Flexible Deviations from FIRE in the Sequence Space

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Motivation

- **Question**: Can we depart from FIRE in a *flexible* and *standardised* manner that is consistent with the empirical literature (macro, micro and behavioral)?
- Recent HANK literature emphasises the importance of matching microeconomic evidence/moments
  - Auclert et al. (2020) also emphasise need to simultaneously match the macro evidence i.e. macro humps.
  - Their solution is sticky expectations which creates *persistence* but tends to kill *amplification*
  - This can be an issue e.g. Albuquerque et al. (2025).
- Growing empirical evidence base on the distance between actual expectations and rational expectations e.g. Adam et al. (2024),Coibion and Gorodnichenko (2015), Kohlhas and Walther (2021).

Today

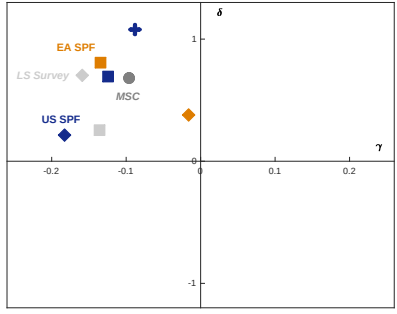
- Go through a proposal and implementation of a structured but flexible departure from rational expectations
- Explore properties in a Canonical HANK model
- Apply in a HANK housing model

Idea/Proposal

- Propose that we can depart from FIRE in a similar way as in Auclert et al. (2020)
- But depart based on the reduced form used in Kohlhas and Walther (2021):

$$p_{t+k} - f_{it} p_{t+k} = \alpha_i + \underbrace{\gamma p_t}_{\textit{Today}} + \underbrace{\delta(\bar{f}_t p_{t+k} - \bar{f}_{t-1} p_{t+k})}_{\textit{News}} + \epsilon_{i,t|t+k}$$

Figure 1: Empirical evidence on δ and γ



Note: Evidence from Kohlhas and Walther (2021)

Idea/Proposal

What are the advantages of this approach:

1. Clear mapping from the empirical literature.
2. Allows for fact agents might simultaneously overreact to outcomes and underreact to news
3. Nests different expectations models:
   - Asymmetric attention ($\gamma, \delta \neq 0$)
   - Sticky / noisy information ($\gamma = 0$,$\delta > 0$)
   - $\approx$ Diagnostic expectations ($\delta = 0$,$\gamma < 0$)
4. Is near-rational expectations with agents behaving rationally conditional on their subjective beliefs.

Sequence Space

- We can implement this expectations process by building up partial equilibrium Jacobians J in the model using the '**Fake News**' matrix F outlined by Auclert et al. (2021).
- This allows us to map an arbitrary price path p to block specific outcomes y.

$$F_{y,p} = \begin{vmatrix} F_{0,0} & F_{0,1} & \dots & F_{0,s} \dots & F_{0,T} \\ \dots & & & & \\ F_{T,0} & F_{T,1} & \dots & F_{T,s} \dots & F_{T,T} \end{vmatrix} \tag{1}$$

Sequence Space: Rep Agent Euler Example

Consider the standard consumption equation in a representative agent model:

$$c_t^{-\sigma} = \beta E_t[(1 + r_{t+1})c_{t+1}^{-\sigma}] = \beta^T E_t[(\prod_{i=1}^{T}(1 + r_{t+i}))c_{t+T}^{-\sigma}]$$

If we linearise around a steady state we get:

$$-\sigma c_{ss}^{-\sigma}\hat{c}_t = \beta^T(1 + r_{ss})^{T-1}c_{ss}^{-\sigma}\sum_{i=1}^{T} dr_{t+i} - \sigma\beta^T(1 + r_{ss})^T\hat{c}_T$$

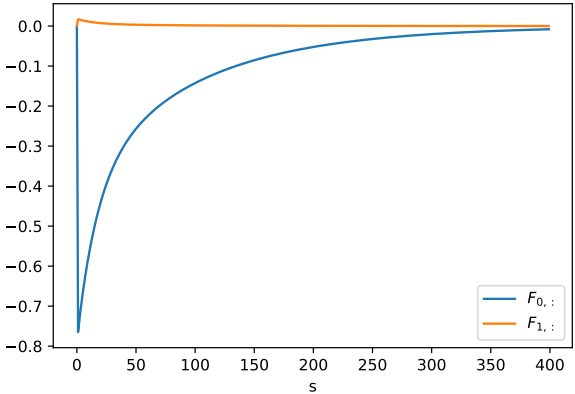
Cleaning up and assume T is large enough such that $\hat{c_T} = 0$, and that $(1 + r_{ss}) = \frac{1}{\beta}$.

$$\hat{c}_t = \frac{-\beta}{\sigma}\sum_{i=1}^{T} dr_{t+i}$$

$$F_{c,r} = \begin{vmatrix} 0 & \frac{-\beta}{\sigma} & \dots & \frac{-\beta}{\sigma}\dots & \frac{-\beta}{\sigma} \\ \dots & & & & \\ F_{T,0} & F_{T,1} & \dots & F_{T,s}\dots & F_{T,T} \end{vmatrix} \tag{2}$$

Sequence Space

Figure Fake News Matrix in a HANK model $F_{C,r}$



Implementation

$$p_{t+k} - f_{it}p_{t+k} = \alpha_i + \gamma p_t + \delta(\bar{f}_t p_{t+k} - \bar{f_{t-1}} p_{t+k}) + \epsilon_{i,t|t+k} \rightarrow$$

$$\bar{f}_t p_{t+k} = c + \frac{1}{1+\delta}(\delta \bar{f_{t-1}} p_{t+k} + E_t^{RE}[p_{t+k}] - \gamma p_t)$$

Consider the evolution of a price forecast initially k periods ahead. This evolves as follows:

$f_{ok} = c + \frac{1}{1+\delta}(\delta p_{ss} + p_{ss} + dp_k - \gamma(p_{ss} + p_o)) = p_{ss} + \frac{1}{1+\delta}(dp_k - \gamma dp_o)$, where we assume $c = \frac{\gamma}{1+\delta} p_{ss}$

....

$$f_{nk-n} = p_{ss} + \underbrace{\sum_{j=o}^{n} \left(\frac{\delta}{1+\delta}\right)^j \frac{1}{1+\delta} dp_k}_{News} - \underbrace{\gamma \sum_{j=o}^{n} \left(\frac{\delta}{1+\delta}\right)^j \frac{1}{1+\delta} dp_j}_{Extrapolation}$$

News effect converges to RE for $\delta > o$. Extrapolative effect fades over time

Implementation - Growth Case

- Can also implement when expectations are made in growth space but model is in levels.
- Just a slightly more complicated forecast formula to follow and map to the Fake News Matrix
- Can do so for arbitrary lag (e.g QoQ or Q4oQ)

$$(y_{t+k} - y_{t+k-h}) - (f_{it}y_{t+k} - y_{t+k-h}) = \alpha_i + \gamma_k(y_t - y_{t-h}) + \delta_k((\bar{f}_t y_{t+k} - \bar{f}_t y_{t+k-h}) - (\bar{f_{t-1}} y_{t+k} - \bar{f_{t-1}} y_{t+k-h})) + \epsilon_{it}$$

Rearranging equation and averaging over agents we get:

$$\bar{f}_t y_{t+k} = c + \tfrac{1}{1+\delta_k}(\delta_k \bar{f_{t-1}} y_{t+k} + y_{t+k} - \gamma_k y_t) + \underbrace{\frac{1}{1+\delta_k}(\gamma_k y_{t-h} + \delta_k(\bar{f}_t y_{t+k-h} - \bar{f_{t-1}} y_{t+k-h}))}_{\text{New Terms}}$$

Implementation

To work out Jacobian just a case of mapping Fake new matrix to where prices show up. For example:

$$J_{0,0} = \underbrace{F_{0,0}}_{Impact} - \underbrace{\frac{\gamma}{1+\delta}(F_{0,1} + F_{0,2} + \ldots + F_{0,s} + F_{0,T})}_{Extrapolation}$$

$$J_{3,2} = \underbrace{F_{1,0}}_{Impact} - \underbrace{\frac{\gamma\delta}{(1+\delta)^2}(F_{0,1} + F_{0,2} + \ldots + F_{0,s} + F_{0,T}) - \frac{\gamma}{1+\delta}(F_{1,1} + F_{1,2} + \ldots + F_{1,s} + F_{1,T})}_{Extrapolation}$$

$$+ \underbrace{\frac{1}{1+\delta}F_{3,2} + (\frac{1}{1+\delta} + \frac{\delta}{(1+\delta)^2})F_{2,1}}_{News}$$

Application: Canonical HANK

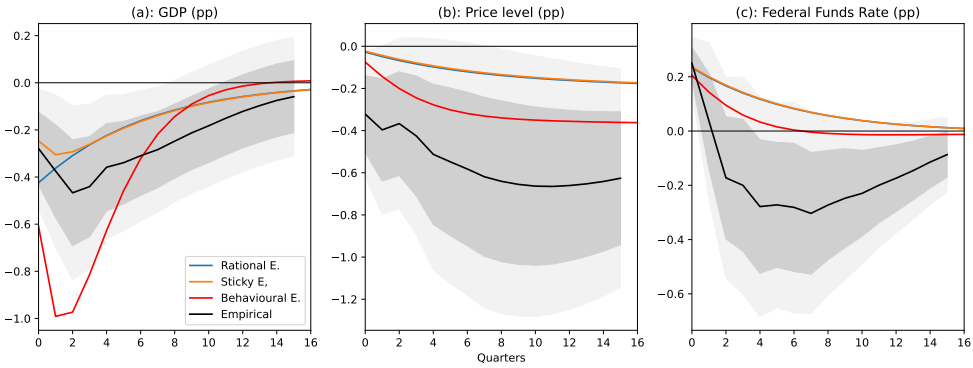
Lets apply the algorithm using common parameters from the literature ($\delta = 0.7, \gamma = -0.15$) in the canonical HANK framework of Auclert et al. (2024)

- One asset HANK model matched to US wealth and income distribution
- Sticky wages
- Income taxes, balanced budget and Taylor Rule
- Compare to empirical evidence from Bauer and Swanson (2023)

Application: Canonical HANK

Lets apply the algorithm using common parameters from the literature ($\delta = 0.7, \gamma = -0.15$) in the canonical HANK framework of Auclert et al. (2024)

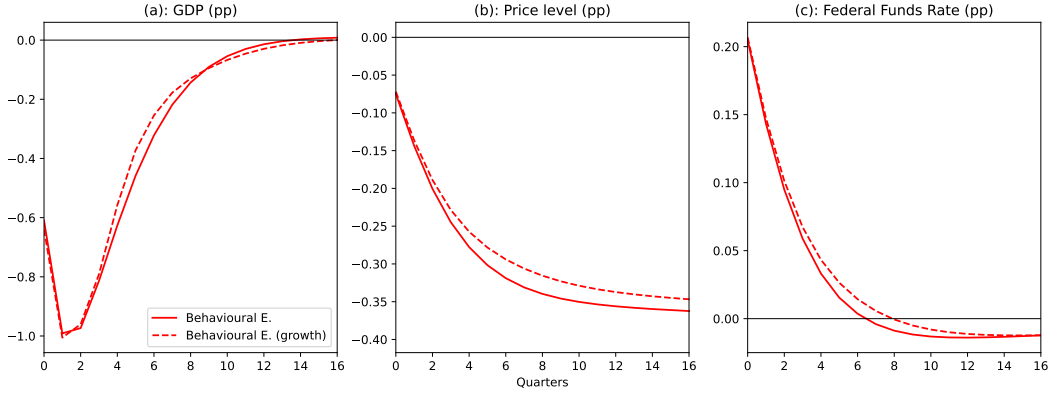
Figure Impulse Response to MP Shock: Model and Evidence



Application: Canonical HANK (levels and growth)

Can also apply growth or levels algorithm depending on how data usually estimated. Income variables expectations assumed to be annual growth space. Again use ($\delta = 0.7, \gamma = -0.15$).

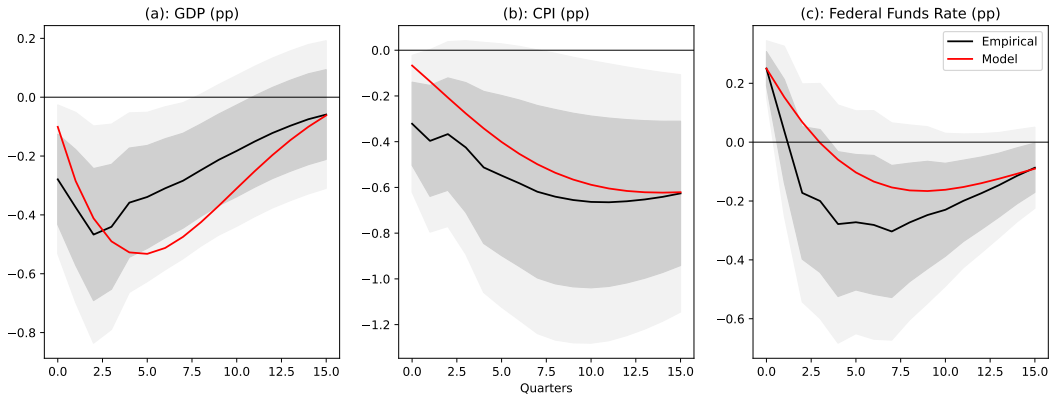
Figure Impulse Response to MP Shock: Model and Evidence



Application: Canonical HANK - estimated

Now what if we estimate $\delta = 10.98$ and $\gamma = -0.29$.

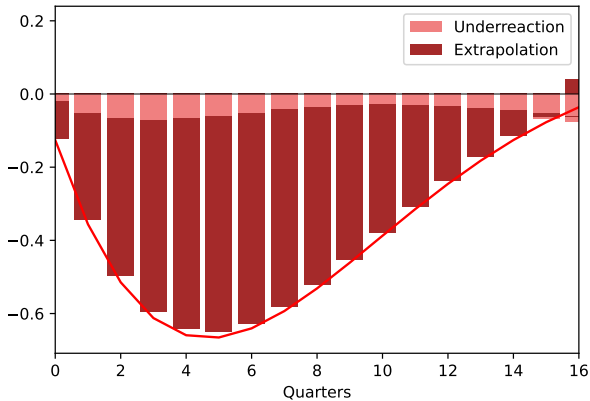
Figure Impulse Response to MP Shock: Model and Evidence



Application: Role of extrapolation

In the estimated version extrapolation plays a big role in generating a large and persistent hump.

Figure Consumption Response Decomposed

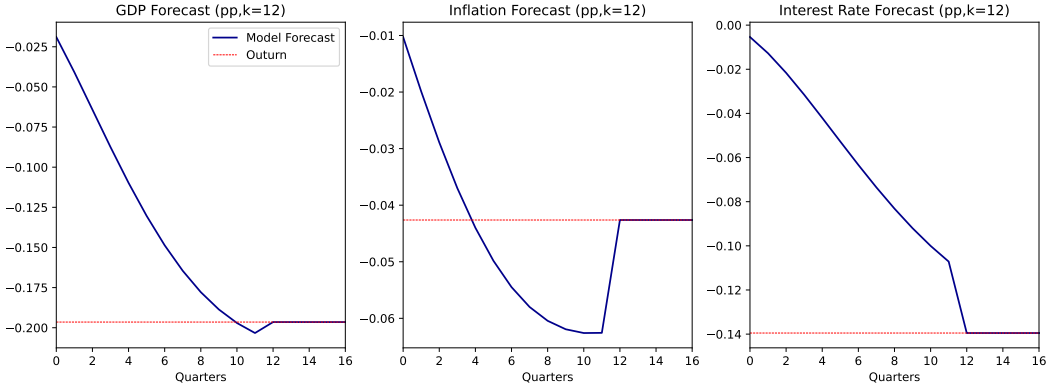


($\delta = 10.98$ and $\gamma = -0.29$.)

Application: Forecast Evolutions

This process also tends to deliver an initial underreaction and subsequent overreaction commonly documented in the empirical literature.

Figure Impulse Response to MP Shock: Price Forecasts



($\delta = 10.98$ and $\gamma = -0.29$.)

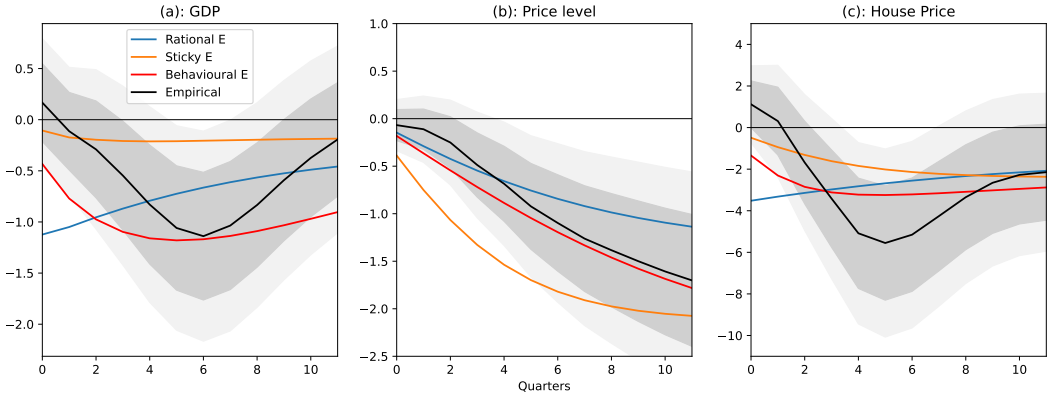
Application: HANK + Housing

- Arguably, the previous empirical evidence and model could have been pretty well matched without introducing extrapolation
- But some empirical evidence is a lot harder and a trade off can emerge between sluggishness and amplification
- This was the case in Albuquerque et al. (2025):
  - Two asset discrete choice HANK model
  - Secured borrowing against houses

Application: HANK + Housing

And what about a model/evidence that needs big humps
($\delta = 2.86$, $\delta_{p_h} = 1.26$, $\gamma = -0.105$, $\gamma_{p_h} = 0.048$)

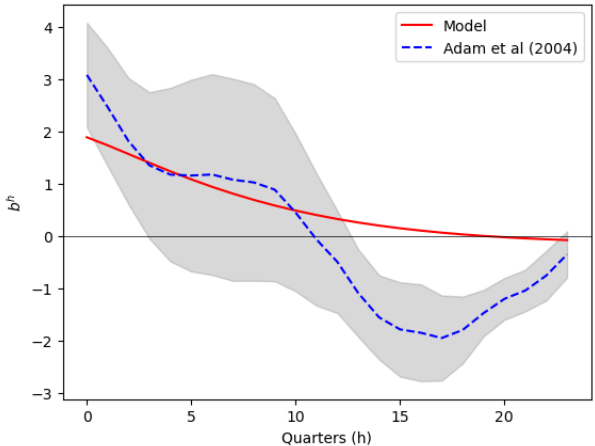
Figure Impulse Response to MP Shock: Model and Evidence



Application: HANK + Housing

And again this approach delivered empirically realistic price forecasts when compared to the evidence of Adam et al. (2024).

Figure Impulse Response to MP Shock: Model and Evidence



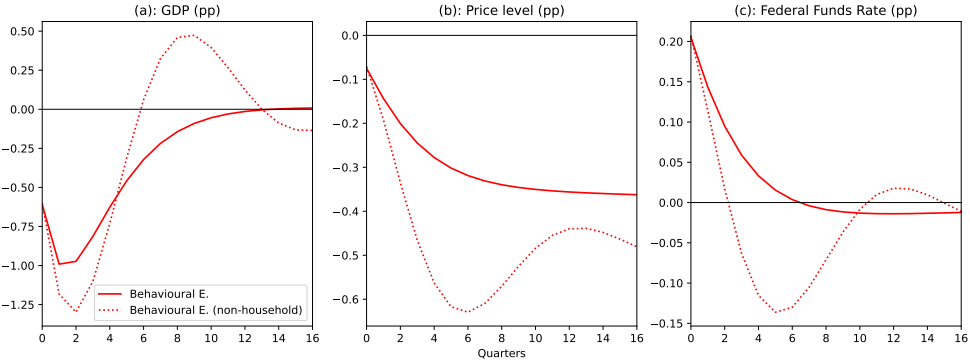
Firms and Financial Markets

- Auclert et al. (2020) focus on the inattentiveness of households
- But our empirical evidence is largely based on professional forecasters
- We can also depart on the firm side or financial market side in exactly the same way

Firms and Financial Markets

Adding behavioral expectations on firm and financial markets side ($\delta = 0.77$, $\gamma = -0.15$).

Figure Impulse Response to MP Shock: Model and Evidence



IRF matching: ($\delta = 9.77$, $\gamma = -0.31$, $\delta_f = 0.00$, $\gamma_f = -0.05$).

Conclusion/next steps

- Proposed a flexible approach to deviating from FIRE in the sequence space.
- Estimated parameters through IRF matching exercises.
  - Measurement supports evidence sluggishness and extrapolation
  - Less so on firm, financial market side but model less rich in this area.

Next steps

- Incorporate into other housing paper.
- More empirical work: e.g horizon varying parameters
- Interest rate expectations of particular interest
  - More information available to household on interest rate path
  - Deviating from FIRE has implications for the elasticity of intertemporal substitution
- New Keynesian model relevant example of asymmetric attention

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