Optimization Homework#2

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Problem#1

Let A = $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 0 \\ 1 & 0 & -2 \end{pmatrix}$

(a) LDL^T decomposition:

$$A = LDL^T = \begin{pmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{pmatrix} \begin{pmatrix} D_1 & 0 & 0 \\ 0 & D_2 & 0 \\ 0 & 0 & D_3 \end{pmatrix} \begin{pmatrix} 1 & L_{21} & L_{31} \\ 0 & 1 & L_{32} \\ 0 & 0 & 1 \end{pmatrix} =$$
$$\begin{pmatrix} D_1 & . & . \\ L_{21}D_1 & L_{21}^2 D_1 + D_2 & . \\ L_{31}D_1 & L_{31}L_{21}D_1 + L_{32}D_2 & L_{31}^2 D_1 + L_{32}^2 D_2 + D_3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 0 \\ 1 & 0 & -2 \end{pmatrix}$$

$\Rightarrow D_1 = A_{11} = 1 \Rightarrow L_{21} \times 1 = 2$, $L_{21} = 2 \Rightarrow 2^2 \times 1 + D_2 = 3,\ D_2 = -1 \Rightarrow \cdots \Rightarrow D_1 = 1, L_{21} = 2, D_2 = -1, L_{31} = 1, L_{32} = 2, D_3 = 1_\square$

(b) eigenvalue decomposition:

$det(\lambda I - A) = 0 \Rightarrow det(\begin{pmatrix} \lambda - 1 & 2 & 1 \\ 2 & \lambda - 3 & 0 \\ 1 & 0 & \lambda + 2 \end{pmatrix}) = 0 \Rightarrow (\lambda - 1)(\lambda - 3)(\lambda + 2) - (\lambda - 3) - 4(\lambda + 2) = 0 \Rightarrow \lambda^3 - 2\lambda^2 - 10\lambda + 1 = 0 \Rightarrow \lambda_1 = -2.3794, \lambda_2 = 0.0982, \lambda_3 = 4.2812_\square$

Problem#2

(a)

$$\|Qx\| = \sqrt{(Qx)^T(Qx)} \overset{Q\ is\ orthogonal}{=} \sqrt{x^T Q^T Q x} = \sqrt{x^T x} = \|x\|_\square$$

(b) Since A is symmetric, eigenvectors matrix is orthogonal.

$Let\ A\ be\ n \times n\ symmetric\ metrix,$

$\|A\| = \underset{\|x\|=1}{max} \|Ax\| = \underset{\|x\|=1}{max} \sqrt{x^T A^T Ax} = \underset{\|x\|=1}{max} \sqrt{x^T VDV^{-1}VDV^{-1}x} = \underset{\|x\|=1}{max} \sqrt{x^T VDDV^{-1}x} \overset{Let\ z=x^T V}{=} \underset{\|z\|=1}{max} \sqrt{zDDz^T} = \underset{\|z\|=1}{max} \sqrt{\lambda_1^2 \|z_1\| + \lambda_2^2 \|z_2\| + \cdots + \lambda_n^2 \|z_n\|} = \underset{i}{max} \sqrt{\lambda_i^2} = \underset{i}{max} |\lambda_i|_\square$

Problem#3

$f = (1-x)^2 + 100(y-x^2)^2$

(a) $\nabla f = (\ 2(1-x)(-1) + 200(y-x^2)(2x)\ ,\ 200(y-x^2)\ )$

$H = \begin{pmatrix} 1200x^2 - 400y + 2 & -400x \\ -400x & 200 \end{pmatrix}_\square$

(b) Code: my_linesearch.m

(c)