IE2110

Signals and Systems Part 1

1.2 Elementary and Singularity Signals

with Instructor:

A/P Teh Kah Chan

Outline of Signals & Systems - Part 1

1. Signals and Systems
    - 1.1 **Classification of Signals**☜ Recap through further examples √
    - 1.2 **Elementary** ☜ and Singularity Signals
    - 1.3 Operations on Signals
    - 1.4 Properties of Systems

2. Linear Time-Invariant (LTI) Systems
    - 2.1 Discrete-Time and Continuous-Time LTI Systems
    - 2.2 Convolution
    - 2.3 LTI System Properties
    - 2.4 Correlation Functions

1.2 Elementary and Singularity Signals

Elementary Signals

1) Exponential ☜
2) Sinusoidal ☜
3) Complex exponential ☜

Singularity Signals

1) Impulse function
2) Step function
3) Signum function
4) Rectangular function
5) Sinc function

1.2 Elementary Signals

1) Exponential signal

$$x(t) = A\exp(at)$$

- $x(t)$ is growing if $a > 0$
- $x(t)$ is decaying if $a < 0$

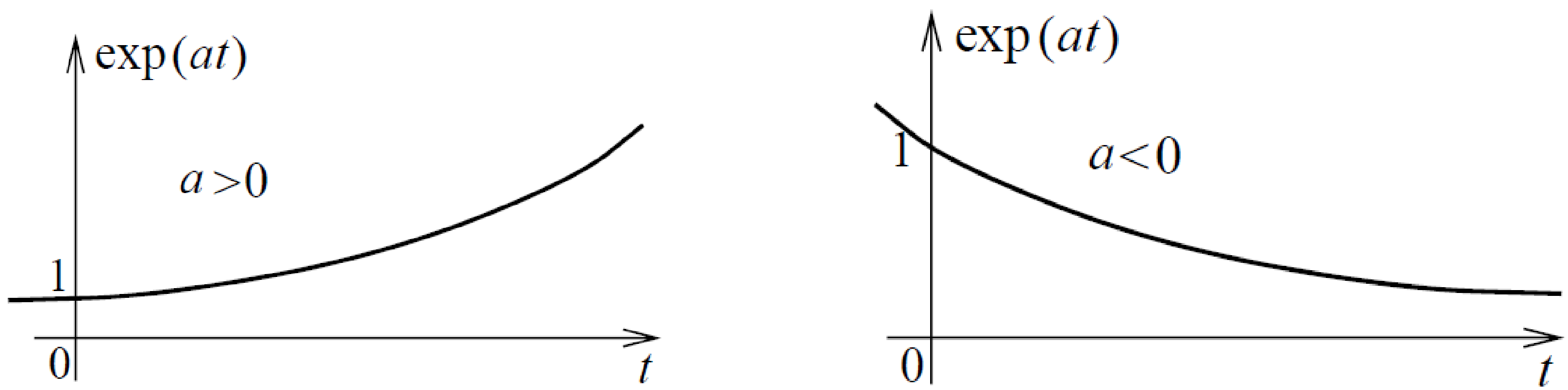

Figure 12: Exponential signal

1.2 Elementary Signals

2) Sinusoidal signal

$$x(t) = A\cos(2\pi f_0 t + \theta) \quad \text{or} \quad A\sin(2\pi f_0 t + \theta)$$

where A is the amplitude, f_0 is the frequency in Hertz, and θ is the phase angle in radians

- A sinusoidal signal is periodic with period $T_0 = 1/f_0$

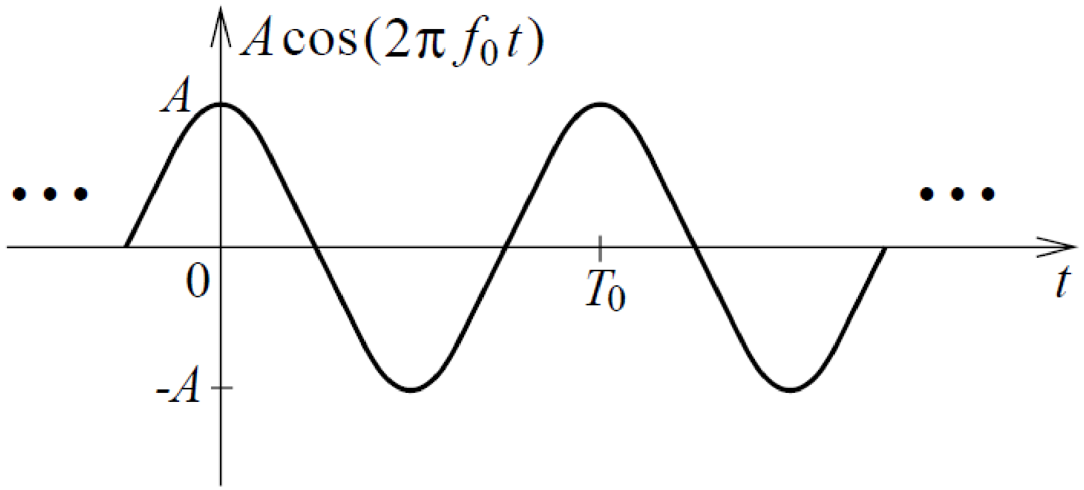

Figure 13: CT sinusoidal signal

1.2 Elementary Signals

- The discrete-time version of the sinusoidal signal is

$$x[n] = A\cos\left(\frac{2\pi n}{K_0} + \theta\right) \quad \text{or} \quad A\sin\left(\frac{2\pi n}{K_0} + \theta\right)$$

where A is the amplitude, K_0 is a positive integer defined as the fundamental period, and θ is the phase angle in radians

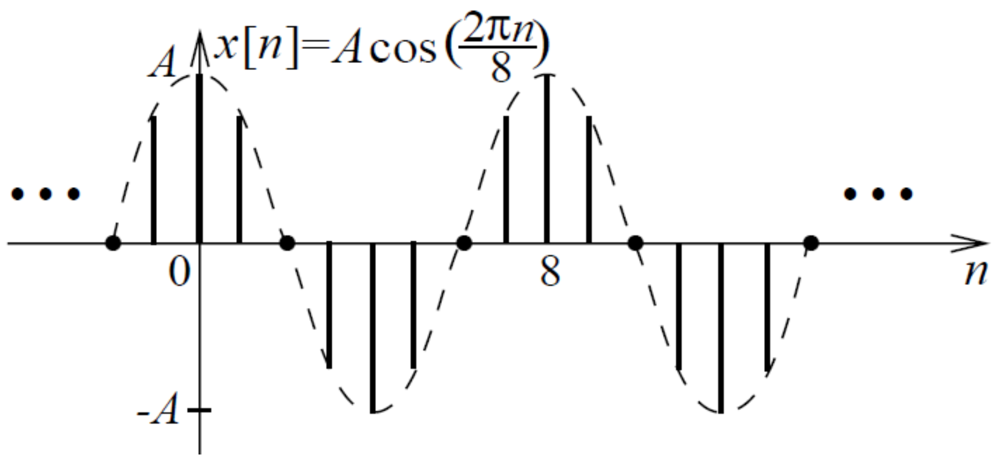

Figure 14: DT sinusoidal signal

1.2 Elementary Signals

3) Complex exponential signal

$$A\exp(j2\pi f_0 t) = A\cos(2\pi f_0 t) + jA\sin(2\pi f_0 t)$$

- The magnitude of complex exponential signal is given by

$$|A\exp(j2\pi f_0 t)| = A$$

- The sinusoidal signal can be expressed as

$$A\cos(2\pi f_0 t + \theta) = \Re\{A\exp(j2\pi f_0 t)\exp(j\theta)\}$$

and

$$A\sin(2\pi f_0 t + \theta) = \Im\{A\exp(j2\pi f_0 t)\exp(j\theta)\}$$

Elementary Signals Summary 3

- Example 9 on Overall Classification of Signals
- Elementary Signals

  1) Exponential Signal: $x(t) = A\exp(at)$

  2) Sinusoidal Signal: $x(t) = A\cos(2\pi f_0 t + \theta)$

  $$x[n] = A\cos\left(\frac{2\pi n}{K_0} + \theta\right)$$

  3) Complex Exponential Signal:

  $$\begin{aligned} x(t) &= A\exp(j2\pi f_0 t) \\ &= A\cos(2\pi f_0 t) + jA\sin(2\pi f_0 t) \end{aligned}$$

Reflect on how much you have understood the lesson so far before proceeding.

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Signals and Systems Part 1

1.2 Elementary and Singularity Signals

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Outline of Signals & Systems - Part 1

1. Signals and Systems
   - 1.1 Classification of Signals
   - 1.2 **Elementary** ✓ and Singularity Signals
   - 1.3 Operations on Signals
   - 1.4 Properties of Systems

2. Linear Time-Invariant (LTI) Systems
   - 2.1 Discrete-Time and Continuous-Time LTI Systems
   - 2.2 Convolution
   - 2.3 LTI System Properties
   - 2.4 Correlation Functions

Recap: 1.2 Elementary Signals

Example 10: Sketch the function $x(t) = 5\exp(-at) \times \cos(2\pi 10t)$ for $t > 0$. Assume that $a > 0$.

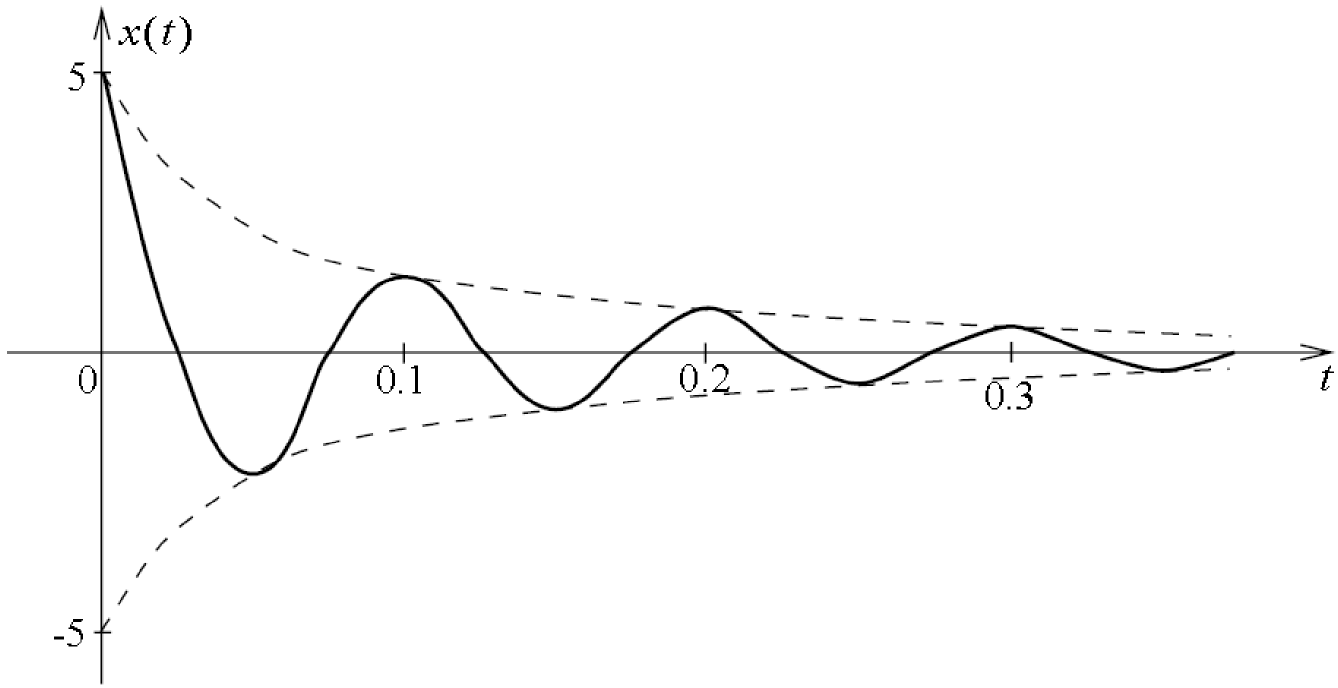

Figure 15: An exponentially damped sinusoidal signal

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1.2 Elementary and Singularity Signals

Elementary Signals

1) Exponential ✓
2) Sinusoidal ✓
3) Complex exponential ✓

Singularity Signals

1) Impulse function ☜
2) Step function ☜
3) Signum function ☜
4) Rectangular function ☜
5) Sinc function ☜

1.2 Singularity Signals

1) The DT unit impulse (or Dirac Delta) function $\delta[n]$ is defined as

$$\delta[n] = \begin{cases} 1, & n = 0, \\ 0, & n \neq 0. \end{cases}$$

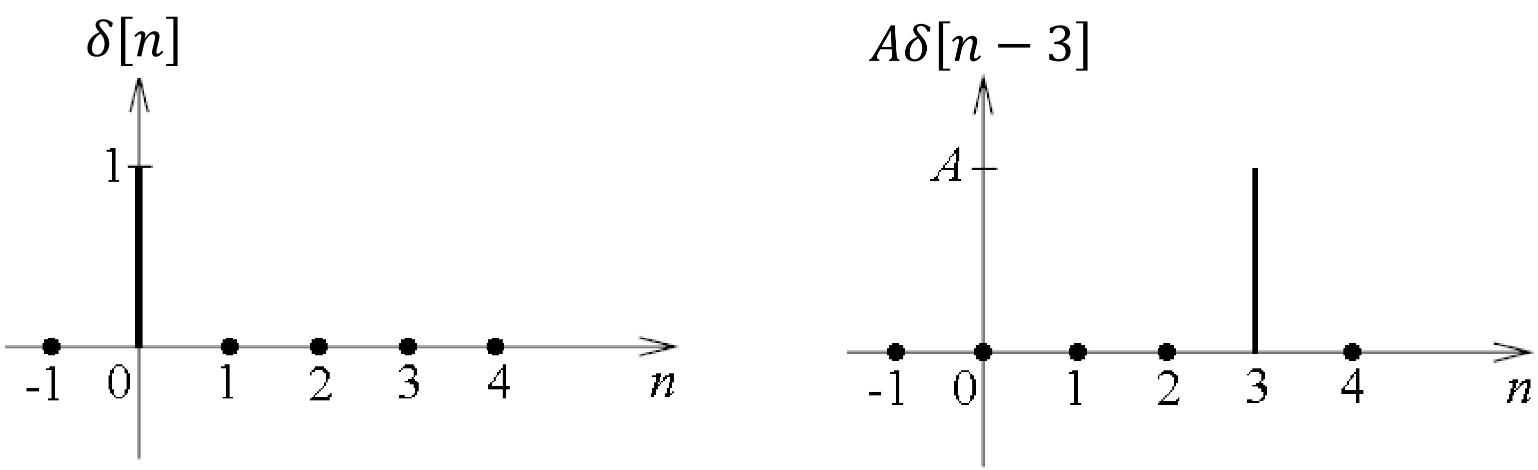

Figure 16: DT impulse functions

1.2 Singularity Signals

1) The CT unit impulse (or Dirac Delta) function $\delta(t)$ is defined as

$$\delta(t) = \begin{cases} \infty, & t = 0, \\ 0, & t \neq 0. \end{cases}$$

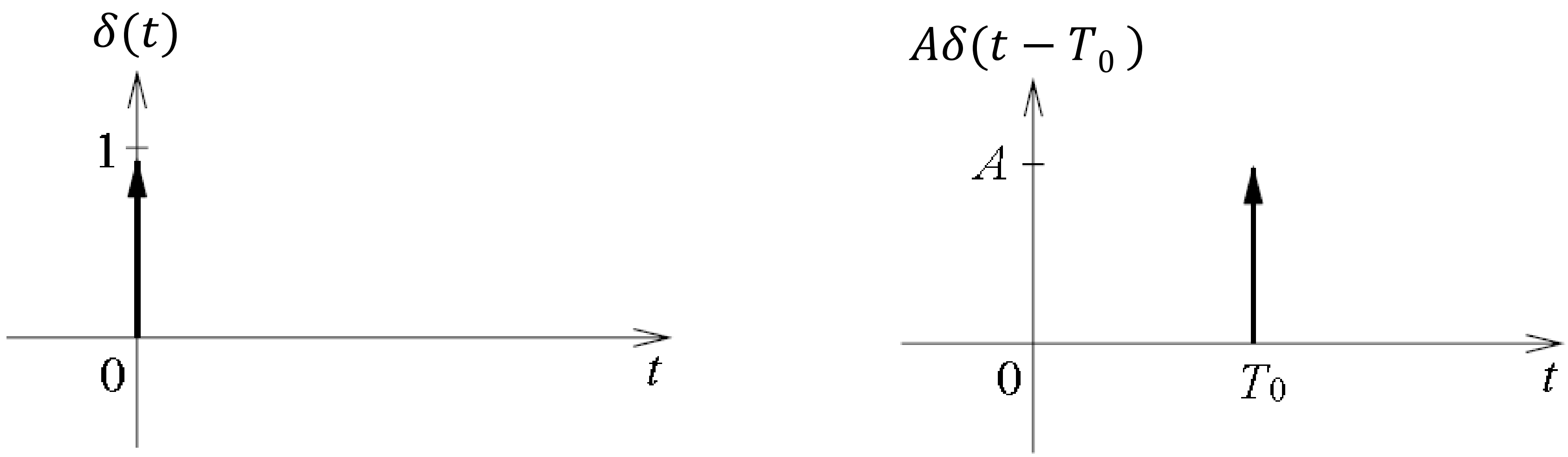

Figure 17: CT impulse functions

1.2 Singularity Signals

Properties of the CT impulse function

- Property One

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

- Property Two

$$x(t) \times \delta\,(t - T_0) \quad = \quad x(T_0) \times \delta\,(t - T_0)$$

- Property Three

$$\int_{-\infty}^{\infty} x(t) \times \delta(t - T_0)\, dt \quad = \quad x(T_0)$$

1.2 Singularity Signals

2) The CT unit step function $u(t)$ is defined as

$$u(t) = \begin{cases} 1, & t \geq 0, \\ 0, & t < 0. \end{cases}$$

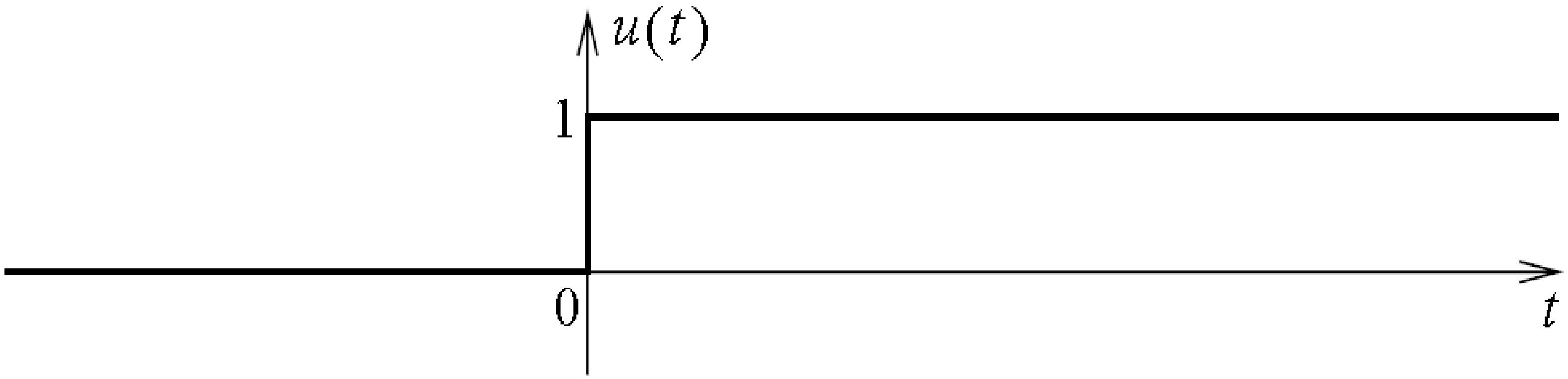

Figure 18: A CT unit step function

1.2 Singularity Signals

2) The DT unit step function $u[n]$ is defined as

$$u[n] = \begin{cases} 1, & n \geq 0, \\ 0, & n < 0. \end{cases}$$

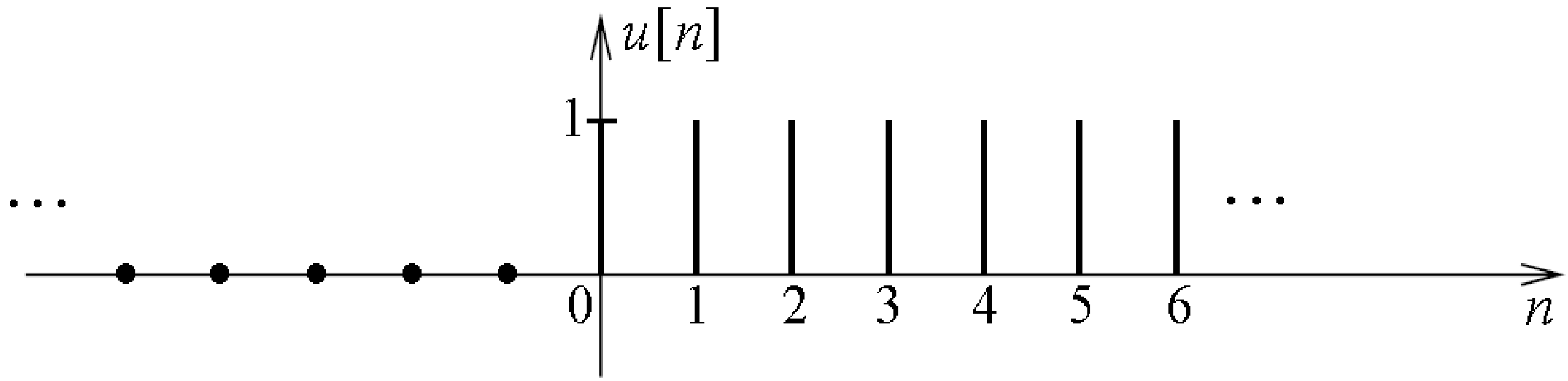

Figure 19: A DT unit step function

1.2 Singularity Signals

3) The CT signum function $\mathrm{sgn}(t)$ is defined as

$$\mathrm{sgn}(t) = \begin{cases} 1, & t > 0, \\ 0, & t = 0, \\ -1, & t < 0. \end{cases}$$

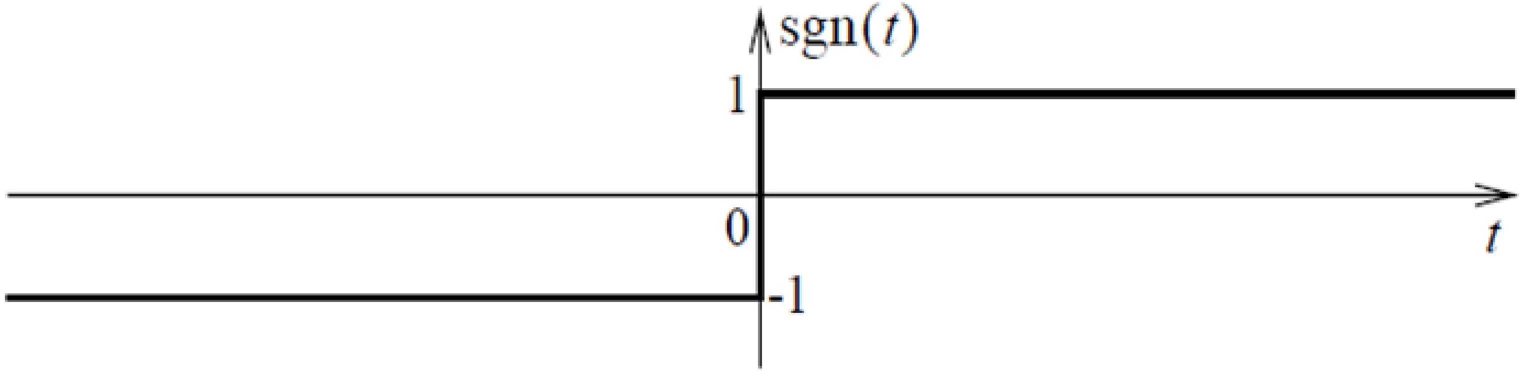

Figure 20: A CT signum function

1.2 Singularity Signals

3) The DT signum function sgn$[n]$ is defined as

$$\text{sgn}[n] = \begin{cases} 1, & n > 0, \\ 0, & n = 0, \\ -1, & n < 0. \end{cases}$$

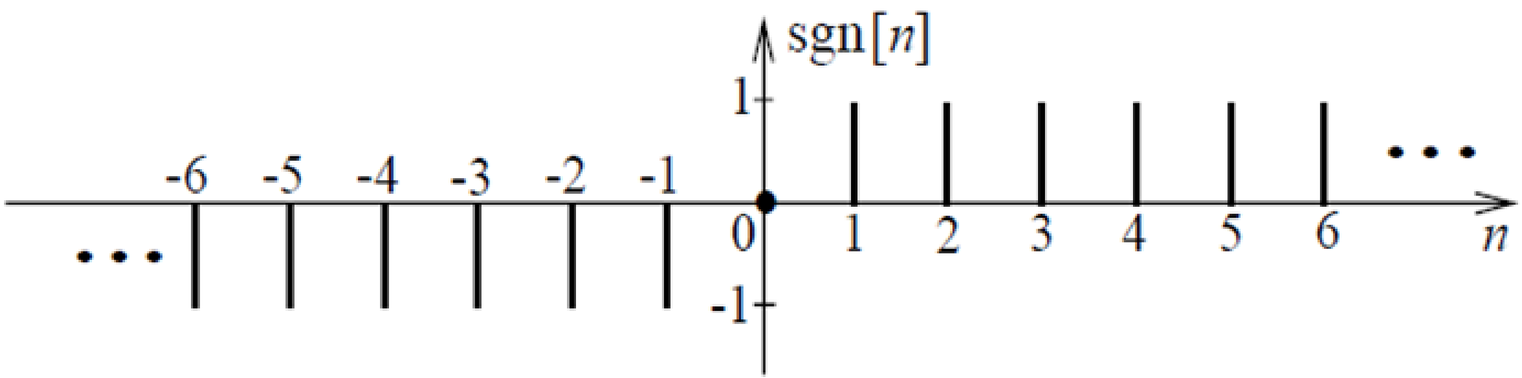

Figure 21: A DT signum function

1.2 Singularity Signals

4) The CT unit rectangular function $\mathrm{rect}\left(\frac{t}{T}\right)$ is defined as

$$\mathrm{rect}\left(\frac{t}{T}\right) = \begin{cases} 1, & |t| \le T/2, \\ 0, & \text{otherwise.} \end{cases}$$

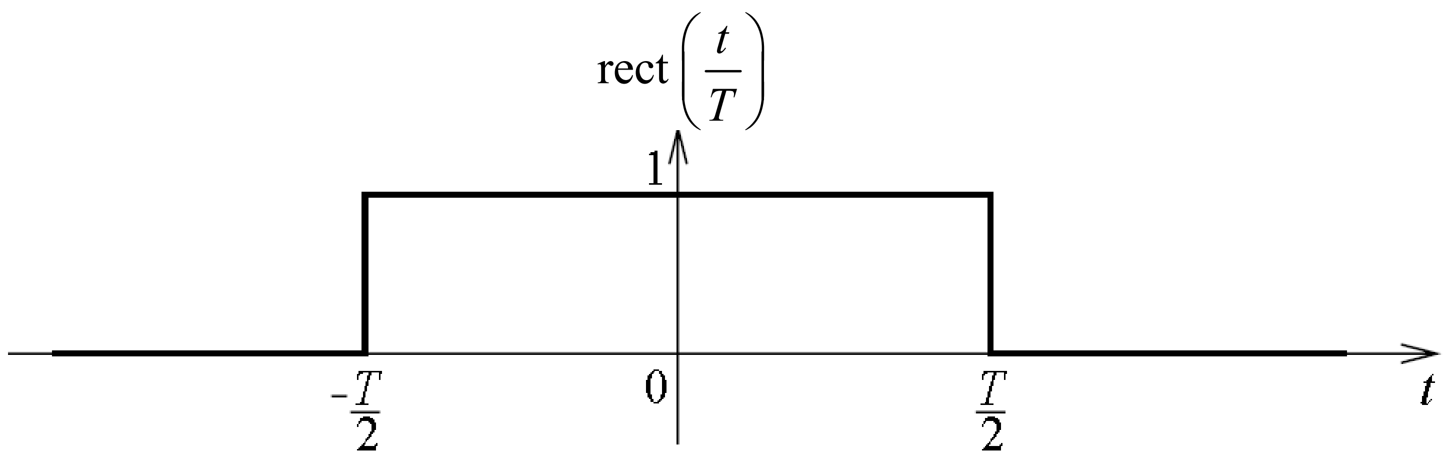

Figure 22: A CT unit rectangular function

1.2 Singularity Signals

4) The DT unit rectangular function $\mathrm{rect}\left[\frac{n}{K}\right]$ (assume that K is even) is defined as

$$\mathrm{rect}\left[\frac{n}{K}\right] = \begin{cases} 1, & |n| \le K/2, \\ 0, & \text{otherwise.} \end{cases}$$

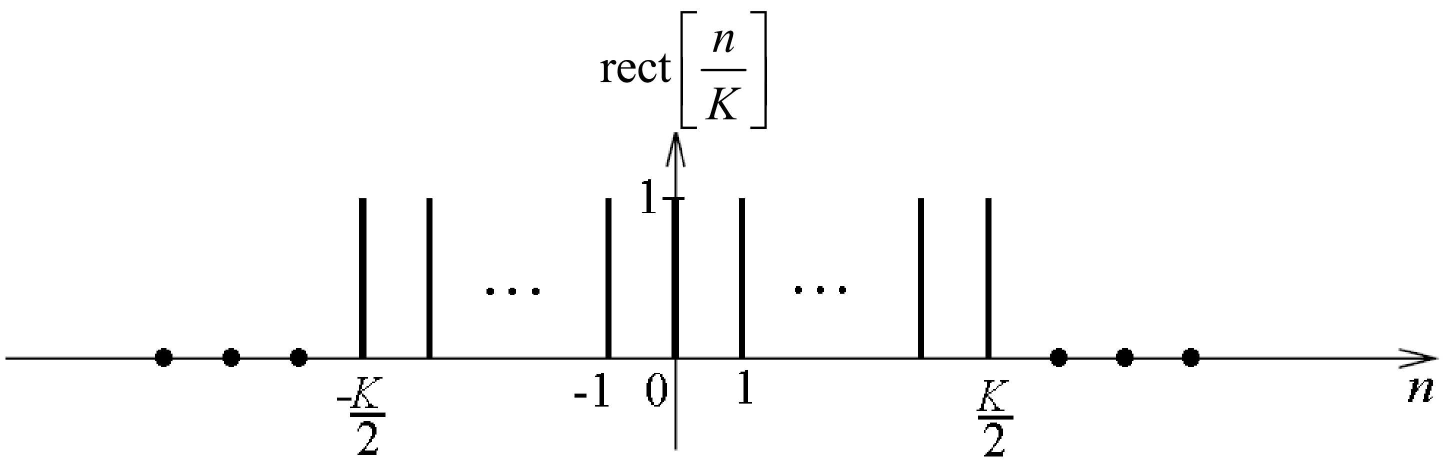

Figure 23: A DT unit rectangular function

1.2 Singularity Signals

5) The sinc function sinc(t) is defined as

$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

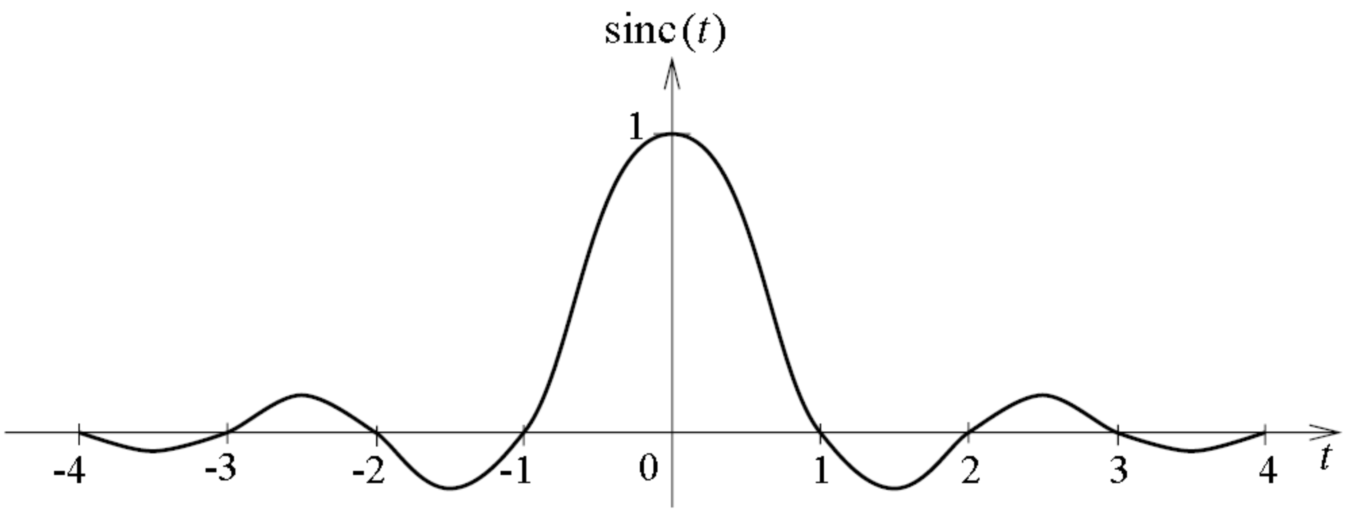

Figure 24: A sinc function

1.2 Singularity Signals

Example 11: The function $x(t) = 5 \times \text{sinc}(t)$is sampled at every T_s= 0.5 second interval to produce the sampled signal $x_s(t)$. Sketch the waveforms for $x(t)$ and $x_s(t)$, respectively.

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(t) \times \delta(t - nT_s)$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \times \delta(t - nT_s)$$

$$= \sum_{n=-\infty}^{\infty} 5 \times \text{sinc}(nT_s) \times \delta(t - nT_s)$$

1.2 Singularity Signals

Example 11: The function $x(t) = 5 \times \text{sinc}(t)$ is sampled at every $T_s = 0.5$ second interval to produce the sampled signal $x_s(t)$. Sketch the waveforms for $x(t)$ and $x_s(t)$, respectively.

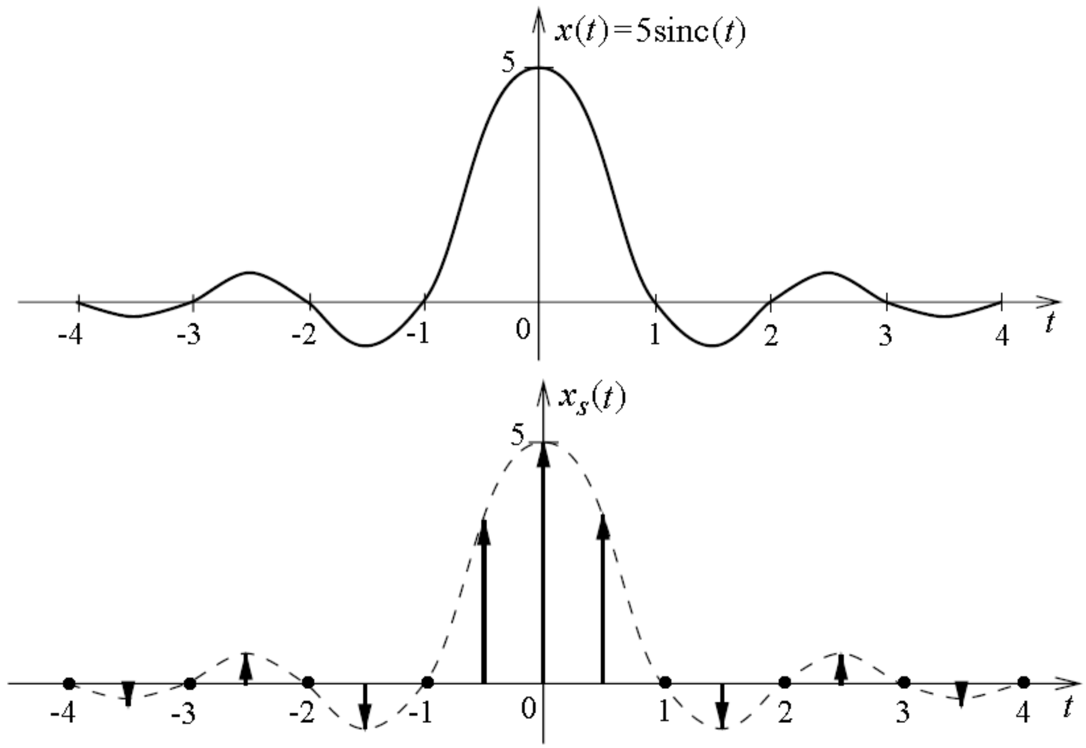

Figure 25: Waveforms for $x(t)$ and $x_s(t)$

Singularity Signals Summary 4

- Singularity Signals

1) Impulse Function: $\delta(t) = \begin{cases} \infty, & t = 0, \\ 0, & t \neq 0. \end{cases}$

2) Step Function: $u(t) = \begin{cases} 1, & t \geq 0, \\ 0, & t < 0. \end{cases}$

3) Signum Function: $\operatorname{sgn}(t) = \begin{cases} 1, & t > 0, \\ 0, & t = 0, \\ -1, & t < 0. \end{cases}$

4) Rectangular Function: $\operatorname{rect}\left(\dfrac{t}{T}\right) = \begin{cases} 1, & |t| \leq T/2, \\ 0, & \text{otherwise.} \end{cases}$

5) Sinc Function: $\operatorname{sinc}(t) = \dfrac{\sin(\pi t)}{\pi t}$

You have reached the end of 1.2: Elementary and Singularity Signals.

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IE2110
Signals and Systems Part 1
1.3 Operations on Signals

with Instructor:

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1.3 Operations on Signals

- Amplitude scaling: The operation $Ax(t)$ (or $Ax[n]$) is to multiply the amplitude of $x(t)$ (or $x[n]$) by an amount A

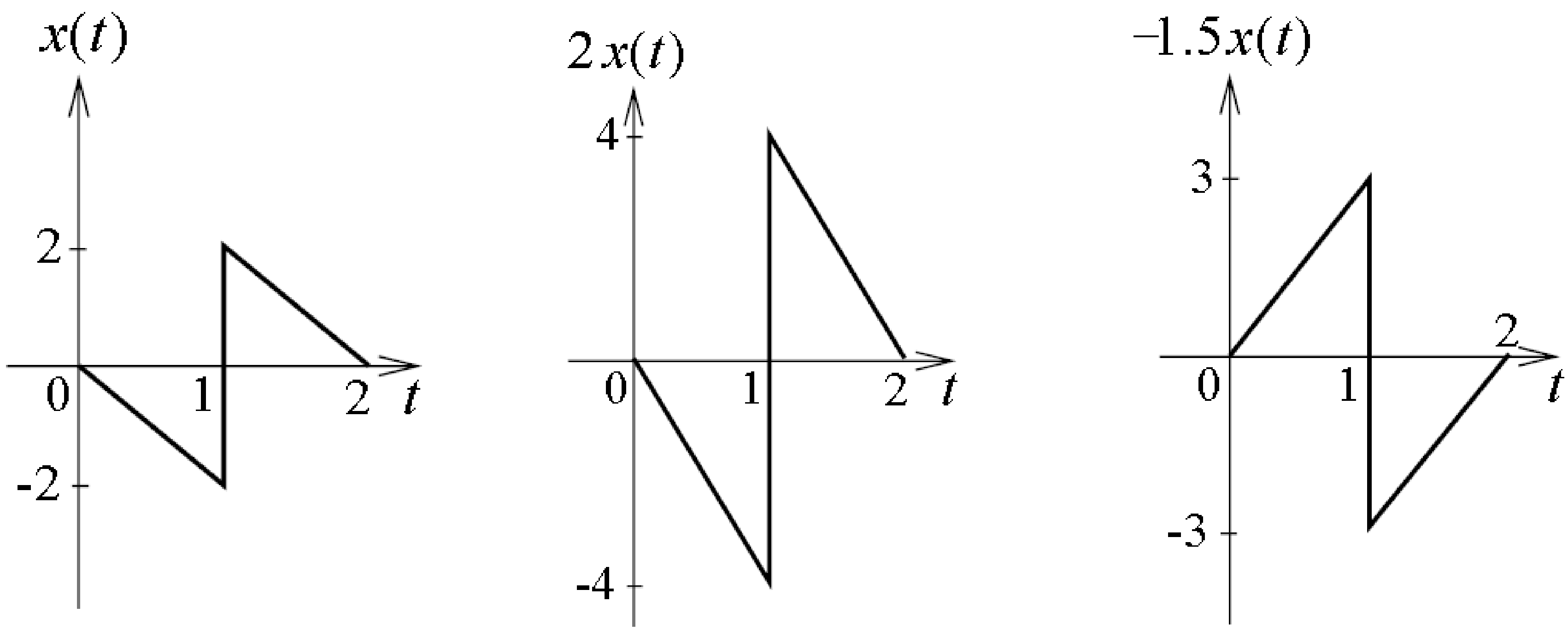

Figure 26: Amplitude scaling of signals

1.3 Operations on Signals

- Time shifting: The operation $x(t - T)$ (or $x[n - K]$) is to shift $x(t)$ (or $x[n]$) by an amount T (or K)

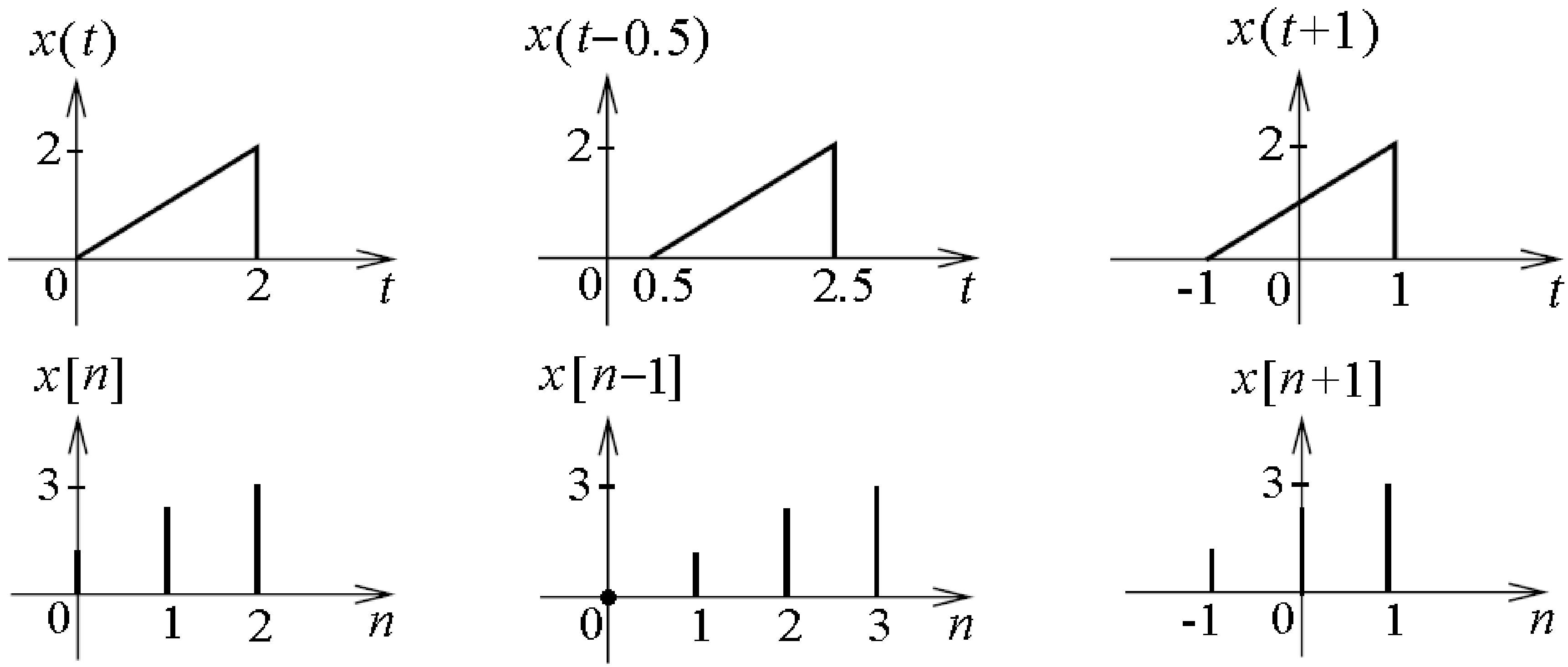

Figure 27: Time shifting of signals

1.3 Operations on Signals

Example 12: Show that $\mathrm{rect}\left(\frac{t}{T}\right) = u\left(t+\frac{T}{2}\right) - u\left(t-\frac{T}{2}\right)$.

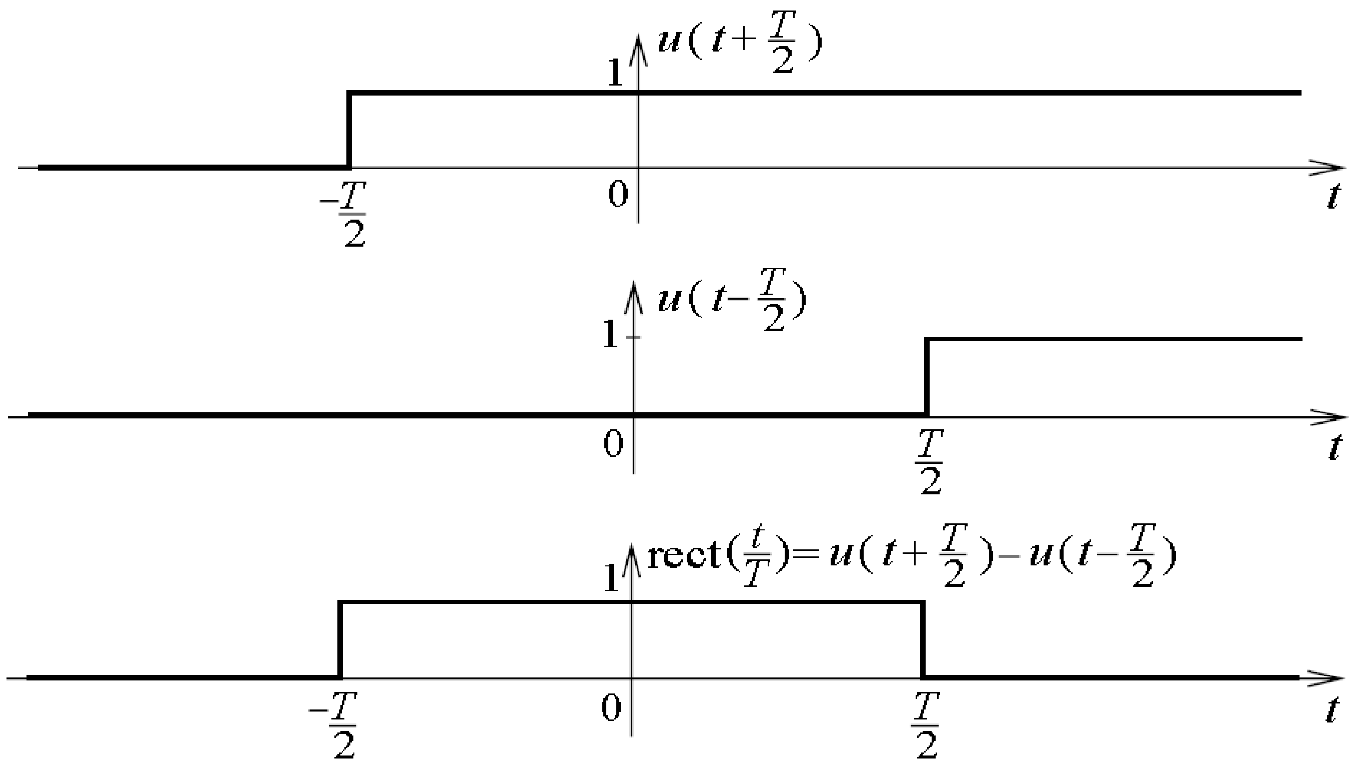

Figure 28: Example on time shifting operation

1.3 Operations on Signals

- CT time scaling: The operation $x(t/a)$ is to scale $x(t)$ by the factor a
  - ➢ It expands the function horizontally by the factor $|a|$
  - ➢ If $a < 0$, the function will be also time inverted

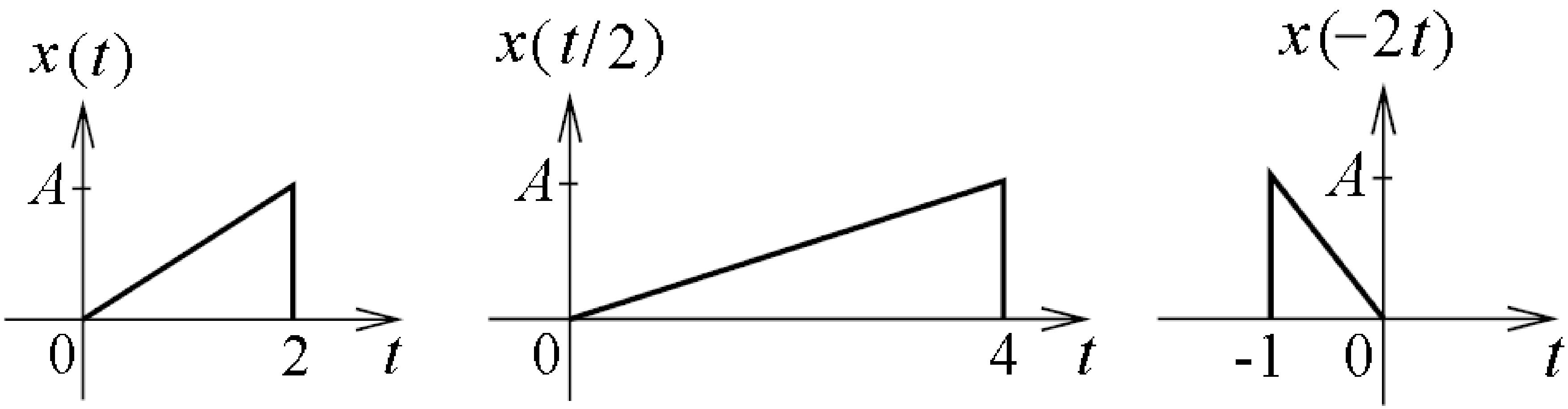

Figure 29: CT time scaling of signals

1.3 Operations on Signals

- DT time scaling: $x[Kn]$ or $x[n/K]$ where K is an integer
  - $x[Kn]$: Time compression or decimation
  - $x[n/K]$: Time expansion

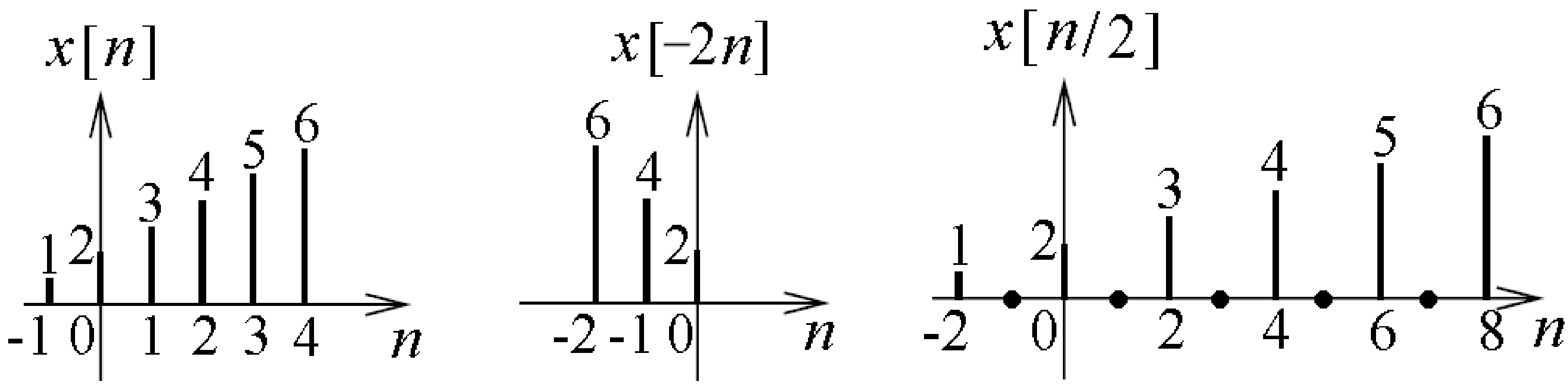

Figure 30: DT time scaling of signals

1.3 Operations on Signals

Example 13: If $x(t) = 0.5 \times \mathrm{rect}\left(\frac{t}{4}\right)$, as shown in Figure 31, sketch the waveform $y(t) = -2x\left(\frac{t-2}{2}\right)$.

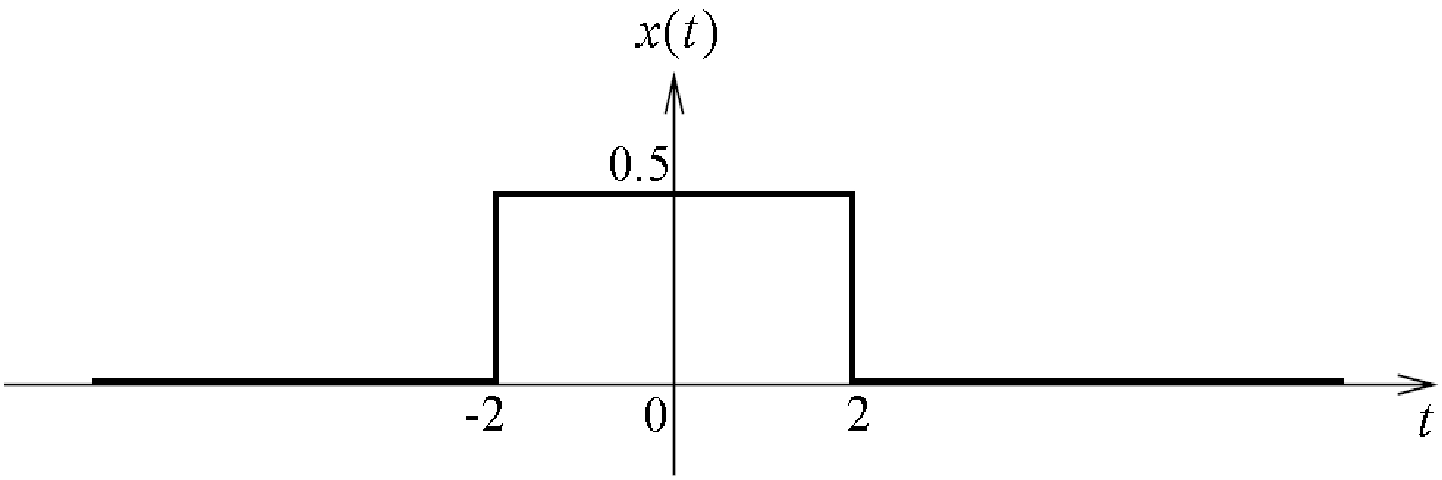

Figure 31: Example of operations on signals

1.3 Operations on Signals

Example 13:

If $x(t) = 0.5 \times \mathrm{rect}\left(\frac{t}{4}\right)$, sketch the waveform $y(t) = -2x\left(\frac{t-2}{2}\right)$.

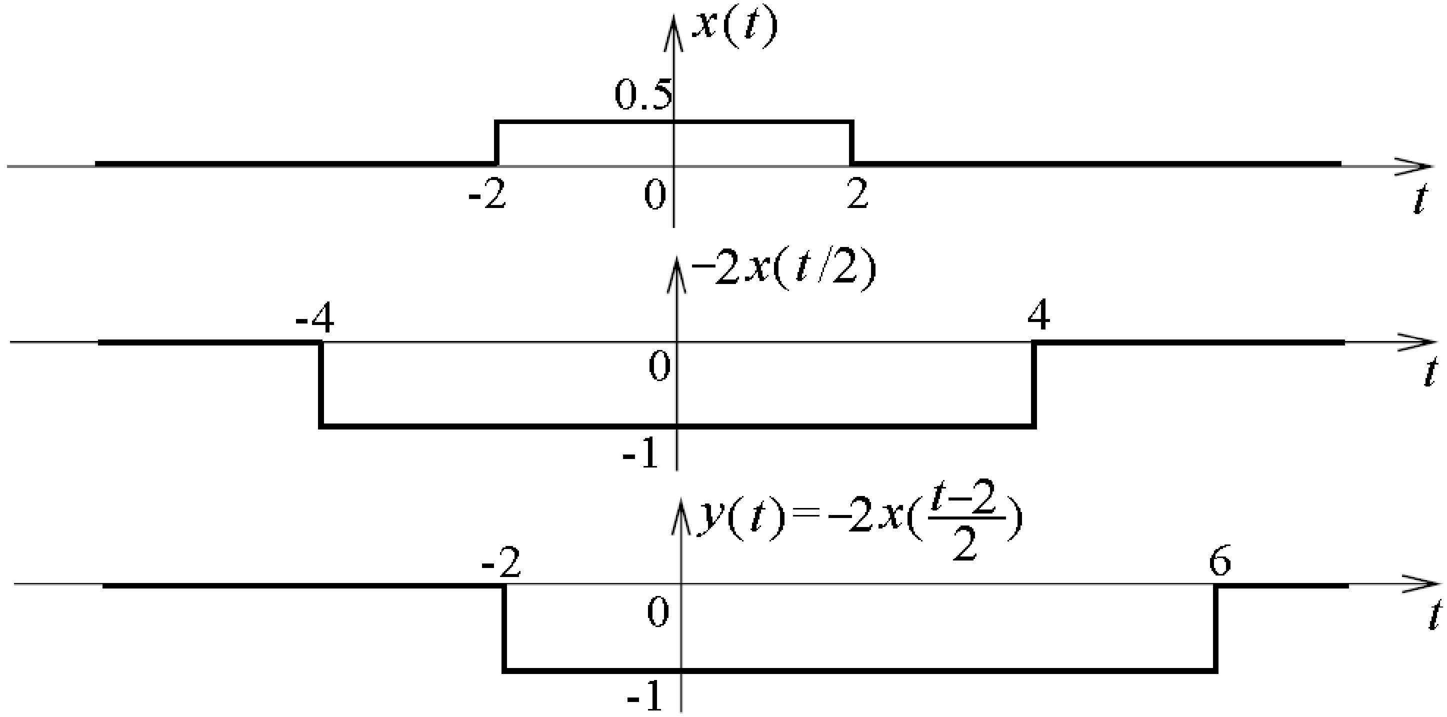

Figure 32: Example of operations on signals

Operations on Signals Summary 5

- Amplitude Scaling: The operation $A\,x(t)$ (or $A\,x[n]$) is to multiply the amplitude of $x(t)$ (or $x[n]$) by an amount A.

- Time Shifting: The operation $x(t\text{-}T)$ (or $x[n\text{-}k]$) is to shift $x(t)$ (or $x[n]$) by an amount T (or K).

- Time Scaling:
  - CT signals: The operation $x(t/a)$ is to scale $x(t)$ by an amount a.
    - It expands the function horizontally by the factor $|a|$.
    - If $a < 0$, the function will be also time inverted.
  - DT signals: $x[Kn]$ or $x[n/K]$ where K is an integer.
    - $x[Kn]$: Time compression or decimation.
    - $x[n/K]$: Time expansion.

You have reached the end of 1.3: Operations on Signals.