IE2110
Signals and Systems

Course Overview

Course Overview	
Academic Units	3
General Structure	Online learning activities & face-to-face tutorials per week
Pre-requisite	MH2810 Mathematics A or (MH1810 Mathematics I & MH1811 Mathematics II)
Description	Signals and Systems provides basic concepts of signals, Fourier analysis and linear time-invariant systems in a generic engineering context. It includes applications in control engineering, communications and signal processing. This course brings continuous-time and discrete-time concepts together in a unified way and relates them through sampling theory and modulation.

Course Overview (cont'd)

Contents	1. Signals and Systems 2. Linear Time-Invariant (LTI) Systems 3. Fourier Representation of Signals and LTI Systems 4. Sampling 5. Modulation
Learning Outcome	Through this course, students should be able to understand the representation of continuous-time and discrete-time signals; their frequency characteristics and Fourier spectrum; representation and characteristics of linear time-invariant systems in both time and frequency domains; the principles of sampling a continuous-time signal to a discrete-time one; and the concepts of modulation. This course is a pre-requisite for IE3012 Communication Principles IE3014 Digital Signal Processing

Course Overview (cont'd)		
Continuous Assessment (CA)	40%	Individual Readiness Assessment (in-class test) (20%) 1 Quiz (10%) 2 Labs (5% x 2 = 10%) - L2110A and L2110B
Final Examination	60%	

Important Notes on Individual Readiness Assessment

- **Individual Readiness Assessment** (closed-book) contributes to 20% of the final marks
- **Individual Readiness Assessment** will be conducted by the respective tutor during tutorial classes on weeks #2, #3, #4, #6, #7, #8, #11 and #12; thus, 8 IRAs in total.
- Students will be asked to solve multiple-choice questions (MCQs) and/or fill-in-the-blank questions, which are related to the topics covered in the LAMS sequence of the previous weeks.
- All IRAs will be conducted at the beginning of tutorial session, with a duration of **15 minutes**. Any tutorial question discussion or possible summary should be conducted ***after*** the completion of IRA.
- IRAs will be returned to students in the subsequent week for learning/feedback purposes
- **No make-up** for any IRA, as the best 6 out of the 8 sessions will be selected to compute the final IRA score.

Important Notes on Quiz

There will be one quiz (closed-book) for the entire semester

- Quiz #1 (10%): Topics to be tested include materials covered in Tutorials #1 to #7, it will be conducted during tutorial class on week #9 (13 Oct 2025 to 17 Oct 2025)
- To answer a few compulsory questions within 50 minutes
- Students can only take this quiz in their respective tutorial group
- Students must present ID (with photo) for taking attendance
- Zero mark will be given for absentees without valid reasons or MCs
- Absentees must write in to the tutor through email within **THE SAME DAY OR EARLIER** of Quiz to request a make-up (Failure to do so will result in a zero mark)

IE2110 WEEKLY STUDY GUIDE
AY2025/2026 Semester 1

Week Number	LAMS Sequence, Note's Pages and Topics	IRA and Tutorial
Part 1: (Video recorded by Prof. Teh Kah Chan)		
#1: (11 Aug–15 Aug)	Module 1 to Module 3 (Pages 1-45) Classification of Signals	General introduction by respective class tutor. Optional: discussion on Preparatory Exercise
#2: (18 Aug–22 Aug)	Module 4 to Module 5 (Pages 46-81) Operations of Signals	Conduct IRA #1 (of previous week #1 LAMS) and Tutorial #1.
#3: (25 Aug–29 Aug)	Module 6 to Module 8 (Pages 82-112) LTI Systems & Properties	Conduct IRA #2 (of previous week #2 LAMS) and Tutorial #2.
#4: (1 Sep – 5 Sep)	Module 9 to Module 10 (Pages 113-134) Convolution	Conduct IRA #3 (of previous week #3 LAMS) and Tutorial #3.
#5: (8 Sep – 12 Sep)	Module 11 to Module 13 (Pages 135-159) Correlation	Conduct Tutorial #4. **No IRA** this week.

Part 2: (Video recorded by Prof. Er Meng Hwa)		
#6: (15 Sep – 19 Sep)	Module 1 (Pages 1-50) Sinusoids	Conduct IRA #4 (of previous week #5 LAMS) and Tutorial #5.
#7: (22 Sep – 26 Sep)	Module 2 (Pages 1-41) Line Spectra and Fourier Series	Conduct IRA #5 (of previous week #6 LAMS) and Tutorial #6.
Recess Week		
#8: (6 Oct –10 Oct)	Module 2 (Pages 42-59) Line Spectra and Fourier Series Module 3 (Pages 1-22) Fourier Transform and Continuous Spectra	Conduct IRA #6 (of previous week #7 LAMS) and Tutorial #7.
#9: (14 Oct –18 Oct)	Module 3 (Pages 23-58) Fourier Transform and Continuous Spectra	Conduct **Quiz** (coverage is from Tutorials 1 to 7) and Tutorial #8. **No IRA** this week.
#10:(20 Oct –24 Oct)	Module 3 (Pages 59-83) Fourier Transform and Continuous Spectra Module 4 (Pages 1-24) Frequency Domain Analysis of LTI Systems	Conduct Tutorial #9 via MS TEAMS or ZOOM. **No IRA** this week.
#11: (27 Oct –31 Oct)	Module 5 (Pages 1-44) Sampling and Aliasing	Conduct IRA #7 (of previous weeks #9 and #10 LAMS) and Tutorial #10.
Part 3: (Video recorded by Prof. Teh Kah Chan)		
#12: (3 Nov – 7 Nov)	Module 1 to Module 3 (Pages 1-34) Modulation	Conduct IRA #8 (of previous week #11 LAMS) and Tutorial #11.
#13: (10 Nov–14 Nov)	Course Revision	Conduct Tutorial #12. **No IRA** this week.

Recommended Readings

Textbook

1. M. J. Roberts, *Fundamentals of Signals and Systems,* McGraw-Hill, International Edition, 2008. (TK5102.9.R646F)

References

1. A. V. Oppenheim, A. S. Willsky, and S. H. Nawab, *Signals and Systems,* Prentice-Hall, 2nd Edition, 1997. (QA402.P62)

2. S. Haykin and B. V. Veen, *Signals and Systems,* Wiley, 2nd Edition, 2003. (TK5102.5.H419)

3. M. K. Mandal and A. Asif, *Continuous and Discrete Time Signals and Systems,* Cambridge University Press, 1st Edition, 2007. (QA402.M271)

4. Hwei P. Hsu, *Schaums Outlines Signals and Systems,* Mc-Graw Hill, 3rd Edition, 2013. (TK5102.92.H873)

Relevant Formulae

Fourier Transform Pairs			Fourier Transform Operations	
$x(t)$	$X(f)$		$x(t)$	$X(f)$
$e^{-at}u(t)$	$\dfrac{1}{a+j2\pi f}$	$a>0$	$kx(t)$	$kX(f)$
$e^{at}u(-t)$	$\dfrac{1}{a-j2\pi f}$	$a>0$	$x_1(t)+x_2(t)$	$X_1(f)+X_2(f)$
$e^{-a\|t\|}$	$\dfrac{2a}{a^2+(2\pi f)^2}$	$a>0$	$x^*(t)$	$X^*(-f)$
$\delta(t)$	1		$X(t)$	$x(-f)$
1	$\delta(f)$		$x(at)$	$\dfrac{1}{\|a\|}X\left(\dfrac{f}{a}\right)$
$e^{j2\pi f_0 t}$	$\delta(f-f_0)$		$x(t-t_0)$	$X(f)e^{-j2\pi f t_0}$
$\cos 2\pi f_0 t$	$\frac{1}{2}[\delta(f-f_0)+\delta(f+f_0)]$		$x(t)e^{j2\pi f_0 t}$	$X(f-f_0)$
$\sin 2\pi f_0 t$	$\frac{1}{j2}[\delta(f-f_0)-\delta(f+f_0)]$		$x_1(t)*x_2(t)$	$X_1(f)X_2(f)$
$u(t)$	$\frac{1}{2}\delta(f)+\dfrac{1}{j2\pi f}$		$x_1(t)x_2(t)$	$X_1(f)*X_2(f)$
$\mathrm{rect}\left(\frac{t}{T}\right)=\begin{cases}1, & \|t\|\le T/2,\\ 0, & \|t\|>T/2.\end{cases}$	$T\,\mathrm{sinc}\,(fT)$		$\dfrac{d^n x(t)}{dt^n}$	$(j2\pi f)^n X(f)$
$\Lambda\left(\frac{t}{T}\right)=\begin{cases}1-\frac{\|t\|}{T}, & \|t\|\le T,\\ 0, & \|t\|>T.\end{cases}$	$T\,\mathrm{sinc}^2\,(fT)$		$\int_{-\infty}^{t} x(u)\mathrm{d}u$	$\dfrac{X(f)}{j2\pi f}+\dfrac{X(0)}{2}\delta(f)$
$\mathrm{sinc}\,(2Wt)=\dfrac{\sin(2W\pi t)}{2W\pi t}$	$\dfrac{1}{2W}\mathrm{rect}\left(\dfrac{f}{2W}\right)$			
$\sum_{n=-\infty}^{\infty}\delta(t-nT_0)$	$\dfrac{1}{T_0}\sum_{n=-\infty}^{\infty}\delta(f-nf_0)$	$f_0=\dfrac{1}{T_0}$		

Useful Trigonometric Identities

$\exp(\pm j\theta)=\cos(\theta)\pm j\sin(\theta)$	$\cos(A)+\cos(B)=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
$\cos(\theta)=\frac{1}{2}[\exp(j\theta)+\exp(-j\theta)]$	$\sin(\theta)=\frac{1}{2j}[\exp(j\theta)-\exp(-j\theta)]$
$2\cos(A)\cos(B)=\cos(A-B)+\cos(A+B)$	$\cos^2(A)=\frac{1}{2}[1+\cos(2A)]$
$2\sin(A)\sin(B)=\cos(A-B)-\cos(A+B)$	$\sin^2(A)=\frac{1}{2}[1-\cos(2A)]$
$2\cos(A)\sin(B)=\sin(A+B)-\sin(A-B)$	$\sin(2A)=2\cos(A)\sin(A)$
$\cos(A\pm B)=\cos(A)\cos(B)\mp\sin(A)\sin(B)$	$\sin(A\pm B)=\sin(A)\cos(B)\pm\cos(A)\sin(B)$

Please note that relevant trigonometric identities will be provided if needed.

COURSE OVERVIEW SUMMARY

- Structure of Course
- Learning Objective
- Contents
- Learning Outcome
- Assessment Weightages
- Important Notes on Quizzes and Assignments
- Recommended Readings
- Relevant Formulae

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

IE2110
Signals and Systems
Part I

Overview

with Instructor:

A/P Teh Kah Chan

Outline of Signals & Systems - Part 1

1. Signals and Systems
   - 1.1 Classification of Signals
   - 1.2 Elementary and Singularity Signals
   - 1.3 Operations on Signals
   - 1.4 Properties of Systems

2. Linear Time-Invariant (LTI) Systems
   - 2.1 Continuous-Time and Discrete-Time LTI Systems
   - 2.2 Convolution
   - 2.3 LTI System Properties
   - 2.4 Correlation Functions

Signals and Systems Overview

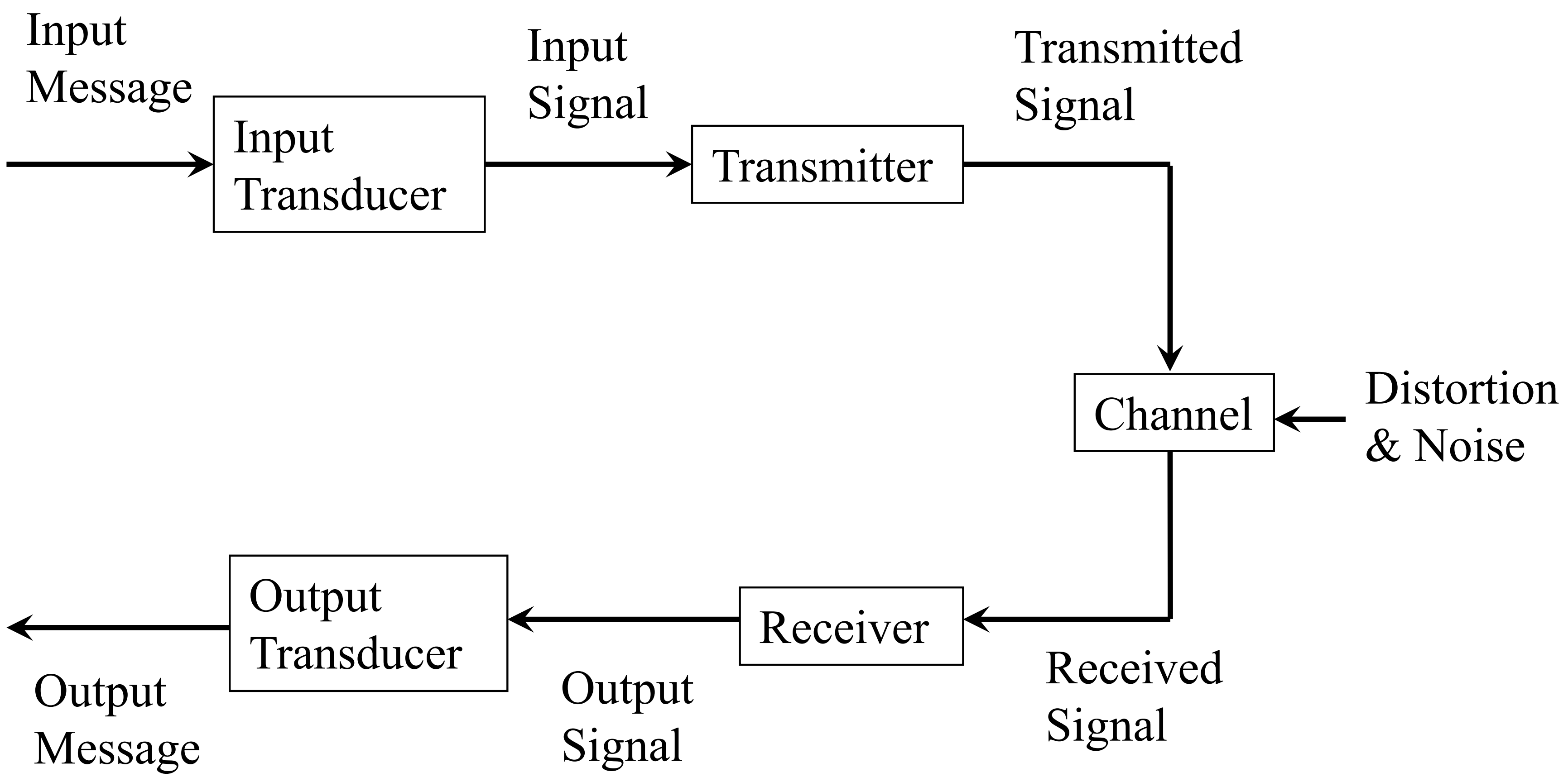


Figure 1: A typical signal and system example

IE2110
Signals and Systems
Part I

1.1 Classification of Signals I

with Instructor:

A/P Teh Kah Chan

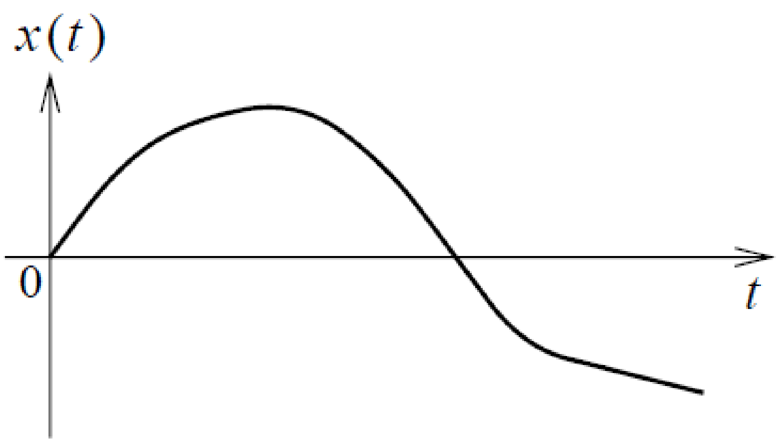
1.1 Classification of Signals

1) **Continuous-Time vs Discrete-Time Signal**
2) **Continuous-Value vs Discrete-Value Signal**
3) **Deterministic vs Random Signal**
4) Even vs Odd Signal
5) Periodic vs Aperiodic Signal
6) Energy-Type vs Power-Type Signal

1) Continuous-Time vs Discrete-Time Signal

- Continuous-Time (CT) Signal: A signal $x(t)$ that is specified for all value of time t
- Discrete-Time (DT) Signal: A signal $y[n]$ that is specified only for integer value of n

Graph of Continuous-Time Signal



Graph of Discrete-Time Signal

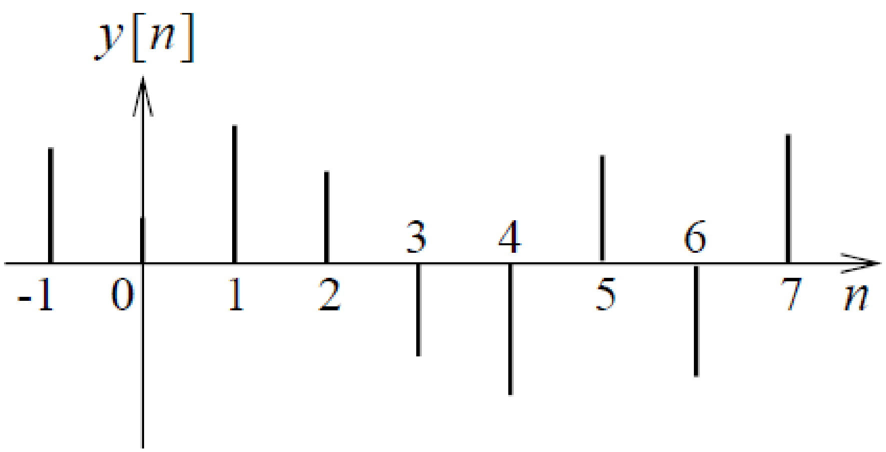
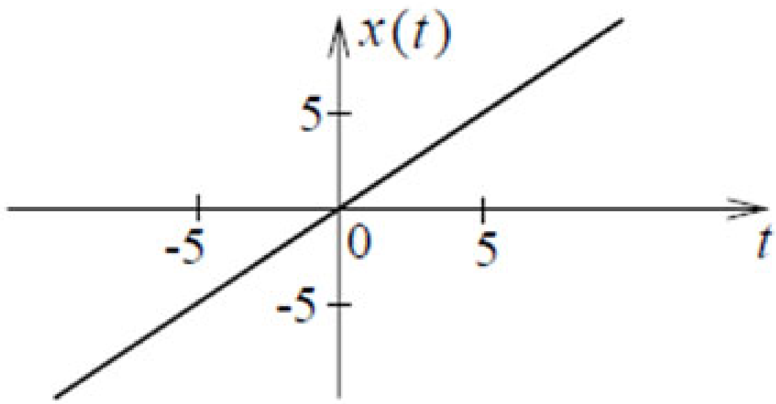


Figure 2: Continuous-Time vs Discrete-Time signal

1) Continuous-Time vs Discrete-Time Signal

Example 1:
Try sketching the waveforms of the CT signal $x(t) = t$ and DT signal $x[n] = n$, respectively.

Graph of CT signal



Graph of DT signal

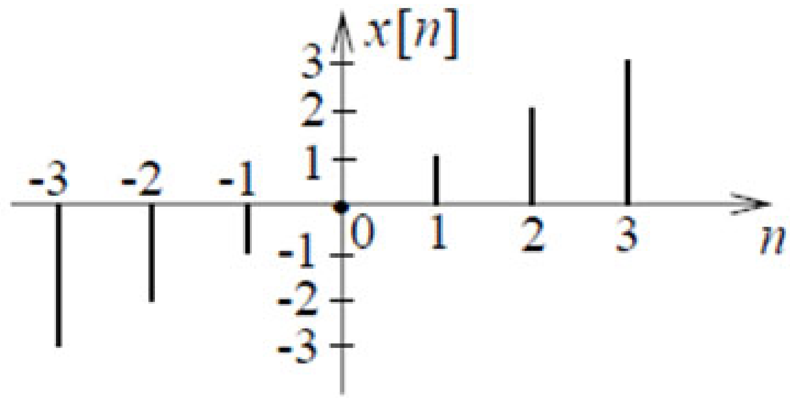
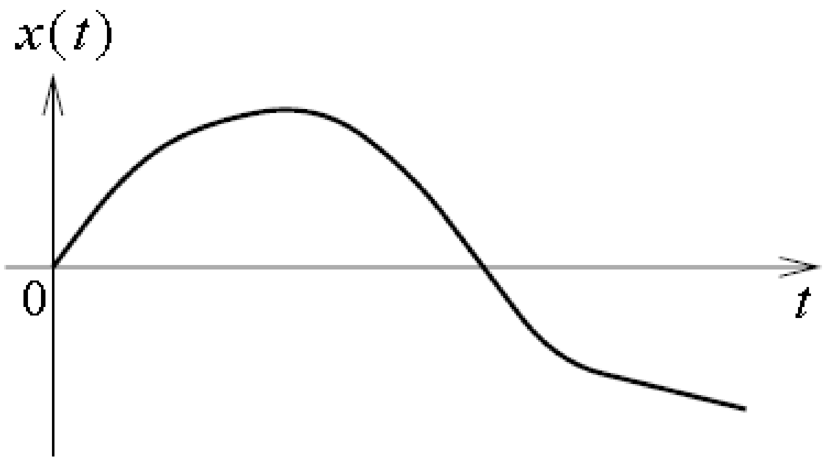


Figure 3: Example of CT and DT signals

2) Continuous-Value vs. Discrete-Value Signal

- Continuous-Value Signal: A signal $x(t)$ whose amplitude can take on any value
- Discrete-Value Signal: A signal $y(t)$ whose amplitude can take on only a finite number of values

Graph of Continuous-Value Signal



Graph of Discrete-Value Signal

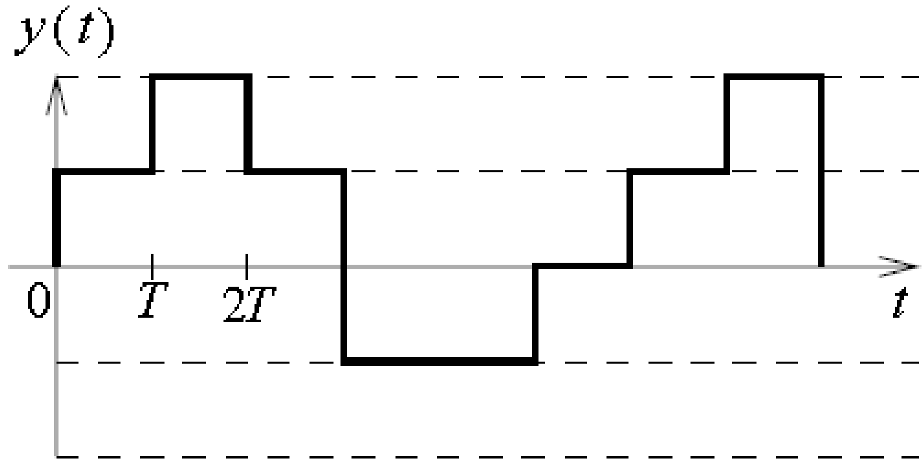


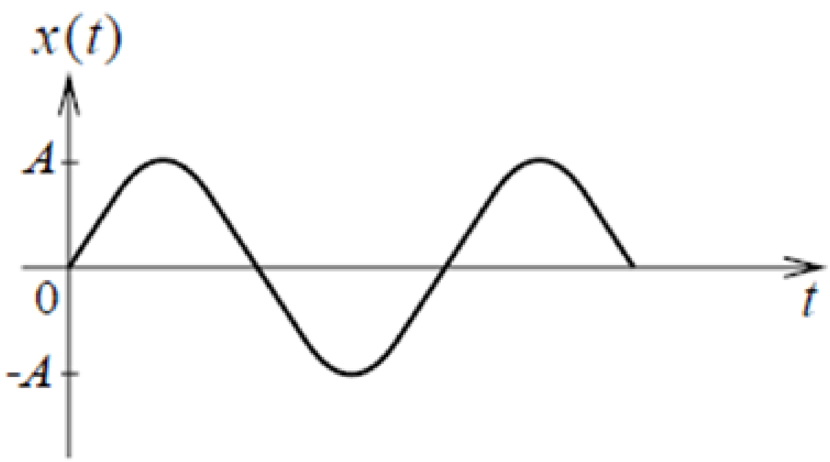
Figure 4: Continuous-Value vs Discrete-Value signal

2) Continuous-Value vs. Discrete-Value Signal

Example 2:

Try sketching the waveforms of the continuous-value signal $x(t) = A\sin(2\pi f_0 t)$ and discrete-value signal $y[n] = (-1)^n$ respectively.

Graph of Continuous-Value Signal



Graph of Discrete-Value Signal

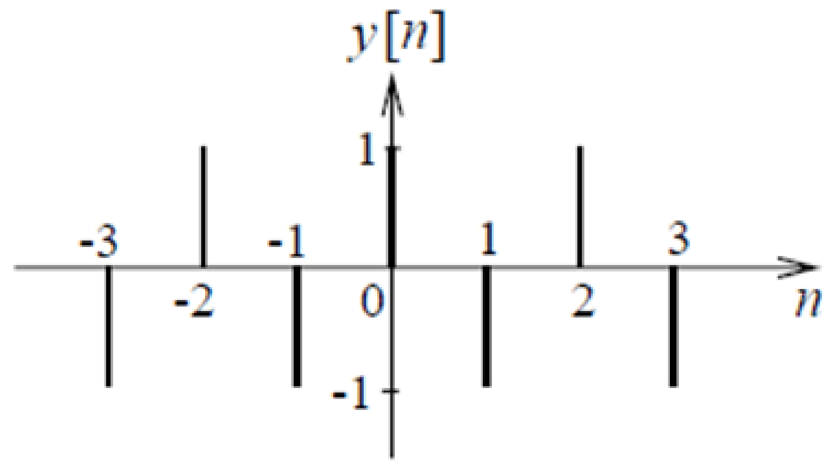
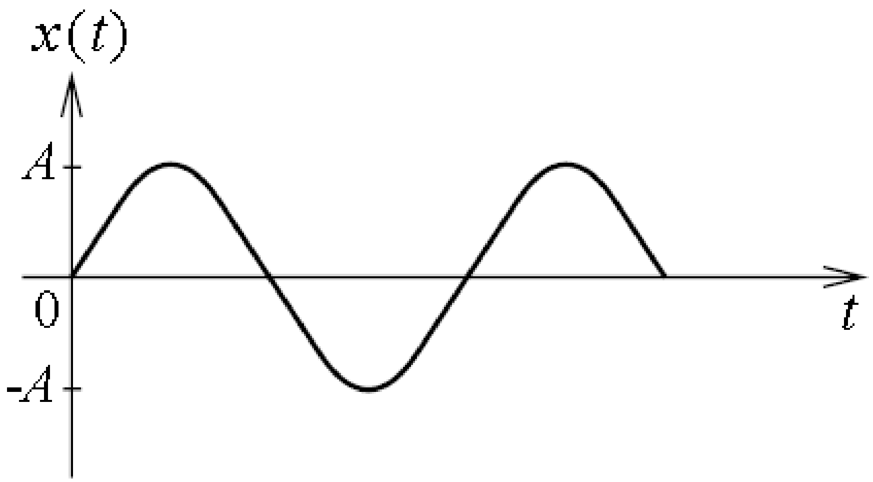


Figure 5: Example of Continuous-Value vs Discrete-Value signals

3) Deterministic vs Random Signal

- Deterministic Signal: A signal $x(t)$ that can be mathematically modeled explicitly as a function of time, i.e., $x(t) = A\sin(2\pi f_0 t)$
- Random Signal: A signal $y(t)$ that is known only in terms of probabilistic description, i.e., noise

Graph of Deterministic Signal



Graph of Random Signal

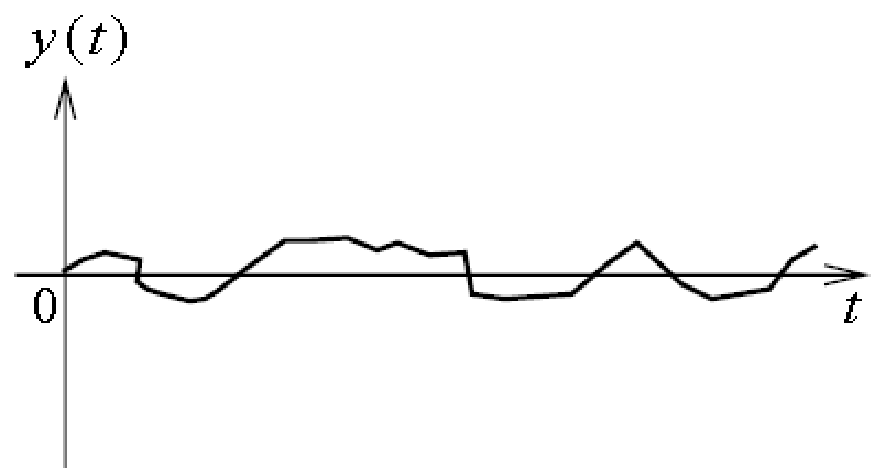


Figure 6: Deterministic vs Random signal

Classification of Signals Summary 1

- ❑ Overview of Signals and Systems
- ❑ 1.1 Classification of Signals
    - 1) Continuous-Time vs Discrete-Time Signal
    - 2) Continuous-Value vs Discrete-Value Signal
    - 3) Deterministic vs Random Signal

You have reached the end of this lesson, you have 3 more types to learn!

Please proceed with the next activity.

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

IE2110
Signals and Systems
Part I

1.1 Classification of Signals II

with Instructor:

A/P Teh Kah Chan

Outline of Signals & Systems - Part 1

1.1 Classification of Signals

1) Continuous-Time vs Discrete-Time Signal ✓
2) Continuous-Value vs Discrete-Value Signal ✓
3) Deterministic vs Random Signal ✓
4) **Even vs Odd Signal**
5) **Periodic vs Aperiodic Signal**
6) **Energy-Type vs Power-Type Signal**

4) Even vs Odd Signal

- Even Signal: A signal $x_e(t)$ that satisfies the condition $x_e(t) = x_e(-t)$
- Odd Signal: A signal $x_o(t)$ that satisfies the condition $x_o(t) = -x_o(-t)$

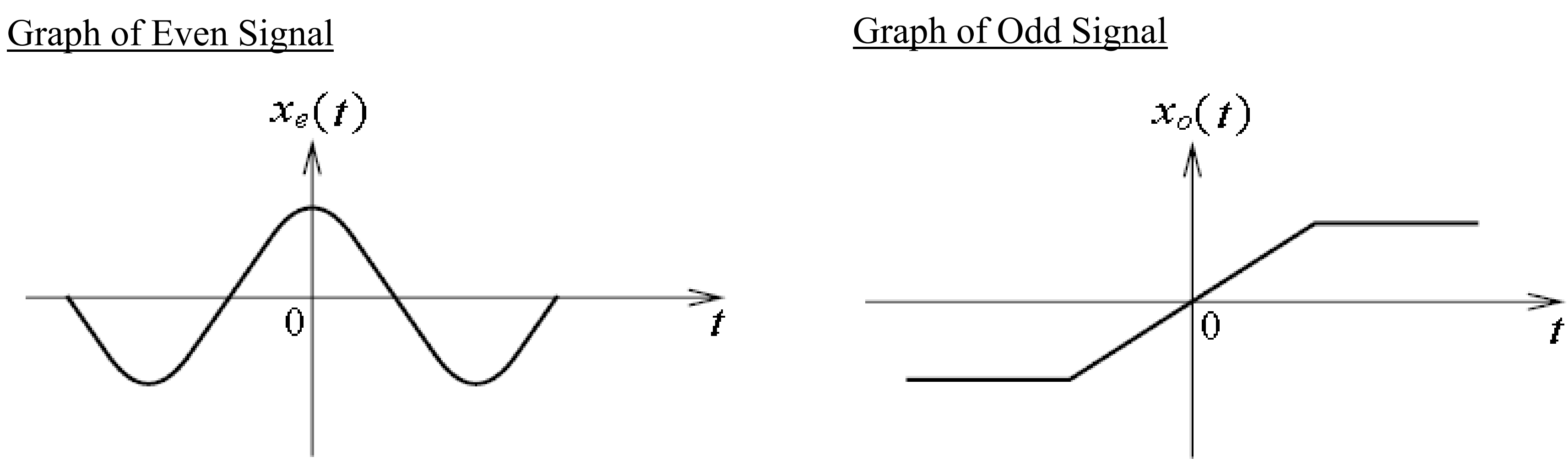


Figure 7: Even vs Odd signal

4) Even vs Odd Signal

- Any deterministic signal $x(t)$ can be decomposed into sum of an even and odd signal

$$x(t) = x_e(t) + x_o(t)$$

where

$$x_e(t) = \frac{1}{2}\{x(t) + x(-t)\}$$

and

$$x_o(t) = \frac{1}{2}\{x(t) - x(-t)\}$$

4) Even vs Odd Signal

- The product of two **even** signals is an **even** signal
- The product of two **odd** signals is an **even** signal
- The product of an **even** signal and an **odd** signal is an **odd** signal
- Note that

$$\int_{-T_0}^{T_0} x_e(t)\, dt = 2\int_0^{T_0} x_e(t)\, dt$$

and

$$\int_{-T_0}^{T_0} x_o(t)\, dt = 0$$

4) Even vs Odd Signal

Example 3:

Show that the signal $x(t) = A\sin(2\pi f_0 t)$ is an odd signal

Since $$\begin{aligned} x(-t) &= A\sin\{2\pi f_0(-t)\} \\ &= -A\sin(2\pi f_0 t) \\ &= -x(t) \end{aligned}$$

hence, $x(t)$ is an odd signal.

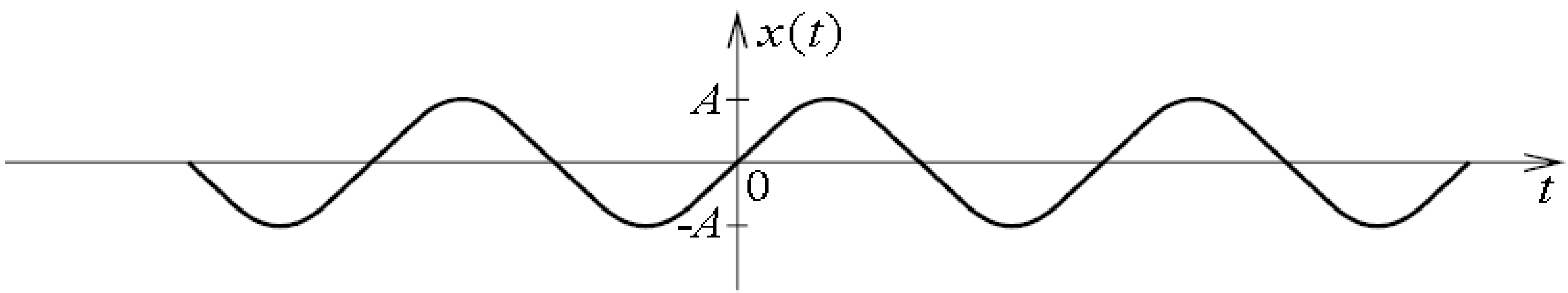


Figure 8: An odd signal example

4) Even vs Odd Signal

Example 4:
Find the even and odd components of the signal $x(t) = \cos(t) + \sin(t)\cos(t)$.

The even component of $x(t)$ is

$$x_e(t) = \frac{1}{2}\{x(t) + x(-t)\}$$

$$= \frac{1}{2}\{\cos(t) + \sin(t)\cos(t) + \cos(-t) + \sin(-t)\cos(-t)\}$$

$$= \cos(t)$$

The odd component of $x(t)$ is

$$x_o(t) = \frac{1}{2}\{x(t) - x(-t)\}$$

$$= \frac{1}{2}\{\cos(t) + \sin(t)\cos(t) - \cos(-t) - \sin(-t)\cos(-t)\}$$

$$= \sin(t)\cos(t)$$

4) Even vs Odd Signal

Example 5: Evaluate $\int_{-T_0}^{T_o} x(t)\,dt$ where $x(t) = t^3\cos^3(10t)$.

Since

$$x(-t) = (-t)^3 \cos^3\{10(-t)\}$$

$$= - t^3 \cos^3(10t)$$

$$= - x(t)$$

hence, $x(t)$ is an odd signal. Thus,

$$\int_{-T_0}^{T_o} x(t)\,dt = 0$$

5) Periodic vs Aperiodic Signal

- Periodic Signal: A signal $x(t)$ with a constant period $0 < T_0 < \infty$ that

$$x(t) = x(t+T_0)\ ,\ -\infty < t < \infty$$

For a discrete-time signal, the constant period is an integer $0 < K_o < \infty$ that

$$x[n] = x[n+K_0]\ ,\ -\infty < n < \infty$$

- Aperiodic Signal: A signal $y(t)$ or $y[n]$ that does not satisfy the above equation

5) Periodic vs Aperiodic Signal

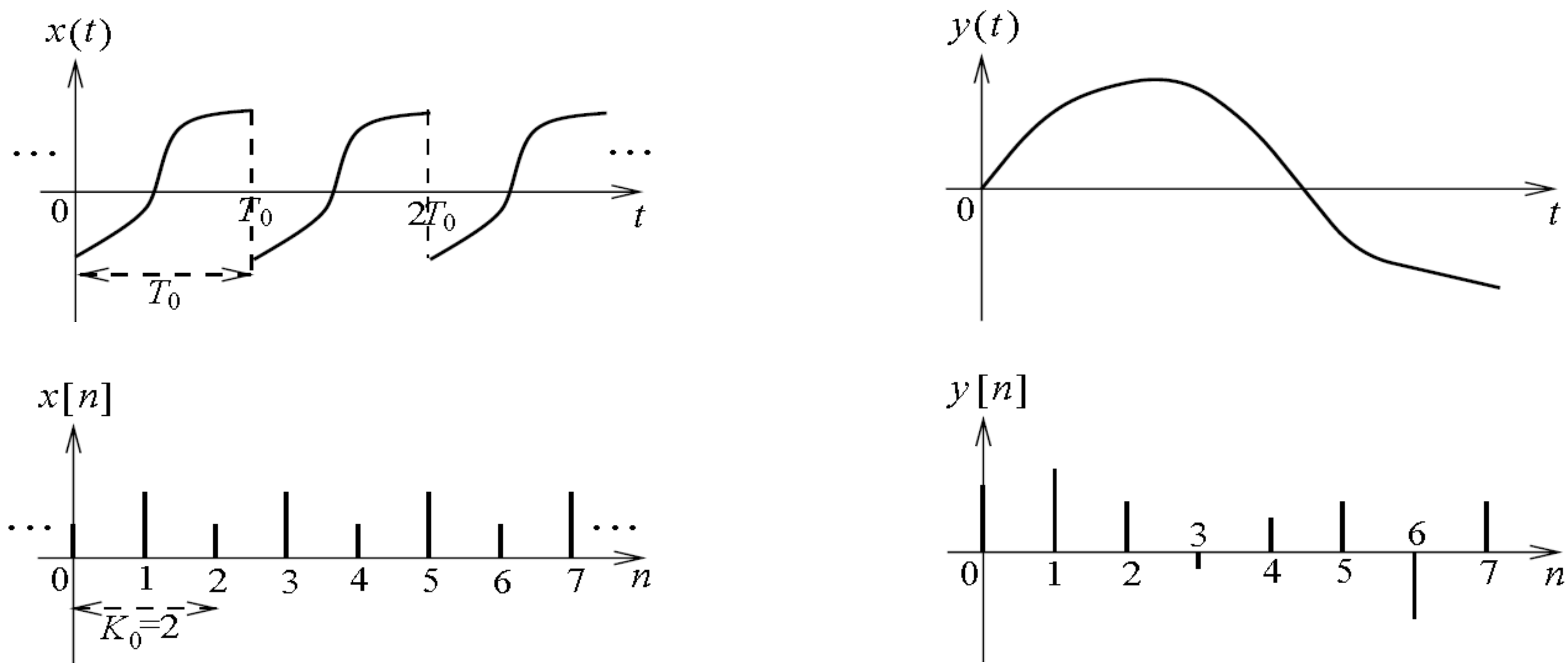


Figure 9: Periodic vs Aperiodic signal

6) Energy-Type vs Power-Type Signal

- Energy-Type Signal

- A signal $x(t)$ or $x[n]$ that has finite energy, i.e., $0 < E_x < \infty$, where

CT signal: $$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

DT signal: $$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

6) Energy-Type vs Power-Type Signal

- Power-Type Signal

  - A signal $x(t)$ or $x[n]$ that has finite power, i.e., $0 < P_x < \infty$, where

CT signal: $$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

DT signal: $$P_x = \lim_{K\to\infty} \frac{1}{2K+1} \sum_{n=-K}^{K} |x[n]|^2$$

6) Energy-Type vs Power-Type Signal

Note that if $x(t)$ or $x[n]$ is a periodic signal with period T_0 or K_0, respectively, then

CT signal: $$P_x = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} |x(t)|^2\, dt$$

DT signal: $$P_x = \frac{1}{K_0}\sum_{n=k}^{k+K_0-1} |x[n]|^2$$

with any real value of t_1 and any integer value of k.

6) Energy-Type vs Power-Type Signal

Example 6: Determine the energy and power of the periodic signal $x(t) = A\cos(2\pi f_0 t)$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

$$= \int_{-\infty}^{\infty} |A\cos(2\pi f_0 t)|^2\, dt$$

$$= \infty$$

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\, dt$$

$$= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2\, dt$$

$$= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |A\cos(2\pi f_0 t)|^2\, dt$$

$$= \frac{A^2}{2}$$

Hence, $x(t)$ is a power-type signal.

In general, power-type signals are periodic signals.

Classification of Signals Summary 2

- Classification of Signals

  4) Even vs Odd Signal

$$x(t) = x_e(t) + x_o(t)$$
$$x_e(t) = \{x(t) + x(-t)\}/2$$
$$x_o(t) = \{x(t) - x(-t)\}/2$$

  5) Periodic vs Aperiodic Signal

  6) Energy-Type vs Power-Type Signal

CT Signals : $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} |x(t)|^2 dt$$

DT Signals : $E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$

$$P_x = \lim_{K\to\infty} \frac{1}{2K+1}\sum_{n=-K}^{K} |x[n]|^2 = \frac{1}{K_0}\sum_{n=k}^{k+K_0-1} |x[n]|^2$$

Classification of Signals Summary 2

1.1 Classification of Signals

- 1) Continuous-Time vs Discrete-Time Signal
- 2) Continuous-Value vs Discrete-Value Signal
- 3) Deterministic vs Random Signal
- 4) Even vs Odd Signal
- 5) Periodic vs Aperiodic Signal
- 6) Energy-Type vs Power-Type Signal

You have reached the end of module 1.1.
Made some mental notes on each classification of signals?
Try it and proceed with the next activity!

IE2110
Signals and Systems
Part 1

with Instructor:

A/P Teh Kah Chan

Outline of Signals & Systems - Part 1

1. Signals and Systems
   - 1.1 **Classification of Signals**☜ Recap through further examples
   - 1.2 Elementary and Singularity Signals
   - 1.3 Operations on Signals
   - 1.4 Properties of Systems

2. Linear Time-Invariant (LTI) Systems
   - 2.1 Discrete-Time and Continuous-Time LTI Systems
   - 2.2 Convolution
   - 2.3 LTI System Properties
   - 2.4 Correlation Functions

Recap: 1.1 Classification of Signals

Example 7:
Determine the energy and power of the signal $y(t) = \exp(-|t|)$.

$$E_y = \int_{-\infty}^{\infty} |y(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} |\exp(-|t|)|^2 \, dt$$

$$= 2 \times \int_0^{\infty} \exp(-2t) dt$$

$$= 1$$

$$P_y = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |y(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \times E_y$$

$$= 0$$

Hence, $y(t)$ is an energy-type signal. In general, energy-type signals are aperiodic signals.

Recap: 1.1 Classification of Signals

Example 8:
Determine the energy and power of the discrete-time periodic signal
$x[n] = A\sin(2\pi n/4)$.

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$= \sum_{n=-\infty}^{\infty} \left|A\sin\left(\frac{2\pi n}{4}\right)\right|^2$$

$$= \infty$$

Hence, $x[n]$ is a power-type signal.

$$P_x = \frac{1}{K_0}\sum_{n=k}^{k+K_0-1} |x[n]|^2$$

$$= \frac{1}{4}\sum_{n=0}^{3} \left|A\sin\left(\frac{2\pi n}{4}\right)\right|^2$$

$$= \frac{A^2}{4} \times [0^2 + 1^2 + 0^2 + (-1)^2]$$

$$= \frac{A^2}{2}$$

Recap: 1.1 Classification of Signals

Example 9:
A simplified transmitter model of a digital communication system is shown below (Figure 10). Determine the classifications of each signal.

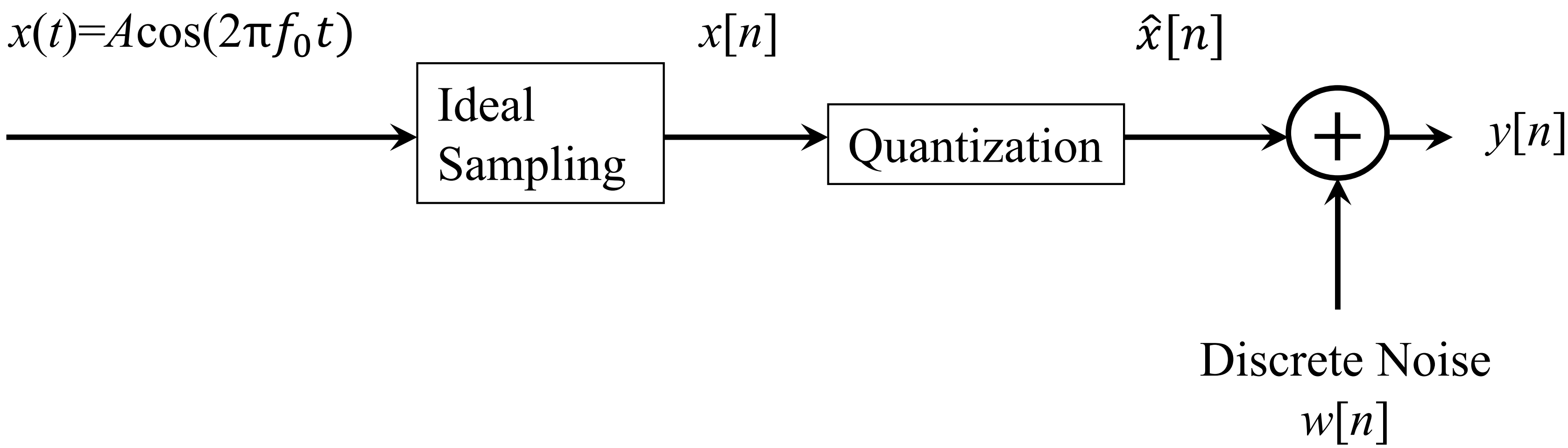
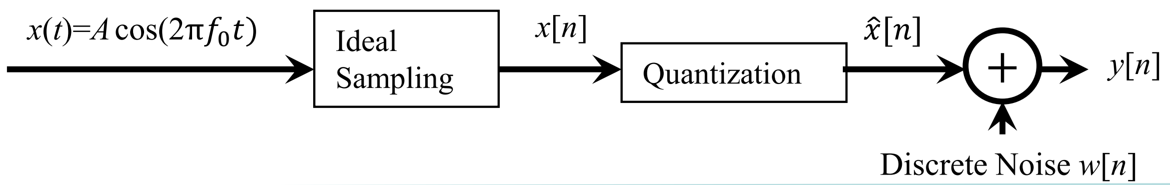


Figure 10: Transmitter model of a digital communication system

Recap: 1.1 Classification of Signals

Example 9:

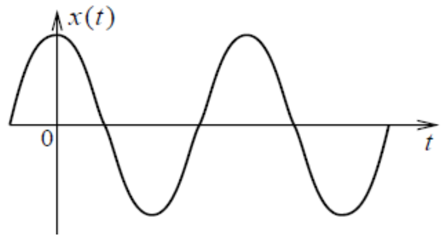


- $x(t)$ is a continuous-time, continuous-value, deterministic, even, periodic, and power-type signal
- $x[n] = x(nT_s)$ is a discrete-time, discrete-value, deterministic, even, periodic, and power-type signal
- $\hat{x}[n]$ is a discrete-time, discrete-value, deterministic, even, periodic, and power-type signal
- $w[n]$ is a discrete-time, continuous-value, random, and aperiodic signal
- $y[n]$ is a discrete-time, continuous-value, random, and aperiodic signal

Recap: 1.1 Classification of Signals

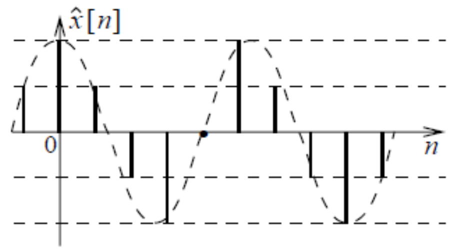
Example 9:

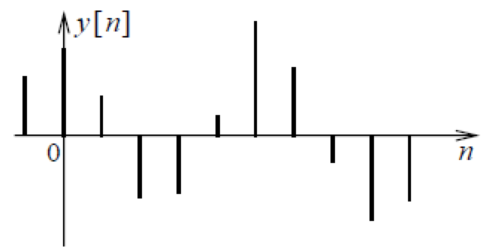
$x(t)$ is a continuous-time, continuous-value, deterministic, even, periodic, and power-type signal



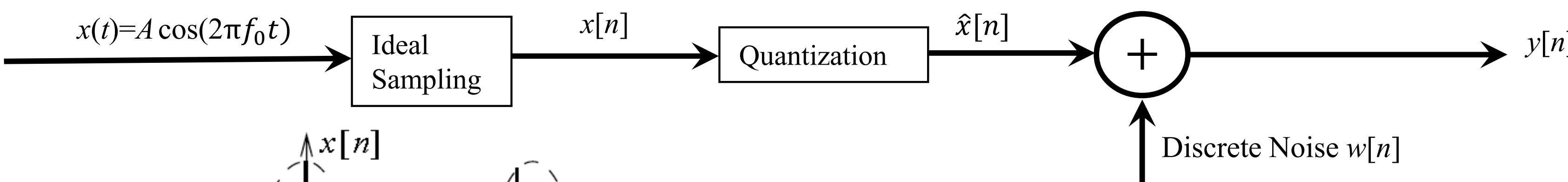
$x[n] = x(nT_s)$ is a discrete-time, discrete-value, deterministic, even, periodic, and power-type signal

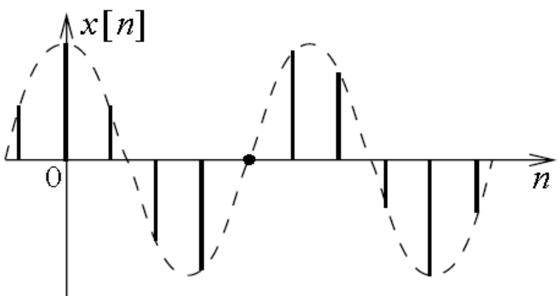
$\hat{x}[n]$ is a discrete-time, discrete-value, deterministic, even, periodic, and power-type signal





$y[n]$ is a discrete-time, continuous-value, random, and aperiodic signal





$w[n]$ is a discrete-time, continuous-value, random, and aperiodic signal

Figure 10: Transmitter model of a digital communication system

Recap: 1.1 Classification of Signals

Example 9:

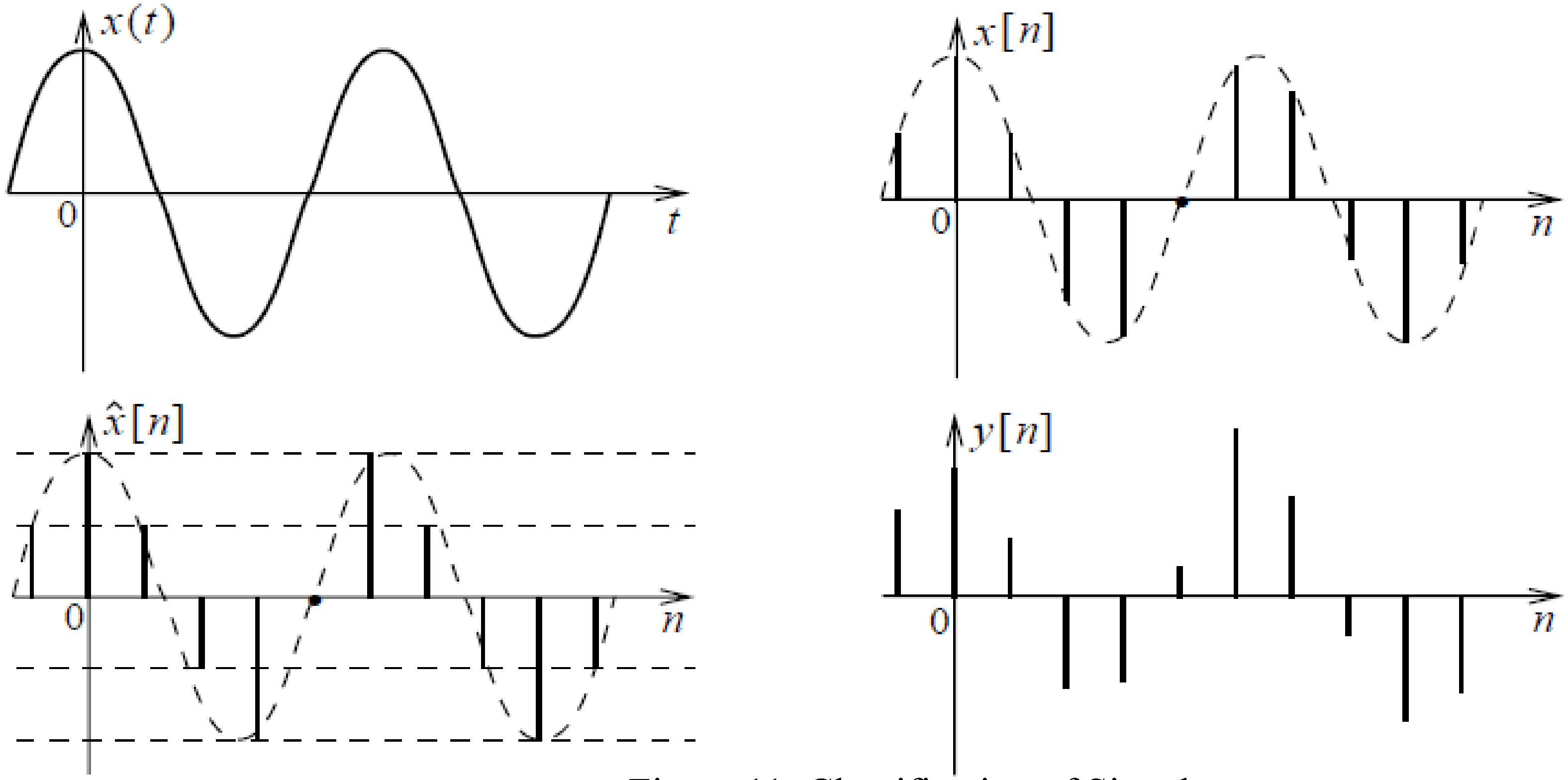


Figure 11: Classification of Signals

IE2110
Signals and Systems Part 1
1.2 Elementary and Singularity Signals

with Instructor:

A/P Teh Kah Chan

Outline of Signals & Systems - Part 1

1. Signals and Systems
   - 1.1 **Classification of Signals**☜ Recap through further examples √
   - 1.2 **Elementary** ☜ and Singularity Signals
   - 1.3 Operations on Signals
   - 1.4 Properties of Systems

2. Linear Time-Invariant (LTI) Systems
   - 2.1 Discrete-Time and Continuous-Time LTI Systems
   - 2.2 Convolution
   - 2.3 LTI System Properties
   - 2.4 Correlation Functions

1.2 Elementary and Singularity Signals

Elementary Signals

1) Exponential ☜
2) Sinusoidal ☜
3) Complex exponential ☜

Singularity Signals

1) Impulse function
2) Step function
3) Signum function
4) Rectangular function
5) Sinc function

1.2 Elementary Signals

1) Exponential signal

$$x(t) = A\exp(at)$$

- $x(t)$ is growing if $a > 0$
- $x(t)$ is decaying if $a < 0$

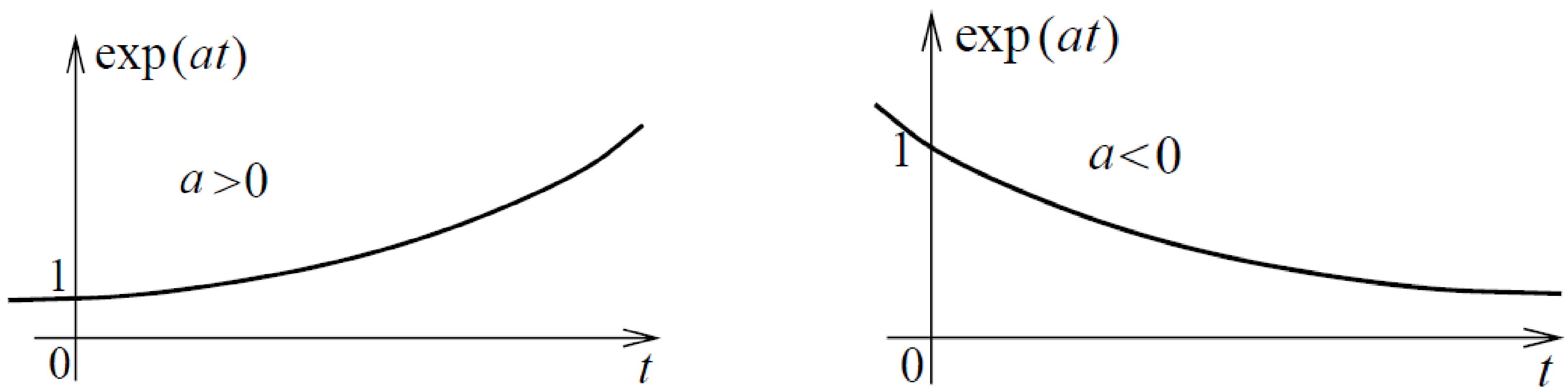


Figure 12: Exponential signal

1.2 Elementary Signals

2) Sinusoidal signal

$$x(t) = A\cos\left(2\pi f_0 t + \theta\right) \quad \text{or} \quad A\sin\left(2\pi f_0 t + \theta\right)$$

where A is the amplitude, f_0 is the frequency in Hertz, and θ is the phase angle in radians

- A sinusoidal signal is periodic with period $T_0 = 1/f_0$

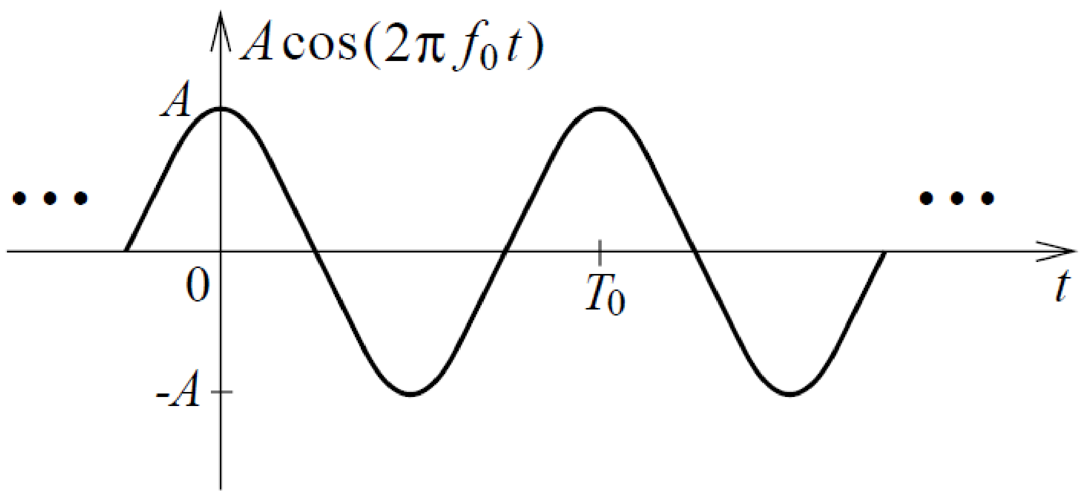


Figure 13: CT sinusoidal signal

1.2 Elementary Signals

- The discrete-time version of the sinusoidal signal is

$$x[n] = A\cos\left(\frac{2\pi n}{K_0} + \theta\right) \quad \text{or} \quad A\sin\left(\frac{2\pi n}{K_0} + \theta\right)$$

where A is the amplitude, K_0 is a positive integer defined as the fundamental period, and θ is the phase angle in radians

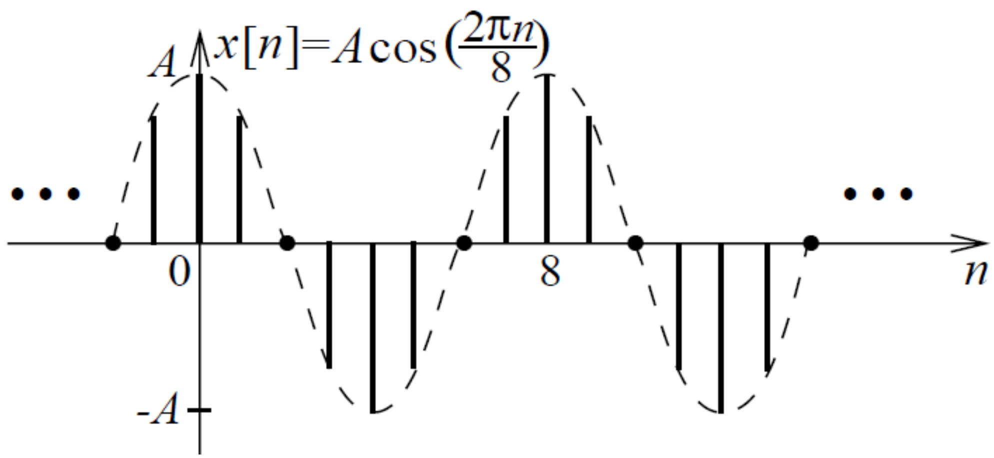


Figure 14: DT sinusoidal signal

1.2 Elementary Signals

3) Complex exponential signal

$$A\exp(j2\pi f_0 t) = A\cos(2\pi f_0 t) + jA\sin(2\pi f_0 t)$$

- The magnitude of complex exponential signal is given by

$$|A\exp(j2\pi f_0 t)| = A$$

- The sinusoidal signal can be expressed as

$$A\cos(2\pi f_0 t + \theta) = \Re\{A\exp(j2\pi f_0 t)\exp(j\theta)\}$$

and

$$A\sin(2\pi f_0 t + \theta) = \Im\{A\exp(j2\pi f_0 t)\exp(j\theta)\}$$

Elementary Signals Summary 3

- Example 9 on Overall Classification of Signals
- Elementary Signals

1) Exponential Signal: $x(t) = A\exp(at)$

2) Sinusoidal Signal: $x(t) = A\cos(2\pi f_0 t + \theta)$

$$x[n] = A\cos\left(\frac{2\pi n}{K_0} + \theta\right)$$

3) Complex Exponential Signal:

$$\begin{aligned} x(t) &= A\exp(j2\pi f_0 t) \\ &= A\cos(2\pi f_0 t) + jA\sin(2\pi f_0 t) \end{aligned}$$

Reflect on how much you have understood the lesson so far before proceeding.

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

IE2110
Signals and Systems Part 1
1.2 Elementary and Singularity Signals

with Instructor:

A/P Teh Kah Chan

Outline of Signals & Systems - Part 1

1. Signals and Systems
   - 1.1 Classification of Signals
   - 1.2 **Elementary** ✓ and Singularity Signals
   - 1.3 Operations on Signals
   - 1.4 Properties of Systems

2. Linear Time-Invariant (LTI) Systems
   - 2.1 Discrete-Time and Continuous-Time LTI Systems
   - 2.2 Convolution
   - 2.3 LTI System Properties
   - 2.4 Correlation Functions

Recap: 1.2 Elementary Signals

Example 10: Sketch the function $x(t) = 5\exp(-at) \times \cos(2\pi 10 t)$ for $t > 0$. Assume that $a > 0$.

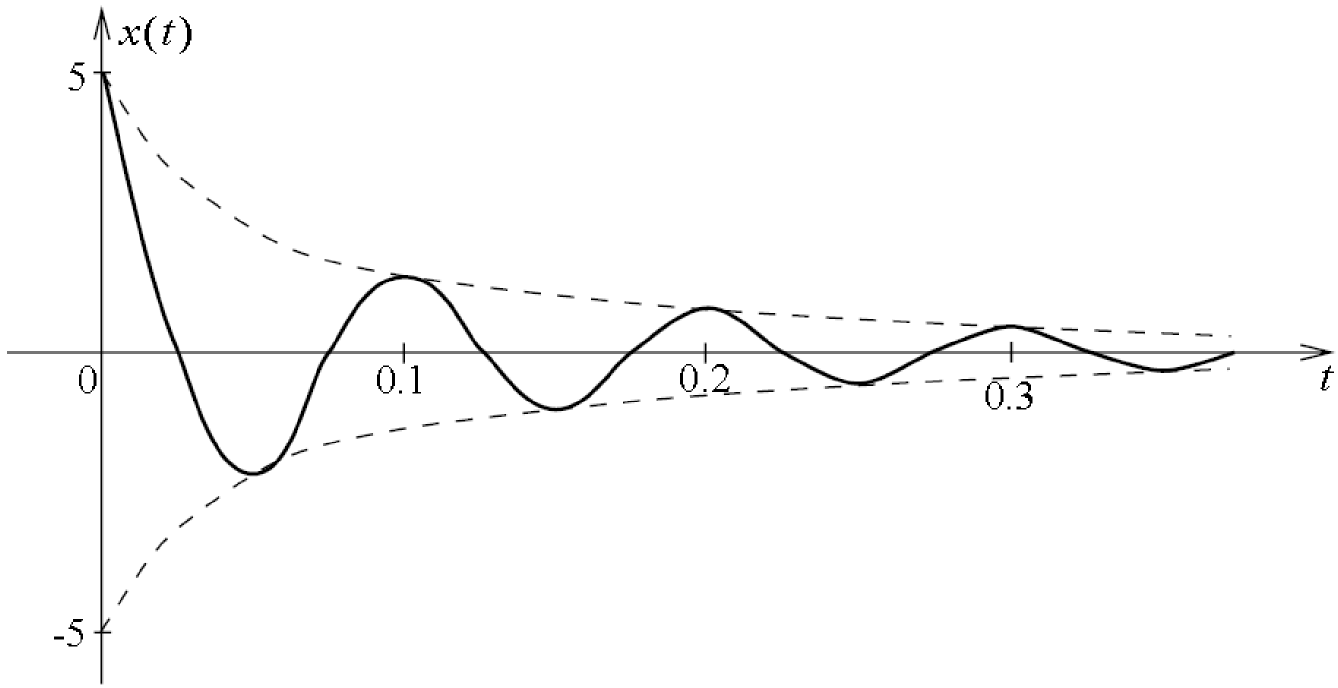


Figure 15: An exponentially damped sinusoidal signal

Outline of Signals & Systems - Part 1

1. Signals and Systems
   - 1.1 Classification of Signals
   - 1.2 **Elementary** ✓ and **Singularity Signals** ☜
   - 1.3 Operations on Signals
   - 1.4 Properties of Systems

2. Linear Time-Invariant (LTI) Systems
   - 2.1 Discrete-Time and Continuous-Time LTI Systems
   - 2.2 Convolution
   - 2.3 LTI System Properties
   - 2.4 Correlation Functions

1.2 Elementary and Singularity Signals

Elementary Signals

1) Exponential ✓
2) Sinusoidal ✓
3) Complex exponential ✓

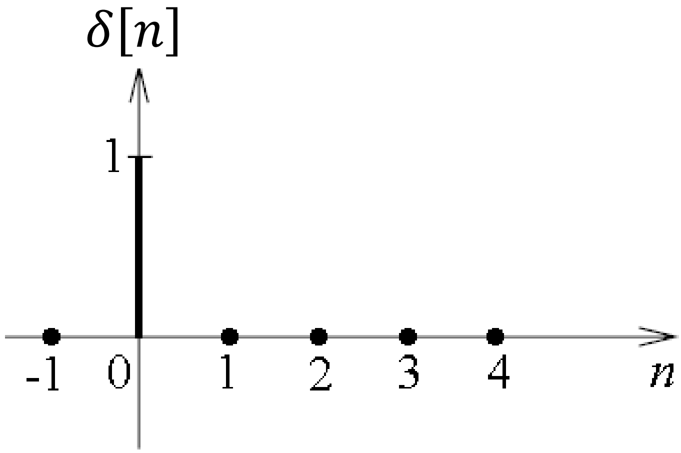
Singularity Signals

1) Impulse function ☜
2) Step function ☜
3) Signum function ☜
4) Rectangular function ☜
5) Sinc function ☜

1.2 Singularity Signals

1) The DT unit impulse (or Dirac Delta) function $\delta[n]$ is defined as

$$\delta[n] = \begin{cases} 1, & n = 0, \\ 0, & n \neq 0. \end{cases}$$



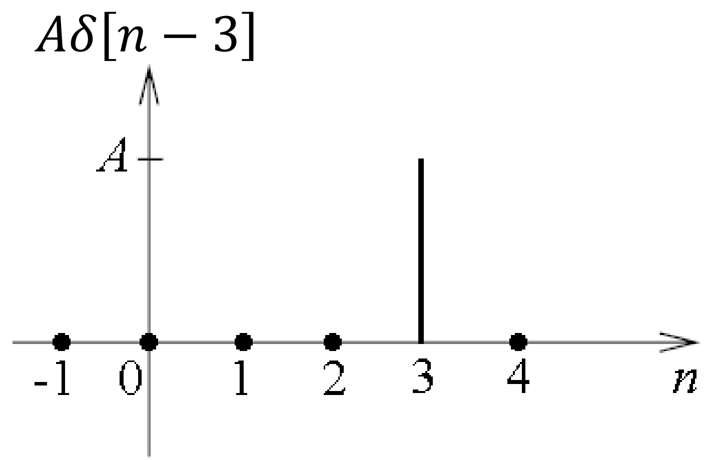


Figure 16: DT impulse functions

1.2 Singularity Signals

1) The CT unit impulse (or Dirac Delta) function $\delta(t)$ is defined as

$$\delta(t) = \begin{cases} \infty, & t = 0, \\ 0, & t \neq 0. \end{cases}$$

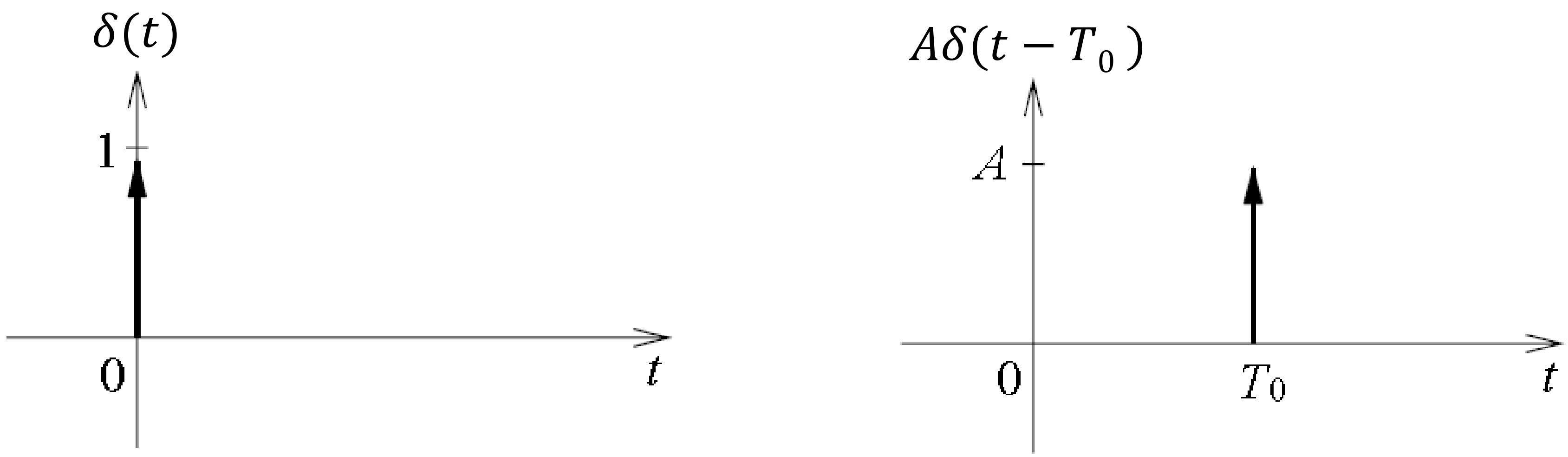


Figure 17: CT impulse functions

1.2 Singularity Signals

Properties of the CT impulse function

- Property One

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

- Property Two

$$x(t) \times \delta\,(t - T_0) \quad = \quad x(T_0) \times \delta\,(t - T_0)$$

- Property Three

$$\int_{-\infty}^{\infty} x(t) \ \times \ \delta(t - T_0)\, dt \quad = \quad x(T_0)$$

1.2 Singularity Signals

2) The CT unit step function $u(t)$ is defined as

$$u(t) = \begin{cases} 1, & t \geq 0, \\ 0, & t < 0. \end{cases}$$

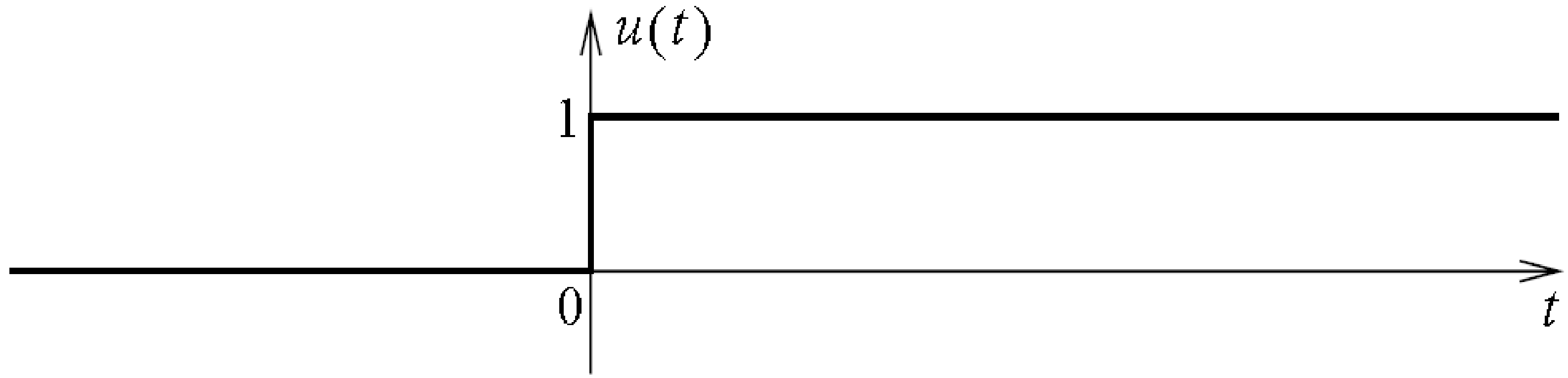


Figure 18: A CT unit step function

1.2 Singularity Signals

2) The DT unit step function $u[n]$ is defined as

$$u[n] = \begin{cases} 1, & n \geq 0, \\ 0, & n < 0. \end{cases}$$

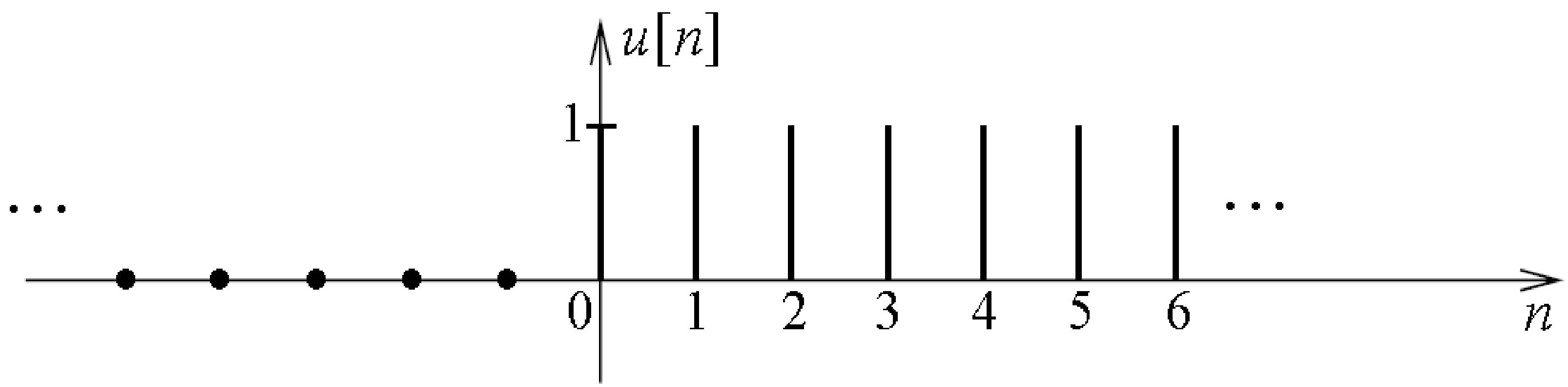


Figure 19: A DT unit step function

1.2 Singularity Signals

3) The CT signum function $\text{sgn}(t)$ is defined as

$$\text{sgn}(t) = \begin{cases} 1, & t > 0, \\ 0, & t = 0, \\ -1, & t < 0. \end{cases}$$

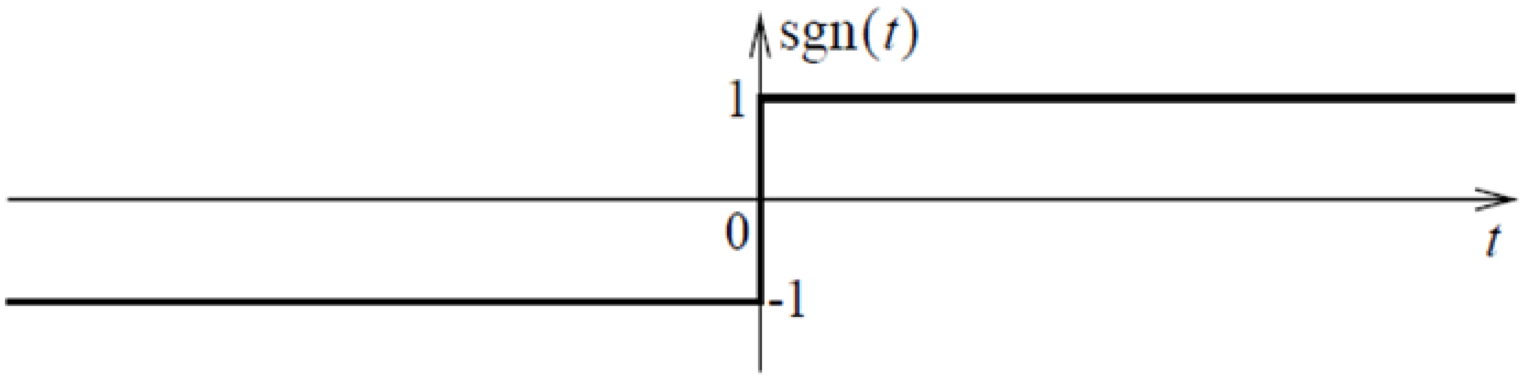


Figure 20: A CT signum function

1.2 Singularity Signals

3) The DT signum function sgn$[n]$ is defined as

$$\operatorname{sgn}[n] = \begin{cases} 1, & n > 0, \\ 0, & n = 0, \\ -1, & n < 0. \end{cases}$$

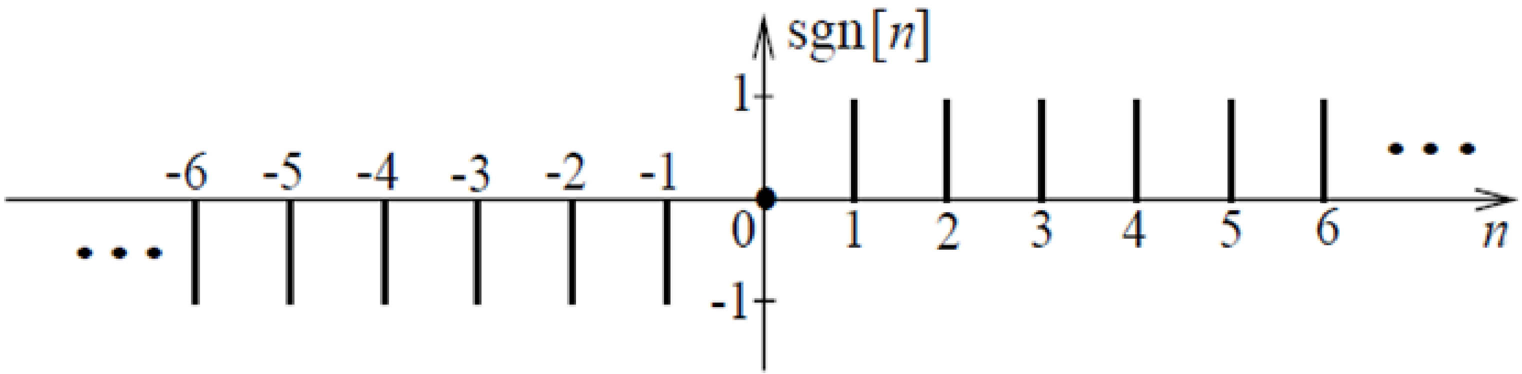


Figure 21: A DT signum function

1.2 Singularity Signals

4) The CT unit rectangular function $\mathrm{rect}\left(\frac{t}{T}\right)$ is defined as

$$\mathrm{rect}\left(\frac{t}{T}\right) = \begin{cases} 1, & |t| \le T/2, \\ 0, & \text{otherwise.} \end{cases}$$

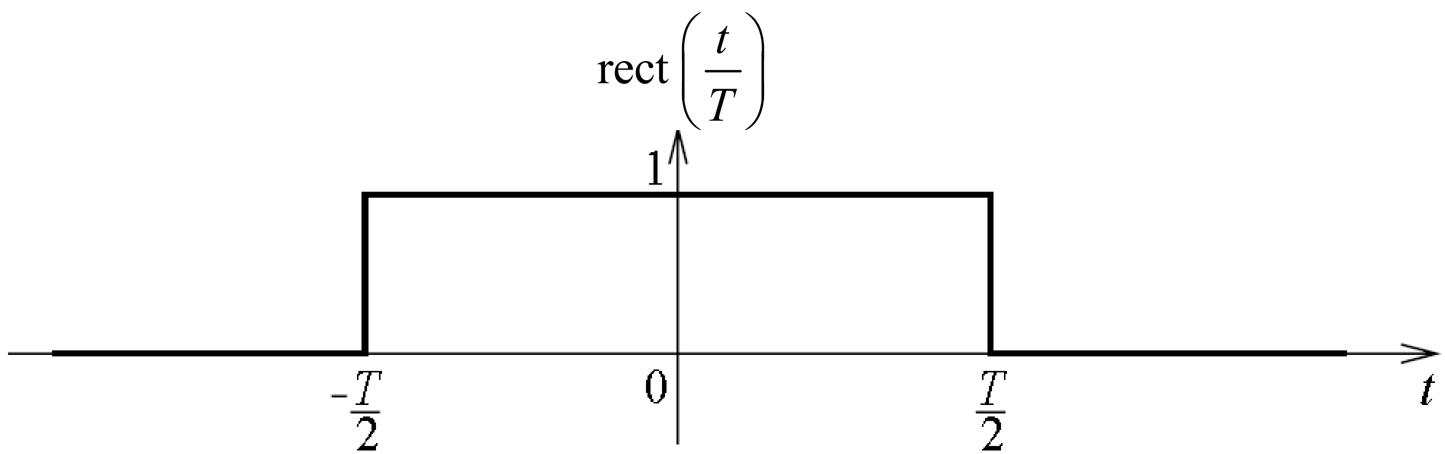


Figure 22: A CT unit rectangular function

1.2 Singularity Signals

4) The DT unit rectangular function $\text{rect}\left[\frac{n}{K}\right]$ (assume that K is even) is defined as

$$\text{rect}\left[\frac{n}{K}\right] = \begin{cases} 1, & |n| \le K/2, \\ 0, & \text{otherwise.} \end{cases}$$

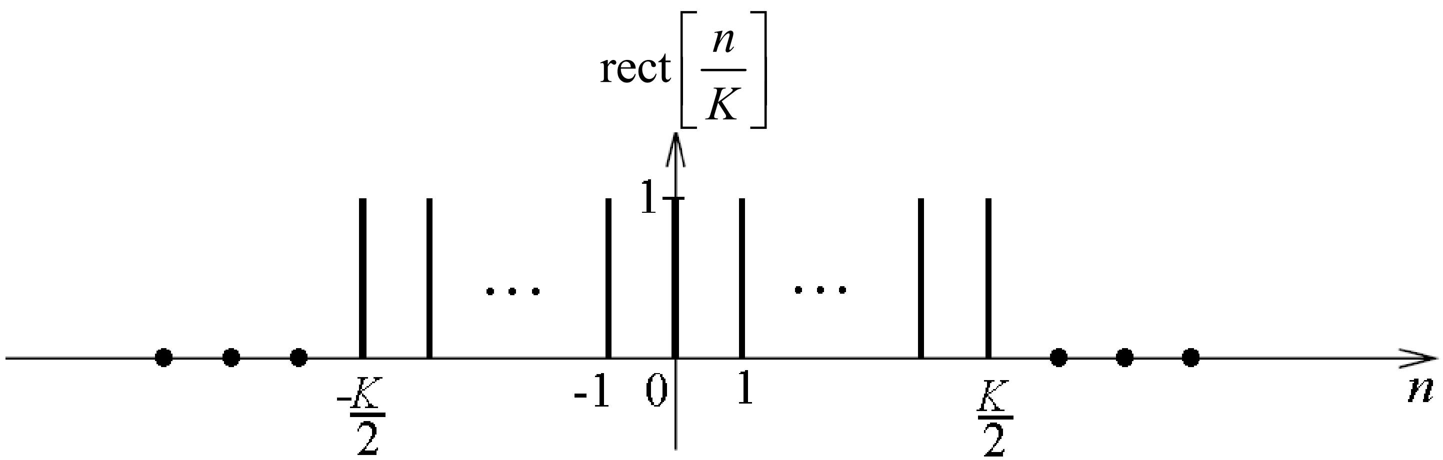


Figure 23: A DT unit rectangular function

1.2 Singularity Signals

5) The sinc function sinc(t) is defined as

$$\operatorname{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

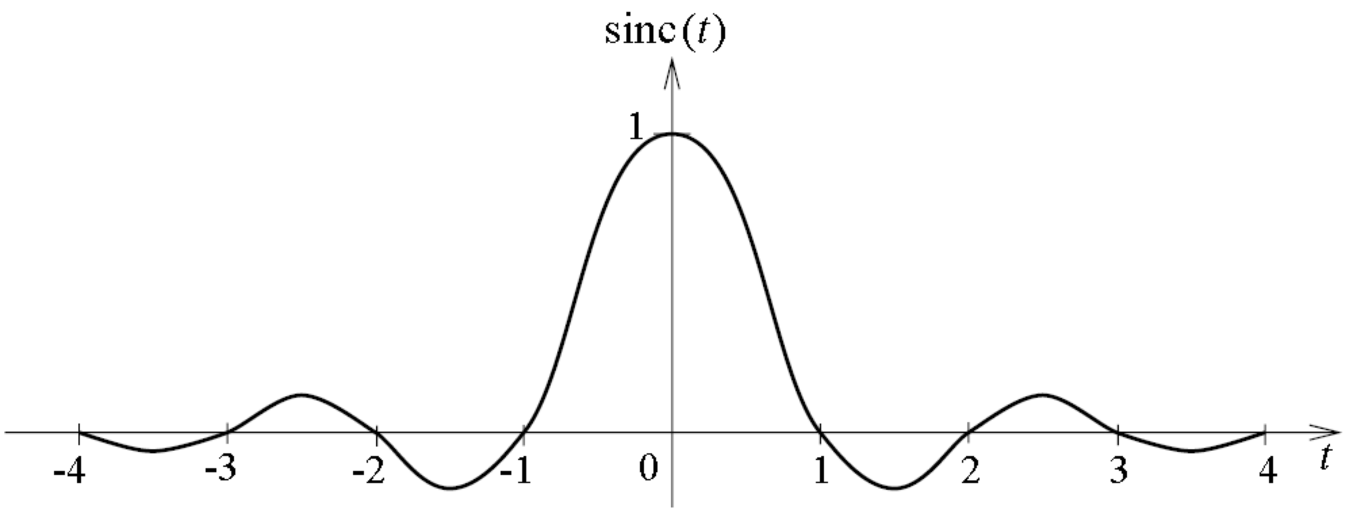


Figure 24: A sinc function

1.2 Singularity Signals

Example 11: The function $x(t) = 5 \times \text{sinc}(t)$is sampled at every T_s= 0.5 second interval to produce the sampled signal $x_s(t)$. Sketch the waveforms for $x(t)$ and $x_s(t)$, respectively.

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(t) \times \delta(t - nT_s)$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \times \delta(t - nT_s)$$

$$= \sum_{n=-\infty}^{\infty} 5 \times \text{sinc}(nT_s) \times \delta(t - nT_s)$$

1.2 Singularity Signals

Example 11: The function $x(t) = 5 \times \mathrm{sinc}(t)$ is sampled at every $T_s = 0.5$ second interval to produce the sampled signal $x_s(t)$. Sketch the waveforms for $x(t)$ and $x_s(t)$, respectively.

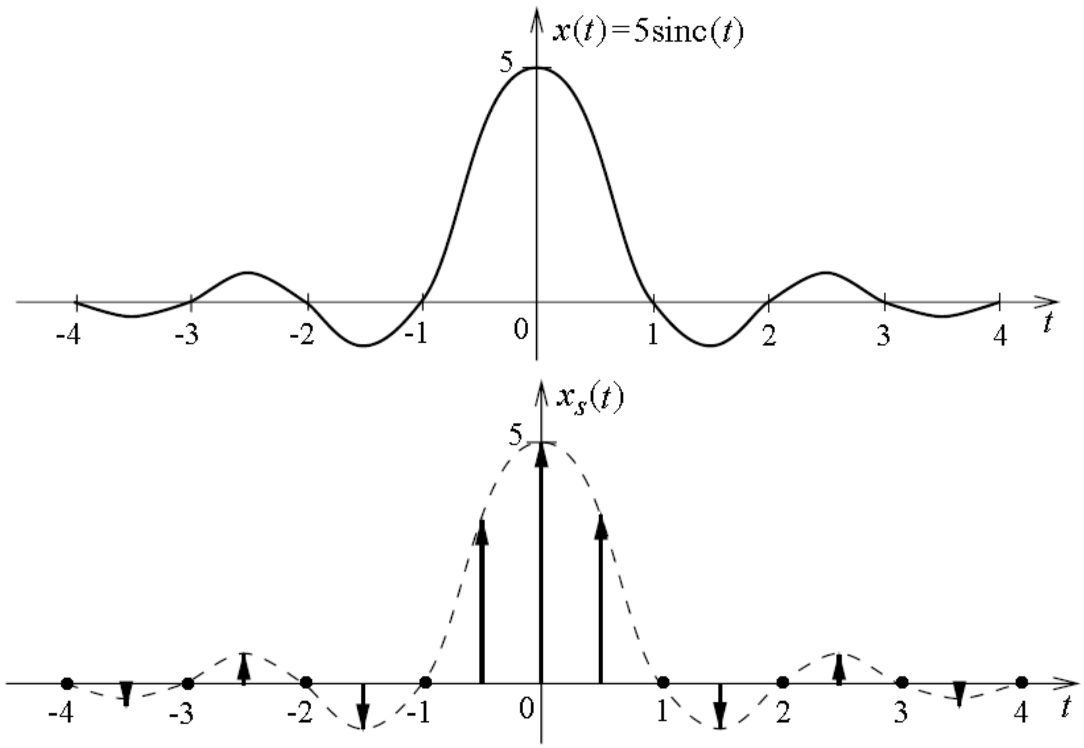


Figure 25: Waveforms for $x(t)$ and $x_s(t)$

Singularity Signals Summary 4

- Singularity Signals

1) Impulse Function: $\delta(t)=\begin{cases}\infty, & t=0,\\ 0, & t\neq 0.\end{cases}$

2) Step Function: $u(t)=\begin{cases}1, & t\geq 0,\\ 0, & t<0.\end{cases}$

3) Signum Function: $\mathrm{sgn}(t)=\begin{cases}1, & t>0,\\ 0, & t=0,\\ -1, & t<0.\end{cases}$

4) Rectangular Function: $\mathrm{rect}\left(\frac{t}{T}\right)=\begin{cases}1, & |t|\leq T/2,\\ 0, & \text{otherwise.}\end{cases}$

5) Sinc Function: $\mathrm{sinc}(t)=\frac{\sin(\pi t)}{\pi t}$

You have reached the end of 1.2: Elementary and Singularity Signals.

IE2110

Signals and Systems Part 1

1.3 Operations on Signals

with Instructor:

A/P Teh Kah Chan

Outline of Signals & Systems - Part 1

1. Signals and Systems
   - 1.1 Classification of Signals ✓
   - 1.2 Elementary and Singularity Signals ✓
   - 1.3 **Operations on Signals**
   - 1.4 Properties of Systems

2. Linear Time-Invariant (LTI) Systems
   - 2.1 Discrete-Time and Continuous-Time LTI Systems
   - 2.2 Convolution
   - 2.3 LTI System Properties
   - 2.4 Correlation Functions

1.3 Operations on Signals

- Amplitude scaling: The operation $Ax(t)$ (or $Ax[n]$) is to multiply the amplitude of $x(t)$ (or $x[n]$) by an amount A

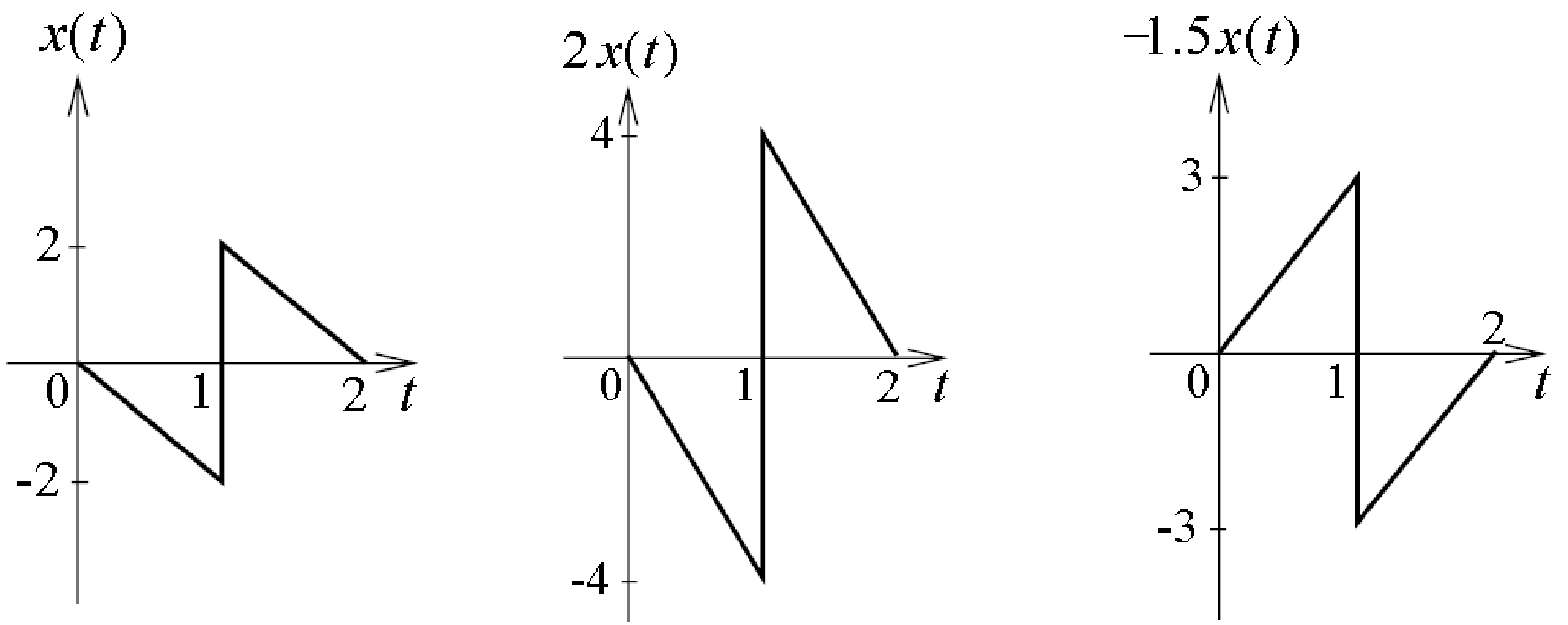


Figure 26: Amplitude scaling of signals

1.3 Operations on Signals

- Time shifting: The operation $x(t - T)$ (or $x[n - K]$) is to shift $x(t)$ (or $x[n]$) by an amount T (or K)

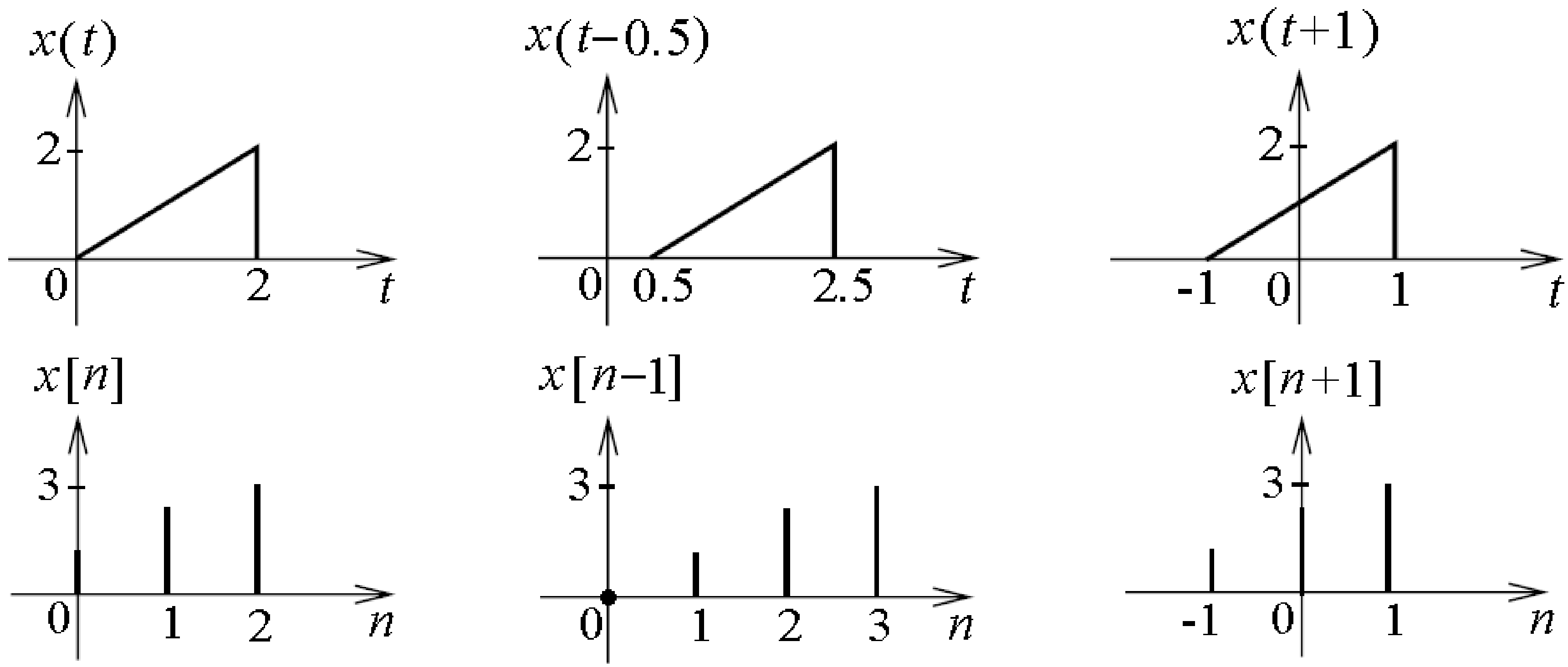


Figure 27: Time shifting of signals

1.3 Operations on Signals

Example 12: Show that $\text{rect}\left(\frac{t}{T}\right) = u\left(t + \frac{T}{2}\right) - u\left(t - \frac{T}{2}\right)$.

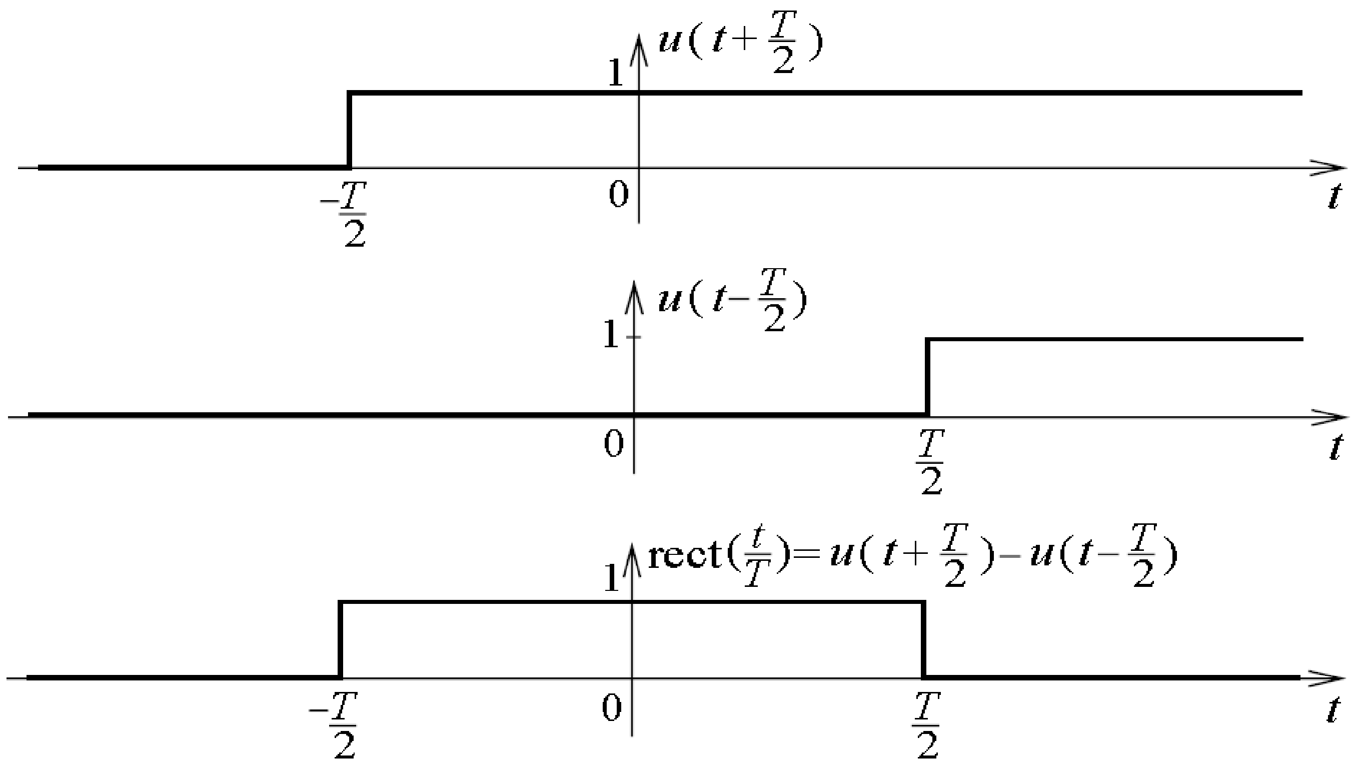


Figure 28: Example on time shifting operation

1.3 Operations on Signals

- CT time scaling: The operation $x(t/a)$ is to scale $x(t)$ by the factor a
  - It expands the function horizontally by the factor $|a|$
  - If $a < 0$, the function will be also time inverted

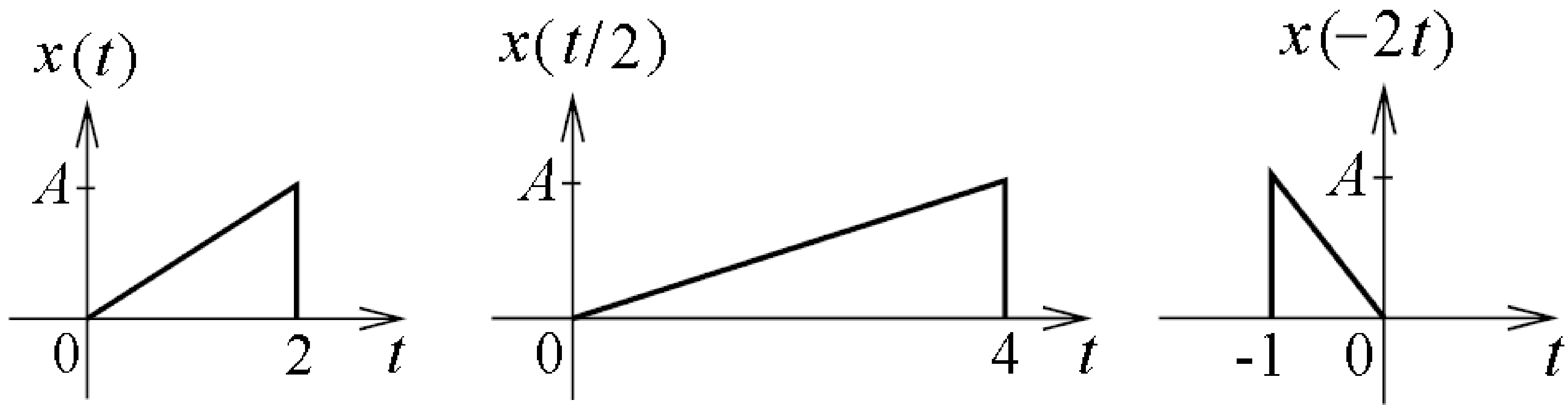


Figure 29: CT time scaling of signals

1.3 Operations on Signals

- DT time scaling: $x[Kn]$ or $x[n/K]$ where K is an integer
  - ➢ $x[Kn]$: Time compression or decimation
  - ➢ $x[n/K]$: Time expansion

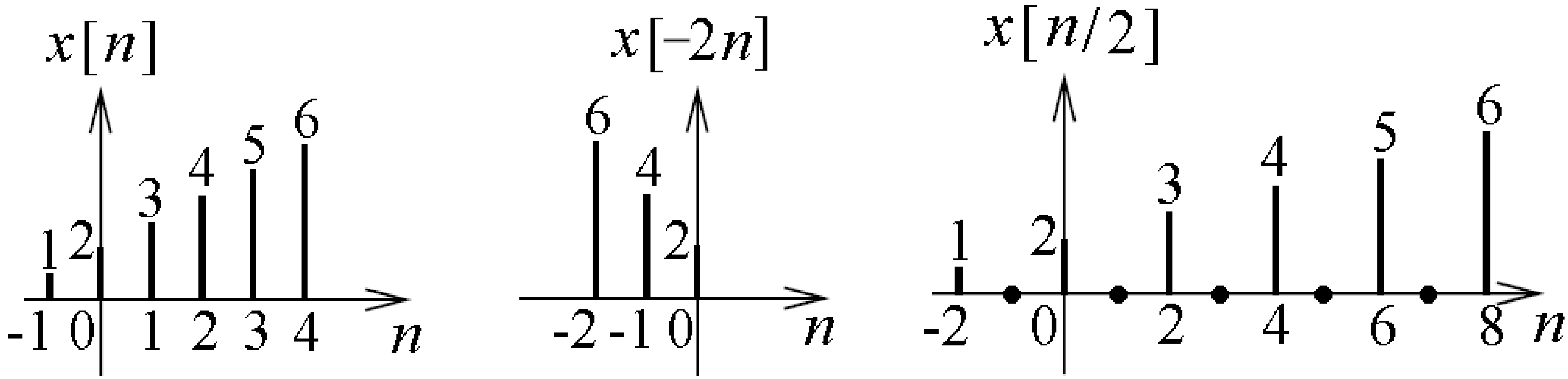


Figure 30: DT time scaling of signals

1.3 Operations on Signals

Example 13: If $x(t) = 0.5 \times \text{rect}\left(\frac{t}{4}\right)$, as shown in Figure 31, sketch the waveform $y(t) = -2x\left(\frac{t-2}{2}\right)$.

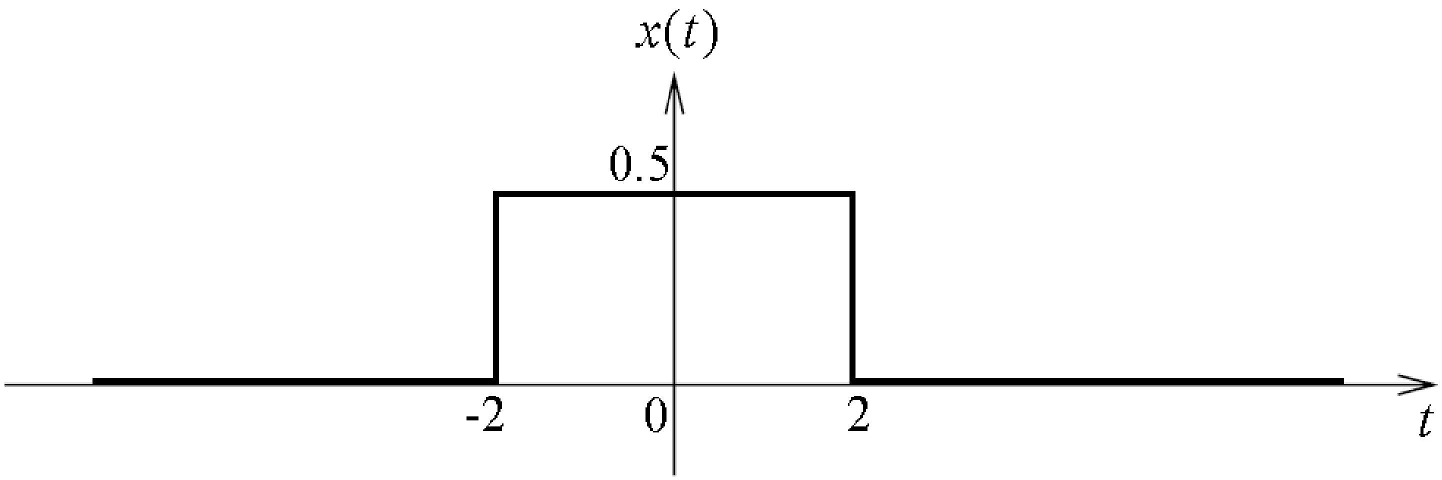


Figure 31: Example of operations on signals

1.3 Operations on Signals

Example 13:

If $x(t) = 0.5 \times \text{rect}\left(\frac{t}{4}\right)$, sketch the waveform $y(t) = -2x\left(\frac{t-2}{2}\right)$.

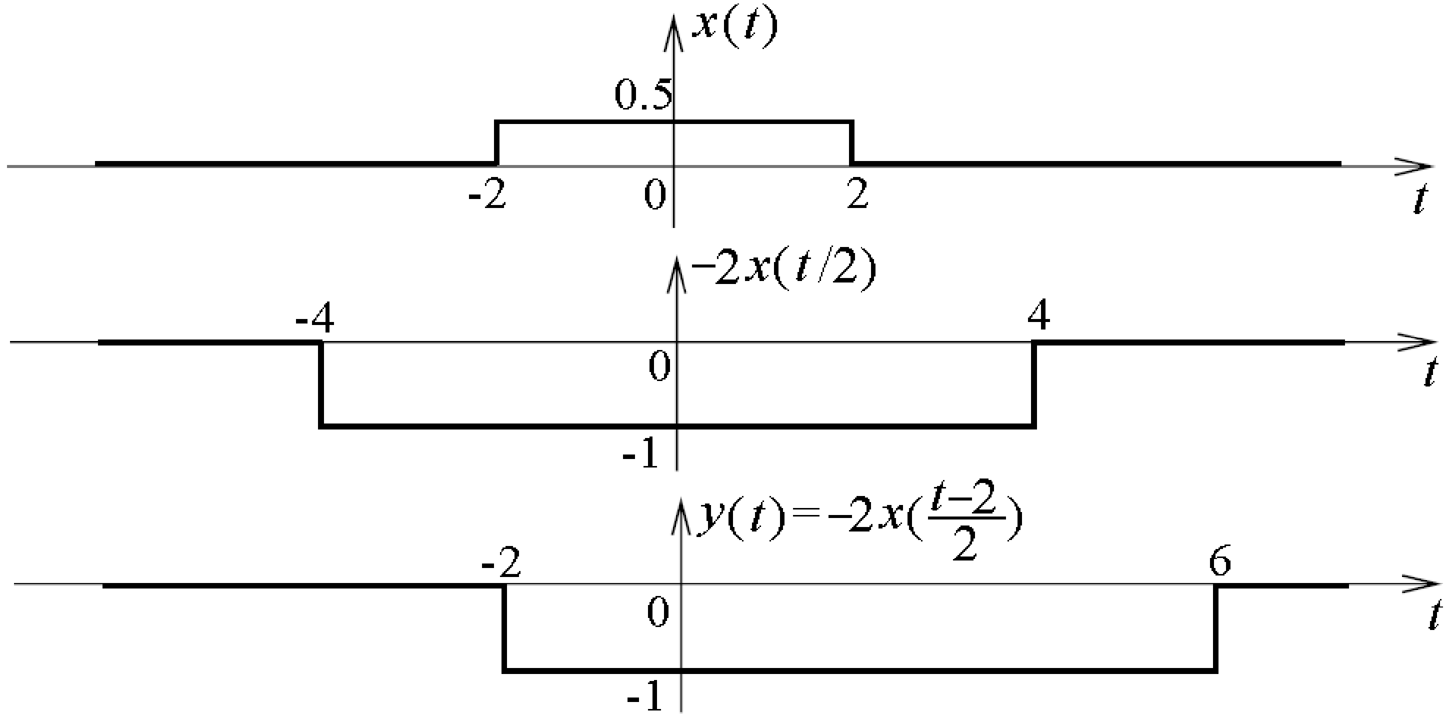


Figure 32: Example of operations on signals

Operations on Signals Summary 5

- Amplitude Scaling: The operation $A\,x(t)$ (or $A\,x[n]$) is to multiply the amplitude of $x(t)$ (or $x[n]$) by an amount A.

- Time Shifting: The operation $x(t\text{-}T)$ (or $x[n\text{-}k]$) is to shift $x(t)$ (or $x[n]$) by an amount T (or K).

- Time Scaling:
  - CT signals: The operation $x(t/a)$ is to scale $x(t)$ by an amount a.
    - It expands the function horizontally by the factor $|a|$.
    - If $a < 0$, the function will be also time inverted.
  - DT signals: $x[Kn]$ or $x[n/K]$ where K is an integer.
    - $x[Kn]$: Time compression or decimation.
    - $x[n/K]$: Time expansion.

You have reached the end of 1.3: Operations on Signals.

IE2110
Signals and Systems Part 1
1.4 Properties of Signals

with Instructor:

A/P Teh Kah Chan

Outline of Signals & Systems - Part 1

1. Signals and Systems
   1.1 Classification of Signals ✓
   1.2 Elementary and Singularity Signals ✓
   1.3 Operations on Signals ✓
   1.4 **Properties of Systems**

2. Linear Time-Invariant (LTI) Systems
   2.1 Discrete-Time and Continuous-Time LTI Systems
   2.2 Convolution
   2.3 LTI System Properties
   2.4 Correlation Functions

1.4 Properties of Systems

1) Stability
2) Memory
3) Causality
4) Linearity
5) Time Invariant

1.4 Properties of Systems

A system refers to any physical device (i.e., communication channels, filters) that produces an output signal $y(t)$ in response to an input signal $x(t)$

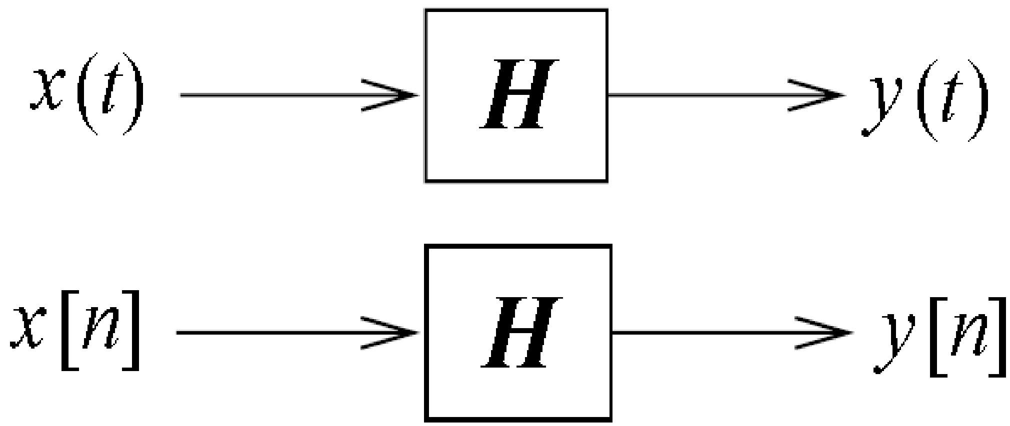


Figure 33: Block diagram representation of a system

1.4 Properties of Systems

1) Stability

- A system is said to be bounded-input bounded-output (BIBO) stable if and only if every bounded input (i.e., $|x(t)| < \infty$ for all t, or $|x[n]| < \infty$ for all n) results in bounded output

An example of a BIBO stable system

$$y[n] = r^n x[n]u[n], \qquad |r| < 1$$

An example of a BIBO unstable system

$$y[n] = r^n x[n]u[n], \qquad |r| > 1$$

1.4 Properties of Systems

2) Memory

- A system is said to possess memory if its output signal depends on past or future values of the input signal

An example of a system with memory

$$y[n] = x[n] + x[n-1] + x[n-2]$$

- A system is memoryless if its output signal depends only on the present value of the input signal

An example of a memoryless system

$$y(t) = x^2(t)$$

1.4 Properties of Systems

3) Causality

- A system is causal if the present value of the output signal depends only on the present or past values of the input signal

An example of a causal system

$$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$$

- A system is noncausal if the present value of the output signal depends on the future values of the input signal
- A noncausal system is not physically realizable in real time

An example of a noncausal system

$$y[n] = \frac{1}{3}(x[n+1] + x[n] + x[n-1])$$

1.4 Properties of Systems

4) Linearity

- A system is linear if the principle of superposition holds, i.e.,
  if input signal is $x_3(t) = a_1x_1(t) + a_2x_2(t)$, then the output signal is
  $y_3(t) = a_1y_1(t) + a_2y_2(t)$ for any constants $a_1$and a_2

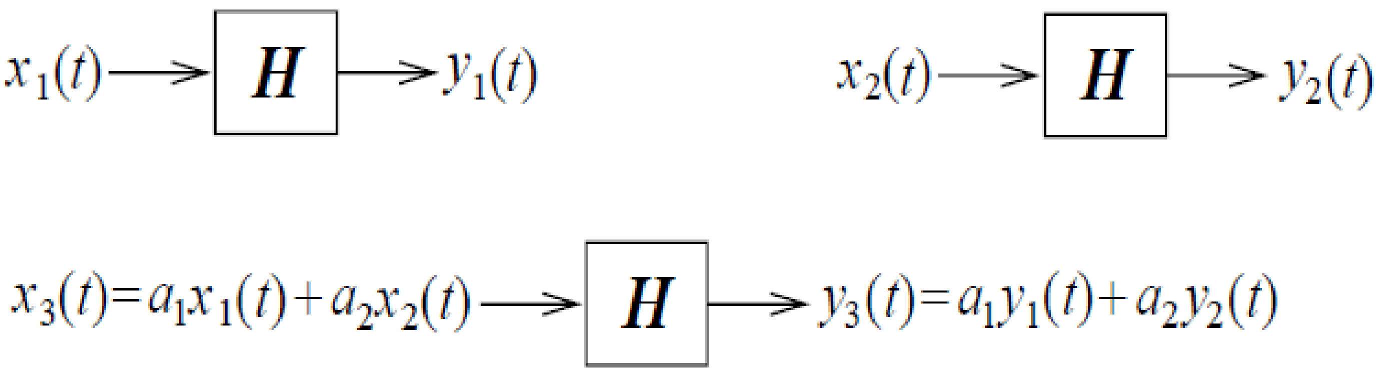


Figure 34: A linear system

1.4 Properties of Systems

Example 14:

A system is shown below (Figure 35). Determine whether it is a linear system.

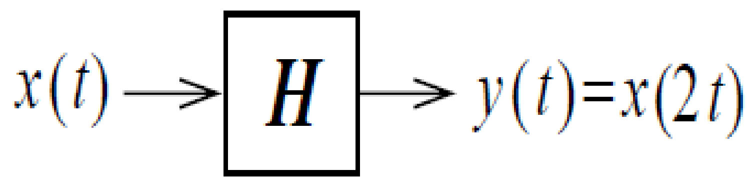


Figure 35: A linear system example

1.4 Properties of Systems

Example 14:
Determine whether it is a linear system.

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t) = x(2t)$$

$$x_1(t) \rightarrow \boxed{H} \rightarrow y_1(t) = x_1(2t) \qquad x_2(t) \rightarrow \boxed{H} \rightarrow y_2(t) = x_2(2t)$$

$$x_3(t) = a_1 x_1(t) + a_2 x_2(t) \rightarrow \boxed{H} \rightarrow y_3(t) = a_1 x_1(2t) + a_2 x_2(2t)$$
$$= a_1 y_1(t) + a_2 y_2(t)$$

Figure 36: A linear system example

In this case, the principle of superposition holds, hence it is a <u>linear system</u>.

1.4 Properties of Systems

5) Time Invariant

- A system is time invariant if for any delayed $x(t - T)$, the output is delayed by the same amount $y(t - T)$

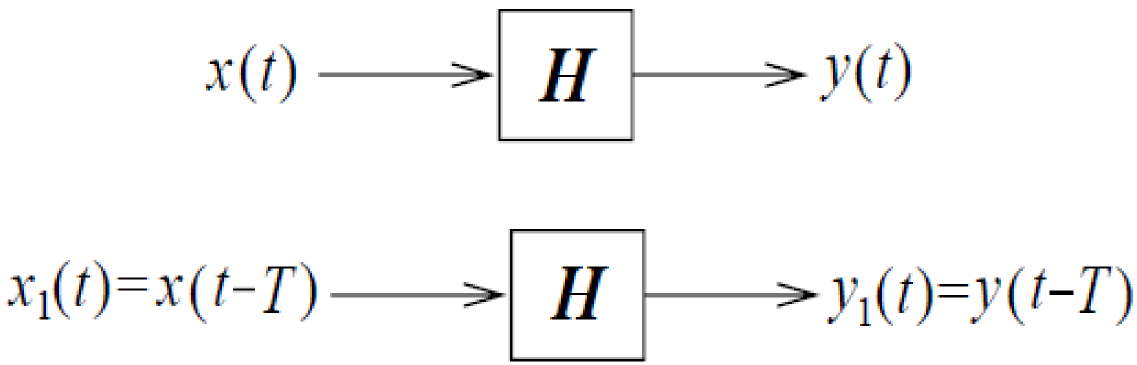


Figure 37: A time invariant system

1.4 Properties of Systems

Example 15:

For the system as shown below (Figure 38) with $y(t) = x(t) + c$, where c is an arbitrary constant, determine whether it is a time invariant system.

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t) = x(t) + c$$

Figure 38: A time invariant system example

1.4 Properties of Systems

Example 15:
Determine whether it is a time invariant system.

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t)=x(t)+c$$

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t)=x(t)+c$$

$$x_1(t)=x(t-T) \longrightarrow \boxed{H} \longrightarrow y_1(t)=x(t-T)+c$$
$$=y(t-T)$$

Figure 39: A time invariant system example

In this case, the system is <u>time invariant</u>.

2. Linear Time-Invariant (LTI) Systems

Linear Time Invariant (LTI)

- A system is linear time invariant if it satisfies both conditions of linear and time invariance
- A LTI system can be analyzed in both time domain and frequency domain

$$x_1(t) \rightarrow \boxed{H} \rightarrow y_1(t) \qquad x_2(t) \rightarrow \boxed{H} \rightarrow y_2(t)$$

$$x_3(t) = a_1 x_1(t-T_1) + a_2 x_2(t-T_2) \rightarrow \boxed{H} \rightarrow y_3(t) = a_1 y_1(t-T_1) + a_2 y_2(t-T_2)$$

Figure 40: An LTI system

2. Linear Time-Invariant (LTI) Systems

Example 16:
Determine whether the system below given by $y(t) = x(2t)$ in Example 14 is an LTI system.

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t) = x(2t)$$

Figure 35

From Example 14, the system is linear.

$$x_1(t) = x(t-T) \longrightarrow \boxed{H} \longrightarrow y_1(t) = x(2t-T) \neq y(t-T)$$

Figure 41: A non-LTI system example

However, the system is not time invariant, hence it is not an LTI system.

Properties of Systems Summary 6

- 1) Stability: Bounded input results in bounded output.
- 2) Memory: Output depends on past and/or future values of input.
- 3) Causality: Output does not depend on future values of input.
- 4) Linearity: Principle of superposition holds.
- 5) Time Invariant: For any delayed input $x(t\text{-}T)$, the output is delayed by the same amount $y(t\text{-}T)$.
  - Linear Time-Invariant (LTI) Systems

You have reached the end of 1.4: Properties of Systems.
Consider mapping out your learning and proceed.

IE2110
Signals and Systems Part 1
2. Linear Time-Invariant (LTI) Systems

with Instructor:

A/P Teh Kah Chan

Outline of Signals & Systems - Part 1

1. Signals and Systems
   - 1.1 Classification of Signals ✓
   - 1.2 Elementary and Singularity Signals ✓
   - 1.3 Operations on Signals ✓
   - 1.4 Properties of Systems

2. **Linear Time-Invariant (LTI) Systems**
   - 2.1 **Discrete-Time**☜ and Continuous-Time **LTI Systems**
   - 2.2 Convolution
   - 2.3 LTI System Properties
   - 2.4 Correlation Functions

2.1 Discrete-Time and Continuous-Time LTI Systems

Analysis of DT and CT LTI Systems:

- Any LTI system can be uniquely defined by its impulse response

$$\delta(t) \longrightarrow \boxed{H} \longrightarrow h(t)$$

$$\delta[n] \longrightarrow \boxed{H} \longrightarrow h[n]$$

Figure 42: Impulse response of an LTI system

2.1 Discrete-Time and Continuous-Time LTI Systems

The output of any LTI system is the convolution of the input signal and its impulse response

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) * h(t)$$

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n] = x[n] * h[n]$$

Figure 43: System response of an LTI system

2.1 Discrete-Time and Continuous-Time LTI Systems

The discrete time convolution (convolution sum) is defined as

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

The continuous time convolution (convolution integral) is defined as

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

2.1a Discrete-Time LTI Systems

Example 17:
Sketch the waveform of $y[n] = x[n] * h[n]$ using the graphical approach for convolution sum.

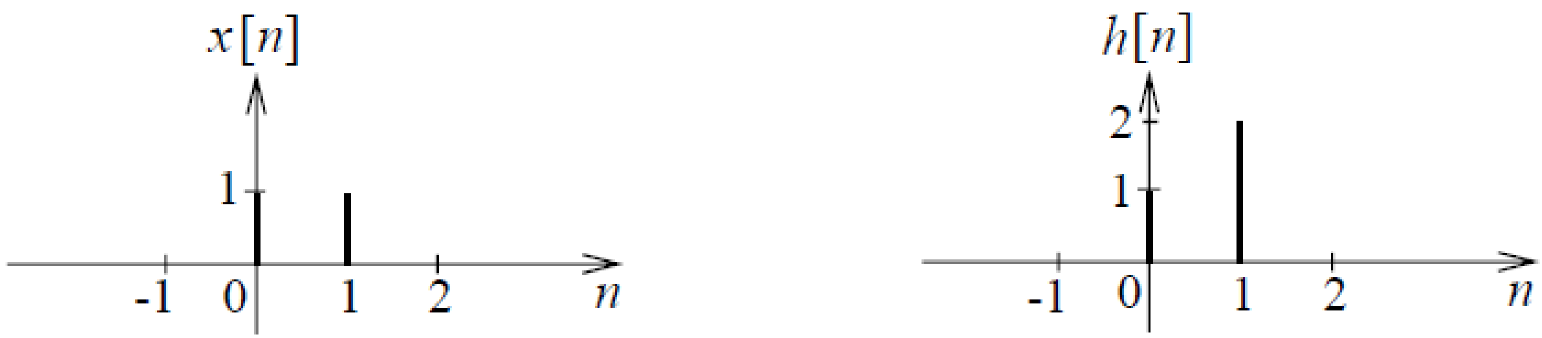
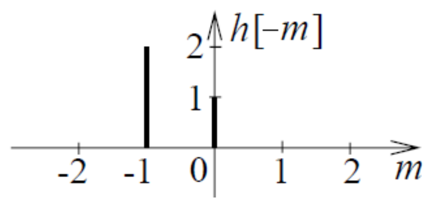


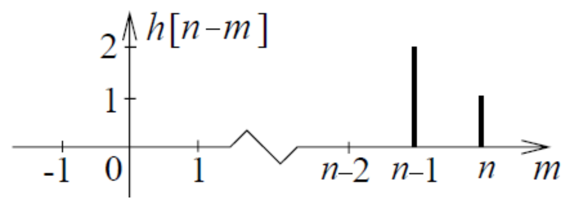
Figure 44: Example on convoluted sum

2.1a Discrete-Time LTI Systems

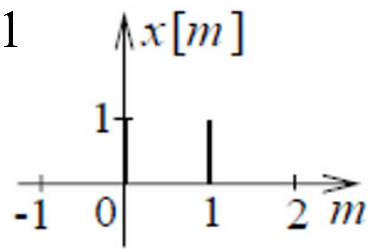
Example 17:
Sketch the waveform of $y[n] = x[n] * h[n]$

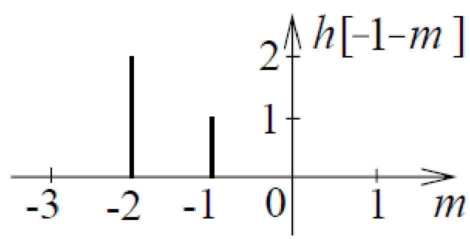
$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

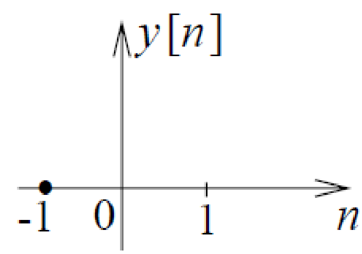




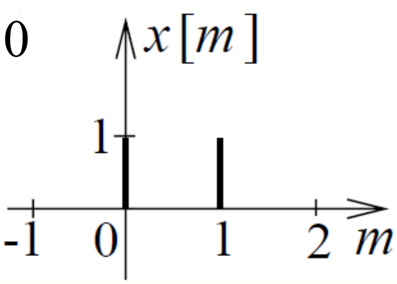
(i) $n = -1$

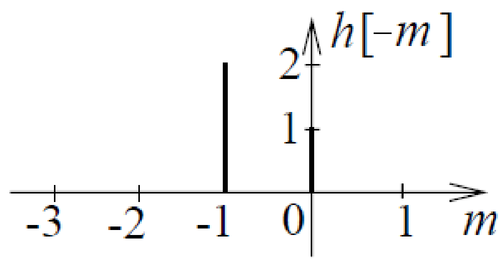


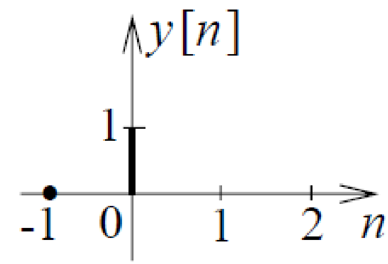




(ii) $n = 0$







2.1a Discrete-Time LTI Systems

Example 17:

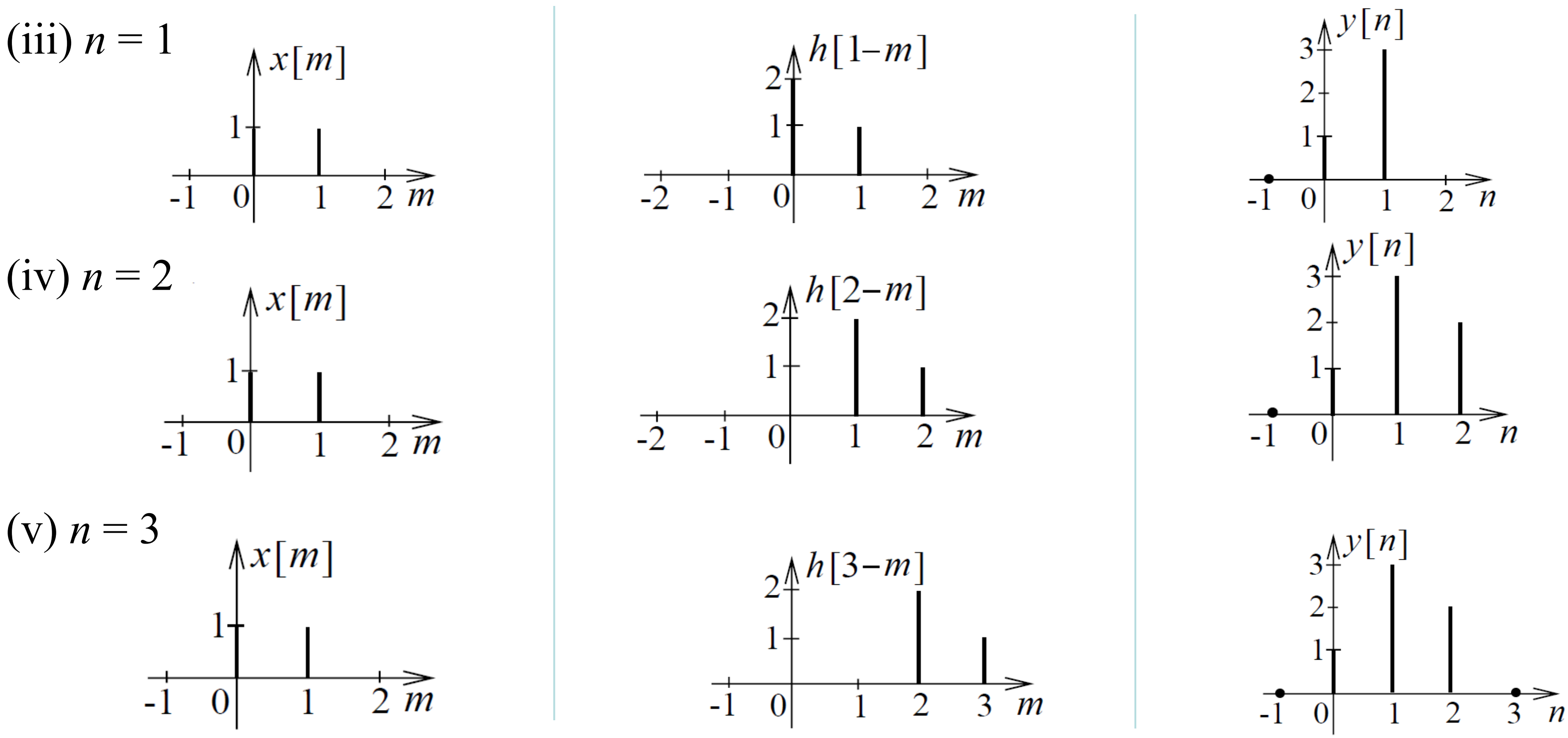


Figure 45: Solution for example on convolution sum

2.1 Discrete-Time and Continuous-Time LTI Systems
Summary 7

- Analysis of DT Systems
  - Any DT LTI system can be uniquely defined by its impulse response, $h[n]$.
  - The output of a DT LTI system is the convolution of the input signal and its impulse response.
  - The DT convolution (or convolution sum) is defined as

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

  - The graphical approach for evaluating the convolution sum.

You have reached the end of 2.1a. Do reflect on your level of understanding. Please proceed to 2.1b Continuous-Time LIT Systems.

IE2110
Signals and Systems Part 1
2.0 Linear Time-Invariant (LTI) Systems

with Instructor:

A/P Teh Kah Chan

Outline of Signals & Systems - Part 1

1. Signals and Systems
   - 1.1 Classification of Signals ✓
   - 1.2 Elementary and Singularity Signals ✓
   - 1.3 Operations on Signals ✓
   - 1.4 Properties of Systems ✓

2. Linear Time-Invariant (LTI) Systems
   - 2.1 Discrete-Time ✓ and **Continuous-Time LTI Systems** ☜
   - 2.2 Convolution
   - 2.3 LTI System Properties
   - 2.4 Correlation Functions

2.1b Continuous-Time LTI Systems

Example 18:
Sketch the waveform of $y(t) = x_1(t) * x_2(t)$ using the graphical approach for convolution integral.

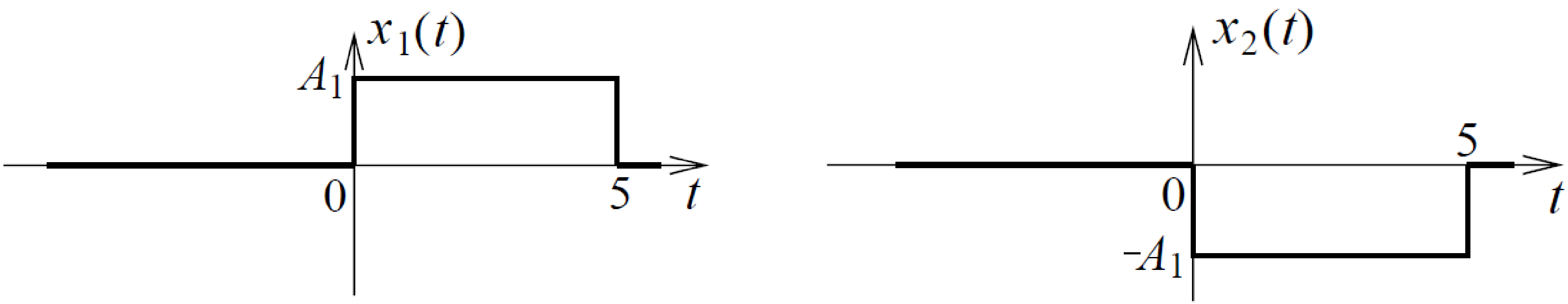
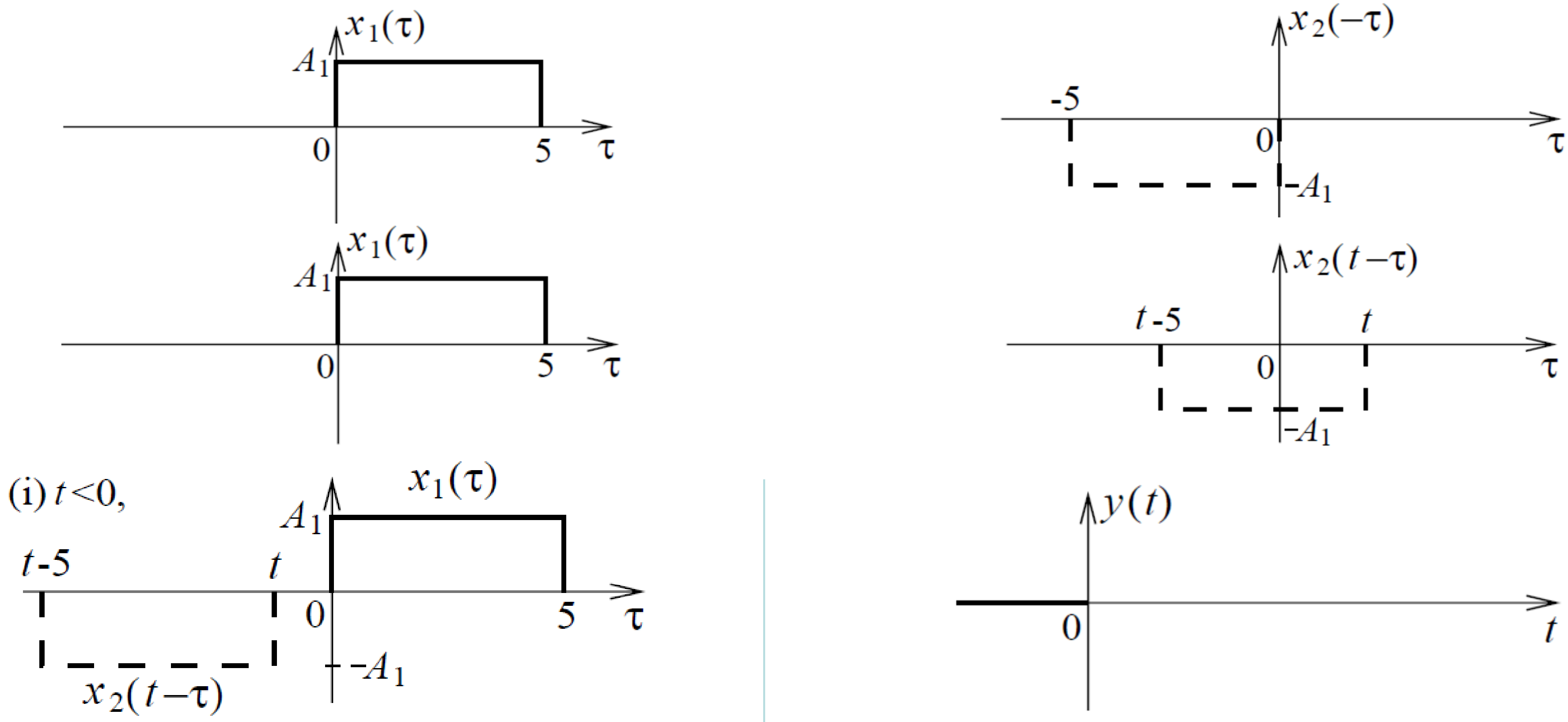


Figure 46: Example on convolution integral

2.1b Continuous-Time LTI Systems

$$y(t) = x(t) * x(t) = \int_{-\infty}^{\infty} x(\tau)x(t-\tau)d\tau$$



2.1b Continuous-Time LTI Systems

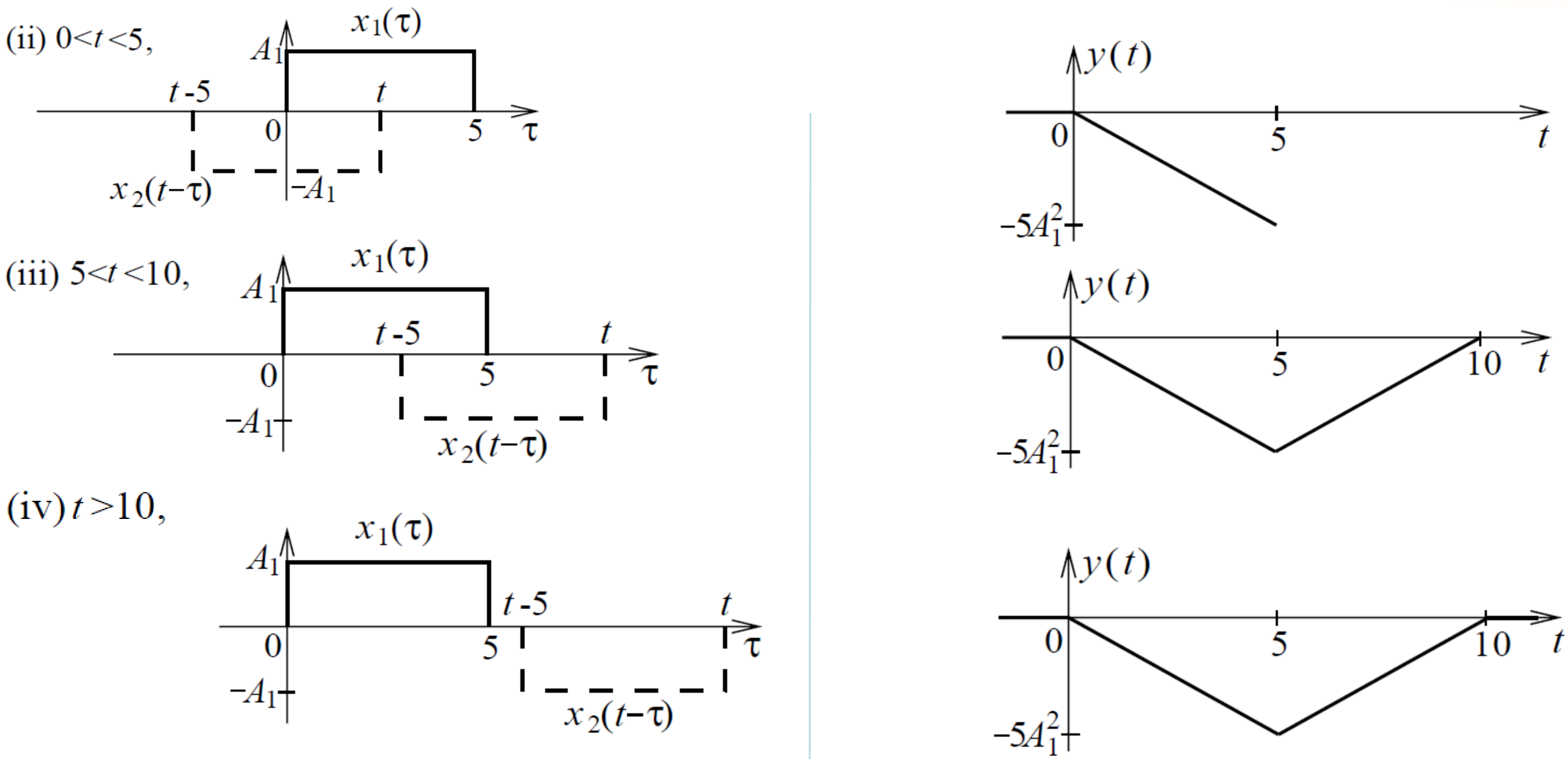


Figure 47: Solution for example on convolution integral

2.1 Discrete-Time and Continuous-Time LTI Systems
Summary 8

❑ Analysis of CT Systems

- Any CT LTI system can be uniquely defined by its impulse response, $h(t)$.
- The output of a CT LTI system is the convolution of the input signal and its impulse response.
- The CT convolution (or convolution integral) is defined as

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

- The graphical approach for evaluating the convolution integral.

You have reached the end of 2.1. Do reflect on your level of understanding. Please proceed to 2.2 Convolution.

IE2110
Signals and Systems Part 1
2.2 Convolution

with Instructor:

A/P Teh Kah Chan

Outline of Signals & Systems - Part 1

1. Signals and Systems
   - 1.1 Classification of Signals ✓
   - 1.2 Elementary and Singularity Signals ✓
   - 1.3 Operations on Signals ✓
   - 1.4 Properties of Systems

2. Linear Time-Invariant (LTI) Systems
   - 2.1 Discrete-Time and Continuous-Time LTI Systems
   - 2.2 **Convolution** ☜
   - 2.3 LTI System Properties
   - 2.4 Correlation Functions

2.2 Convolution

2.2a Properties of Convolution

1) Commutative
2) Distributive
3) Associative
4) Convolution with Delta Function

2.2b Step Response of LTI Systems

2.2a Properties of Convolution

1) Commutative

$$x_1[n] * x_2[n] = x_2[n] * x_1[n]$$

$$x_1(t) * x_2(t) = x_2(t) * x_1(t)$$

2) Distributive

$$x_1[n] * \{x_2[n] + x_3[n]\} = x_1[n] * x_2[n] + x_1[n] * x_3[n]$$

$$x_1(t) * [x_2(t) + x_3(t)] = x_1(t) * x_2(t) + x_1(t) * x_3(t)$$

2.2a Properties of Convolution

3) Associative

$$x_1[n] * \{x_2[n] * x_3[n]\} \quad = \quad \{x_1[n] * x_2[n]\} * x_3[n]$$

$$x_1(t) * [x_2(t) * x_3(t)] \quad = \quad [x_1(t) * x_2(t)] * x_3(t)$$

4) Convolution with Delta function

$$x[n] * \delta[n - K_0] \quad = \quad x[n - K_0]$$

$$x(t) * \delta(t - T_0) \quad = \quad x(t - T_0)$$

2.2a Properties of Convolution

Example 19: Show that $x(t) * \delta(t - T_0) = x(t - T_0)$ using the definition of convolution integral.

Based on the definition of convolution integral, we have

$$\begin{aligned} y(t) &= x(t) * \delta(t - T_0) \\ &= \int_{-\infty}^{\infty} \delta(\tau - T_0) x(t - \tau) d\tau \\ &= \int_{-\infty}^{\infty} \delta(\tau - T_0) x(t - T_0) d\tau \\ &= x(t - T_0) \int_{-\infty}^{\infty} \delta(\tau - T_0) d\tau \\ &= x(t - T_0) \end{aligned}$$

2.2b Step Response of LTI Systems

The step response is defined as the output of the system with the unit step function as input signal

- Step response of a DT system

$$
\begin{aligned}
s[n] &= u[n] * h[n] = \sum_{m=-\infty}^{\infty} h[m]u[n-m] \\
&= \sum_{m=-\infty}^{n} h[m]
\end{aligned}
$$

- Step response of a CT system

$$
\begin{aligned}
s(t) &= u(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)u(t-\tau)d\tau \\
&= \int_{-\infty}^{t} h(\tau)d\tau
\end{aligned}
$$

2.2b Step Response of LTI Systems

Example 20: Find the step response of the one-stage RC filter as shown below (Figure 48), where the impulse response is given by

$$h(t) = \frac{1}{RC} \times \exp\left(-\frac{t}{RC}\right) u(t).$$

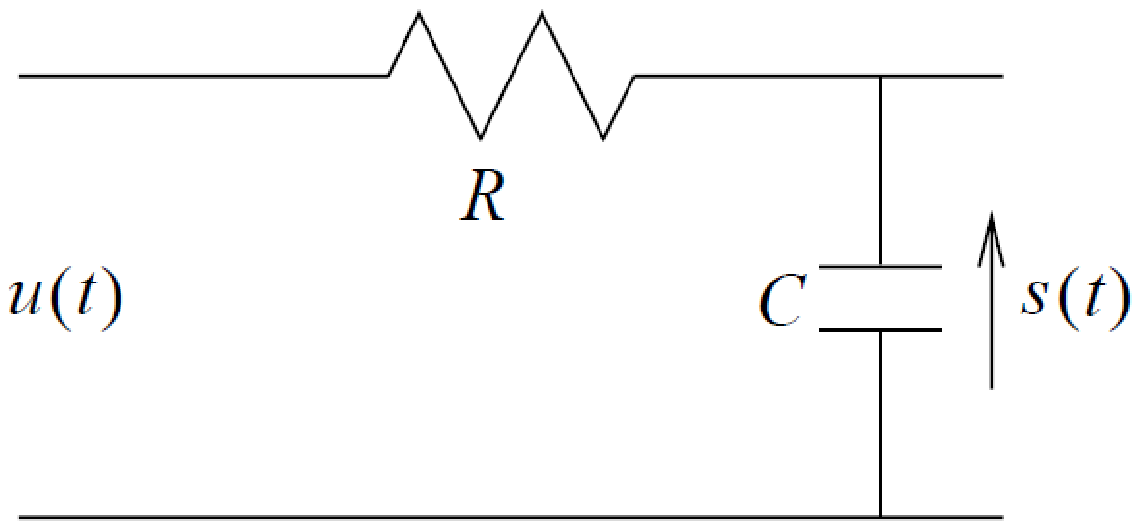


Figure 48: A simple one-stage *RC* filter

2.2b Step Response of LTI Systems

Example 20: Find the step response of the one-stage RC filter as shown below (Figure 48), where the impulse response is given by

$$h(t) = \frac{1}{RC} \times \exp\left(-\frac{t}{RC}\right) u(t).$$

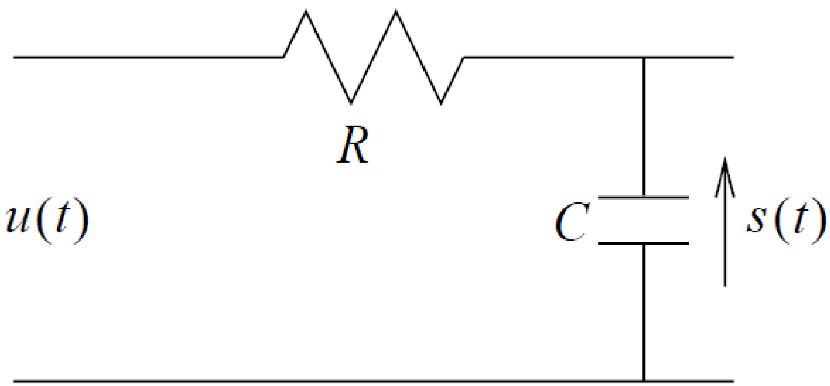


Figure 48: A simple one-stage *RC* filter

$$\begin{aligned} s(t) &= u(t) * h(t) \\ &= \int_{-\infty}^{t} h(\tau) d\tau \\ &= \int_{-\infty}^{t} \frac{1}{RC} \times \exp\left(-\frac{\tau}{RC}\right) u(\tau) d\tau \\ &= \frac{1}{RC} \int_{0}^{t} \exp\left(-\frac{\tau}{RC}\right) d\tau \\ &= \begin{cases} 1 - \exp\left(-\frac{t}{RC}\right), & t \geq 0, \\ 0, & t < 0. \end{cases} \end{aligned}$$

2.2 Convolution Summary 9

- Properties of Convolution
  - Commutative, Distributive, and Associative.
  - Convolution with Impulse Function:

$$x(t) * \delta(t - T_0) = x(t - T_0)$$

$$x[n] * \delta[n - K_0] = x[n - K_0]$$

  - Step Response: Output of the system with the unit step function as the input signal

$$s[n] = u[n] * h[n] = \sum_{m=-\infty}^{n} h[m]$$

$$s(t) = u(t) * h(t) = \int_{-\infty}^{t} h(\tau) d\tau$$

You have reached the end of 2.2. Please proceed.

IE2110
Signals and Systems Part 1
2.3 LTI System Properties

with Instructor:

A/P Teh Kah Chan

Outline of Signals & Systems - Part 1

2.3 Properties of LTI Systems

- 2.3a Properties of LTI Systems
  1) Memoryless
  2) Causal
  3) BIBO Stable

- 2.3b System Interconnections
  1) Parallel Connection: Summation of subsystem impulse responses.
  2) Cascade Connection: Convolution of subsystem impulse responses.

2.3a LTI System Properties

1) Memoryless LTI Systems

- A LTI system is memoryless if and only if its impulse response is given by

$$\text{DT system:} \qquad h[n] = c\delta[n]$$

$$\text{CT system:} \qquad h(t) = c\delta(t)$$

where c is an arbitrary constant

- For all memoryless LTI systems, simply perform scalar multiplication on the input

2.3a LTI System Properties

2) Causal LTI Systems

- A LTI system is causal if and only if its impulse response satisfies the following condition

$$\begin{aligned} \text{DT system:} \qquad & h[n] &=& \ 0, \quad \text{for } n < 0 \\ \text{CT system:} \qquad & h(t) &=& \ 0, \quad \text{for } t < 0 \end{aligned}$$

- A causal LTI system cannot generate an output before the input is applied

2.3a LTI System Properties

3) Stable LTI Systems

- A LTI system is BIBO stable if and only if its impulse response satisfies the following condition

DT system: $$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

CT system: $$\int_{-\infty}^{\infty} |h(t)| dt < \infty$$

- An example of a stable LTI system

$$h[n] = \rho^n u[n], \quad |\rho| < 1$$

2.3a LTI System Properties

Example 21:
Determine whether the system with impulse response
$h(t) = \exp(-at)u(t)$ where $a > 0$ is (i) memoryless, (ii) causal, and (iii) BIBO stable.

(i) The system is not memoryless since $h(t) \neq c\delta\,(t)$

(ii) The system is causal since $h(t) = 0$ for $t < 0$

(iii) The system is BIBO stable since

$$\begin{aligned} \int_{-\infty}^{\infty} |h(t)| dt &= \int_{0}^{\infty} \exp(-at) dt \\ &= 1/a \; < \; \infty \end{aligned}$$

2.3b System Interconnections

1) Parallel Connection

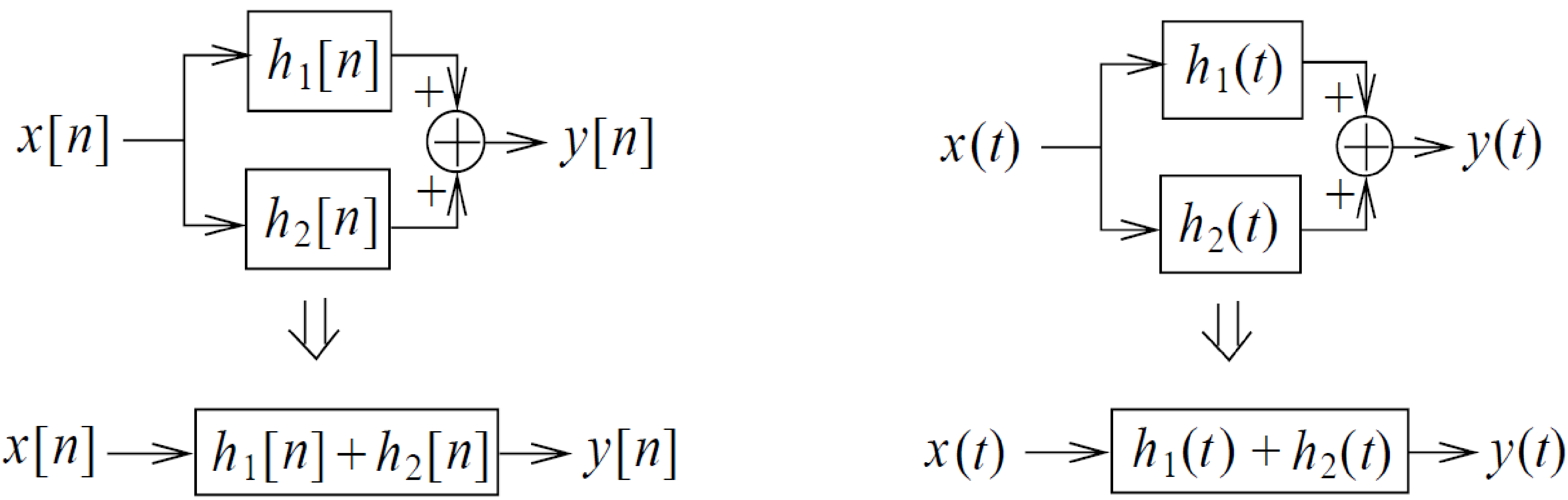


Figure 49: Parallel connection of systems

2.3b System Interconnections

2) Cascade Connection

$$x[n] \rightarrow \boxed{h_1[n]} \rightarrow \boxed{h_2[n]} \rightarrow y[n] \qquad\qquad x(t) \rightarrow \boxed{h_1(t)} \rightarrow \boxed{h_2(t)} \rightarrow y(t)$$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$x[n] \rightarrow \boxed{h_1[n] * h_2[n]} \rightarrow y[n] \qquad\qquad x(t) \rightarrow \boxed{h_1(t) * h_2(t)} \rightarrow y(t)$$

Figure 50: Cascade connection of systems

2.3b System Interconnections

Example 22: Determine the equivalent impulse response $h[n]$ of the overall system as shown below (Figure 51), where

$$h_1[n] = u[n],$$
$$h_2[n] = u[n+2] - u[n],$$
$$h_3[n] = \delta[n-2],$$
$$h_4[n] = \alpha^n u[n]$$

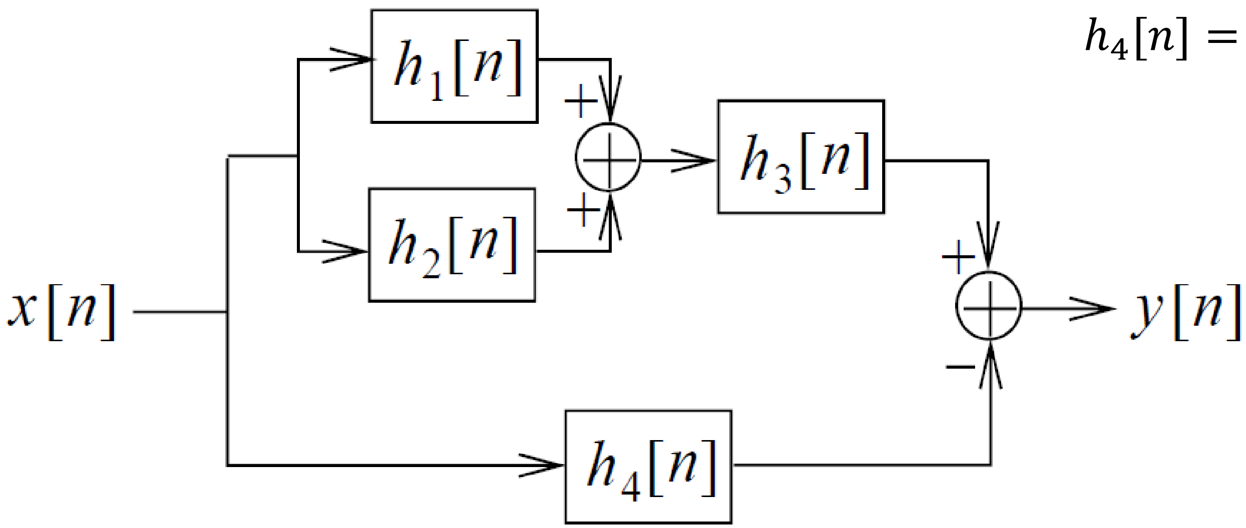


Figure 51: Example on interconnections of systems

2.3b System Interconnections

Example 22: Determine the equivalent impulse response $h[n]$ of the overall system as shown below (Figure 51), where

$h_1[n] = u[n]$,

$h_2[n] = u[n+2] - u[n]$,

$h_3[n] = \delta[n-2]$,

$h_4[n] = \alpha^n u[n]$

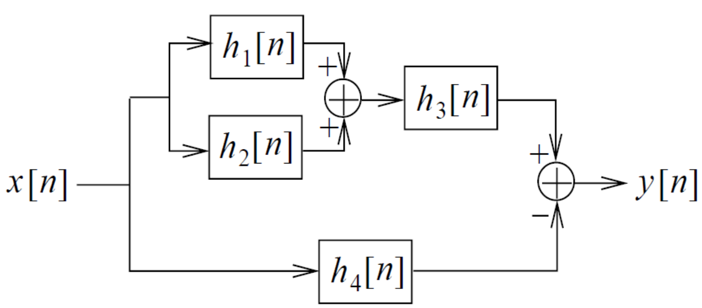


Figure 51: Example on interconnections of systems

The resultant overall system impulse response is

$$
\begin{aligned}
h[n] &= \{h_1[n] + h_2[n]\} * h_3[n] - h_4[n] \\
&= \{u[n] + u[n+2] - u[n]\} * \delta[n-2] - \alpha^n u[n] \\
&= u[n+2] * \delta[n-2] - \alpha^n u[n] \\
&= u[n] - \alpha^n u[n] \\
&= \{1 - \alpha^n\} u[n]
\end{aligned}
$$

2.3 Properties of LTI Systems Summary 10

- 2.3a Properties of LTI Systems
  - 1) Memoryless: $h[n] = c\delta[n]$, or $h(t) = c\delta(t)$
  - 2) Causal: $h[n] = 0$, for $n < 0$.
    $h(t) = 0$, for $t < 0$.
  - 3) BIBO Stable:
    $$\sum_{n=-\infty}^{\infty} |h[n]| < \infty, \text{ or } \int_{-\infty}^{\infty} |h(t)|\, dt < \infty$$

- 2.3b System Interconnections
  - 1) Parallel Connection : Summation of subsystem impulse responses.
  - 2) Cascade Connection: Convolution of subsystem impulse responses.

You have reached the end of 2.3. Please proceed.

IE2110
Signals and Systems Part 1
2.4 Correlation Functions I

with Instructor:

A/P Teh Kah Chan

Outline of Signals & Systems - Part 1

2.4 Correlation Function

- The correlation function is a mathematical expression of how correlated two signals are as a function of how much one of them is shifted

- The correlation function between two functions is a function of the amount of shift

- The Two types of correlation functions are:
  1) Autocorrelation function
  2) Cross correlation function

2.4b Autocorrelation Function

The autocorrelation is the correlation of a function with itself

- 1.1. For an energy-type signal $x[n]$ or $x(t)$

$$\text{DT signal:} \qquad \mathcal{R}_{xx}[m] = \sum_{n=-\infty}^{\infty} x[n]x^*[n+m]$$

$$\text{CT signal:} \qquad \mathcal{R}_{xx}(\tau) = \int_{-\infty}^{\infty} x(t)x^*(t+\tau)dt$$

where $x^*(t)$ denotes the complex conjugation of $x(t)$.

- 1.2. For a power-type signal $x[n]$ or $x(t)$

$$\text{DT signal:} \qquad \mathcal{R}_{xx}[m] = \lim_{K\to\infty} \frac{1}{2K+1} \sum_{n=-K}^{K} x[n]x^*[n+m]$$

$$\text{CT signal:} \qquad \mathcal{R}_{xx}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x^*(t+\tau)dt$$

2.4b Autocorrelation Function

1) Autocorrelation function

- 1.3. For an energy-type signal $x[n]$ or $x(t)$

$$\begin{aligned} \text{DT signal:} \qquad & E_x = \mathcal{R}_{xx}[0] \\ \text{CT signal:} \qquad & E_x = \mathcal{R}_{xx}(0) \end{aligned}$$

- 1.4. For a power-type signal $x[n]$ or $x(t)$

$$\begin{aligned} \text{DT signal:} \qquad & P_x = \mathcal{R}_{xx}[0] \\ \text{CT signal:} \qquad & P_x = \mathcal{R}_{xx}(0) \end{aligned}$$

2.4b Autocorrelation Function

2) Properties of autocorrelation function of a real-valued signal

- The peak of autocorrelation function occurs at the zero shift

DT signal: $$\mathcal{R}_{xx}[0] \geq \mathcal{R}_{xx}[m]$$

CT signal: $$\mathcal{R}_{xx}(0) \geq \mathcal{R}_{xx}(\tau)$$

- Autocorrelation functions are even functions

DT signal: $$\mathcal{R}_{xx}[m] = \mathcal{R}_{xx}[-m]$$

CT signal: $$\mathcal{R}_{xx}(\tau) = \mathcal{R}_{xx}(-\tau)$$

- A time shift in the signal does not change its autocorrelation function, i.e., the autocorrelation functions of $x(t)$ and $x(t-T_1)$ are the same

2.4 Correlation Functions I
Summary 11

- Autocorrelation Function
  - 1.1. For an energy-type signal $x[n]$ or $x(t)$

$$\text{DT signal:} \qquad \mathcal{R}_{xx}[m] = \sum_{n=-\infty}^{\infty} x[n]x^*[n+m]$$

$$\text{CT signal:} \qquad \mathcal{R}_{xx}(\tau) = \int_{-\infty}^{\infty} x(t)x^*(t+\tau)dt$$

  - 1.2. For a power-type signal $x[n]$ or $x(t)$

$$\text{DT signal:} \qquad \mathcal{R}_{xx}[m] = \lim_{K\to\infty} \frac{1}{2K+1} \sum_{n=-K}^{K} x[n]x^*[n+m]$$

$$\text{CT signal:} \qquad \mathcal{R}_{xx}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x^*(t+\tau)dt$$

You have reached the end of 2.4(I). Proceed with more examples ahead.

IE2110

Signals and Systems Part 1

2.4 Correlation Functions II

with Instructor:

A/P Teh Kah Chan

Outline of Signals & Systems - Part 1

1. Signals and Systems ✓✓
   - 1.1 Classification of Signals ✓
   - 1.2 Elementary and Singularity Signals ✓
   - 1.3 Operations on Signals ✓
   - 1.4 Properties of Systems ✓

2. Linear Time-Invariant (LTI) Systems
   - 2.1 Discrete-Time and Continuous-Time LTI Systems ✓
   - 2.2 Convolution ✓
   - 2.3 LTI System Properties ✓
   - 2.4 **Correlation Functions** ☜ **More examples on this…**

2.4b Correlation Functions

Example 23:
Find the autocorrelation function and power of the sinusoidal signal $x(t) = A\sin(2\pi f_0 t)$

Since $x(t)$ is a power-type signal, the autocorrelation function is given by

$$\begin{aligned}
\mathcal{R}_{xx}(\tau) &= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)x^*(t+\tau)dt \\
&= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} A^2 \times \sin(2\pi f_0 t) \times \sin(2\pi f_0 (t+\tau))dt \\
&= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \frac{A^2}{2}\left[\cos(2\pi f_0 \tau) - \cos(2\pi f_0 (2t+\tau))\right] dt \\
&= \frac{A^2}{2} \times \cos(2\pi f_0 \tau)
\end{aligned}$$

2.4b Correlation Functions

Example 23: Find the autocorrelation function and power of the sinusoidal signal $x(t) = A\sin(2\pi f_0 t)$

The power of signal $x(t)$ is given by $P_x = \mathcal{R}_{xx}(0) = \frac{A^2}{2}$

Alternatively, based on the definition of power, we have

$$
\begin{aligned}
P_x &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt \\
&= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 \, dt \\
&= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |A\sin(2\pi f_0 t)|^2 \, dt \\
&= \frac{A^2}{2}
\end{aligned}
$$

2.4b Correlation Functions

Example 24:
Find the autocorrelation function and power of the sinusoidal signal $y(t) = A\sin\{2\pi f_0(t - T_1)\}$, where T_1 is an arbitrary constant delay.

Denote $\theta = 2\pi f_0 T_1$, we have

$$\begin{aligned} y(t) &= A\sin(2\pi f_0 t - 2\pi f_0 T_1) \\ &= A\sin(2\pi f_0 t - \theta) \end{aligned}$$

Since $y(t)$ is a power-type signal, the autocorrelation function is given by

$$\mathcal{R}_{yy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} y(t) y^*(t+\tau) dt$$

2.4b Correlation Functions

Example 24: Find the autocorrelation function and power of the sinusoidal signal $y(t) = A\sin\{2\pi f_0(t - T_1)\}$, where T_1 is an arbitrary constant delay.

Earlier, we have $y(t) = A\sin(2\pi f_0 t - \theta)$

Since $y(t)$ is a power-type signal, the autocorrelation function is given by

$$
\begin{aligned}
\mathcal{R}_{yy}(\tau) &= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} y(t)y^*(t+\tau)dt \\
&= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} A^2 \times \sin(2\pi f_0 t - \theta) \times \sin(2\pi f_0(t+\tau) - \theta)dt \\
&= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} \frac{A^2}{2}\left[\cos(2\pi f_0\tau) - \cos(2\pi f_0(2t+\tau) - 2\theta)\right]dt \\
&= \frac{A^2}{2} \times \cos(2\pi f_0 \tau)
\end{aligned}
$$

Comparing with the results with Example 23, we conclude that the autocorrelation functions of $x(t)$ and $x(t - T_1)$ are the same.

2.4b Correlation Functions

Example 25:

Find the autocorrelation function of the signal $x[n]$ as shown below (Figure 52) using the graphical approach.

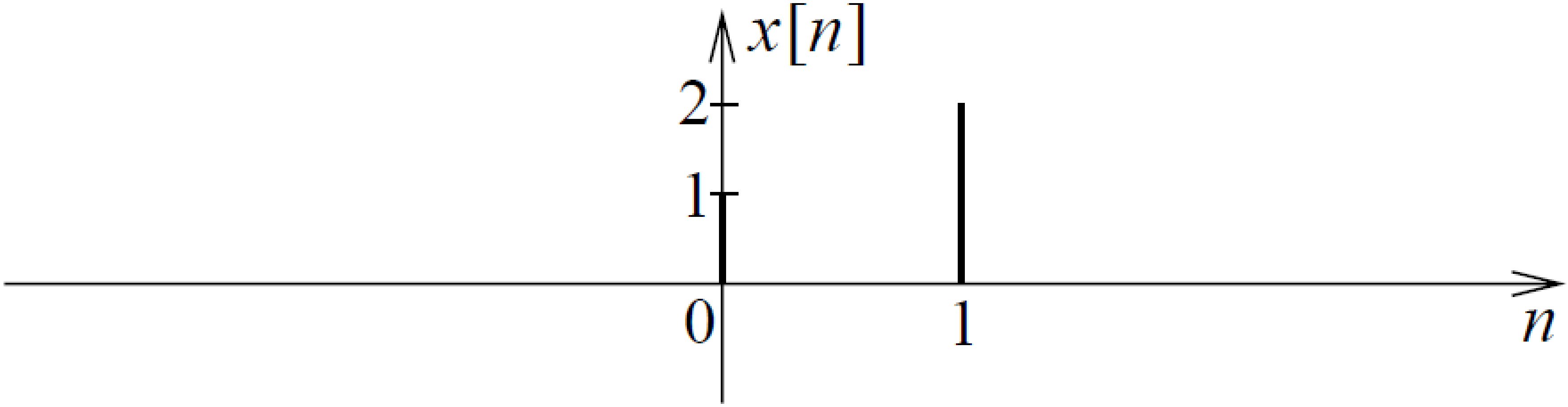


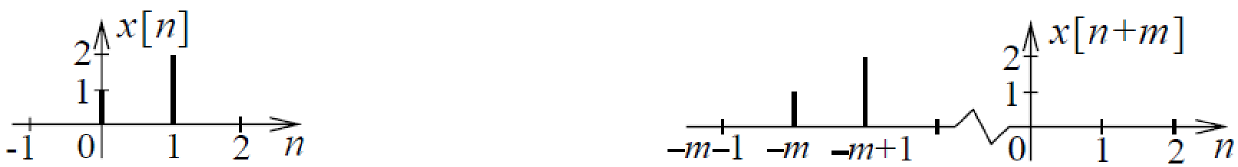
Figure 52: Example on autocorrelation function of a DT signal

2.4b Correlation Functions

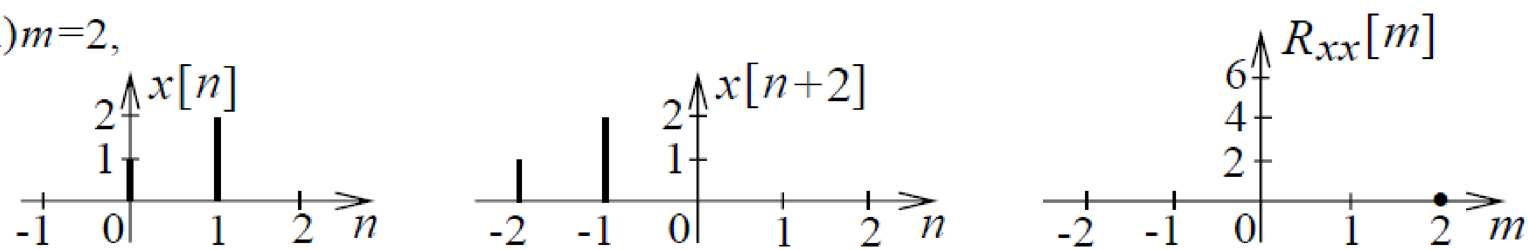
Example 25:

Since $x[n]$ is an energy-type signal,

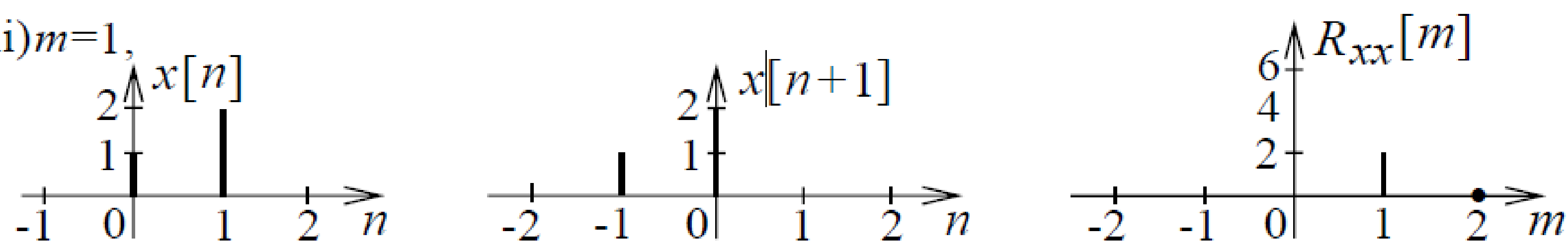
$$\mathcal{R}_{xx}[m] = \sum_{n=-\infty}^{\infty} x[n]x^*[n+m] = \sum_{n=-\infty}^{\infty} x[n]x[n+m]$$



(i) $m=2$,



(ii) $m=1$,



2.4b Correlation Functions

Example 25:

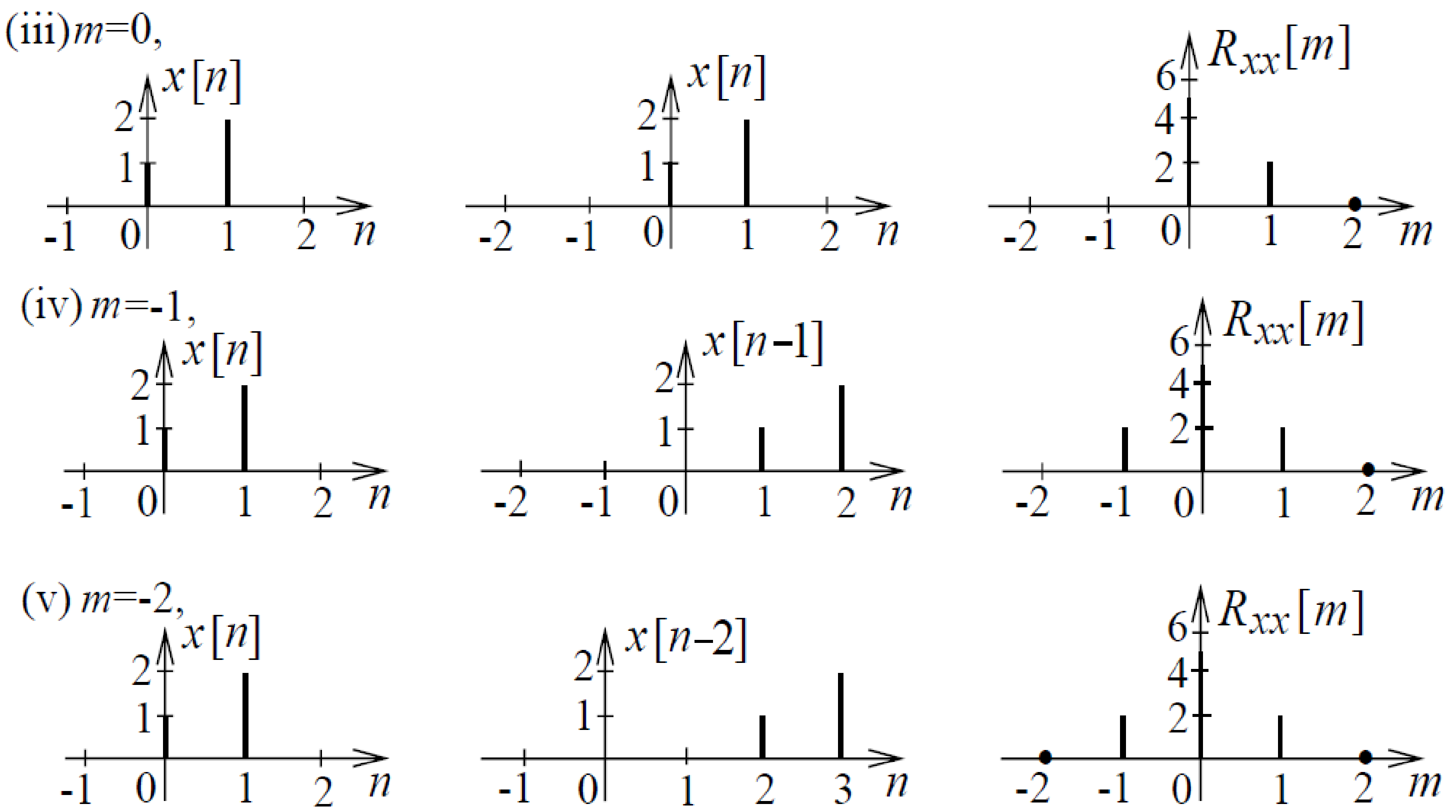


Figure 53: Solution for example on autocorrelation function

2.4 Correlation Functions II
Summary 12

- ❑ Examples on Autocorrelation Functions.
- ❑ The autocorrelation functions of $x(t)$ and $x(t-T_1)$ are the same.
- ❑ The graphical approach for evaluating the autocorrelation function.

You have reached the end of 2.4(II). Take some time to reflect then press on!

IE2110

Signals and Systems Part 1

2.4 Correlation Functions III

with Instructor:

A/P Teh Kah Chan

Outline of Signals & Systems - Part 1

1. Signals and Systems ✓✓
   - 1.1 Classification of Signals ✓
   - 1.2 Elementary and Singularity Signals ✓
   - 1.3 Operations on Signals ✓
   - 1.4 Properties of Systems ✓

2. Linear Time-Invariant (LTI) Systems
   - 2.1 Discrete-Time and Continuous-Time LTI Systems ✓
   - 2.2 Convolution ✓
   - 2.3 LTI System Properties ✓
   - 2.4 **Correlation Functions**☜ **Cross Correlation Function**

2.4c Cross Correlation Functions

The cross correlation is the correlation of two different functions

- For energy-type signals $x[n]$ and $y[n]$ (or $x(t)$ and $y(t)$)

$$\text{DT signal:} \quad \mathcal{R}_{xy}[m] = \sum_{n=-\infty}^{\infty} x[n]y^*[n+m]$$

$$\text{CT signal:} \quad \mathcal{R}_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)y^*(t+\tau)dt$$

- For power-type signals $x[n]$ and $y[n]$ (or $x(t)$ and $y(t)$)

$$\text{DT signal:} \quad \mathcal{R}_{xy}[m] = \lim_{K\to\infty} \frac{1}{2K+1} \sum_{n=-K}^{K} x[n]y^*[n+m]$$

$$\text{CT signal:} \quad \mathcal{R}_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)y^*(t+\tau)dt$$

2.4c Cross Correlation Functions

Example 26: Find the cross correlation function between the two signals
$x(t) = \exp(j2\pi f_0 t)$ and $y(t) = \exp(j2\pi 2 f_0 t)$.

$$
\begin{aligned}
\mathcal{R}_{xy}(\tau) &= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)y^*(t+\tau)dt \\
&= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \exp(j2\pi f_0 t) \times \exp[-j2\pi 2 f_0(t+\tau)]dt \\
&= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \exp(-j2\pi f_0 t) \times \exp(-j4\pi f_0 \tau)dt \\
&= \exp(-j4\pi f_0\tau) \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} [\cos(2\pi f_0 t) - j\sin(2\pi f_0 t)]\, dt \\
&= 0
\end{aligned}
$$

2.4c Cross Correlation Functions

Example 27: Find the cross correlation function between the two signals $x(t)$ and $y(t)$ as shown below (Figure 54) using the graphical approach.

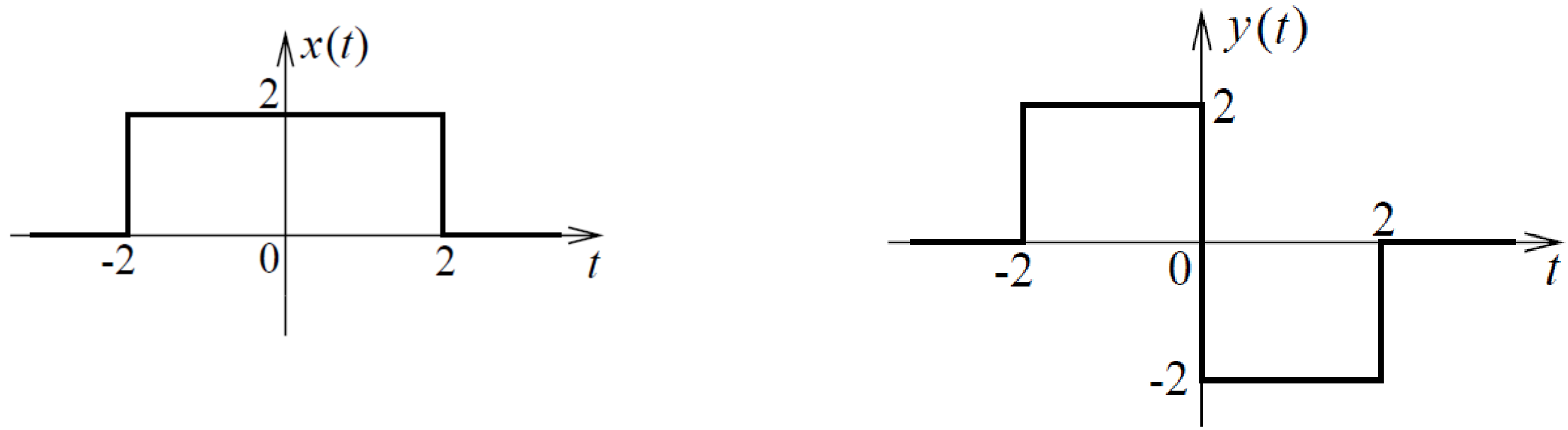


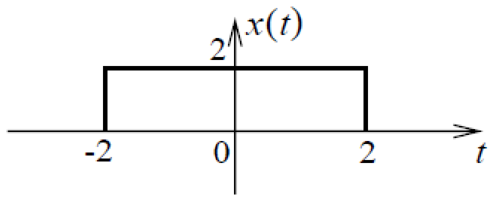
Figure 54: Example on cross correlation function of CT signals

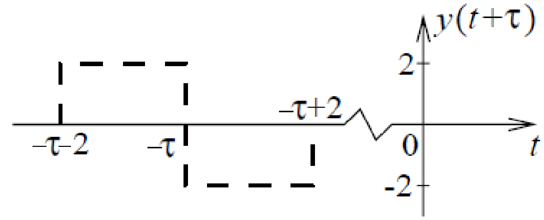
2.4c Cross Correlation Functions

Example 27:

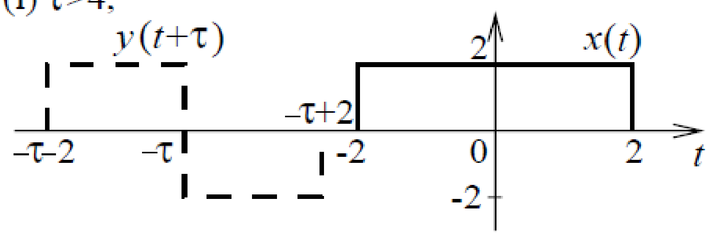
Since $x(t)$ and $y(t)$ are energy-type signals:

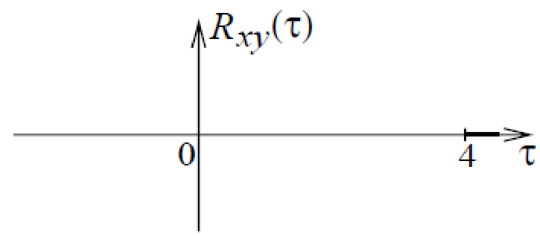
$$\mathcal{R}_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)y^*(t+\tau)dt = \int_{-\infty}^{\infty} x(t)y(t+\tau)dt$$



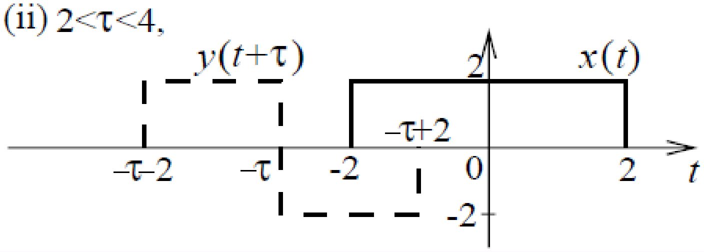


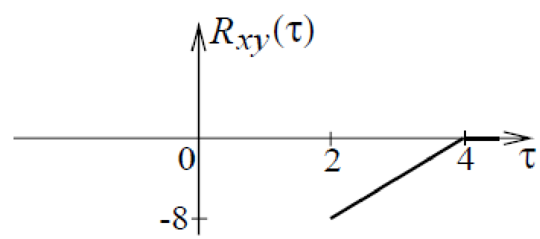
(i) τ>4,





(ii) 2<τ<4,





2.4c Cross Correlation Functions

Example 27:

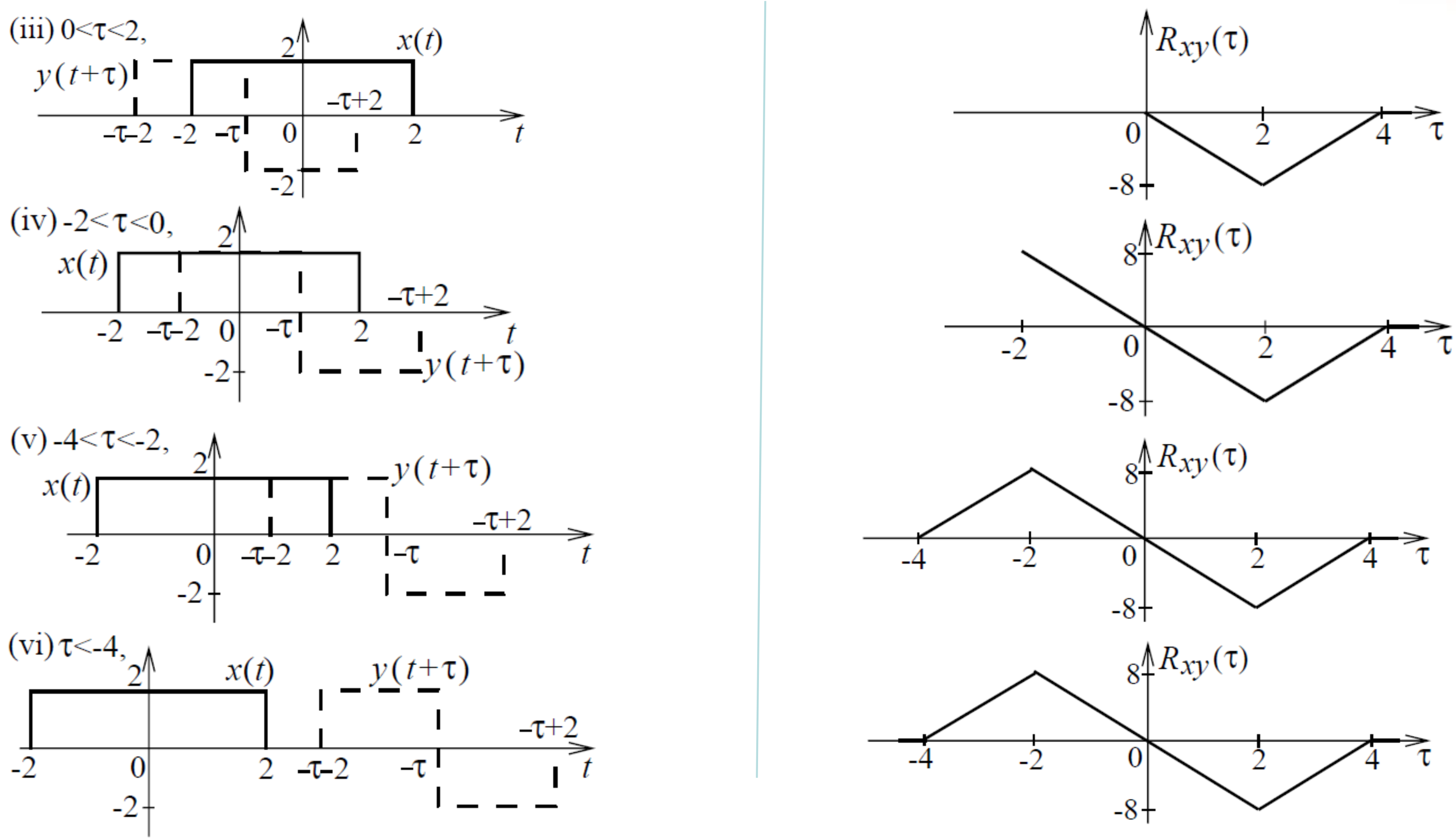


Figure 55: Solution for example on cross correlation function

2.4 Correlation Functions III
Summary 13

- For an energy-type signal $x[n]$ and $y[n]$ (or $x(t)$ and $y(t)$):

$$\text{DT Signals}: \; R_{xy}[m] = \sum_{n=-\infty}^{\infty} x[n] y^*[n+m]$$

$$\text{CT Signals}: \; R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t) y^*(t+\tau) dt$$

- For a power-type signal $x[n]$ and $y[n]$ (or $x(t)$ and $y(t)$):

$$\text{DT Signals}: \; R_{xy}[m] = \lim_{K\to\infty} \frac{1}{2K+1} \sum_{n=-K}^{K} x[n] y^*[n+m]$$

$$\text{CT Signals}: \; R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) y^*(t+\tau) dt$$

You have reached the end of 2.4.