

# **IE2110**

## **Signals and Systems Part 1**

### **1.4 Properties of Signals**

**with Instructor:  
A/P Teh Kah Chan**



# Outline of Signals & Systems - Part 1

1. Signals and Systems
  - 1.1 Classification of Signals ✓
  - 1.2 Elementary and Singularity Signals ✓
  - 1.3 Operations on Signals ✓
  - 1.4 **Properties of Systems**
2. Linear Time-Invariant (LTI) Systems
  - 2.1 Discrete-Time and Continuous-Time LTI Systems
  - 2.2 Convolution
  - 2.3 LTI System Properties
  - 2.4 Correlation Functions

## 1.4 Properties of Systems

- 1) Stability
- 2) Memory
- 3) Causality
- 4) Linearity
- 5) Time Invariant

## 1.4 Properties of Systems

A system refers to any physical device (i.e., communication channels, filters) that produces an output signal  $y(t)$  in response to an input signal  $x(t)$

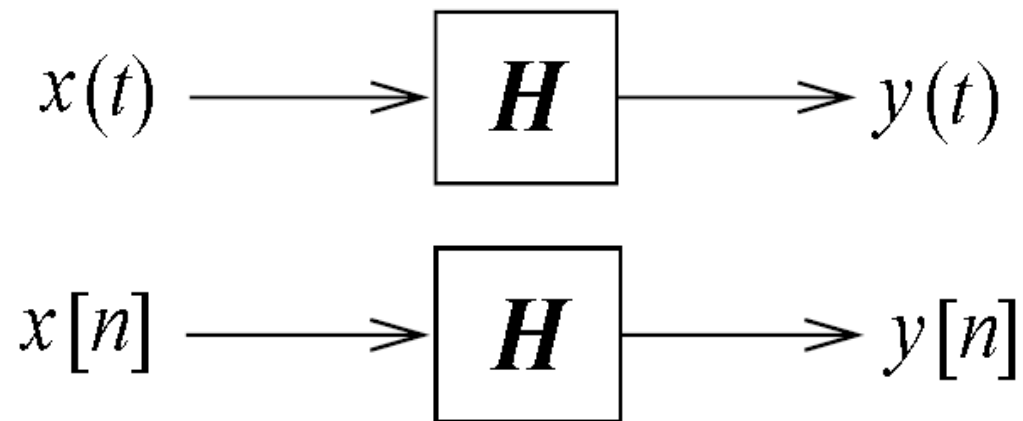


Figure 33: Block diagram representation of a system

## 1.4 Properties of Systems

### 1) Stability

- A system is said to be bounded-input bounded-output (BIBO) stable if and only if every bounded input (i.e.,  $|x(t)| < \infty$  for all  $t$ , or  $|x[n]| < \infty$  for all  $n$ ) results in bounded output

An example of a BIBO stable system

$$y[n] = r^n x[n] u[n], \quad |r| < 1$$

An example of a BIBO unstable system

$$y[n] = r^n x[n] u[n], \quad |r| > 1$$

## 1.4 Properties of Systems

### 2) Memory

- A system is said to possess memory if its output signal depends on past or future values of the input signal

An example of a system with memory

$$y[n] = x[n] + x[n - 1] + x[n - 2]$$

- A system is memoryless if its output signal depends only on the present value of the input signal

An example of a memoryless system

$$y(t) = x^2(t)$$

## 1.4 Properties of Systems

### 3) Causality

- A system is causal if the present value of the output signal depends only on the present or past values of the input signal

An example of a causal system

$$y[n] = \frac{1}{3} (x[n] + x[n-1] + x[n-2])$$

- A system is noncausal if the present value of the output signal depends on the future values of the input signal
- A noncausal system is not physically realizable in real time

An example of a noncausal system

$$y[n] = \frac{1}{3} (x[n+1] + x[n] + x[n-1])$$

## 1.4 Properties of Systems

### 4) Linearity

- A system is linear if the principle of superposition holds, i.e., if input signal is  $x_3(t) = a_1x_1(t) + a_2x_2(t)$ , then the output signal is  $y_3(t) = a_1y_1(t) + a_2y_2(t)$  for any constants  $a_1$  and  $a_2$

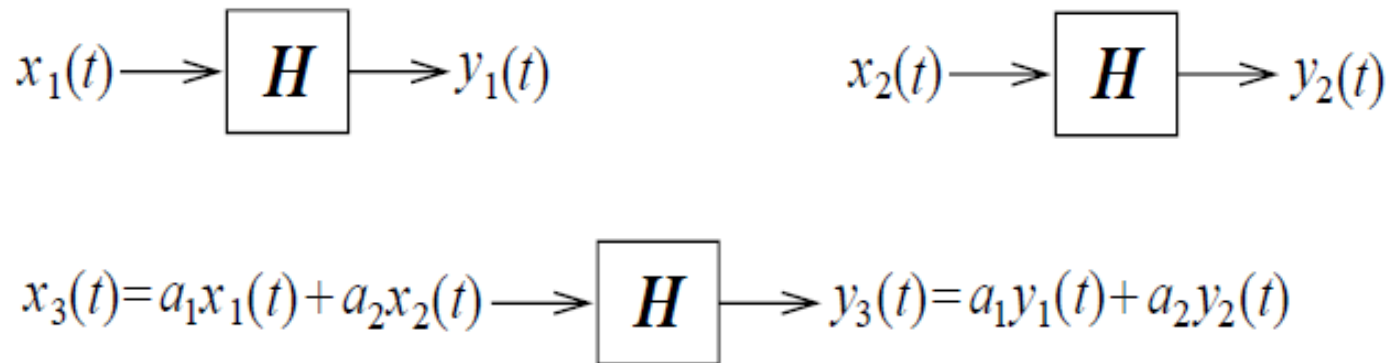


Figure 34: A linear system

## 1.4 Properties of Systems

Example 14:

A system is shown below (Figure 35). Determine whether it is a linear system.

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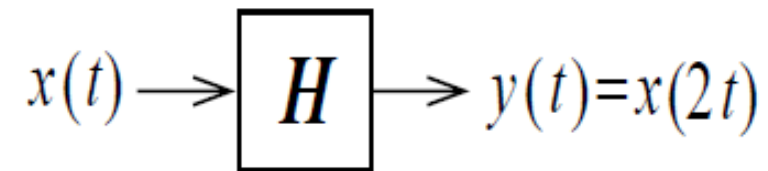


Figure 35: A linear system example

## 1.4 Properties of Systems

Example 14:

Determine whether it is a linear system.

$$x(t) \rightarrow \boxed{H} \rightarrow y(t) = x(2t)$$

$$x_1(t) \rightarrow \boxed{H} \rightarrow y_1(t) = x_1(2t)$$

$$x_2(t) \rightarrow \boxed{H} \rightarrow y_2(t) = x_2(2t)$$

$$\begin{aligned} x_3(t) = a_1 x_1(t) + a_2 x_2(t) \rightarrow \boxed{H} \rightarrow y_3(t) &= a_1 x_1(2t) + a_2 x_2(2t) \\ &= a_1 y_1(t) + a_2 y_2(t) \end{aligned}$$

Figure 36: A linear system example

In this case, the principle of superposition holds, hence it is a linear system.

## 1.4 Properties of Systems

### 5) Time Invariant

- A system is time invariant if for any delayed  $x(t - T)$ , the output is delayed by the same amount  $y(t - T)$

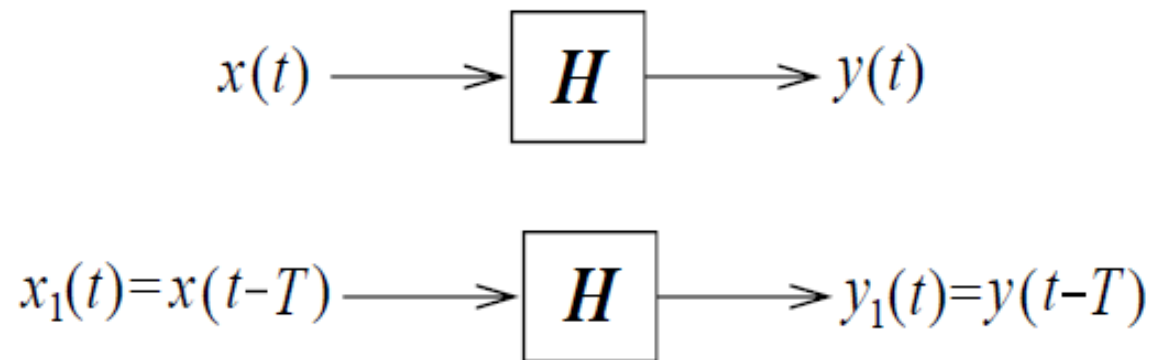


Figure 37: A time invariant system

## 1.4 Properties of Systems

Example 15:

For the system as shown below (Figure 38) with  $y(t) = x(t) + c$ , where  $c$  is an arbitrary constant, determine whether it is a time invariant system.

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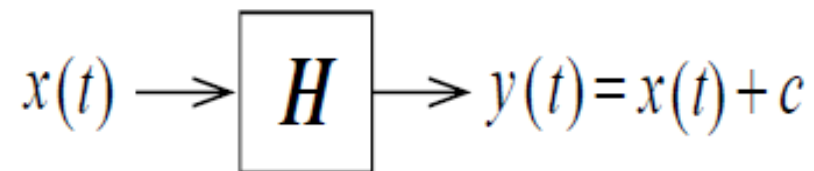


Figure 38: A time invariant system example

## 1.4 Properties of Systems

Example 15:

Determine whether it is a time invariant system.

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t) = x(t) + c$$

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t) = x(t) + c$$

$$x_1(t) = x(t-T) \longrightarrow \boxed{H} \longrightarrow y_1(t) = x(t-T) + c \\ = y(t-T)$$

Figure 39: A time invariant system example

In this case, the system is time invariant.

## 2. Linear Time-Invariant (LTI) Systems

### Linear Time Invariant (LTI)

- A system is linear time invariant if it satisfies both conditions of linear and time invariance
- A LTI system can be analyzed in both time domain and frequency domain

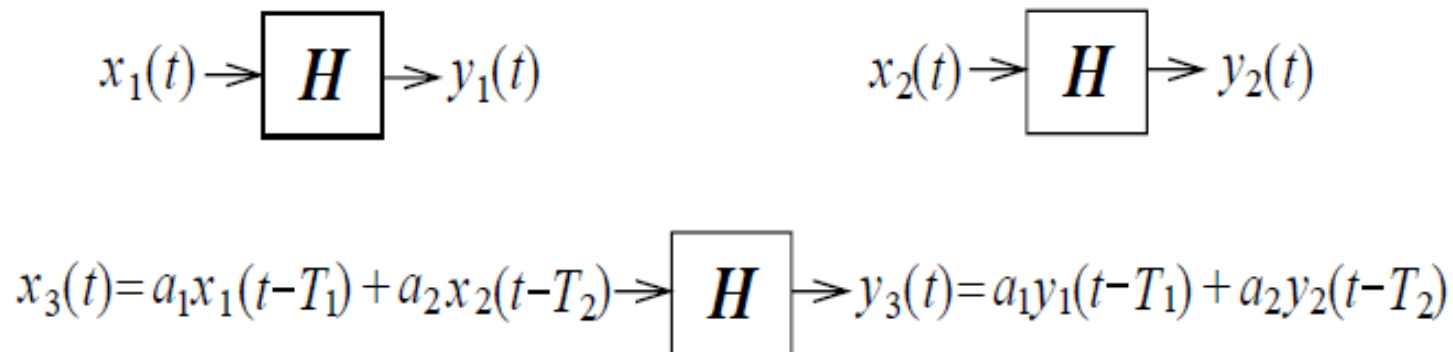


Figure 40: An LTI system

## 2. Linear Time-Invariant (LTI) Systems

Example 16:

Determine whether the system below given by  $y(t) = x(2t)$  in Example 14 is an LTI system.

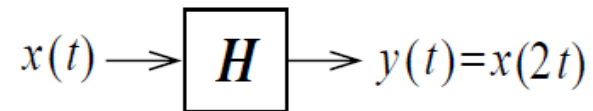


Figure 35

From Example 14, the system is linear.

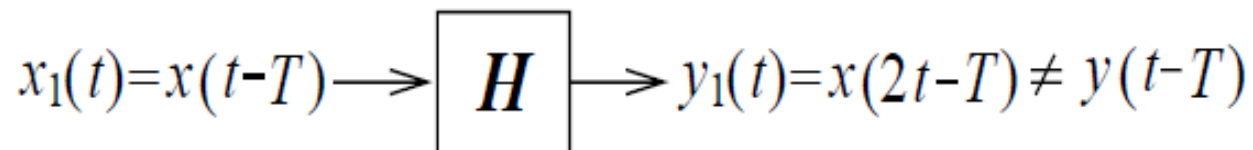


Figure 41: A non-LTI system example

However, the system is not time invariant, hence it is not an LTI system.

# Properties of Systems Summary 6

- ☐ 1) Stability: Bounded input results in bounded output.
- ☐ 2) Memory: Output depends on past and/or future values of input.
- ☐ 3) Causality: Output does not depend on future values of input.
- ☐ 4) Linearity: Principle of superposition holds.
- ☐ 5) Time Invariant: For any delayed input  $x(t-T)$ , the output is delayed by the same amount  $y(t-T)$ .
  - Linear Time-Invariant (LTI) Systems



***You have reached the end of 1.4: Properties of Systems.  
Consider mapping out your learning and proceed.***

# **IE2110**

## **Signals and Systems Part 1**

### **2. Linear Time-Invariant (LTI) Systems**

**with Instructor:  
A/P Teh Kah Chan**



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1. Signals and Systems
  - 1.1 Classification of Signals ✓
  - 1.2 Elementary and Singularity Signals ✓
  - 1.3 Operations on Signals ✓
  - 1.4 Properties of Systems
2. **Linear Time-Invariant (LTI) Systems**
  - 2.1 **Discrete-Time** ➡ and Continuous-Time **LTI Systems**
  - 2.2 Convolution
  - 2.3 LTI System Properties
  - 2.4 Correlation Functions

## 2.1 Discrete-Time and Continuous-Time LTI Systems

Analysis of DT and CT LTI Systems:

- Any LTI system can be uniquely defined by its impulse response

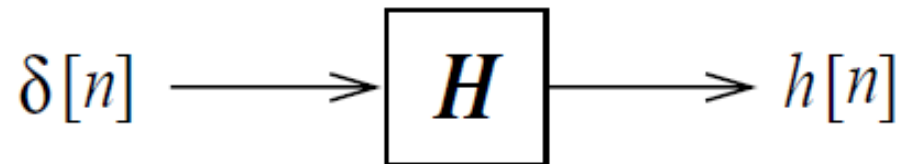
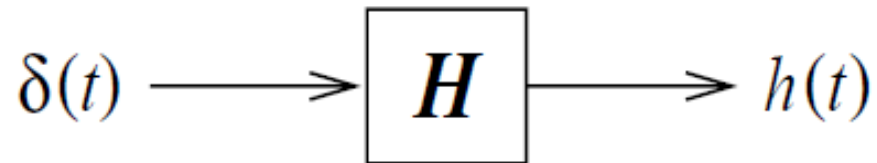


Figure 42: Impulse response of an LTI system

## 2.1 Discrete-Time and Continuous-Time LTI Systems

The output of any LTI system is the convolution of the input signal and its impulse response

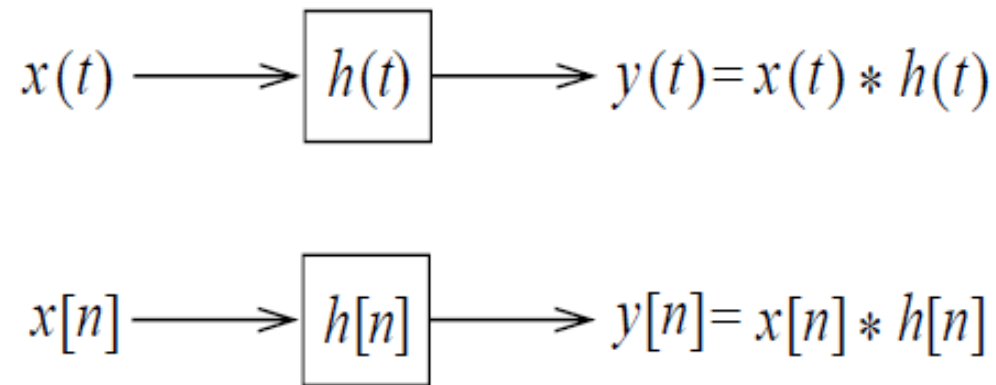


Figure 43: System response of an LTI system

## 2.1 Discrete-Time and Continuous-Time LTI Systems

The discrete time convolution (convolution sum) is defined as

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

The continuous time convolution (convolution integral) is defined as

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

## 2.1a Discrete-Time LTI Systems

Example 17:

Sketch the waveform of  $y[n] = x[n] * h[n]$  using the graphical approach for convolution sum.

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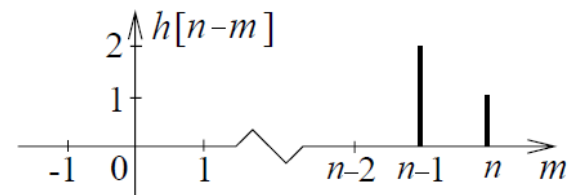
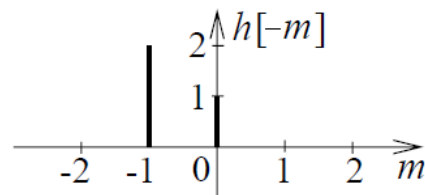


Figure 44: Example on convoluted sum

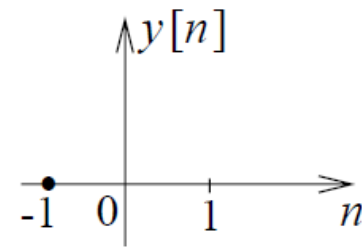
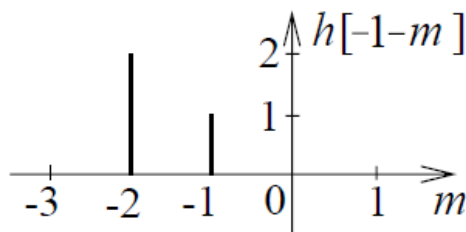
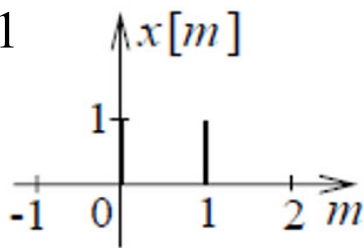
## 2.1a Discrete-Time LTI Systems

Example 17:  
Sketch the waveform  
of  $y[n] = x[n] * h[n]$

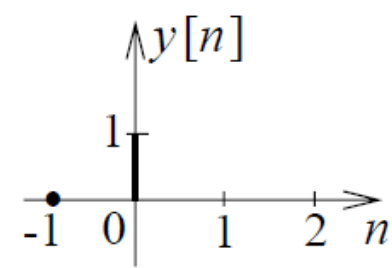
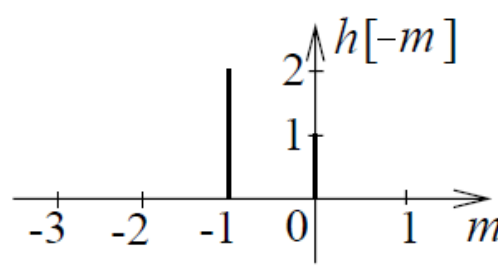
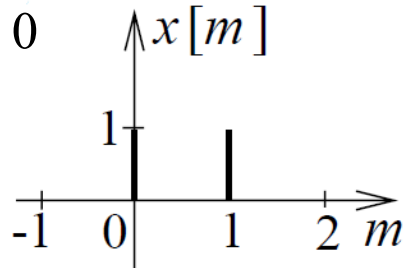
$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$



(i)  $n = -1$



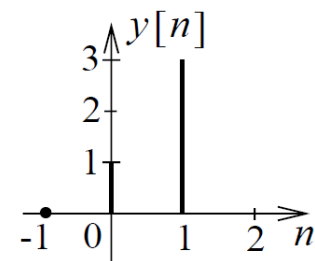
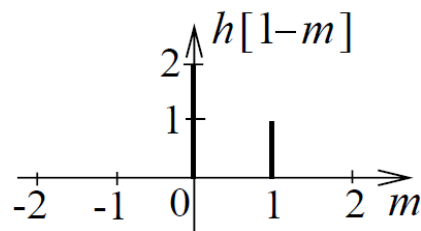
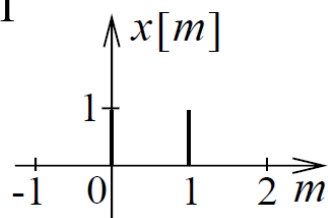
(ii)  $n = 0$



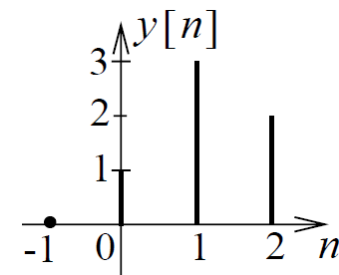
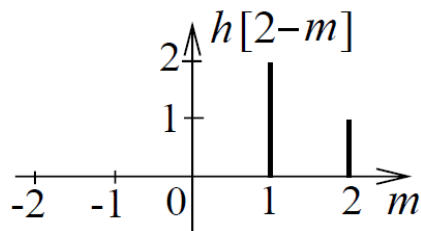
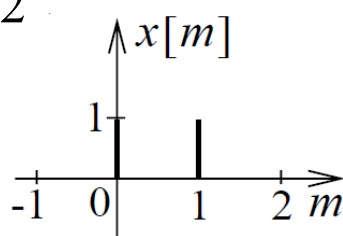
## 2.1a Discrete-Time LTI Systems

Example 17:

(iii)  $n = 1$



(iv)  $n = 2$



(v)  $n = 3$

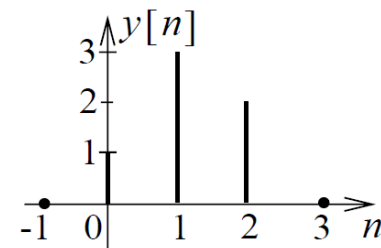
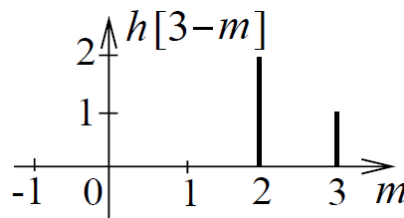
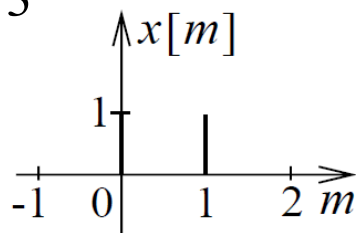


Figure 45: Solution for example on convolution sum

## 2.1 Discrete-Time and Continuous-Time LTI Systems

### Summary 7

#### □ Analysis of DT Systems

- Any DT LTI system can be uniquely defined by its impulse response,  $h[n]$ .
- The output of a DT LTI system is the convolution of the input signal and its impulse response.
- The DT convolution (or convolution sum) is defined as

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

- The graphical approach for evaluating the convolution sum.



***You have reached the end of 2.1a. Do reflect on your level of understanding.  
Please proceed to 2.1b Continuous-Time LIT Systems.***

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## **Signals and Systems Part 1**

### **2.0 Linear Time-Invariant (LTI) Systems**

**with Instructor:  
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  - 1.4 Properties of Systems ✓
2. Linear Time-Invariant (LTI) Systems
  - 2.1 Discrete-Time ✓ and **Continuous-Time LTI Systems** ➡
  - 2.2 Convolution
  - 2.3 LTI System Properties
  - 2.4 Correlation Functions

## 2.1b Continuous-Time LTI Systems

Example 18:

Sketch the waveform of  $y(t) = x_1(t) * x_2(t)$  using the graphical approach for convolution integral.

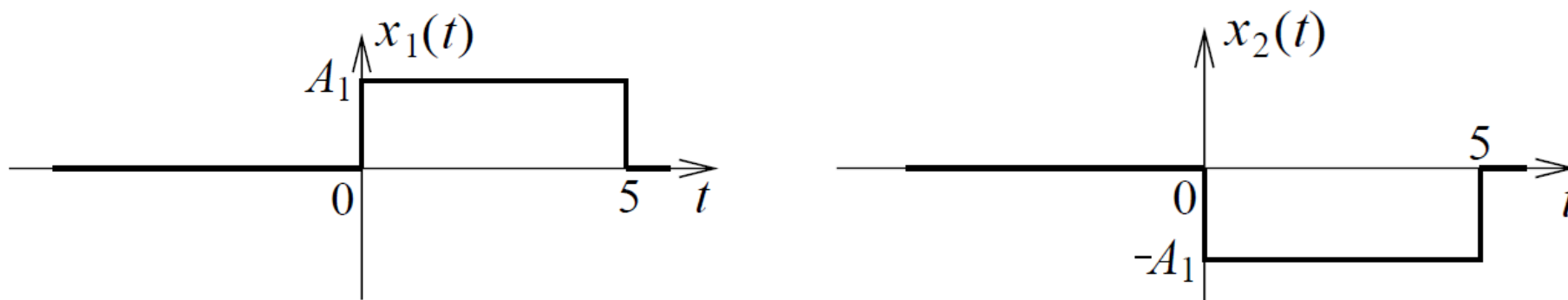
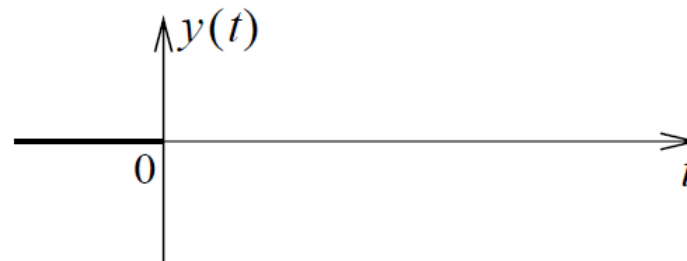
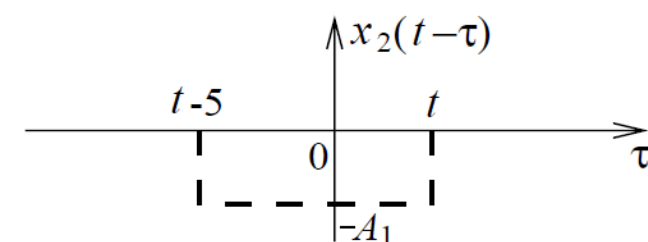
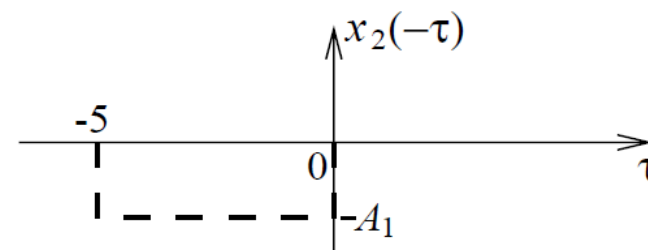
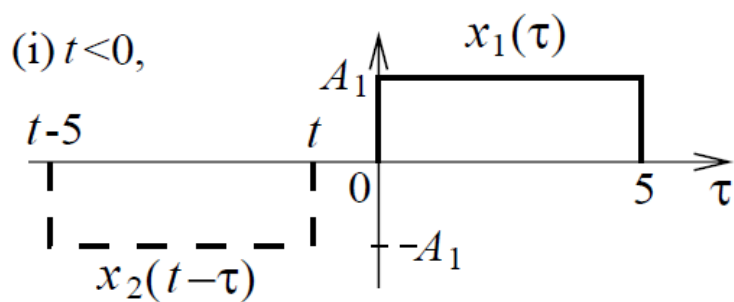
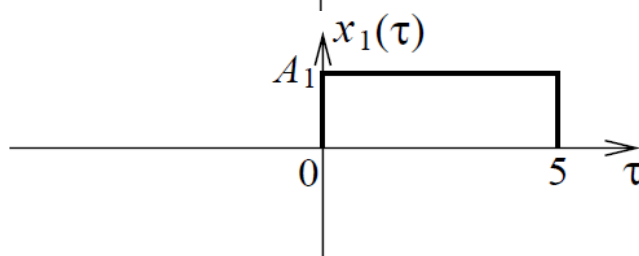
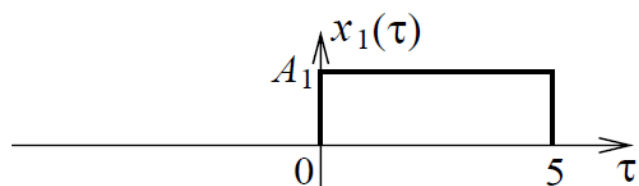


Figure 46: Example on convolution integral

## 2.1b Continuous-Time LTI Systems

$$y(t) = x(t) * x(t) = \int_{-\infty}^{\infty} x(\tau)x(t - \tau)d\tau$$



## 2.1b Continuous-Time LTI Systems

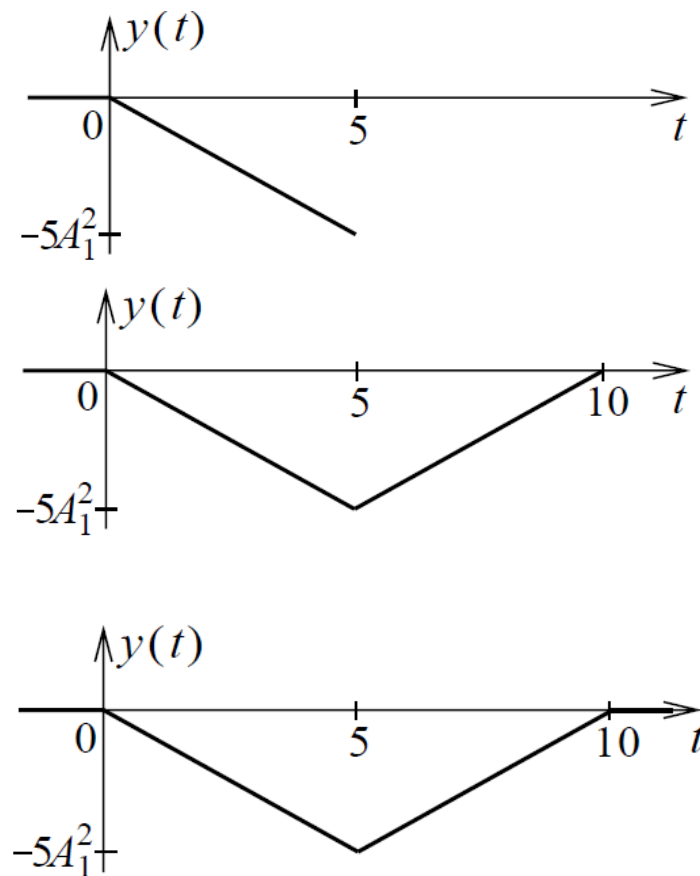
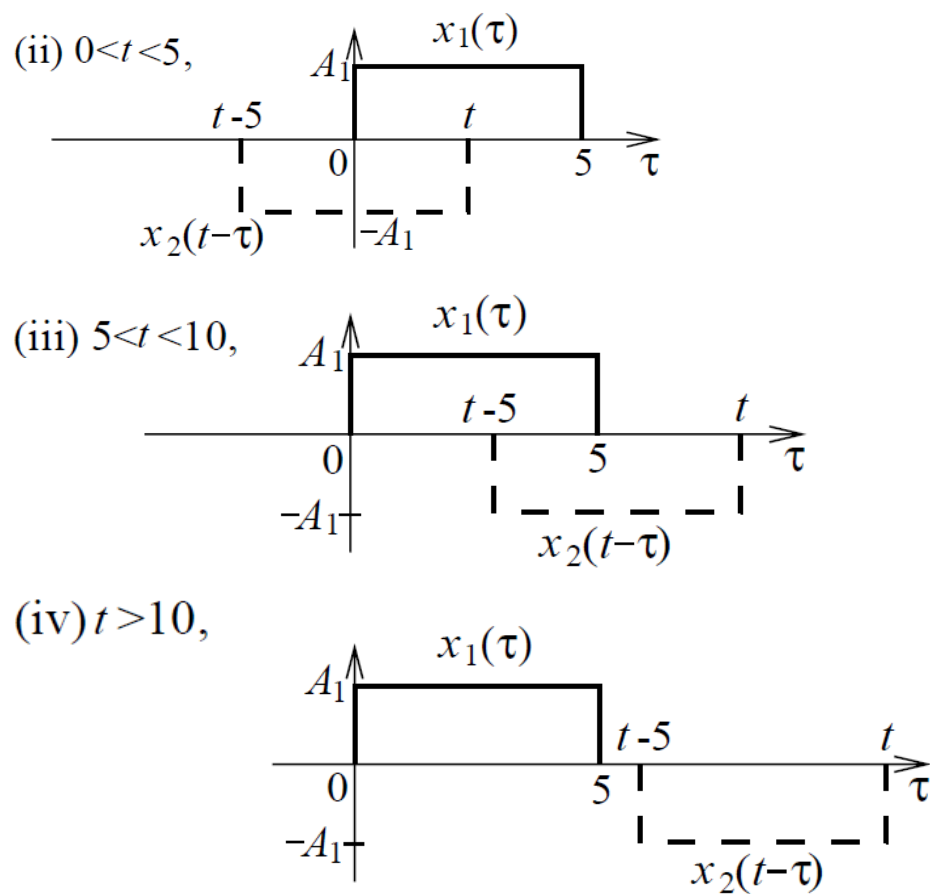


Figure 47: Solution for example on convolution integral

## 2.1 Discrete-Time and Continuous-Time LTI Systems

### Summary 8

#### □ Analysis of CT Systems

- Any CT LTI system can be uniquely defined by its impulse response,  $h(t)$ .
- The output of a CT LTI system is the convolution of the input signal and its impulse response.
- The CT convolution (or convolution integral) is defined as

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$$

- The graphical approach for evaluating the convolution integral.



***You have reached the end of 2.1. Do reflect on your level of understanding.  
Please proceed to 2.2 Convolution.***