# IE2110
# Signals and Systems
## Part I

### Overview

**with Instructor:**

**A/P Teh Kah Chan**

# Outline of Signals & Systems - Part 1

1. Signals and Systems
   - 1.1 Classification of Signals
   - 1.2 Elementary and Singularity Signals
   - 1.3 Operations on Signals
   - 1.4 Properties of Systems

2. Linear Time-Invariant (LTI) Systems
   - 2.1 Continuous-Time and Discrete-Time LTI Systems
   - 2.2 Convolution
   - 2.3 LTI System Properties
   - 2.4 Correlation Functions

# Signals and Systems Overview

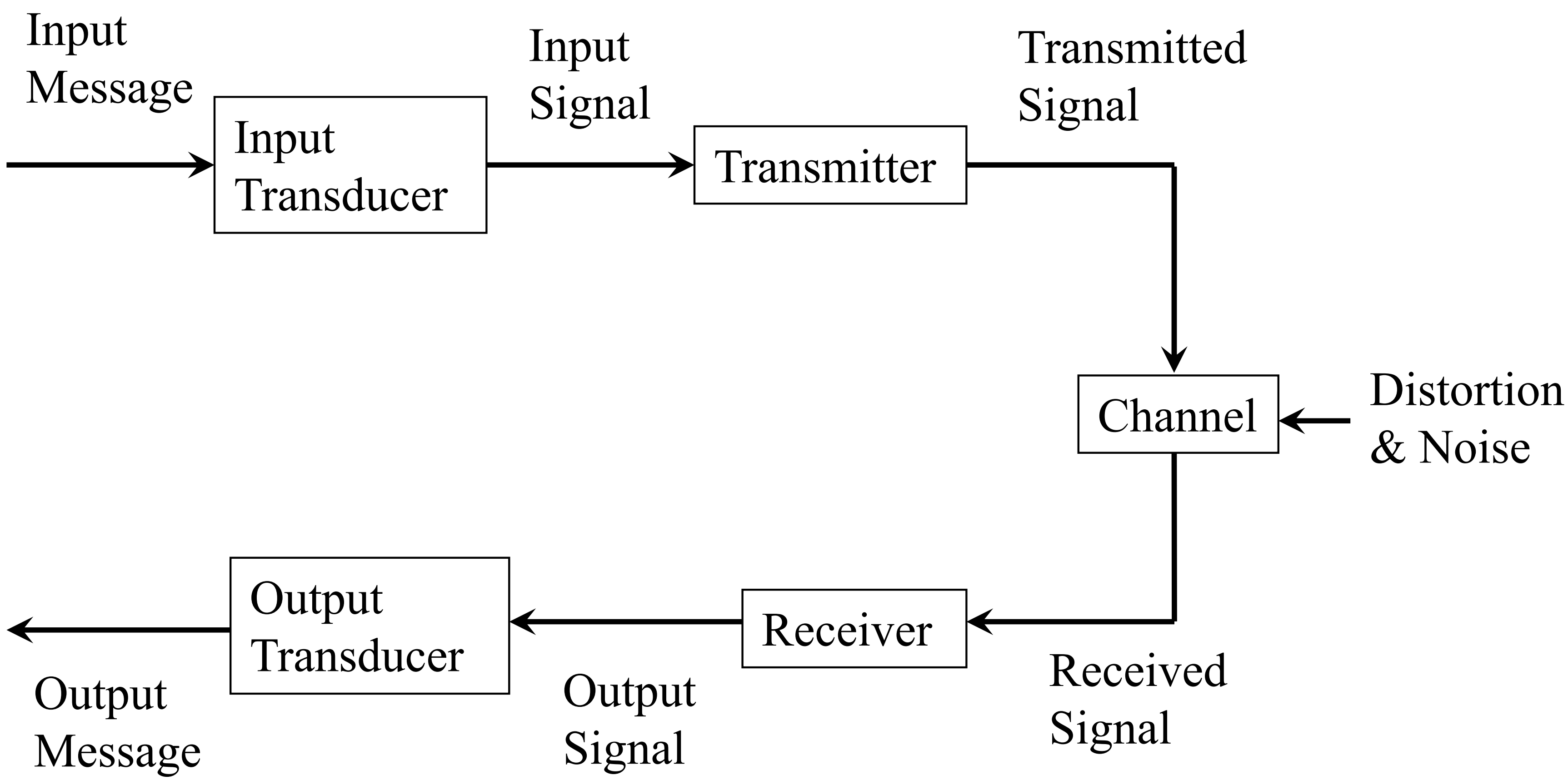


Figure 1: A typical signal and system example

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# IE2110
# Signals and Systems
## Part I

### 1.1 Classification of Signals I

**with Instructor:**

**A/P Teh Kah Chan**

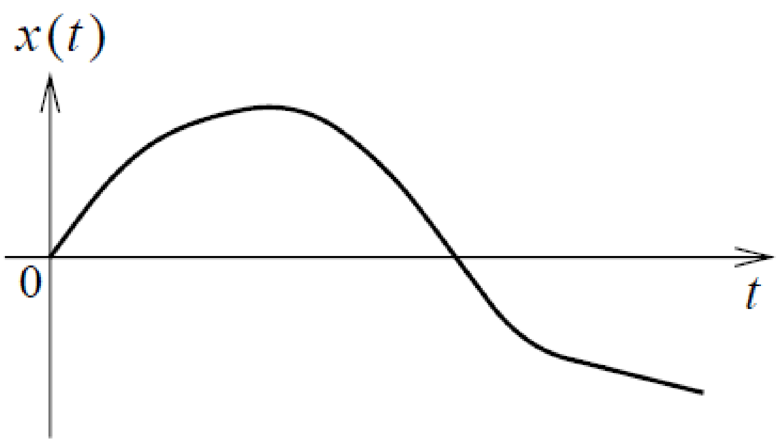
# 1.1 Classification of Signals

1) **Continuous-Time vs Discrete-Time Signal**
2) **Continuous-Value vs Discrete-Value Signal**
3) **Deterministic vs Random Signal**
4) Even vs Odd Signal
5) Periodic vs Aperiodic Signal
6) Energy-Type vs Power-Type Signal

# 1) Continuous-Time vs Discrete-Time Signal

- Continuous-Time (CT) Signal: A signal $x(t)$ that is specified for all value of time $t$
- Discrete-Time (DT) Signal: A signal $y[n]$ that is specified only for integer value of $n$

Graph of Continuous-Time Signal




Graph of Discrete-Time Signal

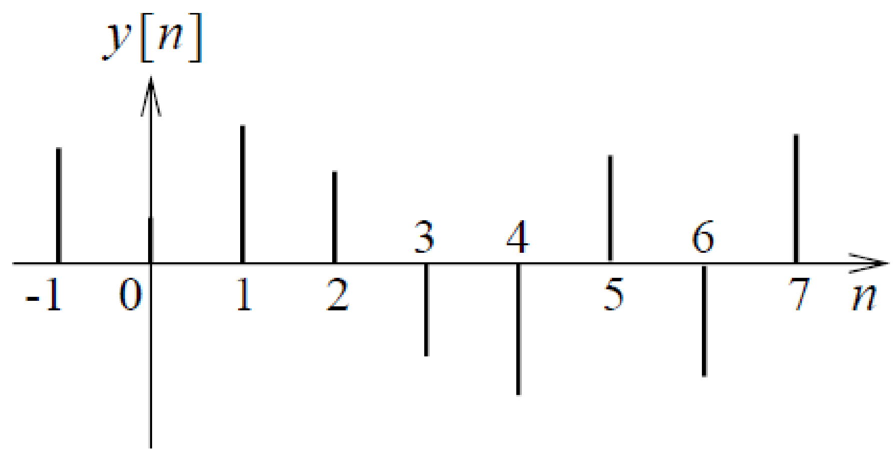
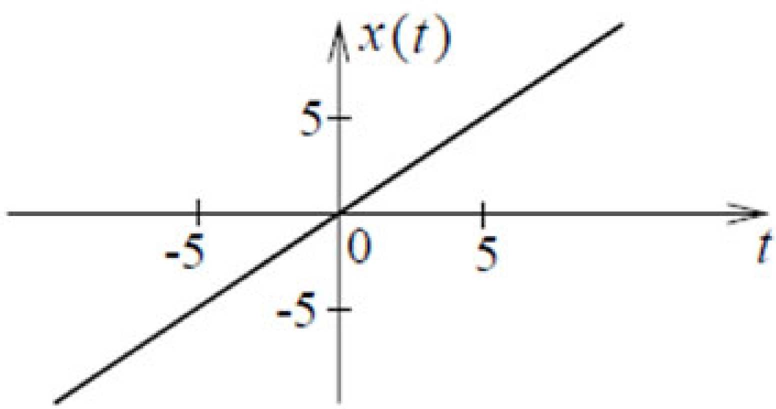



Figure 2: Continuous-Time vs Discrete-Time signal

# 1) Continuous-Time vs Discrete-Time Signal

Example 1:
Try sketching the waveforms of the CT signal $x(t) = t$ and DT signal $x[n] = n$, respectively.

<u>Graph of CT signal</u>




<u>Graph of DT signal</u>

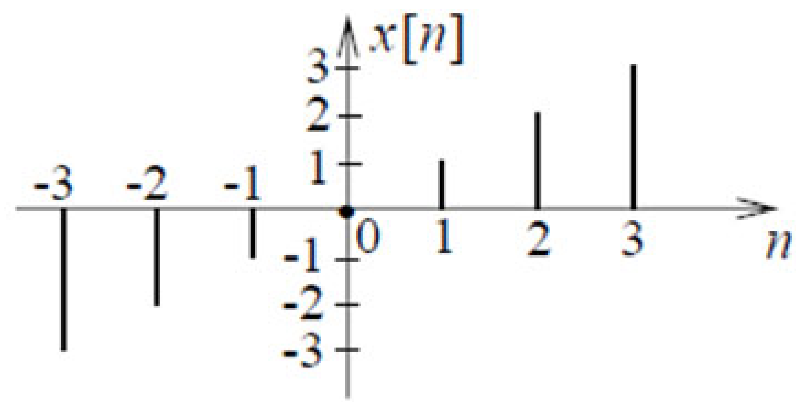
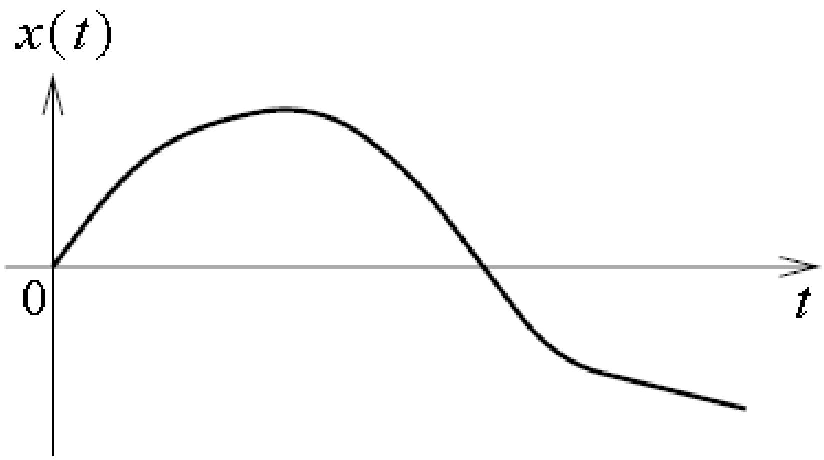



Figure 3: Example of CT and DT signals

# 2) Continuous-Value vs. Discrete-Value Signal

- Continuous-Value Signal: A signal $x(t)$ whose amplitude can take on any value
- Discrete-Value Signal: A signal $y(t)$ whose amplitude can take on only a finite number of values

Graph of Continuous-Value Signal




Graph of Discrete-Value Signal

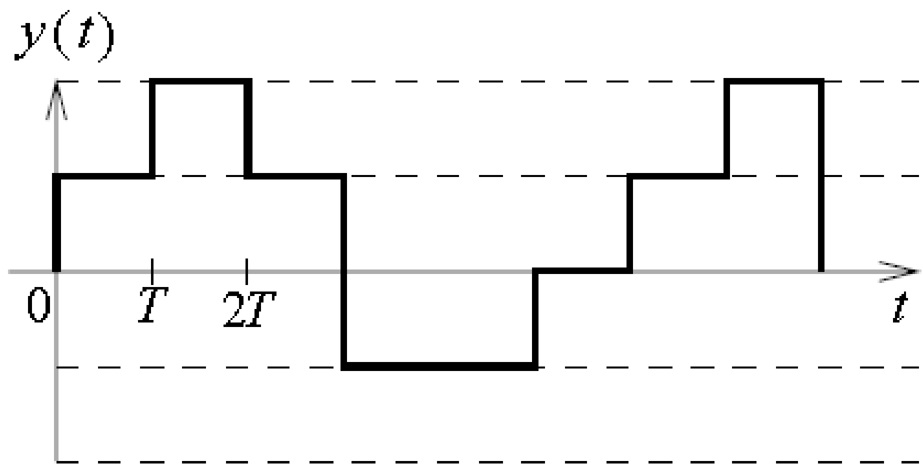



Figure 4: Continuous-Value vs Discrete-Value signal

# 2) Continuous-Value vs. Discrete-Value Signal

Example 2:

Try sketching the waveforms of the continuous-value signal $x(t) = A\sin(2\pi f_0 t)$ and discrete-value signal $y[n] = (-1)^n$ respectively.

Graph of Continuous-Value Signal

Graph of Discrete-Value Signal

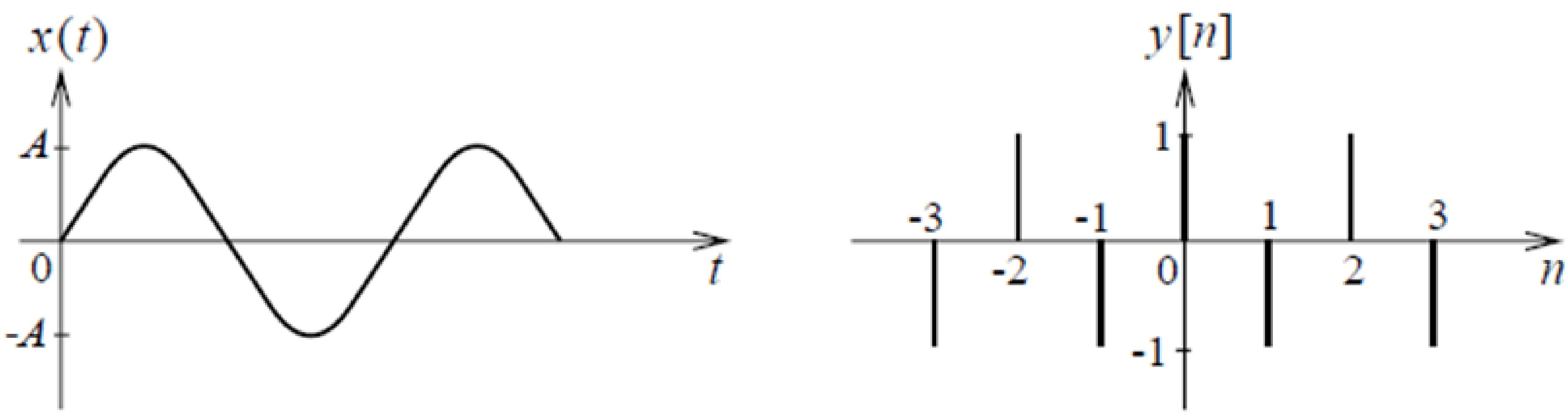
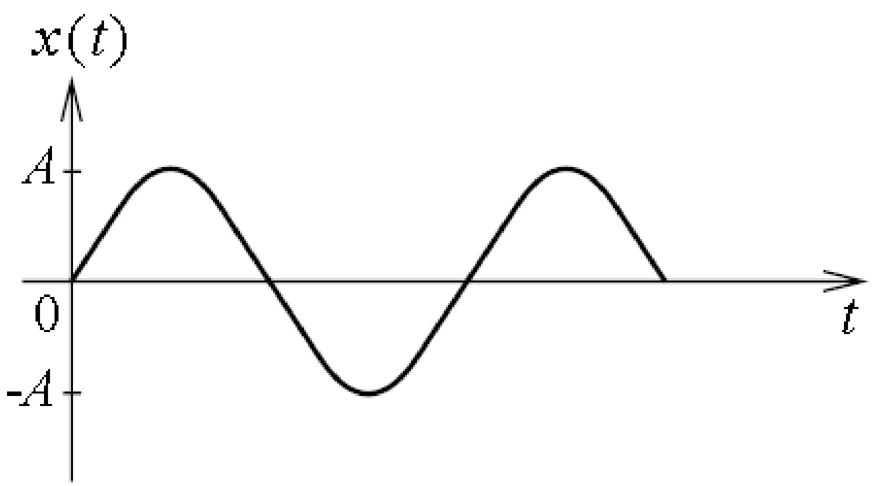



Figure 5: Example of Continuous-Value vs Discrete-Value signals

# 3) Deterministic vs Random Signal

- Deterministic Signal: A signal $x(t)$ that can be mathematically modeled explicitly as a function of time, i.e., $x(t) = A\sin(2\pi f_0 t)$
- Random Signal: A signal $y(t)$ that is known only in terms of probabilistic description, i.e., noise

Graph of Deterministic Signal




Graph of Random Signal

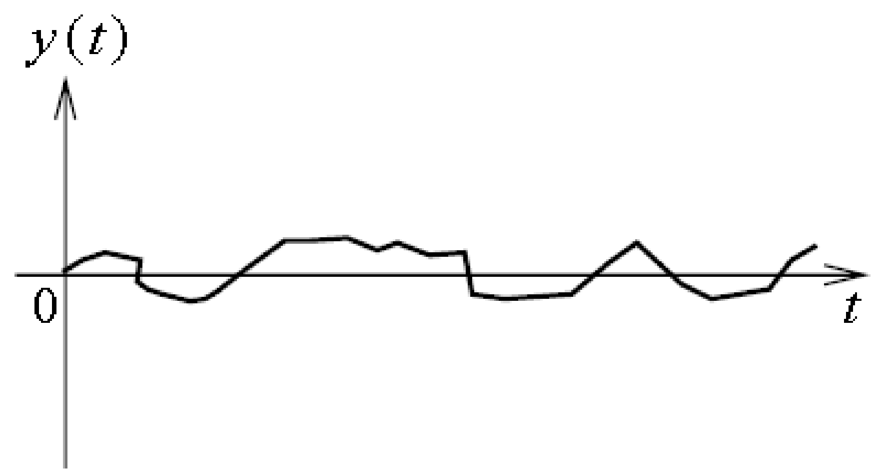



Figure 6: Deterministic vs Random signal

# Classification of Signals Summary 1

- Overview of Signals and Systems
- 1.1 Classification of Signals
  - 1) Continuous-Time vs Discrete-Time Signal
  - 2) Continuous-Value vs Discrete-Value Signal
  - 3) Deterministic vs Random Signal

***You have reached the end of this lesson, you have 3 more types to learn!***

***Please proceed with the next activity.***

# IE2110
# Signals and Systems
## Part I

### 1.1 Classification of Signals II

**with Instructor:**

**A/P Teh Kah Chan**

# Outline of Signals & Systems - Part 1

1. Signals and Systems
   - 1.1 **Classification of Signals**☜
   - 1.2 Elementary and Singularity Signals
   - 1.3 Operations on Signals
   - 1.4 Properties of Systems

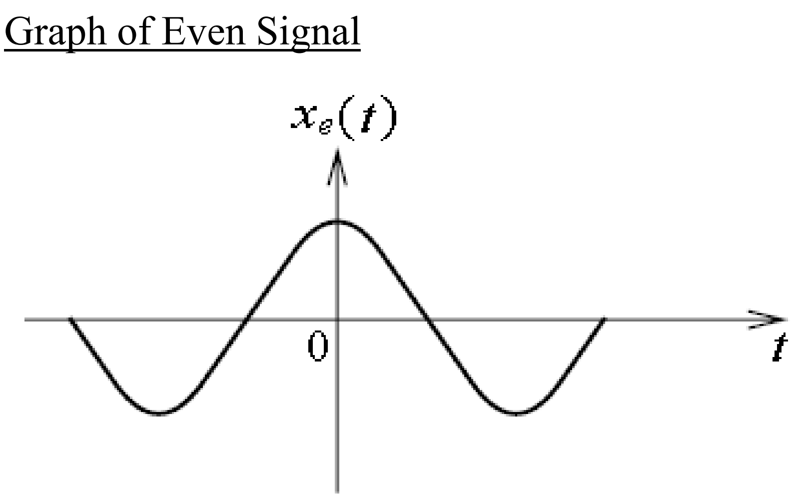
2. Linear Time-Invariant (LTI) Systems
   - 2.1 Discrete-Time and Continuous-Time LTI Systems
   - 2.2 Convolution
   - 2.3 LTI System Properties
   - 2.4 Correlation Functions

# 1.1 Classification of Signals

1) Continuous-Time vs Discrete-Time Signal ✓
2) Continuous-Value vs Discrete-Value Signal ✓
3) Deterministic vs Random Signal ✓
4) **Even vs Odd Signal**
5) **Periodic vs Aperiodic Signal**
6) **Energy-Type vs Power-Type Signal**

# 4) Even vs Odd Signal

- Even Signal: A signal $x_e(t)$ that satisfies the condition $x_e(t) = x_e(-t)$
- Odd Signal: A signal $x_o(t)$ that satisfies the condition $x_o(t) = -x_o(-t)$



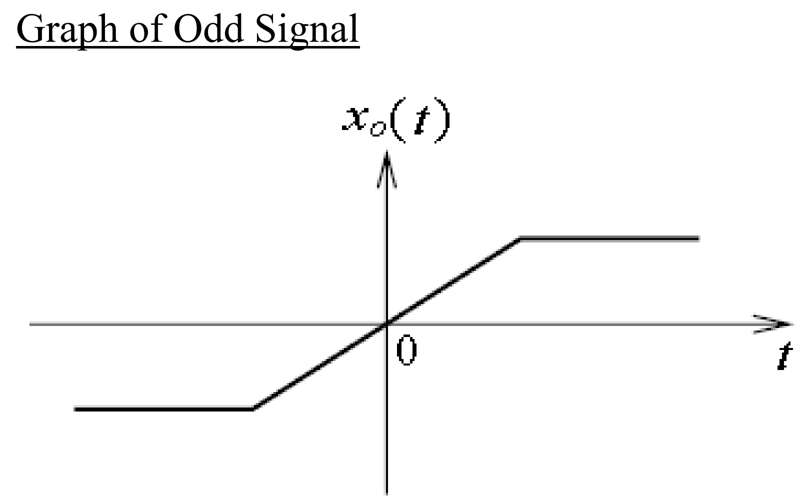




Figure 7: Even vs Odd signal

# 4) Even vs Odd Signal

- Any deterministic signal $x(t)$ can be decomposed into sum of an even and odd signal

$$x(t) = x_e(t) + x_o(t)$$

where

$$x_e\ (t) = \frac{1}{2}\ \{x(t) + x(-t)\}$$

and

$$x_o(t) = \frac{1}{2}\ \{x(t) - x(-t)\}$$

# 4) Even vs Odd Signal

- The product of two **even** signals is an **even** signal
- The product of two **odd** signals is an **even** signal
- The product of an **even** signal and an **odd** signal is an **odd** signal
- Note that

$$\int_{-T_0}^{T_0} x_e(t)\,dt = 2\int_{0}^{T_0} x_e(t)\,dt$$

and

$$\int_{-T_0}^{T_0} x_o(t)\,dt = 0$$

# 4) Even vs Odd Signal

Example 3:

Show that the signal $x(t) = A\sin(2\pi f_0 t)$ is an odd signal

Since $$x(-t) = A\sin\{2\pi f_0(-t)\}$$
$$= -A\sin(2\pi f_0 t)$$
$$= -x(t)$$

hence, $x(t)$ is an odd signal.

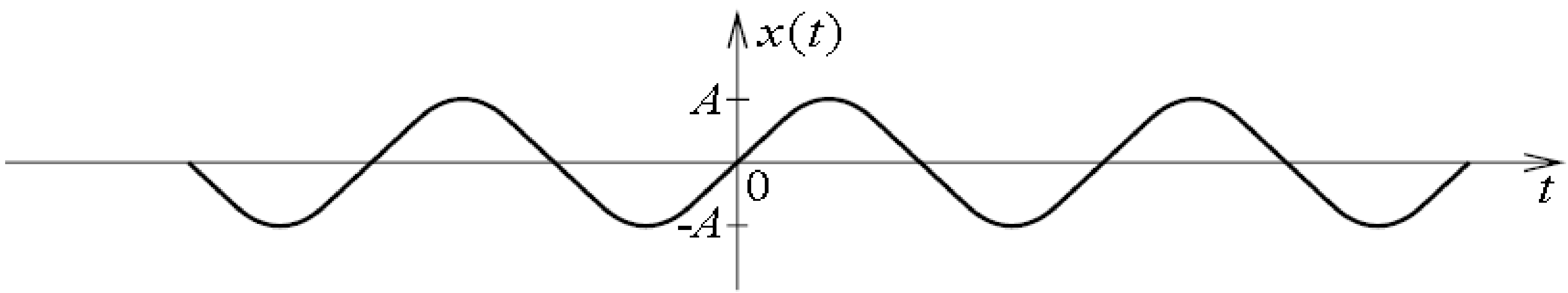



Figure 8: An odd signal example

# 4) Even vs Odd Signal

Example 4:
Find the even and odd components of the signal $x(t) = \cos(t) + \sin(t)\cos(t)$.

The even component of $x(t)$ is

$$x_e(t) = \frac{1}{2}\{x(t) + x(-t)\}$$

$$= \frac{1}{2}\{\cos(t) + \sin(t)\cos(t) + \cos(-t) + \sin(-t)\cos(-t)\}$$

$$= \cos(t)$$

The odd component of $x(t)$ is

$$x_o(t) = \frac{1}{2}\{x(t) - x(-t)\}$$

$$= \frac{1}{2}\{\cos(t) + \sin(t)\cos(t) - \cos(-t) - \sin(-t)\cos(-t)\}$$

$$= \sin(t)\cos(t)$$

# 4) Even vs Odd Signal

Example 5: Evaluate $\int_{-T_0}^{T_o} x(t)\, dt$ where $x(t) = t^3\cos^3(10t)$.

Since

$$x(-t) = (-t)^3 \cos^3\{10(-t)\}$$

$$= - t^3 \cos^3(10t)$$

$$= - x(t)$$

hence, $x(t)$ is an odd signal. Thus,

$$\int_{-T_0}^{T_o} x(t)\, dt = 0$$

# 5) Periodic vs Aperiodic Signal

- Periodic Signal: A signal $x(t)$ with a constant period $0 < T_0 < \infty$ that

$$x(t) = x(t+T_0)\ ,\ -\infty < t < \infty$$

For a discrete-time signal, the constant period is an integer $0 < K_o < \infty$ that

$$x[n] = x[n+K_0]\ ,\ -\infty < n < \infty$$

- Aperiodic Signal: A signal $y(t)$ or $y[n]$ that does not satisfy the above equation

# 5) Periodic vs Aperiodic Signal

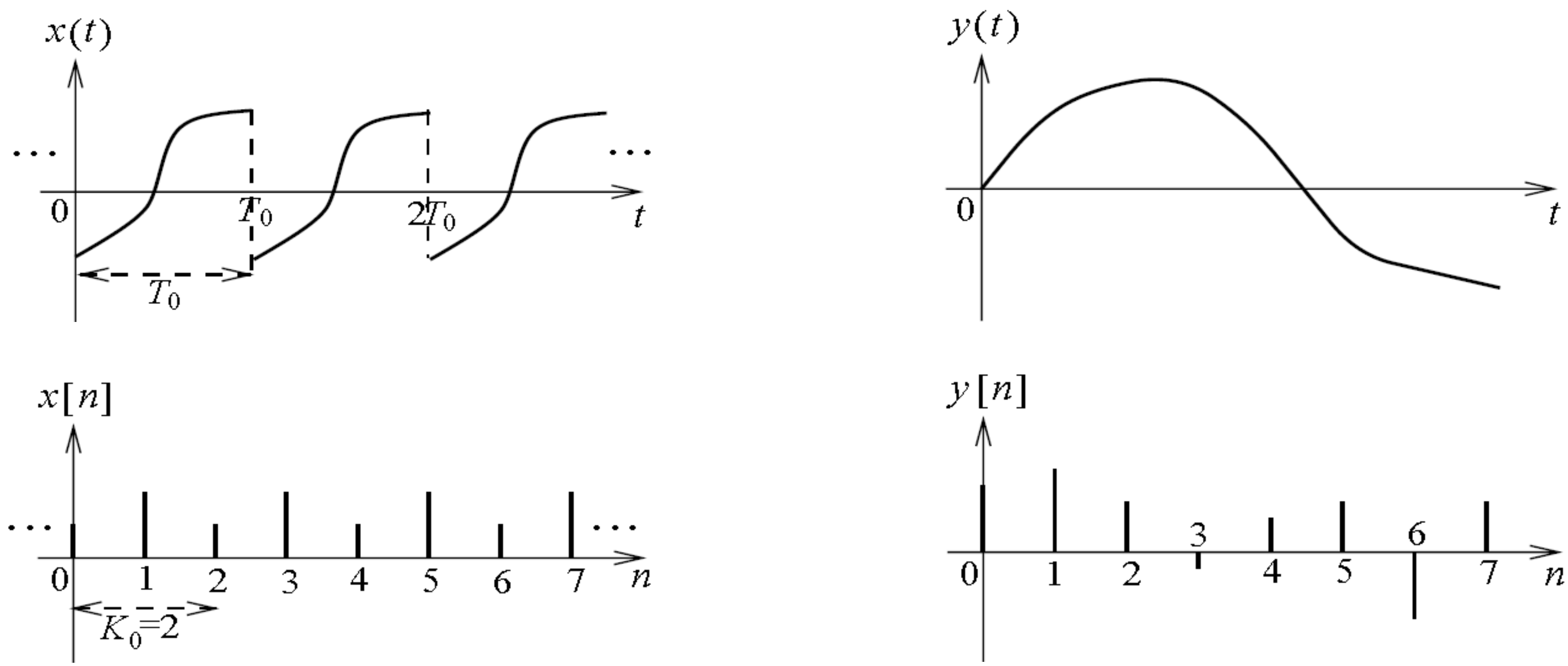



Figure 9: Periodic vs Aperiodic signal

# 6) Energy-Type vs Power-Type Signal

- Energy-Type Signal

  - A signal $x(t)$ or $x[n]$ that has finite energy, i.e., $0 < E_x < \infty$, where

CT signal: $$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

DT signal: $$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

# 6) Energy-Type vs Power-Type Signal

- Power-Type Signal

  - A signal $x(t)$ or $x[n]$ that has finite power, i.e., $0 < P_x < \infty$, where

CT signal: $$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt$$

DT signal: $$P_x = \lim_{K\to\infty} \frac{1}{2K+1}\sum_{n=-K}^{K} |x[n]|^2$$

# 6) Energy-Type vs Power-Type Signal

Note that if $x(t)$ or $x[n]$ is a periodic signal with period $T_0$ or $K_0$, respectively, then

CT signal: $$P_x = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} |x(t)|^2 \, dt$$

DT signal: $$P_x = \frac{1}{K_0}\sum_{n=k}^{k+K_0-1} |x[n]|^2$$

with any real value of $t_1$ and any integer value of $k$.

# 6) Energy-Type vs Power-Type Signal

Example 6: Determine the energy and power of the periodic signal $x(t) = A\cos(2\pi f_0 t)$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} |A\cos(2\pi f_0 t)|^2 \, dt$$

$$= \infty$$

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

$$= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 \, dt$$

$$= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |A\cos(2\pi f_0 t)|^2 \, dt$$

$$= \frac{A^2}{2}$$

Hence, $x(t)$ is a power-type signal.

In general, power-type signals are <u>periodic</u> signals.

# Classification of Signals Summary 2

- Classification of Signals

  4) Even vs Odd Signal
  $$x(t) = x_e(t) + x_o(t)$$
  $$x_e(t) = \{x(t) + x(-t)\}/2$$
  $$x_o(t) = \{x(t) - x(-t)\}/2$$

  5) Periodic vs Aperiodic Signal

  6) Energy-Type vs Power-Type Signal

  CT Signals : $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$

  $$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} |x(t)|^2 dt$$

  DT Signals : $E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$

  $$P_x = \lim_{K\to\infty} \frac{1}{2K+1}\sum_{n=-K}^{K} |x[n]|^2 = \frac{1}{K_0}\sum_{n=k}^{k+K_0-1} |x[n]|^2$$

# Classification of Signals Summary 2

1.1 Classification of Signals

- 1) Continuous-Time vs Discrete-Time Signal
- 2) Continuous-Value vs Discrete-Value Signal
- 3) Deterministic vs Random Signal
- 4) Even vs Odd Signal
- 5) Periodic vs Aperiodic Signal
- 6) Energy-Type vs Power-Type Signal

***You have reached the end of module 1.1.***
***Made some mental notes on each classification of signals?***
***Try it and proceed with the next activity!***

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# IE2110
# Signals and Systems
## Part 1

**with Instructor:**

**A/P Teh Kah Chan**

# Outline of Signals & Systems - Part 1

1. Signals and Systems
   - 1.1 **Classification of Signals**☜ Recap through further examples
   - 1.2 Elementary and Singularity Signals
   - 1.3 Operations on Signals
   - 1.4 Properties of Systems

2. Linear Time-Invariant (LTI) Systems
   - 2.1 Discrete-Time and Continuous-Time LTI Systems
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# Recap: 1.1 Classification of Signals

Example 7:
Determine the energy and power of the signal $y(t) = \exp(-|t|)$.

$$E_y = \int_{-\infty}^{\infty} |y(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} |\exp(-|t|)|^2 \, dt$$

$$= 2 \times \int_0^{\infty} \exp(-2t) dt$$

$$= 1$$

$$P_y = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |y(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \times E_y$$

$$= 0$$

Hence, $y(t)$ is an <u>energy-type</u> signal. In general, energy-type signals are <u>aperiodic</u> signals.

# Recap: 1.1 Classification of Signals

Example 8:
Determine the energy and power of the discrete-time periodic signal
$x[n] = A\sin(2\pi n/4)$.

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$= \sum_{n=-\infty}^{\infty} \left| A\sin\left(\frac{2\pi n}{4}\right) \right|^2$$

$$= \infty$$

$$P_x = \frac{1}{K_0} \sum_{n=k}^{k+K_0-1} |x[n]|^2$$

$$= \frac{1}{4} \sum_{n=0}^{3} \left| A\sin\left(\frac{2\pi n}{4}\right) \right|^2$$

$$= \frac{A^2}{4} \times [0^2 + 1^2 + 0^2 + (-1)^2]$$

$$= \frac{A^2}{2}$$

Hence, $x[n]$ is a <u>power-type</u> signal.

# Recap: 1.1 Classification of Signals

Example 9:
A simplified transmitter model of a digital communication system is shown below (Figure 10). Determine the classifications of each signal.

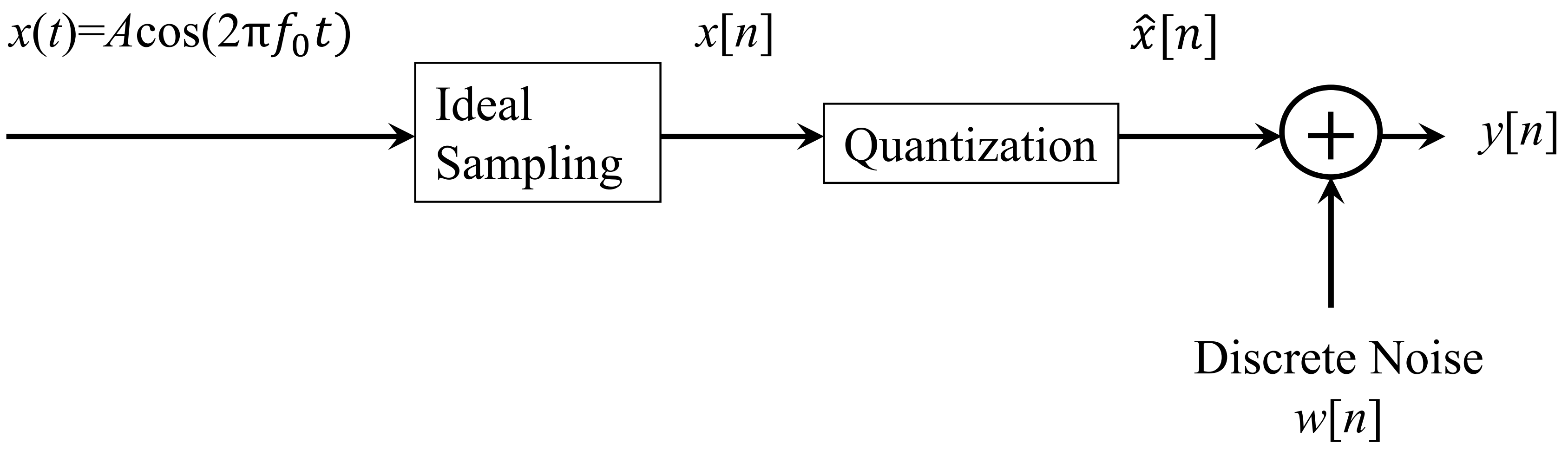
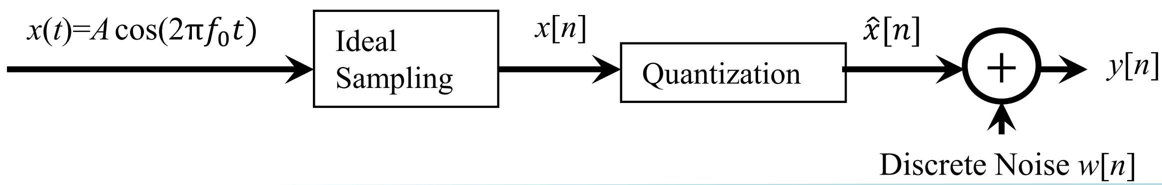



Figure 10: Transmitter model of a digital communication system

# Recap: 1.1 Classification of Signals

Example 9:

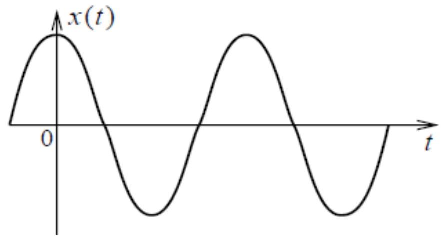



- $x(t)$ is a continuous-time, continuous-value, deterministic, even, periodic, and power-type signal
- $x[n] = x(nT_s)$ is a discrete-time, discrete-value, deterministic, even, periodic, and power-type signal
- $\hat{x}[n]$ is a discrete-time, discrete-value, deterministic, even, periodic, and power-type signal
- $w[n]$ is a discrete-time, continuous-value, random, and aperiodic signal
- $y[n]$ is a discrete-time, continuous-value, random, and aperiodic signal

# Recap: 1.1 Classification of Signals

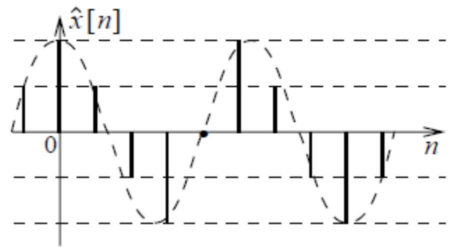
Example 9:

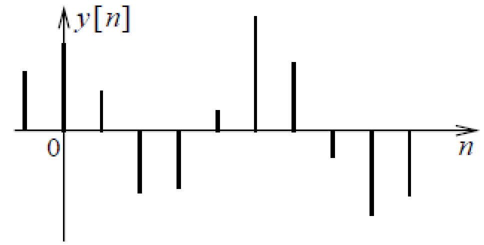
$x(t)$ is a continuous-time, continuous-value, deterministic, even, periodic, and power-type signal



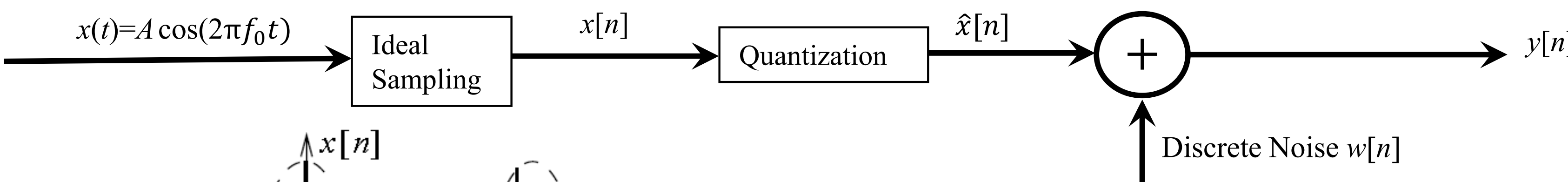

$x[n] = x(nT_s)$ is a discrete-time, discrete-value, deterministic, even, periodic, and power-type signal

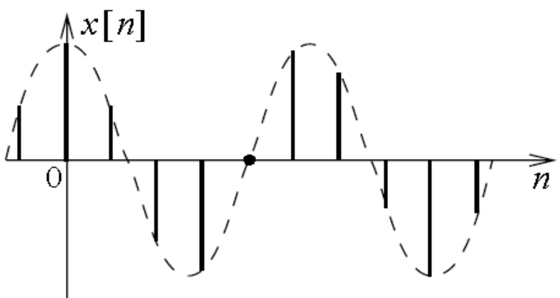
$\hat{x}[n]$ is a discrete-time, discrete-value, deterministic, even, periodic, and power-type signal




$y[n]$ is a discrete-time, continuous-value, random, and aperiodic signal




$w[n]$ is a discrete-time, continuous-value, random, and aperiodic signal

Figure 10: Transmitter model of a digital communication system

# Recap: 1.1 Classification of Signals

Example 9:

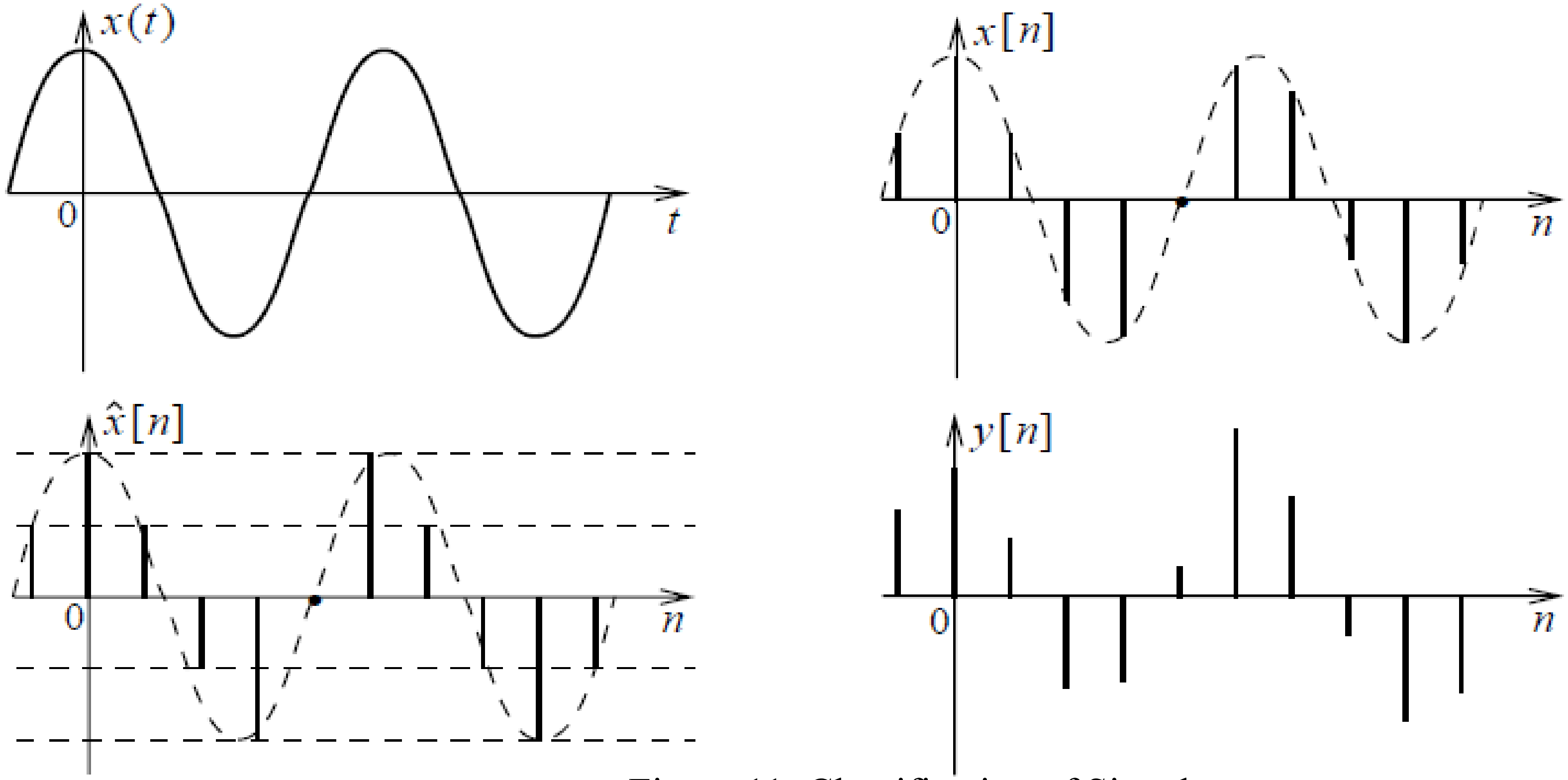



Figure 11: Classification of Signals