tool of modern economic policy analysis yet in recent years it's underlying transmission mechanism has come under increasing scrutiny and criticism e.g. Nekarda & Ramey (2020) or Broer et al. (2020). In response to a demand shock, such as a monetary or fiscal tightening[1], the textbook sticky price models sees markups rise and a relative redistribution of income away from labour to capital. Consequently the labour share is usually pro-cyclical in response to demand shocks in these models, and at odds with the econometric evidence e.g. Cantore et al. (2021) that robustly estimates the labour share as counter-cyclical to such shocks. Figure 1 reproduces some of this VAR evidence for the US. The VAR provides evidence of a counter-cyclical response of the labour share and pro-cyclical response of productivity to monetary policy shocks using US data from 1984-2007.

New Keynesian models in the literature generally produce impulse responses in contradiction to figure 1. In this paper, based on the suggested framework of Kaplan & Zoch (2020), I analyse a model that can produce responses consistent with figure 1 and ask what that implies for the transmission of demand shocks particularly in relation to the wealth distribution. This is

[1]We will consider contractionary shocks in this manuscript and assume the opposite sign response in the case of an expansionary shock.

achieved through the introduction of an alternate non-production form of labour 'expansionary labour' into otherwise standard and popular business cycle models used for policy analysis. This form of labour is nested in the consumer facing sector and focuses on expanding the firms measure (variety) of goods available to customers as opposed to labours more traditional role in the direct production of goods. Practically this form of labour could fall under a number of headers. Research and development/product development fit this definition quite closely as they are forms of labour directly employed to innovate and expand firms product offerings. But the definition is broader as roles including sales and marketing, supply chain management and general management might also fit this definition of expansionary labour, as these roles do not directly contribute to the production of goods and services but are indirectly crucial for efficiently delivering and expanding the range of goods and services on offer.

I demonstrate the introduction of this form of labour can deliver a counter-cyclical response of the labour share to demand shocks. As in the standard model, sticky prices combine with a fall in demand to raise markups as prices do not fall enough to close the output gap. However, in a model with expansionary labour higher markups leads to higher demand for expansionary labour offsetting the fall in demand for production labour and raising the labour share. In doing so the model endogenously delivers a data consistent pro-cyclical response of labour productivity without significantly altering the impulse responses of other other key economic aggregates. Thus expansionary labours acts as a dynamic form of overhead labour. The key to this mechanism is an inefficient over allocation of expansionary labour relative to production labour in response to rising markups. The inefficiency occurs when consumer facing firms do not internalise cost pressures in the production sector when the measure of goods is increased. For example management may fail to account for the extra costs and pressures placed on the production line when they offer consumers more bespoke products.

Given the focus of this analysis on the distribution of income and labour heterogeneity, I compare and contrast the transmission of monetary policy shocks between the standard (NK) and augmented model (NK-YN) under various levels of household wealth heterogeneity[2]. Comparing the models highlights the important role of capital income in the monetary policy transmission mechanism in New Keynesian models, particularly models with wealth heterogeneity. In the textbook New Keynesian model rising markups partially insure richer households in response to a demand shock. In the model with expansionary labour where the labour share rises this link is broken as demand for expansionary labour absorbs the higher markups and in

[2] I compare transmission of monetary policy shocks in a medium scale DSGE RANK model, a two agent worker/capitalist (WCNK) variant and a two asset HANK model with a fully endogenous wealth distribution.

doing so reverses the role of capital income in response to a monetary shock such that capital incomes now drags on consumption particularly for richer capital owning households. As a result in partial equilibrium decomposition's of monetary policy transmission, capital income plays a significant contractionary role in the fall in aggregate demand, accounting for nearly half of the fall in consumption on impact. The rise in consumption inequality following the shock is also dampened, and reversed on impact, as wealthier households whose wealth is predominantly illiquid are now more exposed to demand shocks. This finding highlights the importance of having the correct micro foundations when partial equilibrium decomposition's are used to asses policy implications across the wealth distribution.

Finally the model analysed in this paper provides an attractive means by which to afford a richer and more realistic role for labour while at the same time producing more data consistent co-movements of the labour share in response to demand shocks. However the improvements on the demand side may come at a cost of less data consistency on the supply side as the augmented models response to investment specific technology shocks and markup shocks would appear to be inconsistent with the empirical literature, though the evidence on cyclicality is less clear on the supply side than on the demand side. Furthermore the counter-cyclical labour share in this model is delivered by altering the relationship between markups and the labour share but the model maintains counter-cyclical markups at odds with the data (though harder to measure accurately).

1.1 Literature

This work is related to several distinct but related literature's that touch on the transmission of shocks in New Keynesian models. A recent and growing literature has documented the apparent shortcomings of the New Keynesian model concerning the transmission of demand shocks. Cantore et al. (2021) conduct an exhaustive VAR based empirical exercise that documents a robust counter-cyclical response of the labour share to demand shocks, alongside a robust pro-cyclical response of productivity. They demonstrate this across five currency areas[3] and under several different[4] identification schemes. In all but one[5] of their empirical exercises the labour share rose in response to a contractionary monetary policy shock. Using impulse response function matching they further go on to demonstrate that the well cited medium scale DSGE models in the literature are unable to jointly match the response of the labour share alongside

[3] Australia, Canada, Eurozone, UK and US.

[4] Causal orderings, sign restrictions and instrumental variables.

[5] Australia under a instrumental variable identification.

other key macroeconomic aggregates like inflation and output. This includes model with sticky wages, CES production functions, search and matching, mechanisms to separate markups and the labour share (e.g. overhead labour) or models capable of generating pro-cyclical markups (e.g. Ravenna & Walsh (2006)).

Nekarda & Ramey (2020) study the shock conditional-cyclicality of the markup in US data. They estimate the markup to be pro-cyclical in response to TFP shocks and counter-cyclical in response to investment specific technology shocks. In response to expansionary fiscal and monetary policy shocks they find the markup to be pro-cyclical, particularly when the markup is proxied by the inverse of the labour share. Thus they find a demand shock contingent counter-cyclical response of the labour share. As in Cantore et al. (2021) they compare their empirical results to that delivered by the popular policy focused DSGE models and find the models predictions in contrast to the data.

This work also relates to papers that study the business cycle implications of the redistribution of income including the broader HANK literature. Broer et al. (2020) highlight the relative importance of wage and price rigidity for the transmission of monetary policy shocks in a tractable HANK model relative to a representative agent (RANK) model. They demonstrate the importance of counterfactual counter-cyclical profits to delivering a fall in output in response to a monetary tightening in the RANK model. Under a rigid price flexible wage setup a decline in labour supply from higher capital income drives the fall in output. When agents are separated into workers and capitalists this labour supply channel no longer exists and nullifies the effect of monetary policy on output in their calibration. They conclude that rigid wages which dampen this redistribution between labour and capital income are essential to realistic monetary policy transmission. Building on the insight of Broer et al. (2020), Cantore & Freund (2021) develop a worker capitalist model to abstract from the income effects on labor supply induced by counter-cyclical profits and show that it delivers more realistic and data consistent transmission of fiscal policy.

There is a growing literature that develops business cycle models to study the implications of inequality and redistribution, and contrasts the aggregate predictions with their representative agent counterparts. Papers including Alves et al. (2020), Bayer et al. (2020), Kaplan et al. (2018) and McKay & Reis (2016) to name but a few have developed numerical techniques to practically solve heterogeneous agent models and study the transmission of business cycle shocks. Kaplan et al. (2018) highlight the importance in HANK models of heterogeneous wealth holdings, the general equilibrium effect of wages and fiscal policy in explaining the response of major economic aggregates. These channels are small or missing in RANK models. McKay

& Reis (2016) highlight the importance of using HANK models to study the redistributive properties of fiscal policy and automatic stabilizers. Bayer & Luetticke (2020) estimate a HANK version close to that of Smets & Wouters (2007) and find a significant role for business cycle shocks in the evolution of US inequality. Relatedly Coibion et al. (2017) find that contractionary monetary policy shocks have historically increased consumption and income inequality.

Finally, this paper relates to the growing literature that focuses on the changing ways in which we work and technology. Acemoglu & Autor (2011) forcefully argue the importance of jointly modelling the interaction of workers skills, tasks, technology and exposure to trade in order to understand the evolution of the income distribution and returns to education. Along similar lines, and the jumping off point for this paper, is the work of Kaplan & Zoch (2020). Like Acemoglu & Autor (2011) they focus on a richer modelling of labours role in production but focus on a broader distinction between expansionary labour and production labour as opposed to a richer modeling of the tasks that go into production. They model expansionary labour as labour devoted to the expansion of the number of product lines available to retailers and demonstrate theoretically that demand for such labour should increase in response to rising markups. They use this identifying assumption to qualitatively and quantitatively identify expansionary labour in the US labour market which they estimate at about 20 percent of overall labour compensation spread broadly over the task, skill and wage distribution. Like this paper, Chu (2020) leverages the insights of Kaplan & Zoch (2020) to generate a positive correlation between the labour share and price markup, and then studies the business cycle implications of monetary policy in a medium scale RANK DSGE model with durable and non-durable consumption.

The next section 2 details a general model around which the analysis in this paper is built. Section 3 examines the labour share in the model in closer detail. Section 4 details a specific model calibration and studies the impulse response of various versions of the model. Section 5 concludes.

2 Model

The analysis is built around a standard medium scale closed economy New Keynesian model like that of Smets & Wouters (2007) or Bayer et al. (2020) in the case of heterogeneous households. All variants[6] of the model include sticky[7] prices, sticky wages, capital and investment

[6]Model parameters are as in table 2 unless otherwise stated.

[7]Wages can be renegotiated and prices re-set subject to convex adjustment costs

adjustment costs. The labour market is organised around a labour union that aggregates labour services and sells them to firms.

The government taxes labour, purchases final output and issues debt subject to fiscal rules that stabilise long run debt. The inflation targeting central bank set the nominal interest rate on government debt subject to a Taylor rule.

Households maximise their lifetime utility subject to idiosyncratic productivity shocks, earn income through labour market participation and capital income from returns to saving in liquid government bonds or an illiquid investment fund which owns the firms.

There are two types of firms in the economy. Wholesale firms operate in a perfectly competitive market and sell their production to retail firms at marginal cost. Retail firms convert wholesale goods into numerous differentiated product lines over which they are monopolists and able to sell each line at a markup over marginal cost.

2.1 Firms

2.1.1 Wholesale firms

A unit mass of wholesale firms j produce under a Cobb Douglas production function by hiring production focused labour services $n_{y,j}$ and renting capital services k_j based on solving the following maximisation problem in each period:

$$\Pi_{w,j} = Max_{y_j, n_{j,y}, k_j} \quad p_w y_j - w_y n_{y,j} - r_k k_j, \quad s.t. \quad y_j = z_{TFP} \left(k_j^{\alpha_y} n_{y,j}^{1-\alpha_y} \right)^{\theta_y} \tag{1}$$

This yields the following first order conditions:

$$w_y = p_w \theta_y (1-\alpha_y) z_{TFP} k_j^{\alpha_y \theta_y} n_{y,j}^{(1-\alpha_y)\theta_y - 1} = p_w \theta_y (1-\alpha_y) \frac{y_j}{n_{y,j}} \tag{2}$$

$$r_k = p_w \theta_y \alpha_y z_{TFP} k_j^{\alpha_y \theta_y - 1} n_{y,j}^{(1-\alpha_y)\theta_y} = p_w \theta_y \alpha_y \frac{y_j}{k_j} \tag{3}$$

In the symmetric equilibrium and assuming perfect competition, the wholesale market price p_w equals marginal cost mc, and all firms hire the same amount of labour and rent the same amount of capital.

2.1.2 Retail firms

The major departure in this model from the literature is that of the retail firms problem. In each period retailers j employ expansionary labour services $n_{e,j}$ and rent capital k_j to manage M_j product lines s. Product lines are created by purchasing and differentiating homogeneous wholesale goods at the wholesale price. The retailers enjoy a monopoly on each product line and set the price p_s subject to convex adjustment costs[8] $\Phi(.)$ and households demand elasticity ϵ_p. Adopting the discount rate of their investors (the investment fund) the retail firms problem has the following form:

$$\Pi_{r,j} = Max_{M_{j,t},n_{e,j,t},k_{j,t},\{p_{s,t},y_{s,t}\}} \quad \sum_{t=\tau}^{t=\infty} \frac{1+r_{a,\tau}}{p_\tau \prod_{z=\tau}^{z=t}(1+r_{a,z})}[\int_0^{M_{j,t}} (p_{s,t} - p_{w,t})y_{s,t} - \Phi(p_{s,t}, p_{s,t-1})ds$$
$$-w_{e,t}n_{e,j,t} - r_{k,t}k_{j,t}]$$

$$\text{s.t.} \qquad M_{j,t} = z_{TFP,t}\left(k_{j,t}^{\alpha_e} n_{e,j,t}^{1-\alpha_e}\right)^{\theta_e}, \qquad y_{s,t} = \left(\frac{p_{s,t}}{P_t}\right)^{-\epsilon_p} Y_t \tag{4}$$

This yields the following first order conditions[9]:

$$w_e = \Pi_{M_j}\theta_e(1-\alpha_e)z_{TFP}k_j^{\alpha_e\theta_e}n_{e,j}^{(1-\alpha_e)\theta_e-1} = \Pi_{M_j}\theta_e(1-\alpha_e)\frac{M_j}{n_{e,j}} \tag{5}$$

$$r_k = \Pi_{M_j}\theta_e\alpha_e z_{TFP}k_j^{\alpha_e\theta_e-1}n_{e,j}^{(1-\alpha_e)\theta_e} = \Pi_{M_j}\theta_e\alpha_e\frac{M_j}{k_j} \tag{6}$$

$$0 = \frac{y_{s,t}}{P_t} - \phi\frac{1}{p_{s,t-1}}(\frac{p_{s,t}}{p_{s,t-1}} - 1)Y_t - \epsilon_p\frac{\Pi_{M_s}}{P_t y_{s,t}}\frac{p_{s,t}^{-\epsilon_p-1}}{P_t^{-\epsilon_p}}Y_t + \phi\mathbf{E_t}\left[\frac{1}{1+r_{a,t+1}}\frac{p_{s,t+1}}{p_{s,t}^2}(\frac{p_{s,t+1}}{p_{s,t}} - 1)Y_{t+1}\right] \tag{7}$$

where $\Pi_s = y_s(p_s - p_w)$ are the profits from product line s, $\{Y_t, P_t\}$ is total economy output and prices, and the functional form $\Phi_t = \frac{\phi}{2}(\frac{p_{s,t}}{p_{s_t-1}} - 1)^2 Y_t$ has been adopted for the price adjustment costs. In words the retail firms hire labour and rent capital up until the point the marginal profit from an extra product line equals the marginal cost of an extra input unit. Prices are adjusted to hit the firms target markup $\mu_p = \frac{\epsilon_p}{\epsilon_p - 1}$ taking into account the present and future state of the economy. In the symmetric equilibrium where all firms choose the same price, and

[8] For new products retailers assume the average economy wide price P_{t-1} as a reference point for $p_{s,t}$.
[9] Dropping the t subscript where appropriate.

dropping higher order terms, equation 7 reduces to the familiar Philips curve around a zero inflation steady state:

$$\pi_t = \frac{1}{1 + r_{a,ss}}\mathbf{E}[\pi_{t+1}] + \frac{\epsilon_p - 1}{\phi}\hat{mc} + z_\pi \tag{8}$$

where π is retail price inflation, $\hat{mc}$ is the log deviation of marginal cost from steady state, $r_{a,ss}$ is the steady state investment fund return and z_π is a markup shock[10] the log of which follows an AR1 process.

2.2 Investment fund

As in Kaplan et al. (2018) households can pay into an investment funds which owns and rents out the capital stock K as well as the shares of all firms X, and receives rental payments r_k and dividends Π_d. The investment fund objectives is to invest in capital and shares to maximize it's value subject to rate of return[11] r_a, capital depreciation rate δ and investment adjustment costs $\Psi(.)$:

$$\begin{aligned}
&A_\tau = Max_{I_t,K_t,X_t} \textstyle\sum_{t=\tau}^{\infty} \frac{z_{rp,\tau}(1+r_{a,\tau})}{\prod_{j=\tau}^{j=t} z_{rp,t}(1+r_{a,j})} \left(r_{k,t}K_{t-1} + \Pi_{d,t}X_{t-1} + q_{s,t}(X_{t-1} - X_t) - I_t - \Psi(I_t)\right) \\
&\text{s.t.} \quad K_t = (1-\delta)K_{t-1} + I_t z_{I,t} \quad , ln(z_{I,t}) = \rho_I ln(z_{I,t-1}) + \epsilon_{I,t}, \\
&\qquad ln(z_{rp,t}) = \rho_{rp} ln(z_{rp,t-1}) + \epsilon_{rp,t}
\end{aligned} \tag{9}$$

Equation 9 tells us that the funds value is the present discounted value of future dividends minus the costs of investments in capital and stocks at stock price q_s. Assuming a functional form of $\Psi(I_t) = \frac{\Delta}{2} log(\frac{I_t}{I_{t-1}})^2 I_t$ and denoting the shadow value of capital q_k, yields the following first order conditions for the investment funds problem:

$$q_{s,t} = \mathbf{E}\left[\frac{q_{s,t+1} + \Pi_{d,t+1}}{z_{rp,t+1}(1 + r_{a,t+1})}\right] \tag{10}$$

$$q_{k,t}z_{I,t} = 1 + \Delta log(\frac{I_t}{I_{t-1}}) + \frac{\Delta}{2} log(\frac{I_t}{I_{t-1}})^2 - \mathbf{E}\left[\frac{\Delta}{z_{rp,t+1}(1 + r_{a,t+1})}\frac{I_{t+1}}{I_t} log(\frac{I_{t+1}}{I_t})\right] \tag{11}$$

[10] To allow for parameter estimation and IRF analysis later, the model is augmented with seven exogenous shock processes corresponding to TFP, investment specific technology (IST), government spending, monetary policy, risk premia, price markups and wage markups.

[11] In the final section I include a stochastic AR1 risk premia shock z_{rp}.

$$q_{k,t} = \mathbf{E}\left[\frac{r_{k,t+1} + (1-\delta)q_{k,t+1}}{z_{rp,t+1}(1+r_{a,t+1})}\right] \tag{12}$$

The investment fund therefore invests in capital and stocks in each period up until the expected returns are equalised. The value of the investment fund in equilibrium in each period is given by $A = q_s X + q_k K$, where X is the total amount of shares issued[12], and the investment fund follows the law of motion $A_t = (1 + r_{a,t})A_{t-1} + d_t$ where d is total inflows into the fund from households.

2.3 Government

The government is composed of a fiscal authority and central bank. The government purchases final output funded through taxation and borrowing according to the following budget constraint.

$$B_t = \frac{1 + r_{b,t-1}}{1 + \pi_t} B_{t-1} + G_t - T_t \tag{13}$$

where tax revenues are funded by a proportional tax τ_t on labour income. Government spending is modeled as $G_t = G_{ss} + z_{g,t}$ where G_{ss} is a constant percent of steady state output Y_{ss} and z_g is a stationary AR1 processes. In order to accommodate this policy it adjusts the tax rate according to the following rule:

$$\frac{\tau_t}{\tau_{ss}} = \left(\frac{\tau_{t-1}}{\tau_{ss}}\right)^{\rho_\tau} \left(\frac{B_t}{B_{ss}}\right)^{\gamma_{\tau_B}(1-\rho_\tau)} \left(\frac{Y_t}{Y^*}\right)^{\gamma_{\tau_y}(1-\rho_\tau)} \tag{14}$$

which ensures long run debt stability but allows the deficit to adjust in the short run in response to the output gap[13]. The central bank smoothly adjusts the nominal interest rate on government bonds r_b to hit it's inflation target according to a Taylor rule that also takes into account the output gap.

$$r_{b,t} = \rho_r r_{b,t-1} + (1-\rho_r)(r_b^* + \gamma_\pi(\pi_t - \pi^*) + \gamma_y log(\frac{Y_t}{Y^*})) + \epsilon_{r,t} \tag{15}$$

[12] I normalise X equal to 1.

[13] The output gap is defined in this model as the difference between period output Y_t and what output would be under flexible prices and wages Y^*.

2.4 Household

2.4.1 Portfolio choice

The household problem closely follows that of Bayer et al. (2019). Households seek to maximise their lifetime utility through consumption c[14] and the compliment of work (leisure) h^c. Households can self-insure themselves against idiosyncratic income risk by saving in liquid government bonds b or the illiquid investment fund a. Illiquidity is captured in a Calvo like fashion by assuming that households can only adjust their holdings in the investment fund with some probability ω in each period.

The household problem can be summarised by two Bellman equations that depend on a GHH preference based utility function (Greenwood et al. (1988)), idiosyncratic states (a, b, z, s) and the aggregate state of the economy λ:

$$V_{adj}(b,a,z,s,\lambda) = Max_{a',b',c,h} \ \frac{1}{1-\sigma}(c - \frac{\kappa}{1+\psi}h^{1+\psi})^{1-\sigma} + \beta[(\omega V_{adj}(b',a',h',s',\lambda')+ \\ +(1-\omega)V_{nadj}(b',a',z',s',\lambda')] \tag{16}$$

$$V_{nadj}(b,a,z,s,\lambda) = Max_{b',c,h} \ \frac{1}{1-\sigma}(c - \frac{\kappa}{1+\psi}h^{1+\psi})^{1-\sigma} + \beta[\omega V_{adj}(b',a,z',s',\lambda')+ \\ +(1-\omega)V_{nadj}(b',a,z',s',\lambda')] \tag{17}$$

Subject to the constraints:

$a' + b' + c = (1-\tau)w_s hz + a(1+r_a) + \frac{1+r_b+\mathbf{1_{b<0}}\bar{r}}{1+\pi}b, \quad b \geq -\bar{B}, \quad a \geq 0$

Households can only borrow in the liquid funds market up to an amount $\bar{B}$ and pay a penalty rate[15] above the risk free rate of $\bar{r}$ when they do.

Labour income is composed of the aggregate sector wage w_s, aggregate hours h and the persons individual productivity z. Individual productivity is subject to a stochastic AR(1) process such that:

$$ln(z') = \rho_z ln(z) + \epsilon_z, \quad \epsilon_z \sim N(0, \sigma_z^2) \tag{18}$$

[14]Here c is the composite of over the measure of product lines such that $c = \left(\int_0^M c_j^{\frac{\epsilon_p - 1}{\epsilon_p}} dj\right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$ from which we can derive the demand for each product line as $c_s = \left(\frac{p_s}{P}\right)^{-\epsilon_p} C$.

[15]The revenue from the higher borrowing rate is assumed to be lost through intermediation costs and therefore does not enter the government budget constraint.

There is also a rare superstar state $\bar{z}$ (e.g. Castaneda et al. (2003)) that household transition to with low probability. This state captures households in the top 1 percent of the income distribution and helps deliver realistic wealth and income inequality.

The household problem can be boiled down to and solved[16] using the following Euler equations for the non-borrowing constrained households that describe the trade off between consumption and leisure today versus investment in the illiquid fund or liquid bonds in the adjustment and non-adjustment state respectively:

$$(x_{adj}(a,b,z,s,\lambda))^{-\sigma} = \beta\, \mathbf{E}\left[\omega(1+r'_a)\,(x_{adj}(a',b',z',s',\lambda'))^{-\sigma} + (1-\omega)\frac{dV_{nadj}(a',b',z',s',\lambda')}{da}\right] \tag{19}$$

$$(x_{adj}(a,b,z,s,\lambda))^{-\sigma} = \beta\, \mathbf{E}\left[\omega\frac{1+r'_b}{1+\pi'}\,(x_{adj}(a',b',z',s',\lambda'))^{-\sigma} + (1-\omega)\frac{1+r'_b}{1+\pi'}\,(x_{nadj}(a',b',z',s',\lambda'))^{-\sigma}\right] \tag{20}$$

$$(x_{nadj}(a,b,z,s,\lambda))^{-\sigma} = \beta\, \mathbf{E}\left[\omega\frac{1+r'_b}{1+\pi'}\,(x_{adj}(a,b',z',s',\lambda'))^{-\sigma} + (1-\omega)\frac{1+r'_b}{1+\pi'}\,(x_{nadj}(a,b',z',s',\lambda'))^{-\sigma}\right] \tag{21}$$

where $x_{adj/nadj}$ is the household choice of $c - \frac{\kappa}{1+\psi}h^{1+\psi}$ in the adjustment or non-adjustment case.

2.4.2 Aggregate wages and labour supply

Households rely on labour unions to negotiate hours/wages on their behalf as in Schmitt-Grohé & Uribe (2005). There is a continuum of labour unions, each with a monopoly over the labour services it sells to firms in each sector (expansionary or production). Labour unions negotiate on the basis of their members average utility subject to convex adjustments costs. Demand for the labour unions services i in sector s at time t is given by:

$$h_{i,s,t} = \left(\frac{w_{i,s,t}}{w_{s,t}}\right)^{-\epsilon_W} \tag{22}$$

In the symmetric equilibrium the labour unions optimisation problem simplifies after lin-

[16] See appendix A2.

earisation to the following and standard wage Phillips curve in each sector:

$$\pi_{w,s,t} = \beta \mathbf{E_t}[\pi_{w,s,t+1}] - \frac{(1-\tau_t)(\epsilon_w - 1)}{\phi_w}\hat{\mu_w} + z_{\pi_w} \tag{23}$$

where under GHH preferences, $\hat{\mu_w} = \hat{w}_t - \hat{p}_t - \psi \hat{h}_t$ is the log deviation from steady state of households marginal rate of substitution between consumption and leisure, and z_{π_w} is a shock to the wage markup with the log modeled as an AR1 process.

2.5 Equilibrium

Equilibrium in the model above is characterised by a set of value functions $\{V_{adj}, V_{nadj}\}_t$, household policy functions $\{x_{adj}, x_{nadj}, b'_{adj}, b'_{nadj}, a'_{adj}\}_t$, quantities $\{N_y, N_e, M, y, K, B, A, G\}_t$, set of prices $\{w_y, w_e, \pi, \pi_{w,e}, \pi_{w,y}, r_b, r_a, q_k, q_s, r_k, \tau_t\}_t$, aggregate stochastic states $\{z_{TFP}, z_I, z_g, z_\pi, z_{\pi_w}, z_{rp}\}_t$ and an aggregate distribution over (a, b, z, s) $\{\chi\}_t$ such that:

1. The household policy functions solve[17] the household planning problem (eq 16 and 17) given period t prices and expected $t+1$ prices.

2. Firms profit maximise such that equations 2, 3, 5, 6 and 8 hold in each period.

3. The investment fund maximises it's value in accordance with equations 10 - 12.

4. Unions negotiate wages such that the wage Philips equations hold (eq. 22).

5. The market for government bonds B and the investment fund clears in each period. i.e. $A_t = q_{s,t} + q_{k,t}K_t = \int_i a'_t di$ and $B_t = \int_i b'_t di$ and the aggregate distribution χ_t evolves according to the household policy functions and the stochastic process z.

6. The government budget constraint holds (eq. 13) and government follows it's fiscal and monetary rules (eq. 14 and 15).

7. Aggregate stochastic processes follow stationary AR1 processes.

In the subsequent sections we shall study the dynamics of the generalised model laid out above in three principle settings:

1. **RANK** In the RANK equilibrium households fully insure each other such that consumption and labour is the same between households. Free movement of labour and capital between sectors equates $w_y = w_e$. Therefore this models boils down to the textbook

[17] See appendix A2 for more detail.

medium scale closed economy New Keynesian model with the exception of role of expansionary labour in the retail sector.

2. **WCNK** In this setting households are one of two types, workers or capitalists as in e.g. Broer et al. (2020). Capitalists do not work but instead derive income from capital rental income and monopolistic profits. For tractability as in Ravn & Sterk (2021) I assume $\bar{B} = 0$ and net zero issuance of government bonds B in each period such that the wealth distribution in the economy is degenerate. This means that the interest rate on government bond's adjusts such that the highest productivity households $\bar{z}$ consumes all of their income and by extension the lower productivity households $z < \bar{z}$ do so as well due to the borrowing constraint. Capitalists, who are out of the labour market and therefore not subject to idiosyncratic labour income risk opt out of the government bond market as interest rates are too low. They instead consume and invest in capital out of their capital income. Finally, free movement of labour and capital between sectors again equates $w_y = w_e$.

3. **HANK** In this setting households must self insure against idiosyncratic income risk using liquid and illiquid assets producing a rich non-degenerate wealth distribution as in Bayer et al. (2020). I also consider the case where workers idiosyncratic state include their sector y/n and drop free movement of workers between sectors.

3 The labour share and demand shocks

The labour share in the symmetric equilibrium of the model is defined as $\frac{w_y N_y M + w_e N_e}{pY}$. Where in the equilibrium M product lines are sold by all retailers such that total production labour services demand is MN_y and total output is $Y = My$. Substituting for the first order conditions for w_e, w_y yields the following expression for the labour share:

$$s_l = \frac{p_w \frac{dy}{dN_y} N_y M + y(p - p_w)\frac{dM}{dN_e} N_e}{pMy} = \frac{1}{\mu_p}\xi_{y,N_y} + (1 - \frac{1}{\mu_p})\xi_{M,N_e} \tag{24}$$

where we've also made used of the fact that the retail markup is $\mu_p = \frac{p}{p_w}$. Equation 24 reveals the labour share in the model as an inverse markup weighted average of the elasticity of labour in each of the sectors. For the particular Cobb Douglas framework of section 2 and using equations 2 and 5 this can be rewritten as:

$$s_l = \frac{1}{\mu_p}\theta_y(1-\alpha_y) + (1-\frac{1}{\mu_p})\theta_e(1-\alpha_e) \tag{25}$$

In the textbook New Keynesian model with no overhead labour the labour share is simply the left hand side term. The right hand side term accounts for the fact that labour now contributes by expanding the measure of goods M on offer. Therefore for a given markup the labour share is strictly larger in the model with expansionary labour which provides a useful break to the relationship between markups and the overall labour share because markups now need to cover the cost of expansionary labour. Now consider the derivative of the labour share with respect to the markup:

$$\frac{ds_l}{d\mu} = -\frac{1}{\mu_p^2}\theta_y(1-\alpha_y) + \frac{1}{\mu_p^2}\theta_e(1-\alpha_e) \tag{26}$$

This shows the direction in which the labour share moves in response to a change in the markup is pinned down by the relative returns to scale of labour in the two sectors. Therefore in the framework of section 2 it suffices to set $\theta_e(1-\alpha_e) > \theta_y(1-\alpha_y)$ to create a positive correlation between movements in the markup and the labour share, or negative[18] correlation between the output gap and the labour share. The intuition for this is rising markups cause a substitution away from production labour and towards expansionary labour. This put more weight on expansionary labour in the overall labour share calculation, and if returns to scale are higher for expansionary labour, then this will raise the entire economy labour share because the labour share is a weighted average of returns to scale over the two sectors.

Turning to the response of productivity. The textbook New Keynesian model with decreasing returns to scale in labour produces a rise in productivity in response to negative demand shocks[19]. In the model with expansionary labour this need no longer be the case. The reason for this is the rise in productivity from the decline in employment in the production sector is now offset by a rise in employment in the expansionary sector. This rise in employment in the expansionary sector reduces productivity in that sector and the overall fall in employment. As we see in figures 2 - 4 this offsetting effect dominates creating a data consistent positive correlation between productivity and the output gap.

The important distinction in terms of the model is that some workers contribute to expand-

[18]In the New Keynesian model pricing frictions prevent firms from adjusting prices immediately to their target markup. For example in the face of a negative demand shock, production costs fall due to a decline in demand for the factors of production. This causes the markup to rise as prices do not fall one for one with marginal costs.

[19]Consider for example the derivative with respect to N of productivity in Cobb Douglas production framework with $\frac{d}{dN}\left(\frac{Y}{N}\right) = -\alpha K^{\alpha} N^{-(1+\alpha)} < 0$.

ing the measures of goods on offer where as others produce those goods. As already discussed there are numerous categories one might think of as falling under the header of expansionary such as R&D or marketing. It could also be reasonably argued that the share of labour in these activities is higher than that of production activities which will be more reliant on capital to assist with routine tasks.

One could consider the division of expansionary and production labour along a task based instead of sector or role based dimension. For example individual workers might divide their time between product development and production of existing goods and services. In the face of a negative demand shock, where demand for existing product lines falls it would seem optimal to devote more time to expansionary activities. However a task based version of the above framework while still supporting this optimal shift to expansionary activities would not deliver the desired counter-cyclical movement in the labour share. One can verify[20] that in a one sector model, where production and expansionary activities occur within the same sector, firms will internalise the fact that expansionary activity leads to cost pressures on new and preexisting production activities. Firms will therefore allocate labour in such a way that the benefits of the rise in markups continue to result in a higher capital share. Therefore key to expansionary labour providing a mechanism for a counter-cyclical labour share are frictions that prevents firms from internalising this trade off.

4 Impulse response to a monetary policy shock

In this section we consider the response of the economy to a demand shock delivered via a monetary[21] policy shock and focus on movement of the labour share and dampening/amplification of major economy aggregates in response to the shock in the model with expansionary labour (NK-YN model) in the retail sector relative to one without a role for expansionary labour (NK model).

4.1 Calibration and computation

The impulse response function to the monetary shock is computed at a quarterly frequency using the perturbation method of Schmitt-Grohé & Uribe (2004) around the models deterministic

[20] See the appendix of Kaplan & Zoch (2020).

[21] We shall see the analysis in the model is robust to other types of demand shocks such as government spending shocks or risk premium shocks produce similar results.

steady state up to first order. This requires expressing the model in SGU[22] form:

$$\mathbf{E}\left[F(X_{ss}, X'(X_{ss}), Y(X_{ss}), Y'(X'))\right] = 0 \tag{27}$$

$$\mathbf{E}\left[F_{X_{|X=X_{SS}}}(X, X'(X), Y(X), Y'(X'))\right] = 0 \tag{28}$$

and solving for the policy functions $Y(X)$ such that the above holds. Here X are the models state variables and Y the control variables. In the RANK and WCNK case this is a relatively undemanding and well understood solution method. In the HANK case the dimensionality of the problem quickly become infeasible as the state vector X includes the entire wealth distribution.

To simplify the problem I follow the procedures of Bayer & Luetticke (2020) who demonstrate the dynamics of these high dimension heterogeneous agent models can be well approximated under significant dimensionality reduction. They solve for the steady state (eq. 27) using a rich discretisation of the household state space (a, b, z) before reducing the dimensions of the problem when solving for the dynamics (eq. 28). When solving for the dynamics the problem is reduced by assuming households take into account only the marginal distributions of household states $\{b, a, z\}_t$ as opposed to the full joint distribution. Solving for the household problem is further simplified by approximating the household value functions solved for in the steady state using a discrete cosine transformation (DCT). The DCT coefficients then become control variables in the SGU form and I only perturb the most important[23] coefficients when solving for the models dynamics. Finally as the state vector has been reduced to the marginal distributions, a mapping (fixed copula assumption) is assumed in each period between the marginal and full joint distributions based on the mapping in steady state. For the steady state, a grid of 60 nodes for illiquid assets a, 60 nodes of liquid assets b and 15 nodes for the productivity process z are used. The household policy functions are solved using an endogenous grid point method on the households first order conditions (eq. 19 - 21) at each of the 60*60*15 nodes (see appendix A2 for more details).

The key model parameters are detailed in table 2. The household parameters are largely taken from the recent literature. The superstar state is calibrated to reflect the top 1 percent of labour income earners. The probability of exiting the state is taken from Guvenen et al. (2014) at 6.5 percent and the level of $\bar{z}$ is set such that the top 1 percent take home 12 percent of

[22] See appendix A2 for more detail.

[23] Importance as defined by the minimum number of coefficients needed to retain 99.9 percent of the household policy functions information.

total labour income consistent with data from IRS. Overall the household calibration delivers a capital to output ratio of 11.5 and debt to output ratio of 1.67 as in Bayer et al. (2020).

The labour share and profit share are kept consistent across all models. In the version of the model (NK) without the expansionary sector this is achieved by calibrating the demand elasticity $\epsilon_p = 20$ and the capital elasticity $\alpha_y = 0.35$. For the model with expansionary labour (NK-YN) there are 5 unknowns $(\theta_y, \theta_e, \alpha_y, \alpha_e, \epsilon_p)$ that are calibrated to achieve the profit share, overall labour share and relative labour shares in each sector, see table 1. The share of type e workers is calibrated at 20 percent in line with the empirical findings of Kaplan & Zoch (2020). I assume consistent with the NK model constant returns to production such that $\theta_y = 1$ and I further assume a zero[24] capital share α_e. Note that in the NK-YN model under this calibration the equation for the profit share $\left[s_\Pi = (1 - \frac{1}{\mu_p})(1 - \theta_e(1 - \alpha_e))\right]$ leads to higher steady state markups in that version of the model ($\epsilon_p = 5.75$). The overall calibration leads to a higher returns to scale in labour in the expansionary sector relative to the production sector. In the WCNK model I assume a 10 percent measure of capitalists which means the top 10 percent receive 38 percent of total income in steady state, lower but not far from what the data would suggest at 46 percent.

The pricing and wage frictions adjustment cost parameters ϕ are calibrated in line with average price and wage resetting occurring every 4 quarters by exploiting the linear equivalence with Calvo adjustment frictions as mapped in Born & Pfeifer (2020). This is relatively neutral assumption around the relative stickiness of prices and wages and produces Philips curve coefficients (κ) in line with those estimated in the literature. In the HANK environment I take the investment adjustment cost $\Delta = 0.23$ as estimated in Bayer et al. (2020). In the RANK environment, which does not contain the portfolio adjustment frictions, I scale up the investment adjustment cost parameter in order to obtain the same fall in investment on impact between the NK-RANK and NK-HANK model.

The central bank reacts to inflation and the output gap in line with the parameter estimates from Smets & Wouters (2007) based on the period 1984-2004. The governments tax adjustment parameters are taken from the estimates of Bayer et al. (2020). This calibration results in temporary tax cuts in response to demand shocks before being unwound and raised to bring the debt to GDP ratio back to target.

[24] This is really more of a normalisation as what matters is the relative returns to scale in labour $\theta_e(1 - \alpha_e)$ and $\theta_y(1 - \alpha_y)$ which combine the capital share and overall returns to scale in the sectors.

Table 1: Factor shares

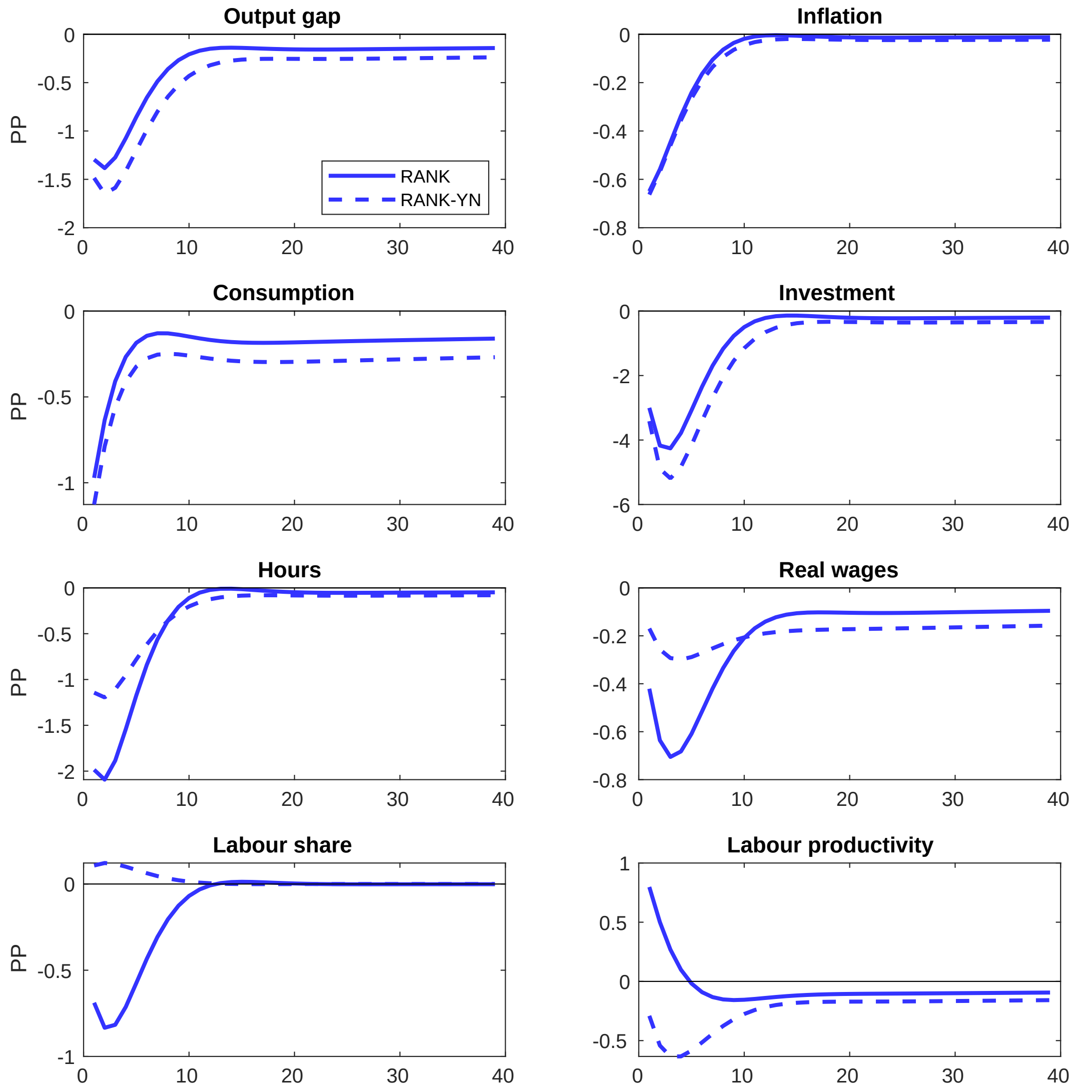
Target	Value	Calibration parameter
Labour share s_y	62%	α_y
Profit share s_Π	5%	ϵ_p
Share of type e workers s_e	20%	$\frac{\theta_y(1-\alpha_y)}{\theta_e(1-\alpha_e)}$

Note: Above calibration set $\theta_e = 0.71$, $\theta_y = 1$, $\alpha_e = 0$ & $\alpha_y = 0.4$.

Table 2: Model parameters

Parameter	Value	Calibration
Household		
CRRA σ	4	Kaplan et al. (2018)
Inverse Frisch elasticity ψ	2	Chetty et al. (2011)
Interest rate r^*	2.5%	Bayer & Luetticke (2020)
Labour income persistence ρ_z	0.98	Storesletten et al. (2004)
Labour income std σ_z	0.12	Storesletten et al. (2004)
Prob of exiting top 1 pct	6.5%	Guvenen et al. (2014)
Top 1 pct labour income share	12%	IRS
Portfolio adjustment prob. ω	0.065	Bayer et al. (2020)
Borrowing premium $\bar{R}$	1.65%	Bayer et al. (2020)
Borrowing limit $\bar{B}$		$\frac{1}{3}$ average labour income
Firm		
Depreciation δ	1.75%	
Investment adj. costs Δ	$1.8^{RANK}/0.23^{HANK}$	Bayer et al. (2020)
Wage Philips curve κ_w	0.09	Wage adjustment every 4 quarters
Price Philips curve κ_p	0.09	Price adjustment every 4 quarters
Government		
Purchases $\frac{G}{Y}$	0.18	NIPA
Debt $\frac{B}{Y}$	1.6	NIPA
Tax persistence ρ_τ	0.55	Bayer et al. (2020)
Debt reaction γ_{τ_b}	0.78	Bayer et al. (2020)
Output reaction γ_{τ_y}	2.65	Bayer et al. (2020)
CB inflation reaction γ_π	1.8	Smets & Wouters (2007)
CB Output reaction γ_y	0.1	Smets & Wouters (2007)
CB inflation target π^*	0	Price stability
Interest rate smoothing ρ_{r_b}	0.8	Smets & Wouters (2007)

Figure 2: RANK - IRF to a monetary policy shock.



Note: Figure shows response of selected aggregate variables to a 100 basis point monetary policy shock over an initial 40 quarters. Variables are expressed as percentage point deviations from steady state. RANK refers to the response of an economy with only production labour and RANK-YN is the response of a similarly calibrated economy with the inclusion of expansionary labour as described in section 2.

4.2 Results

Let us now examine the aggregate response of the economy to a monetary policy shock in the RANK, WCNK and HANK setting.

4.2.1 RANK

Figure 2 shows the response to a 100 basis point monetary policy shock in the more textbook RANK model (solid line) and the RANK-YN model augmented with the expansionary labour sector (dashed line). Through the Euler equation the hike in interest rates in both models increases demand for saving and reduces consumption demand causing the usual aggregate response of labour and wages. The responses of the major components of GDP and consumer prices are qualitatively and quantitatively similar although the RANK-YN model has a mildly amplified response. Noticeably different is the response of the labour market variables which are significantly dampened in the RANK-YN model due to the substitution between production and expansionary labour. This leads to the key observable differences between the models on row 4 whereby we see a more empirically consistent rise in the labour share labour share and fall in productivity in the RANK-YN model.

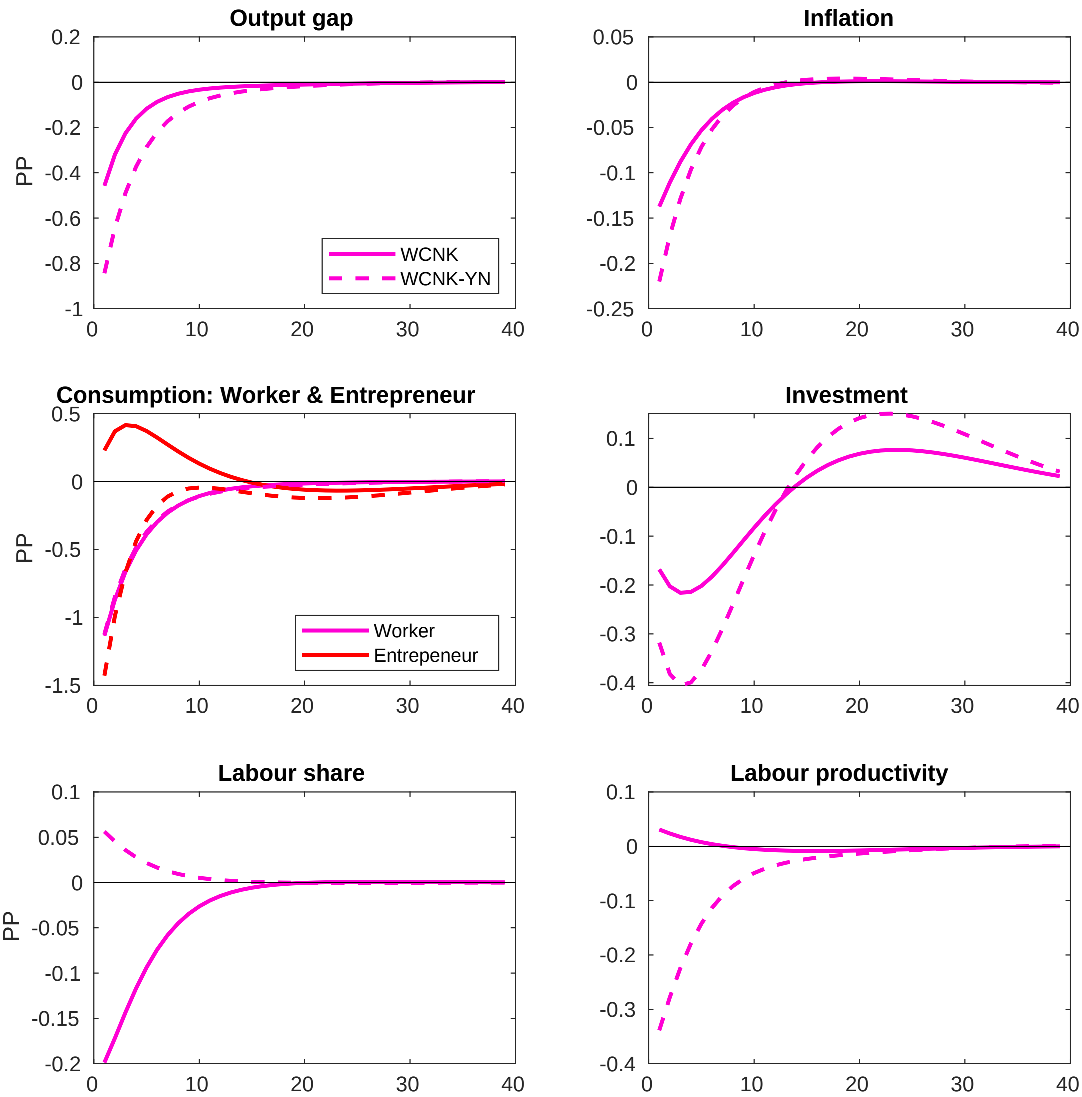
4.2.2 WCNK

To gain greater insight into the transmission mechanism figure 3 plots the response of the worker capitalist economy to the same 100 basis point monetary policy shock. Like in the RANK model the shock is deflationary and reduces aggregate investment and consumption. Like the RANK-YN model, we see the WCNK-YN model (dashed lines) delivering a rise in the labour share and fall in productivity. Of particular interest is the consumption response broken out on the LHS of row 2. Here we see an important implication of the movement in factor shares between the two versions of the model. In both versions of the model the workers consumption (pink lines) fall by almost the same magnitude and drive the aggregate response of the economy. In the WCNK model the entrepreneurs consumption (solid red line) increases in response to the monetary policy shock as the rise in markup's actually increases their incomes. However in the WCNK-YN model the increase in markups is offset by cost of increased labour demand in the expansionary sector such that the consumption of entrepreneurs (dashed red line) falls to a greather extent than that of the workers. As a consequence we get an amplification of the monetary shock as now, unlike the textbook model, all parties lose out from the contraction in demand.

4.2.3 HANK

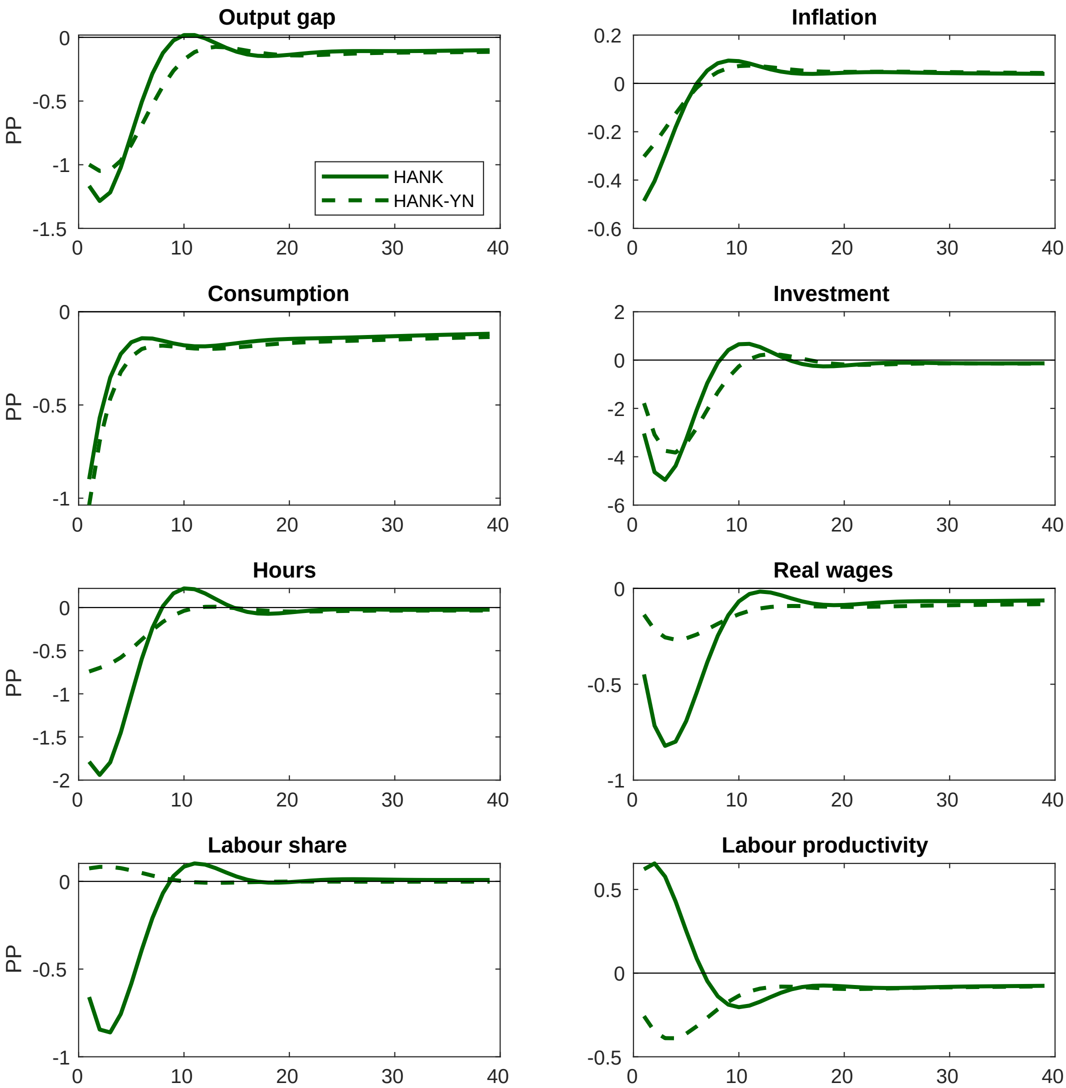
We now drop the stronger assumptions of complete markets, entrepreneurs and zero liquidity by turning to the response of the HANK model in figure 4 which for now maintains a flexible labour market such that $w_e = w_y$. Not inconsistent with the literature the aggregate response of consumption and the output gap is similar between the RANK and HANK versions of the model. But we now have some noticeable differences emerging between the HANK and HANK-YN model. Like in the RANK or WCNK model consumption falls more in the model with expansionary labour as richer capital holding households are less insured by the the rise in

Figure 3: WCNK- IRF to a monetary policy shock.



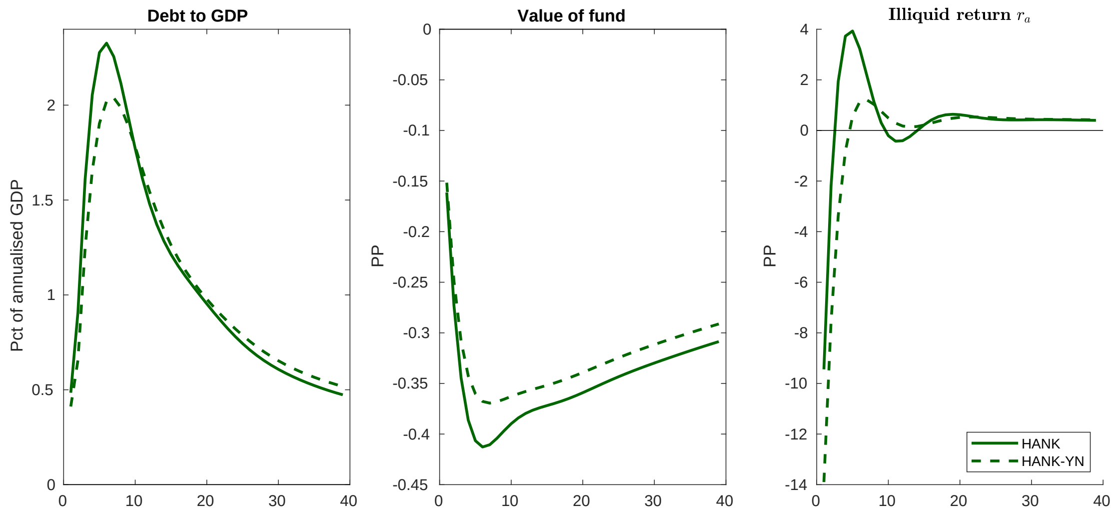
Note: Figure shows response of selected aggregate variables to a 100 basis point monetary policy shock over an initial 40 quarters. Variables are expressed as percentage point deviations from steady state. WCNK refers to the response of an economy with only production labour and WCNK-YN is the response of a similarly calibrated economy with the inclusion of expansionary labour as described in section 2. In the Consumption: Worker & Entrepreneur panel the entrepreneur response is coloured in red. The response of workers consumption in both variants of the model is indistinguishable.

Figure 4: HANK - IRF to a monetary policy shock.



Note: Figure shows response of selected aggregate variables to a 100 basis point monetary policy shock over an initial 40 quarters. Variables are expressed as percentage point deviations from steady state. HANK refers to the response of an economy with only production labour and HANK-YN is the response of a similarly calibrated economy with the inclusion of expansionary labour as described in section 2.

Figure 5: Saving after a monetary policy shock.



Note: Figure shows response of selected aggregate variables to a 100 basis point monetary policy shock over an initial 40 quarters. Debt to GDP is the change in the level of government debt divided by output (annualised). Value of the fund refers to the percentage change in the overall value of the investment fund (eq 9). HANK refers to the response of an economy with only production labour and HANK-YN is the response of a similarly calibrated economy with the inclusion of expansionary labour as described in section 2.

markups induced by the shock. However in terms of overall demand, the HANK model, on impact, now experiences a larger fall in output than the HANK-YN model owing to a slightly larger fall in investment.

To think this through consider the responses plotted in figure 5. Due to the larger fall in labour demand in the HANK model, government deficits and debt rises by more than the HANK-YN model, as tax revenues fall to a greater extent. At the same time the illiquid return falls by less in the HANK model, and despite the larger fall in investment, the value of the investment funds fall by similar amounts on impact in both models due to the relatively higher stock price in the HANK model. Putting this all together we can deduce that household consumption and total household saving (bonds and fund) fall by less in the HANK model but the composition of saving is such that real investment initially falls more in the HANK model than HANK-YN model. As the illiquid return recovers in the HANK model and actually turns positive after a few periods the differences in overall demand reverse as real investment recovers more quickly in the HANK model. Finally smaller deficits that require a lower real rate to clear the bond market and higher real wages combine to dampen the deflationary impact of the monetary policy shock in the HANK-YN model relative to the HANK model.

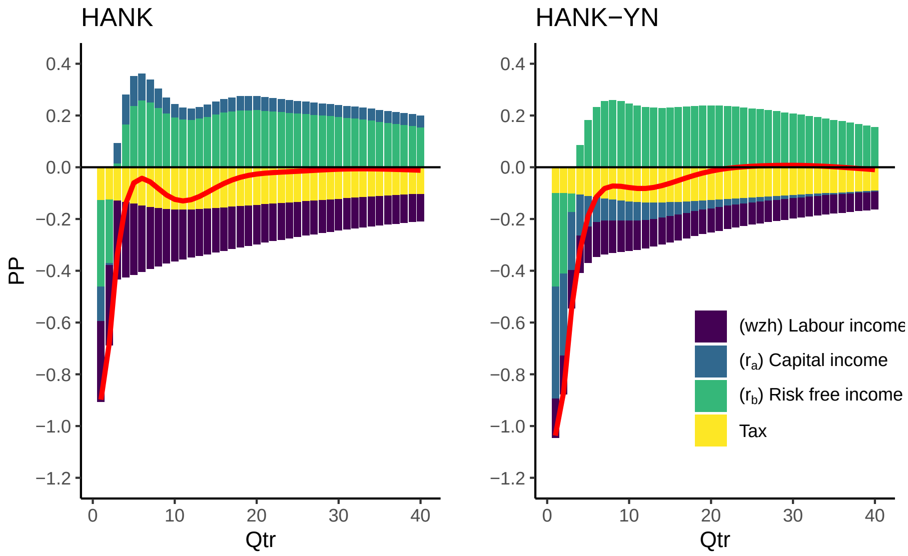
4.2.4 Household consumption response

As demonstrated in Kaplan et al. (2018) while the aggregate response of economic variables may be qualitatively or quantitatively similar between RANK and HANK models the underlying transmissions channels differ considerably. In RANK models the response of consumption is almost completely explained by the change in the policy rate r_b where as in the HANK model the policy rate plays a much smaller role in the direct response with the general equilibrium effects of wages, fiscal policy and the illiquid return playing a much larger role. Figure 6 conducts a similar decomposition to that of Kaplan et al. (2018) by resolving the household policy functions and distributional dynamics after feeding the household relevant equilibrium price and quantity levels from figure 4 one at a time into the dynamic household decision making problem while holding the other relevant prices and quantities fixed.

The LHS and RHS of figure 6 shows the partial equilibrium effect of the liquid return r_b (adjusted for inflation) explaining only around a third of the initial fall in consumption before becoming a positive contributor once the real rate returns to near it's steady state value and households rundown their extra savings. In the LHS (HANK model), the rest of the aggregate response is largely explained by falling real incomes (hours and wages), anticipated higher taxes dragging on consumption and the dynamics of the illiquid return r_a. Comparing this response to the HANK-YN model augmented with expansionary labour on the RHS we see two key differences. The first is that the illiquid return is now a pure drag on consumption in each period and explains a significant (almost half) of the initial fall in consumption. The difference in the contribution of the illiquid return reflects the difference in the dynamic response of the return shown on the RHS of figure 5, where in the HANK-YN model rising markups are offset by higher expansionary labour demand. This offsetting labour demand also diminishes the role of labour income in the overall consumption response.

In the WCNK models we observed (figure 3) that the sign of the consumption response of workers and entrepreneurs who owned the economies capital differed depending on how the monetary policy shock affected the distribution of income between capital and labour. In the HANK model we can conduct a similar exercise by comparing consumption responses across the distribution of illiquid wealth. This is what is shown in figure 7 which plots the consumption response of households across the distribution of illiquid holdings. In both models the bottom half of the distribution response largely resembles the dynamics of labour income. The 80th percentile resembles something of a representative consumer tracking the aggregate consumption response. At the top of the wealth distribution where more than half of total wealth

Figure 6: Decomposing the consumption response

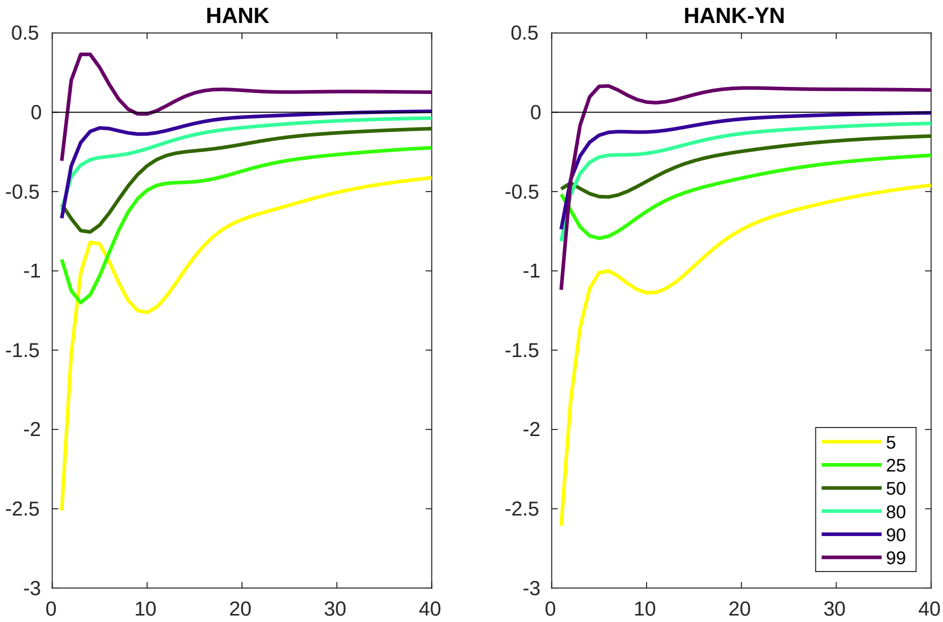


Note: Figure decomposes the aggregate response of consumption to a 100 basis point monetary policy shock. The decomposition is constructed by fixing each price/quantity to it's equilibrium path as in figure 4 while holding the others constant at their steady state value and resolving the household policy functions and dynamics of the aggregate wealth distribution. The red line is the aggregate consumption response.

is owned we see the dynamics of the illiquid return become apparent. Here the consumption response of the top 10 percent is smaller than the rest of the wealth distribution and turns positive for the top 1 percent after the first period. On the RHS in the HANK-YN model the story is different. Here the large fall in the illiquid return drives the consumption response of the richer households below that of households outside of the top 10 percent on impact. After which their consumption recovers but not nearly to the same extent as in the regular HANK model. And what positive contribution there is from the top 1 percent comes from the liquid return (as in figure 6) as these households also hold substantial liquid savings. This results in a fall on impact and overall smaller rise in consumption inequality in the HANK-YN model relative the HANK model (figure 8). The overall magnitude of the response of the consumption GINI in both models is in line with the estimates of Coibion et al. (2017), however the fall on impact is inconsistent with their findings in the case of the HANK-YN model.

The comparison in figure 7 throws up the interesting questions as to whether we'd expect the consumption of the top of the wealth distribution to be more or less volatile than the middle. Basic consumption smoothing would imply that the LHS ordering be more reasonable as the more wealth a household has the more they are able to smooth their consumption. However if wealthy households hold a lot of illiquid wealth and their income is particularly volatile then the dynamics on the RHS may in fact be more reasonable. A recent study by Dany-Knedlik et al. (2021) highlights a U shape relationship for the variance of income across the income

Figure 7: Consumption response across the wealth distribution.



Note: Figure shows the response of household consumption in percent at selected percentiles of the illiquid wealth distribution to a 100 basis point monetary policy shock. The income and liquid wealth holdings are set at the median within each percentile. The responses are estimated by including the consumption levels of the selected households in the F vector when solving for the models dynamics (eq 28).

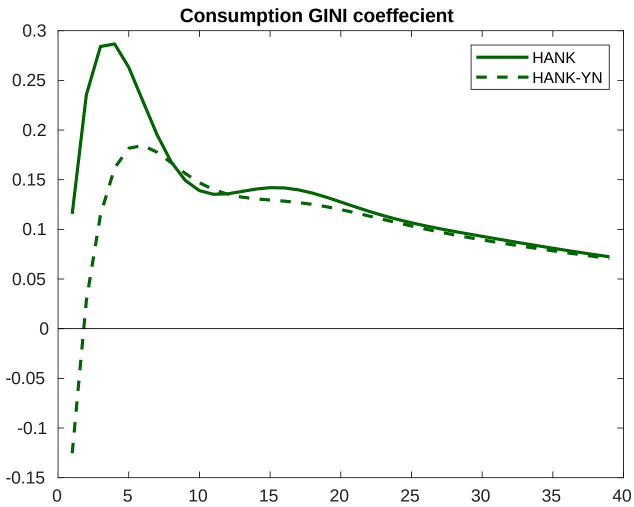
distribution over the business cycle on US data. With income variance highest at the bottom and top of the income distribution. Verifying this relationship for consumption and wealth is an interesting avenue for future work.

4.2.5 Segmented labour markets

All the analysis so far has taken place under the assumption of flexible labour markets between sectors. i.e. labour can frictionlessly move between production and expansionary activities and therefore $w_e = w_y$. The green lines in figure 9 drop this assumption and replace it with the assumption that households are constrained to remain in their sector. This assumptions can be motivated by assuming households are specialised in a particular type of tasks that might be difficult to substitute between at business cycle frequencies.

The implications of this for the impulse response to a monetary policy shock are to exaggerate the features of the HANK-YN model already discussed. As before when markups rise there is a relative rise in demand for expansionary labour and fall for production labour. Hours and wages rise for those in the expansionary sector and this is reflected in the consumption response on row 2 of figure 9, where we see the consumption of the expansionary workers rising on impact compared to a sharp fall for those in the production sector. The sharper fall in consumption of the production sector households (80 percent of households) contributes to the slightly larger

Figure 8: Consumption inequality after a monetary policy shock



Note: Figure shows the response of consumption inequality to a 100 basis point monetary policy shock. The response is estimated by including the GINI coefficient (x100) in the F vector when solving for the models dynamics (eq 28).

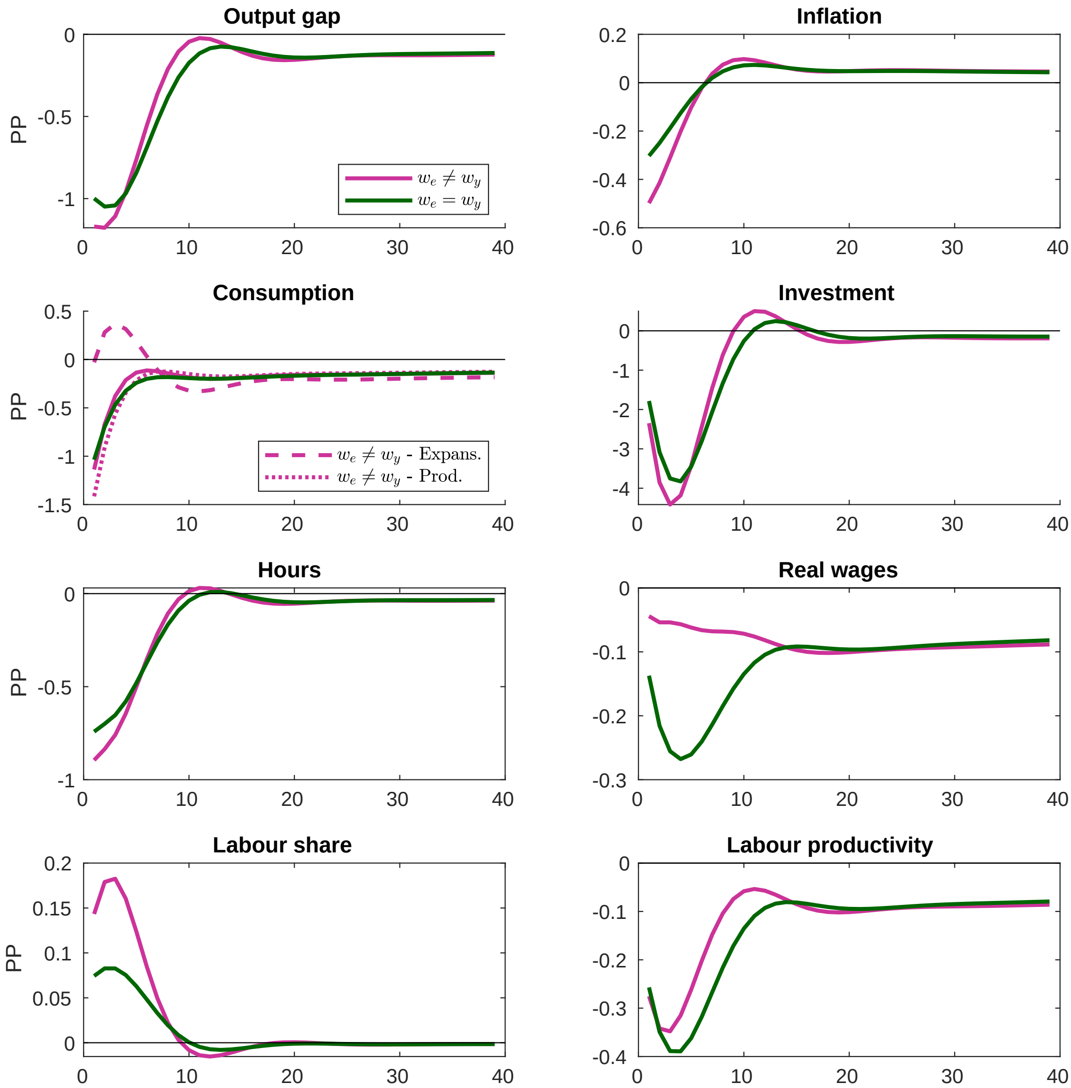
fall in overall demand in the segmented model. The wage pressure in the expansionary sector produces a significantly flatter profile for real wages, an observationally flatter wage Philip's curve and higher labour share.

4.3 Other shocks

Figure 10 plots the estimated IRF's of the labour share to seven common shocks in both versions of the model. In the standard HANK model the labour share is counter-cyclical conditional on two shocks in the 'textbook' model; the TFP shock and the wage markup shock. These are contractionary shocks that decrease markups. The contractionary shocks that increase markups are conditionally pro-cyclical and include the demand shocks (solid lines), the investment specific technology shock and the markup shock. Following from section 3, the inverse is true for the HANK-YN model. Perhaps the more striking distinction between the two models is the overall magnitude of the labour shares response to shocks which is significantly dampened in the HANK-YN model.

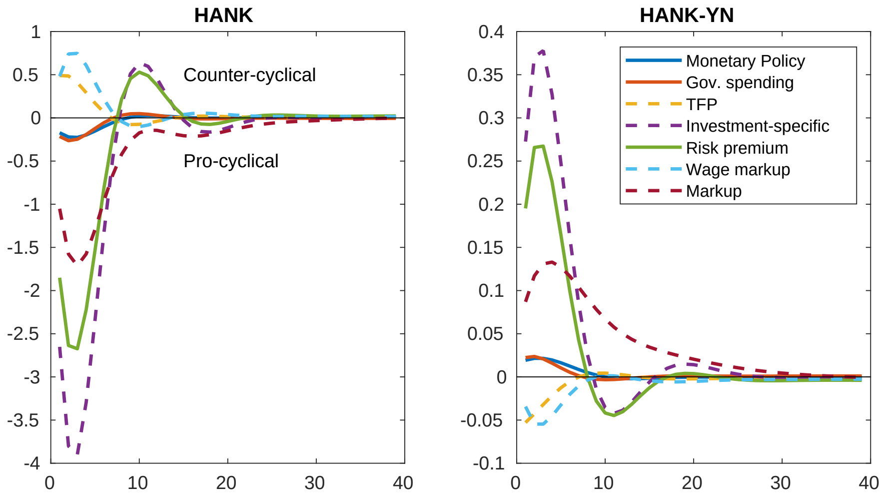
How do the profiles in figure 10 compare to the data/evidence? Table 3 summarises this evidence and reports correlations for all shocks in the final row. Based on the output data used to estimate the VAR in figure 1, and consistent with the broader evidence from Cantore et al. (2021), the overall correlation between the labour share and output is negative. Simulations of the two models show a positive correlation for the HANK model and a more data consistent negative one for the HANK-YN model. However the memo line tells us that the improvement

Figure 9: Segmented labour markets - IRF to a monetary policy shock



Note: Figure shows response of selected aggregate variables to a 100 basis point monetary policy shock over an initial 40 quarters. Variables are expressed as percentage point deviations from steady state. The green line repeats the IRF's in figure 4 and the purple line shows the IRF's when households are constrained to work in a specific sector.

Figure 10: Estimated impulse response of labour share to other shocks



Note: Figure plots the estimated impulse response of the labour share in percentage points to seven persistent one standard deviations shocks. The shocks are modelled as AR1 processes parameterised using the estimates from Bayer et al. (2020). Dashed lines classify supply shocks and solid classify demand shocks.

in labour share correlation has come at the expense of excessively dampened volatility with the HANK model a much closer fit in terms of the overall volatility of the labour share than the HANK-YN model.

What does the literature say? When it comes to specific shocks, for demand shocks such as the government spending shock Nekarda & Ramey (2020) find a counter-cyclical response of the labour share using quarterly US data for the second half of the 20th century. Under a different identification that more rigorously controls for anticipation effects, Cantore & Freund (2021) find that the response is pro-cyclical to government spending shocks. Cantore et al. (2021) find a mildly pro-cyclical labour share in response to a TFP shock based on the measure of Fernald. Nekarda & Ramey (2020) using a similar SVAR based identification strategy on their longer sample back to the 1950's Nekarda & Ramey (2020) find a mildly counter-cyclical labour share in response to TFP shock. Rios-Rull & Santaeulalia-Llopis (2010) find the labour share response to productivity shock to be counter-cyclical on impact but significantly pro-cyclical after 4-5 quarters. Therefore the strong counter-cyclical response in the estimated NK model does not fit well with the stories above but the completely pro-cyclical response of the NK-YN model under the given calibration is also potentially at odds with the data. For the investment specific technology shock Nekarda & Ramey (2020), estimate a pro-cyclical labour share response in line with the estimated NK model prediction. Thus unfortunately the NK-YN model is no silver bullet for the New Keynesian framework as any improvements made with regards to comovements on the demand side come with potential sacrifice of inconsistency on the supply side, and perhaps even to government spending shocks.

Table 3: Labour share cyclicality conditional on different shocks

Shock	Evidence	NK model	NK-YN model
Monetary policy	Counter	Pro	Counter
Government spending	Mixed	Pro	Counter
TFP	Mixed	Counter	Pro
Investment specific technology	Pro	Pro	Counter
All shocks	Counter [−0.15]	Pro [0.24]	Counter [−0.24]
Memo: HP-filter Labour shr. Std. Dev. (pp)	0.84	1.00	0.10

Note: For evidence see Cantore et al. (2021), Cantore & Freund (2021), Nekarda & Ramey (2020), Rios-Rull & Santaeulalia-Llopis (2010) and references therein. Numbers in brackets are HP-filter ($\lambda = 1600$) correlations between the labour share and the log of GDP. The evidence columns show the correlation between the output data and the labour share used for estimating the VAR in figure 1. The model columns show results from 1000 period simulations.

5 Conclusion

In response to evidence documenting a counter-cyclical labour share contingent on monetary policy shocks this paper considered the transmission of such shocks in the New Keynesian model environment. We focused on two versions of the New Keynesian model. A version consistent with most models in the literature that delivers a demand shock contingent pro-cyclical response and a version that delivers a counter-cyclical response. Comparing the two models highlights the importance of the role of capital income and illiquid investments in understanding the transmission of shocks across the wealth distribution and throughout the economy. In the model with a pro-cyclical labour share, relative capital income movements dampen contractionary demand shocks and partially insure wealthy capital owning households. Under the counter-cyclical labour share, capital income plays a significant contractionary role and reduces consumption inequality, as those at the top of the wealth distributions income and consumption fall by more in relative terms on impact than those towards the middle of the wealth distribution. HANK models provide the opportunity for realistic and useful decomposition's of the economic channels through which policy transmits. But that brings an increased need to scrutinise the micro-foundations on which our models are built. Given recent evidence[25] that documents a U shaped profile across the income distribution for income variance over the US business cycle an interesting future avenue for this strand of work will be to test whether that U shaped profile exists for consumption as it does in the NK-YN model in figure 7.

The counter-cyclical labour share in the NK-YN model was achieved by introducing a novel set of workers into the economy that focus on expanding the measure of goods available to consumers instead of directly engaging in production themselves. This paper provides a nice

[25] Dany-Knedlik et al. (2021).

example of how simple additional micro-foundations can not only add increased realism to macro-models but also deliver more data-consistent macro behaviour with implications across the wealth distribution. However more empirical work is required to properly calibrate the role of such workers. And this particular framework is no silver bullet as the shock conditional response to other shocks in this model would still appear inconsistent with the data. Furthermore while the NK-YN model delivers a counter-cyclical labour share, it does so by altering the relationship between the labour share and markups which are still counter-cyclical in the NK-YN model and at odds with the data. Future work should look to assess what further or alternate micro-foundations could be introduced to better match the movements of factor incomes shares and markups, in aggregate and across the wealth distribution, contingent on all shocks.

References

Acemoglu, D. & Autor, D. (2011), Skills, tasks and technologies: Implications for employment and earnings, *in* ‘Handbook of labor economics’, Vol. 4, Elsevier, pp. 1043–1171.

Alves, F., Kaplan, G., Moll, B. & Violante, G. L. (2020), ‘A further look at the propagation of monetary policy shocks in hank’, *Journal of Money, Credit and Banking* **52**(S2), 521–559.

Bayer, C., Born, B. & Luetticke, R. (2020), ‘Shocks, frictions, and inequality in us business cycles’.

Bayer, C. & Luetticke, R. (2020), ‘Solving discrete time heterogeneous agent models with aggregate risk and many idiosyncratic states by perturbation’, *Quantitative Economics* **11**(4), 1253–1288.

Bayer, C., Lütticke, R., Pham-Dao, L. & Tjaden, V. (2019), ‘Precautionary savings, illiquid assets, and the aggregate consequences of shocks to household income risk’, *Econometrica* **87**(1), 255–290.

Born, B. & Pfeifer, J. (2020), ‘The new keynesian wage phillips curve: Calvo vs. rotemberg’, *Macroeconomic Dynamics* **24**(5), 1017–1041.

Broer, T., Harbo Hansen, N.-J., Krusell, P. & Öberg, E. (2020), ‘The new keynesian transmission mechanism: A heterogeneous-agent perspective’, *The Review of Economic Studies* **87**(1), 77–101.

Cantore, C., Ferroni, F. & León-Ledesma, M. (2021), ‘The missing link: monetary policy and the labor share’, *Journal of the European Economic Association* **19**(3), 1592–1620.

Cantore, C. & Freund, L. B. (2021), ‘Workers, capitalists, and the government: fiscal policy and income (re)distribution’, *Journal of Monetary Economics* **119**, 58–74.
URL: *https://www.sciencedirect.com/science/article/pii/S0304393221000040*

Carroll, C. D. (2006), ‘The method of endogenous gridpoints for solving dynamic stochastic optimization problems’, *Economics letters* **91**(3), 312–320.

Castaneda, A., Diaz-Gimenez, J. & Rios-Rull, J.-V. (2003), ‘Accounting for the us earnings and wealth inequality’, *Journal of political economy* **111**(4), 818–857.

Chetty, R., Guren, A., Manoli, D. & Weber, A. (2011), 'Are micro and macro labor supply elasticities consistent? a review of evidence on the intensive and extensive margins', *American Economic Review* **101**(3), 471–75.

Chu, S.-Y. (2020), 'A note on labor share, price markup and monetary policy', *Economics Letters* **192**, 109169.
URL: *https://www.sciencedirect.com/science/article/pii/S0165176520301300*

Coibion, O., Gorodnichenko, Y., Kueng, L. & Silvia, J. (2017), 'Innocent bystanders? monetary policy and inequality', *Journal of Monetary Economics* **88**, 70–89.

Dany-Knedlik, G., Kriwoluzky, A. & Pasch, S. (2021), 'Income business cycles'.

Gomme, P. & Rupert, P. (2004), 'Measuring labor's share of income', *FRB of Cleveland Policy Discussion Paper* (7).

Greenwood, J., Hercowitz, Z. & Huffman, G. W. (1988), 'Investment, capacity utilization, and the real business cycle', *The American Economic Review* pp. 402–417.

Guvenen, F., Ozkan, S. & Song, J. (2014), 'The nature of countercyclical income risk', *Journal of Political Economy* **122**(3), 621–660.

Jentsch, C. & Lunsford, K. G. (2016), 'Proxy svars: asymptotic theory, bootstrap inference, and the effects of income tax changes in the united states'.

Kaplan, G., Moll, B. & Violante, G. L. (2018), 'Monetary policy according to hank', *American Economic Review* **108**(3), 697–743.

Kaplan, G. & Zoch, P. (2020), Markups, labor market inequality and the nature of work, Technical report, National Bureau of Economic Research.

McKay, A. & Reis, R. (2016), 'The role of automatic stabilizers in the us business cycle', *Econometrica* **84**(1), 141–194.

Mertens, K. & Ravn, M. O. (2014), 'A reconciliation of svar and narrative estimates of tax multipliers', *Journal of Monetary Economics* **68**, S1–S19.

Miranda-Agrippino, S. & Ricco, G. (2021), 'The transmission of monetary policy shocks', *American Economic Journal: Macroeconomics* **13**(3), 74–107.

Nekarda, C. J. & Ramey, V. A. (2020), 'The cyclical behavior of the price-cost markup', *Journal of Money, Credit and Banking* **52**(S2), 319–353.

Ravenna, F. & Walsh, C. E. (2006), 'Optimal monetary policy with the cost channel', *Journal of Monetary Economics* **53**(2), 199–216.

Ravn, M. O. & Sterk, V. (2021), 'Macroeconomic fluctuations with hank & sam: An analytical approach', *Journal of the European Economic Association* **19**(2), 1162–1202.

Rios-Rull, J.-V. & Santaeulalia-Llopis, R. (2010), 'Redistributive shocks and productivity shocks', *Journal of Monetary Economics* **57**(8), 931–948.

Romer, C. D. & Romer, D. H. (2004), 'A new measure of monetary shocks: Derivation and implications', *American Economic Review* **94**(4), 1055–1084.

Schmitt-Grohé, S. & Uribe, M. (2004), 'Solving dynamic general equilibrium models using a second-order approximation to the policy function', *Journal of economic dynamics and control* **28**(4), 755–775.

Schmitt-Grohé, S. & Uribe, M. (2005), 'Optimal fiscal and monetary policy in a medium-scale macroeconomic model', *NBER Macroeconomics Annual* **20**, 383–425.

Smets, F. & Wouters, R. (2007), 'Shocks and frictions in us business cycles: A bayesian dsge approach', *American economic review* **97**(3), 586–606.

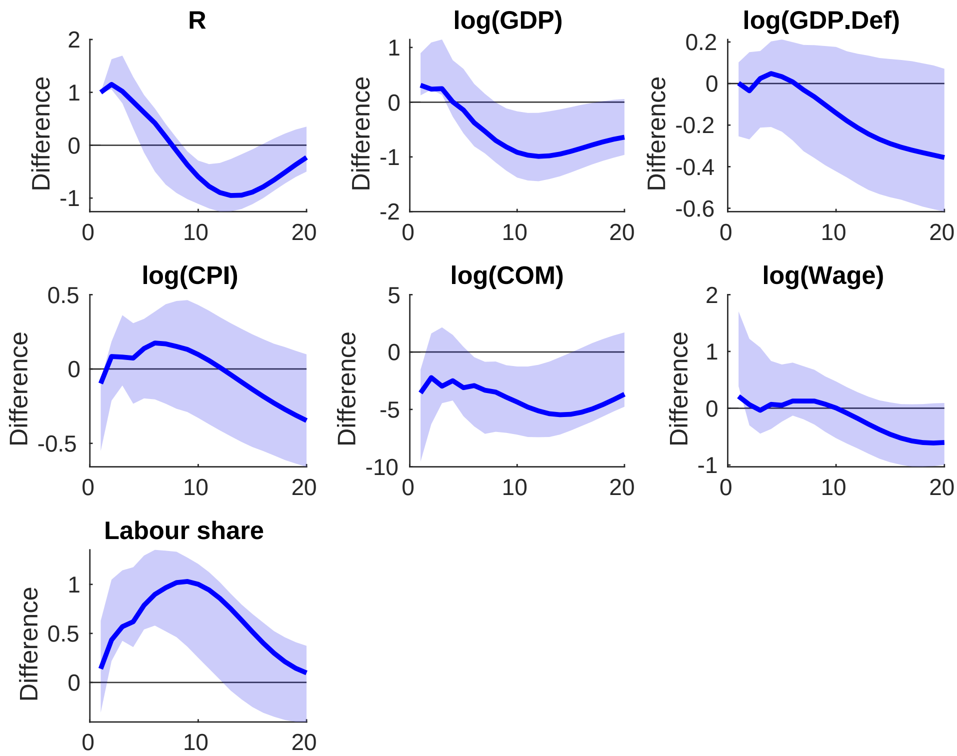
Storesletten, K., Telmer, C. I. & Yaron, A. (2004), 'Cyclical dynamics in idiosyncratic labor market risk', *Journal of political Economy* **112**(3), 695–717.

Tauchen, G. (1986), 'Finite state markov-chain approximations to univariate and vector autoregressions', *Economics letters* **20**(2), 177–181.

Appendix

A1: Empirical VAR

Figure A1: VAR impulse response to a monetary policy shock



Note: Y axis shows differences multiplied by 100. Labour share is expressed as a percentage point difference. The shaded area represents a 68 percent confidence interval based on the the moving block bootstrap routine of Jentsch & Lunsford (2016) with 5000 draws. Blocks are of size 19 quarters. Solid line is the point estimate.

The above figure shows the impulse response of a seven variable three lag VAR to a monetary policy shock. The responses are estimated using quarterly US data from 1984-2007 including:

1. Federal Funds Rate: nominal average over quarter.
2. GDP: Log of real GDP.
3. GDP Deflator: Log of real GDP deflator.
4. CPI: Log of all urban consumer all item CPI index.
5. Wages: Log of NIPA wages and salaries divided by BLS total hours worked and the GDP deflator.
6. Commodity prices: Log of average of CRB spot index.
7. Labour share: $1 - \frac{CorporateProfits + NetInterest - Tax}{NetValueAdded}$. As recommended by Gomme & Rupert (2004) using NIPA data.

The IRF's are identified using the proxy-SVAR external instruments method of Mertens & Ravn (2014) and the instruments sourced from Romer & Romer (2004) [pre-1991] and Miranda-Agrippino & Ricco (2021) [from 1991].

Table A1: Fed Funds instrument first stage regression

	Estimate	SE	tstat	pvalue
(Intercept)	0.003	0.0257	0.118	0.906
Instrument	1.03	0.223	4.52	1.82e-05

Note: LHS variable are reduced form VAR residuals for the Federal Funds rate from the OLS estimate. Number of observations: 93. Error degrees of freedom: 91. Root Mean Squared Error: 0.247. R-squared: 0.184, Adjusted R-Squared: 0.175. F-statistic vs. constant model: 20.5. p-value = 1.82e-05.

A2: Computational appendix

Steady state

The procedures to solve for the steady state and dynamics of the HANK model closely follow that of Bayer et al. (2019). The solution method relies on a discretisation of the household state space with household decisions evaluated on a $60 * 60 * 15$ grid that represents the joint distribution of liquid assets, capital and individual productivity. The size of the grid was selected based on the stability of the observed aggregate impulse response functions. The individual productivity process z is discretised using the method of Tauchen (1986). Below I provide a broad overview of the procedure to solve for the steady state:

1. Set a desired steady state real interest rate for nominal bonds
2. Guess a steady state level of real aggregate capital.
3. Compute labour demand, wage level, the return on capital and stock price from the aggregate first order conditions.
4. Solve the household problem given prices:
   (a) At each point on the joint household distribution (b,a,z) guess household policy functions (x_{adj}, x_{nadj}) in the adjustment and non-adjustment state and the shadow value of illiquid wealth $\frac{dV_{nadj}}{da}$.
   (b) Update x_{nadj} using the constrained budget constraint and equation 21 using the endogenous grid point method (Carroll (2006)).
   (c) Create a mapping from total household resources $(a+b) \to (a,b)$ by solving for a for each b node by equating the RHS's of equations 19 and 20.
   (d) Update x_{adj} using the budget constraint, equation 20 and mapping for $(a+b) \to (a,b)$
   (e) Update the marginal value of capital using the following formula: $\frac{dV_{nadj}}{da}(a,b,z)^{new} = r_a x_{nadj}^{-\sigma} + \beta\, \mathbf{E}\left[\omega(1+r'_a)\left(x_{adj}(a,b',z')\right)^{-\sigma} + (1-\omega)\frac{dV_{nadj}(a,b',z')}{da}^{old}\right]$
   (f) Repeat until policy functions converge.
5. Compute steady state household joint distribution using solved policy functions and transition probabilities.

6. Compare aggregate capital supply from household to initial capital demand embedded in factor prices. Update capital demand guess using a bi-section method.

7. Repeat until convergence of capital demand and capital supply.

Household Euler Equations

$$(x_{adj}(a,b,z,\lambda))^{-\sigma} = \beta\ \mathbf{E}\left[\omega(1+r_a')\,(x_{adj}(a',b',z',\lambda'))^{-\sigma} + (1-\omega)\frac{dV_{nadj}(a',b',z',s',\lambda')}{da}\right] \tag{19}$$

$$(x_{adj}(a,b,z,s,\lambda))^{-\sigma} = \beta\ \mathbf{E}\left[\omega\frac{1+r_b'}{1+\pi'}\,(x_{adj}(a',b',z',s',\lambda'))^{-\sigma} + (1-\omega)\frac{1+r_b'}{1+\pi'}\,(x_{nadj}(a',b',z',s',\lambda'))^{-\sigma}\right] \tag{20}$$

$$(x_{nadj}(a,b,z,s,\lambda))^{-\sigma} = \beta\ \mathbf{E}\left[\omega\frac{1+r_b'}{1+\pi'}\,(x_{adj}(a,b',z',s',\lambda'))^{-\sigma} + (1-\omega)\frac{1+r_b'}{1+\pi'}\,(x_{nadj}(a,b',z',s',\lambda'))^{-\sigma}\right] \tag{21}$$

Figure A2.1: Steady state wealth distributions

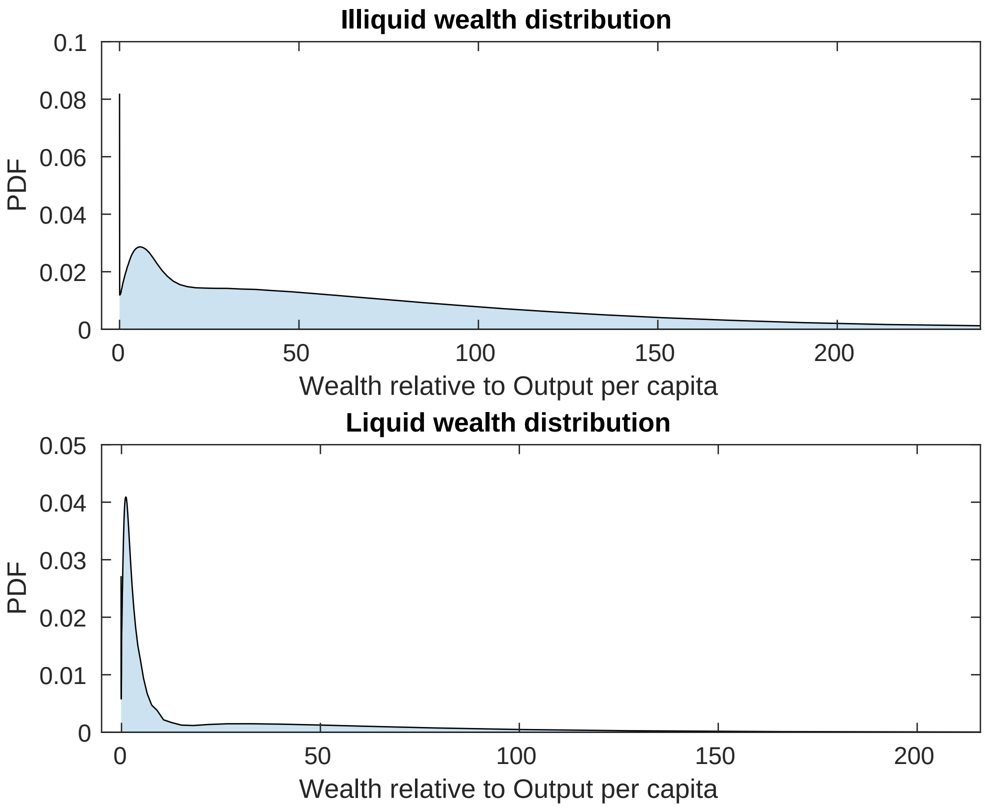


Table A2.1: Steady state moments

Moment	Value
Liquid wealth /Debt to GDP	1.67
Illiquid wealth to Output	16.0
Capital to Output	11.5
Top 10pct wealth share	56 pct
Top 1pct income share	9.8 pct
Wealth GINI	0.71
Gov. spending to Output	17.8 pct
Investment to Output	20 pct

Note: Ratios are expressed relative to quarterly output.

Dynamics

As with the steady state I follow the procedures of Bayer et al. (2019) to solve for the models dynamic response to shocks up to first order using the perturbation approach of Schmitt-Grohé & Uribe (2004). Two shortcuts are taken to overcome the curse of dimensionality. Firstly the model only tracks the marginal distributions of illiquid wealth (a), liquid wealth (b) and individual productivity (z). And in each period the marginal distributions are mapped to the joint distribution based on the relative mapping established for the steady state. This reduces the number of state variables needed to track the distribution from $n_a * n_b * n_z$ to $n_a + n_b + n_z$. The second step is to approximate the household marginal utilities and value functions using a discrete cosine transformation (DCT). A DCT transformation of the below vectors over all nodes in the joint distribution:

$$mutil = \omega x_{adj}(a,b,z,s)^{-\sigma} + (1-\omega)x_{nadj}(a,b,z,s)^{-\sigma}$$

$$v_k = \omega(1+r_a)x_{adj}(a,b,z,s)^{-\sigma} + (1-\omega)(r_a x_{nadj}(a,b,z,s)^{-\sigma} + \beta v_{k,+})$$

is conducted. And only the nodes required to explain 99.9 percent of the original vectors are kept. The controls for the household problem that enter the F vector $(\theta_{mutil,dct}, \theta_{v_k,dct}, \theta_{mutil,dct,+}, \theta_{v_k,dct,+})$ are then perturbations to the selected transformed nodes required to re-capture $mutil$ and v_k for the current and one period ahead, such that they are consistent with the households Euler equations 19-21 at each node on the full joint distribution. This reduces the number of controls from $60 * 60 * 15$ plus aggregate controls to something like 126 plus aggregate controls. A huge reduction in the dimension of the problem.

The household policy functions consistent with the control vectors deliver the law of motion for the marginal distributions, which are combined with the economies aggregate first order conditions to produce an F[26] vector in SGU form.

[26] Below I print the vector for the HANK model with expansionary labour and differing wages between sectors.

$$
F(X, X_+, Y, Y_+) = \begin{pmatrix}
\textbf{Marginal value fns / utilities} \\
DCT[mutil^{\frac{1}{-\sigma}} - mutil_{ss}^{\frac{1}{-\sigma}}] - \theta_{mutil,dct} \\
DCT[v_k - v_{k,ss}] - \theta_{v_k,dct} \\
\textbf{Marginal Distributions} \\
\vdots \\
p(b_j)_+ - \sum_{b_i} p(b_i) \sum_{(a,z)} p(a,z|b_i) \mathbf{1}_{b'=b_j} \\
\vdots \\
\\
\vdots \\
p(a_j)_+ - \sum_{a_i} p(a_i) \sum_{(b,z)} p(b,z|a_i) \mathbf{1}_{a'=a_j} \\
\vdots \\
\\
\vdots \\
p(z_j)_+ - \sum_{z_i} p(z_i) p(z_j^{'}|z_i) \\
\vdots \\
\textbf{Aggregate economy} \\
mc\theta_y(1-\alpha_y)\frac{y}{n_y} - w_y \\
mc\theta_y\alpha_y\frac{y_j}{k} - r_k \\
y(1-mc)\theta_e\frac{M}{n_e} - w_e \\
\frac{1}{1+r_{a,ss}}\mathbf{E}[\pi_+] + \kappa_p \hat{mc} - \pi \\
1 + \Delta log(\frac{I}{I_-}) + \frac{\Delta}{2} log(\frac{I}{I_-})^2 - \mathbf{E}\left[\frac{\Delta}{1+r_{a,+}}\frac{I_+}{I} log(\frac{I_+}{I})\right] - q_k \\
\mathbf{E}\left[\frac{r_{k,+} + (1-\delta)q_{k,+}}{1+r_{a,+}}\right] - q_k \\
\frac{1+r_b}{1+\pi} B + G - T - B_+ \\
\left(\frac{\tau_{t-1}}{\tau_{ss}}\right)^{\rho_\tau} \left(\frac{B_+}{B_{ss}}\right)^{\gamma_{\tau_B}(1-\rho_\tau)} \left(\frac{Y}{Y^*}\right)^{\gamma_{\tau_y}(1-\rho_\tau)} - \frac{\tau}{\tau_{ss}} \\
\rho_r r_b + (1-\rho_r)(r_b^* + \gamma_\pi(\pi - \pi^*) + \gamma_y log(\frac{Y}{Y^*})) + \epsilon_{r,t} - r_{b,+} \\
\beta\mathbf{E}[\pi_{w,y,+}] - \kappa_w \hat{\mu_{w,y}} - \pi_{w,y} \\
\beta\mathbf{E}[\pi_{w,e,+}] - \kappa_w \hat{\mu_{w,e}} - \pi_{w,e} \\
(1-\delta)K + I - K_+ \\
q_s + q_k K_+ - A_+ \\
Y - y * M \\
K - k * M \\
r_k K + \Pi_d + (1-\delta) q_k K - K q_{k,-} + q_s - q_{s,-} - \frac{\Delta}{2} log(I/I_-)^2 I - r_a A \\
\textbf{Exogenous AR1 shock processes} \\
ln(z_{.,+}) - \rho ln(z_.) - \epsilon_.
\end{pmatrix}
$$