



NANYANG
TECHNOLOGICAL
UNIVERSITY
SINGAPORE

IE2110

Signals and Systems Part 1

1.4 Properties of Signals

with Instructor:

A/P Teh Kah Chan



Outline of Signals & Systems - Part 1

1. Signals and Systems
 - 1.1 Classification of Signals ✓
 - 1.2 Elementary and Singularity Signals ✓
 - 1.3 Operations on Signals ✓
 - 1.4 **Properties of Systems**
2. Linear Time-Invariant (LTI) Systems
 - 2.1 Discrete-Time and Continuous-Time LTI Systems
 - 2.2 Convolution
 - 2.3 LTI System Properties
 - 2.4 Correlation Functions

1.4 Properties of Systems

- 1) Stability
- 2) Memory
- 3) Causality
- 4) Linearity
- 5) Time Invariant

1.4 Properties of Systems

A system refers to any physical device (i.e., communication channels, filters) that produces an output signal $y(t)$ in response to an input signal $x(t)$

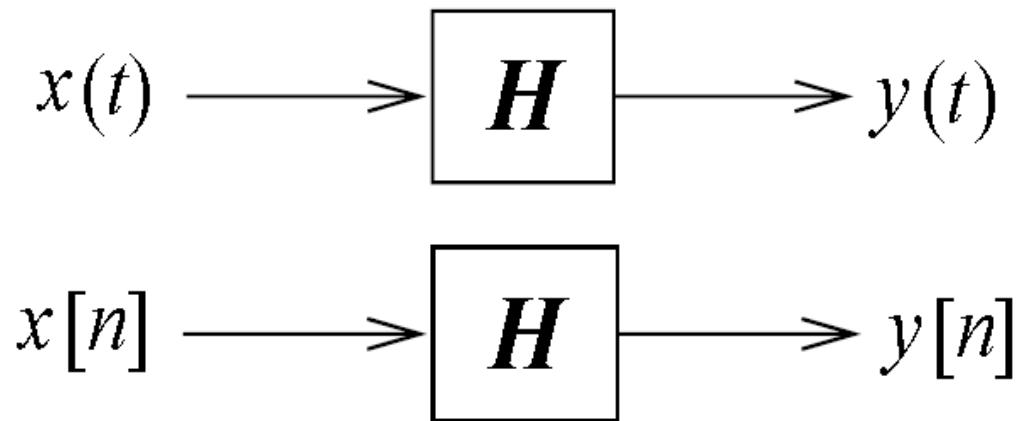


Figure 33: Block diagram representation of a system

1.4 Properties of Systems

1) Stability

- A system is said to be bounded-input bounded-output (BIBO) stable if and only if every bounded input (i.e., $|x(t)| < \infty$ for all t , or $|x[n]| < \infty$ for all n) results in bounded output

An example of a BIBO stable system

$$y[n] = r^n x[n] u[n], \quad |r| < 1$$

An example of a BIBO unstable system

$$y[n] = r^n x[n] u[n], \quad |r| > 1$$

1.4 Properties of Systems

2) Memory

- A system is said to possess memory if its output signal depends on past or future values of the input signal

An example of a system with memory

$$y[n] = x[n] + x[n - 1] + x[n - 2]$$

- A system is memoryless if its output signal depends only on the present value of the input signal

An example of a memoryless system

$$y(t) = x^2(t)$$

1.4 Properties of Systems

3) Causality

- A system is causal if the present value of the output signal depends only on the present or past values of the input signal

An example of a causal system

$$y[n] = \frac{1}{3}(x[n] + x[n - 1] + x[n - 2])$$

- A system is noncausal if the present value of the output signal depends on the future values of the input signal
- A noncausal system is not physically realizable in real time

An example of a noncausal system

$$y[n] = \frac{1}{3}(x[n + 1] + x[n] + x[n - 1])$$

1.4 Properties of Systems

4) Linearity

- A system is linear if the principle of superposition holds, i.e., if input signal is $x_3(t) = a_1x_1(t) + a_2x_2(t)$, then the output signal is $y_3(t) = a_1y_1(t) + a_2y_2(t)$ for any constants a_1 and a_2

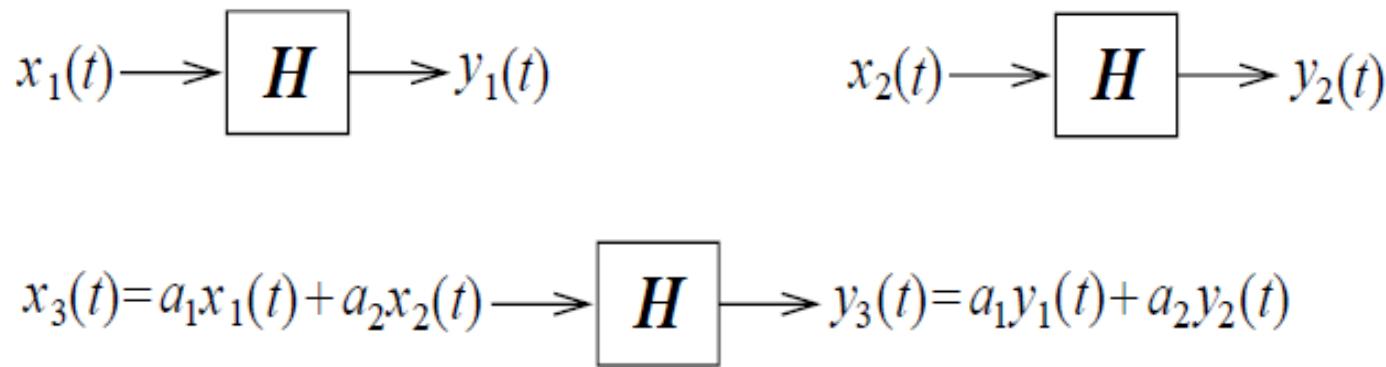


Figure 34: A linear system

1.4 Properties of Systems

Example 14:

A system is shown below (Figure 35). Determine whether it is a linear system.

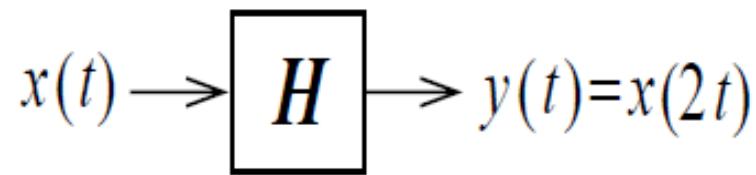


Figure 35: A linear system example

1.4 Properties of Systems

Example 14:

Determine whether it is a linear system.

$$x(t) \rightarrow \boxed{H} \rightarrow y(t) = x(2t)$$

$$x_1(t) \rightarrow \boxed{H} \rightarrow y_1(t) = x_1(2t)$$

$$x_2(t) \rightarrow \boxed{H} \rightarrow y_2(t) = x_2(2t)$$

$$\begin{aligned} x_3(t) = a_1x_1(t) + a_2x_2(t) &\rightarrow \boxed{H} \rightarrow y_3(t) = a_1x_1(2t) + a_2x_2(2t) \\ &= a_1y_1(t) + a_2y_2(t) \end{aligned}$$

Figure 36: A linear system example

In this case, the principle of superposition holds, hence it is a linear system.

1.4 Properties of Systems

5) Time Invariant

- A system is time invariant if for any delayed $x(t - T)$, the output is delayed by the same amount $y(t - T)$

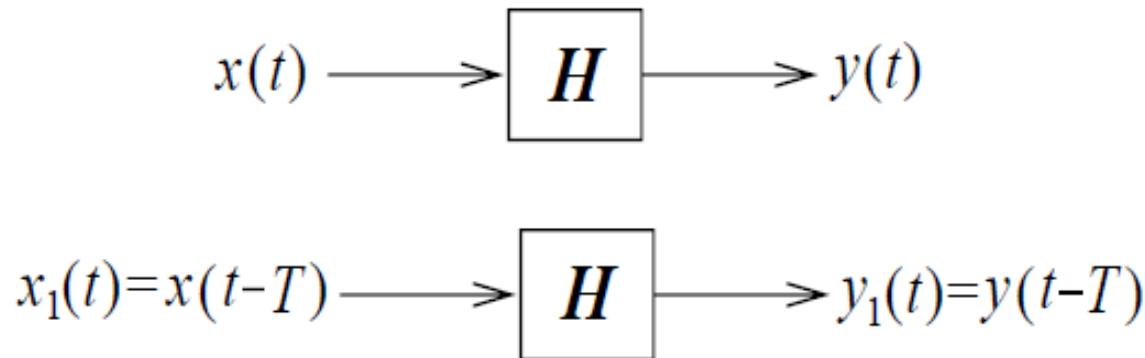


Figure 37: A time invariant system

1.4 Properties of Systems

Example 15:

For the system as shown below (Figure 38) with $y(t) = x(t) + c$, where c is an arbitrary constant, determine whether it is a time invariant system.

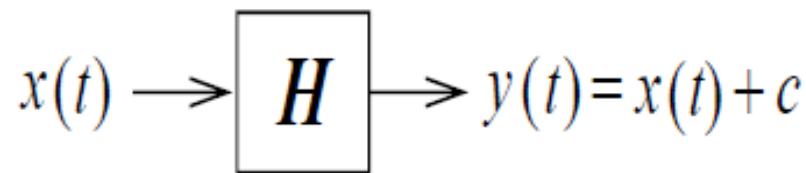
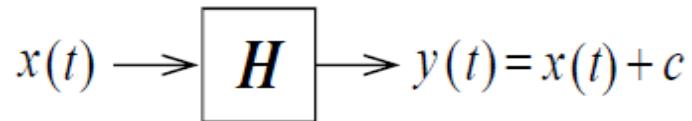


Figure 38: A time invariant system example

1.4 Properties of Systems

Example 15:

Determine whether it is a time invariant system.



$$x(t) \rightarrow \boxed{H} \rightarrow y(t) = x(t) + c$$

$$\begin{aligned} x_1(t) = x(t-T) &\rightarrow \boxed{H} \rightarrow y_1(t) = x(t-T) + c \\ &= y(t-T) \end{aligned}$$

Figure 39: A time invariant system example

In this case, the system is time invariant.

2. Linear Time-Invariant (LTI) Systems

Linear Time Invariant (LTI)

- A system is linear time invariant if it satisfies both conditions of linear and time invariance
- A LTI system can be analyzed in both time domain and frequency domain

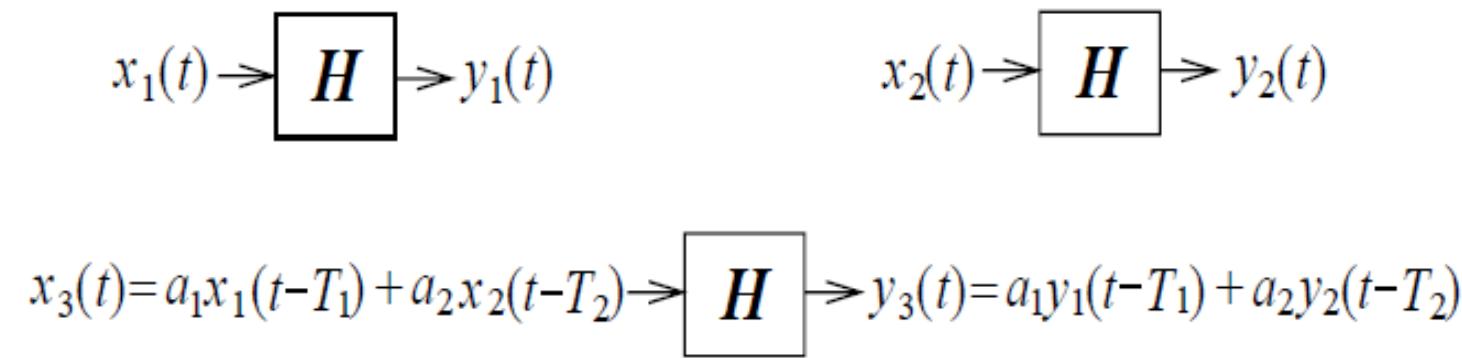


Figure 40: An LTI system

2. Linear Time-Invariant (LTI) Systems

Example 16:

Determine whether the system below given by $y(t) = x(2t)$ in Example 14 is an LTI system.

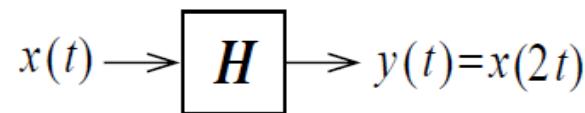


Figure 35

From Example 14, the system is linear.

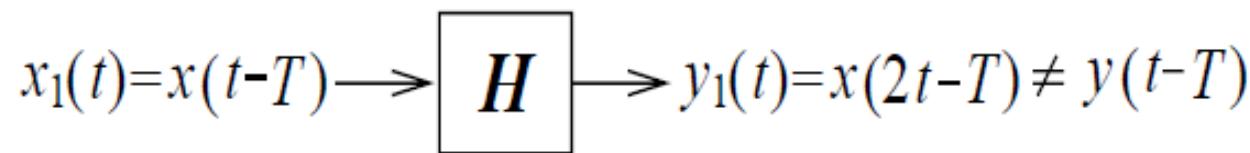


Figure 41: A non-LTI system example

However, the system is not time invariant, hence it is not an LTI system.

Properties of Systems Summary 6



NANYANG
TECHNOLOGICAL
UNIVERSITY
SINGAPORE

- ❑ 1) Stability: Bounded input results in bounded output.
- ❑ 2) Memory: Output depends on past and/or future values of input.
- ❑ 3) Causality: Output does not depend on future values of input.
- ❑ 4) Linearity: Principle of superposition holds.
- ❑ 5) Time Invariant: For any delayed input $x(t-T)$, the output is delayed by the same amount $y(t-T)$.
 - Linear Time-Invariant (LTI) Systems



*You have reached the end of 1.4: Properties of Systems.
Consider mapping out your learning and proceed.*



NANYANG
TECHNOLOGICAL
UNIVERSITY
SINGAPORE

IE2110

Signals and Systems Part 1

2. Linear Time-Invariant (LTI) Systems

with Instructor:
A/P Teh Kah Chan



Outline of Signals & Systems - Part 1

1. Signals and Systems
 - 1.1 Classification of Signals ✓
 - 1.2 Elementary and Singularity Signals ✓
 - 1.3 Operations on Signals ✓
 - 1.4 Properties of Systems
2. **Linear Time-Invariant (LTI) Systems**
 - 2.1 Discrete-Time \Rightarrow and Continuous-Time **LTI Systems**
 - 2.2 Convolution
 - 2.3 LTI System Properties
 - 2.4 Correlation Functions

2.1 Discrete-Time and Continuous-Time LTI Systems

Analysis of DT and CT LTI Systems:

- Any LTI system can be uniquely defined by its impulse response

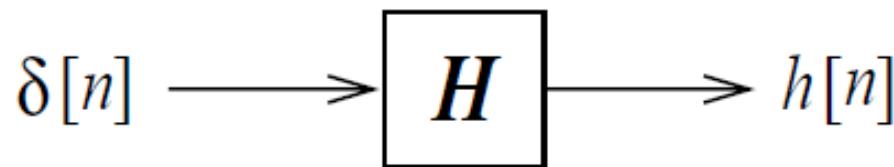
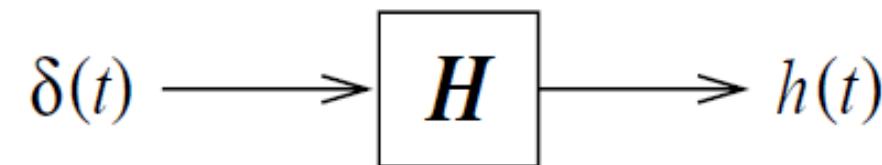


Figure 42: Impulse response of an LTI system

2.1 Discrete-Time and Continuous-Time LTI Systems

The output of any LTI system is the convolution of the input signal and its impulse response

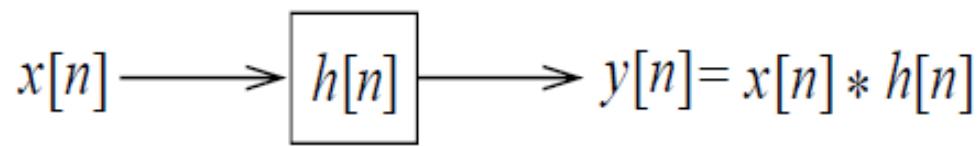
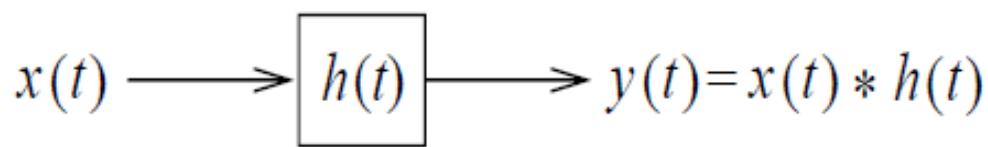


Figure 43: System response of an LTI system

2.1 Discrete-Time and Continuous-Time LTI Systems

The discrete time convolution (convolution sum) is defined as

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

The continuous time convolution (convolution integral) is defined as

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$$

2.1a Discrete-Time LTI Systems

Example 17:

Sketch the waveform of $y[n] = x[n]* h[n]$ using the graphical approach for convolution sum.

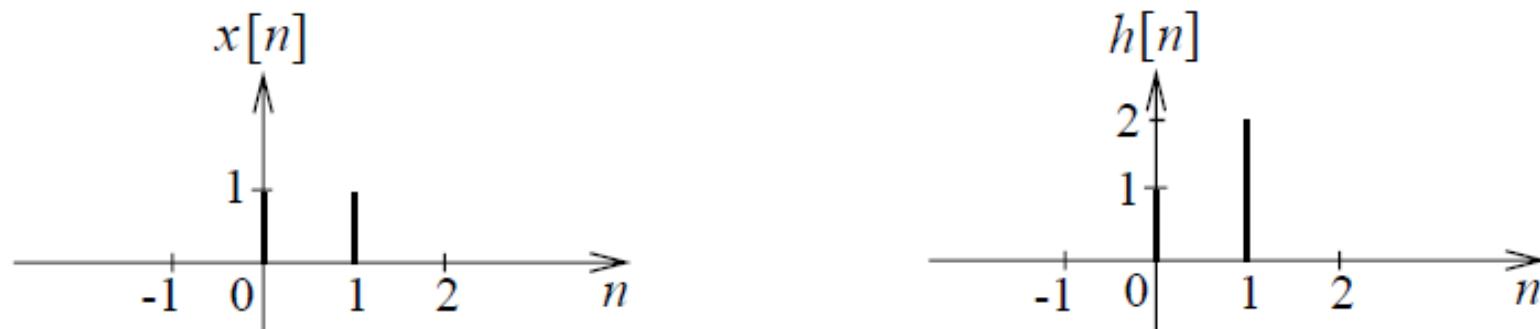
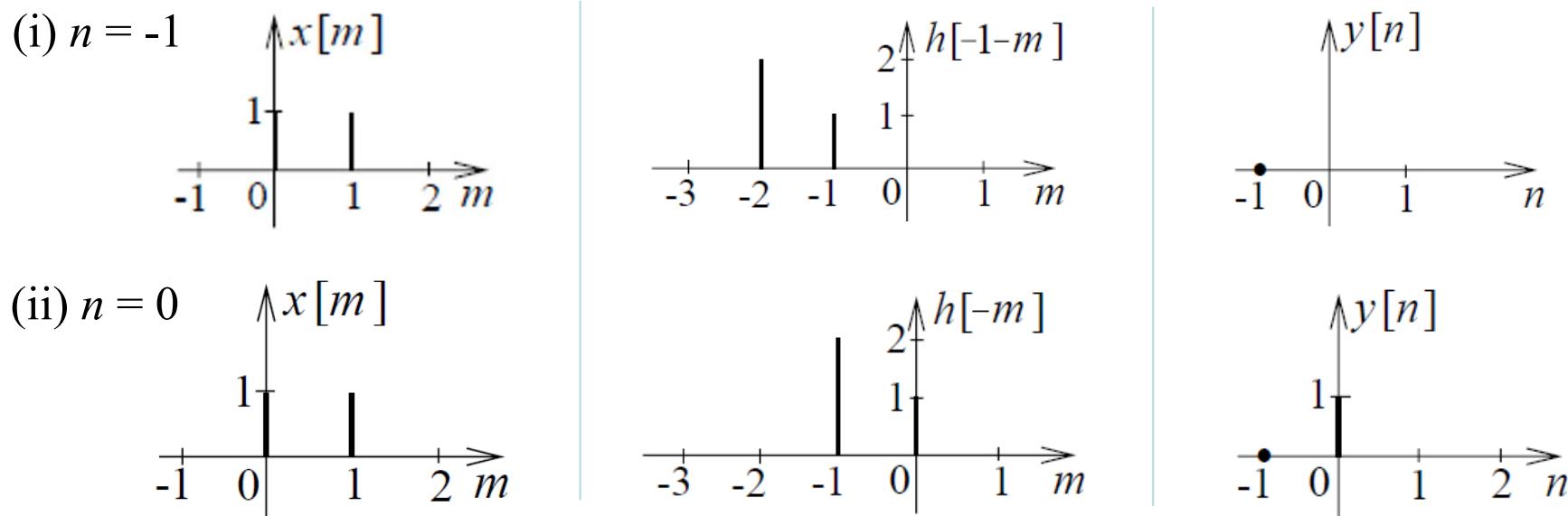
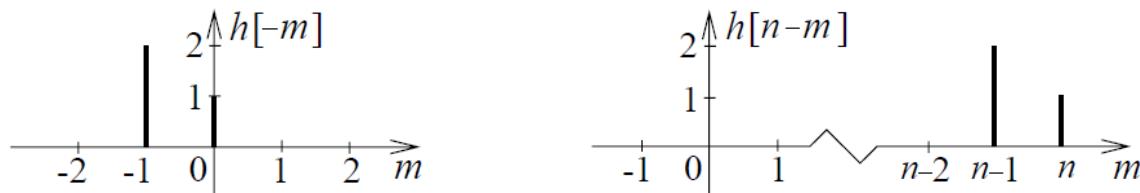


Figure 44: Example on convoluted sum

2.1a Discrete-Time LTI Systems

Example 17:
Sketch the waveform
of $y[n] = x[n] * h[n]$

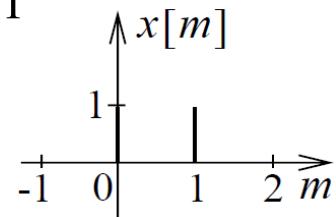
$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$



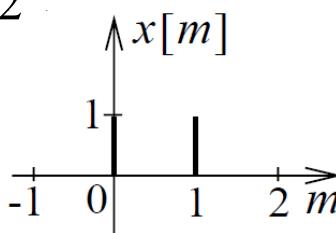
2.1a Discrete-Time LTI Systems

Example 17:

(iii) $n = 1$



(iv) $n = 2$



(v) $n = 3$

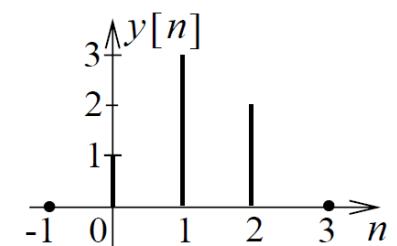
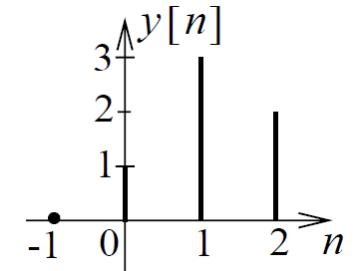
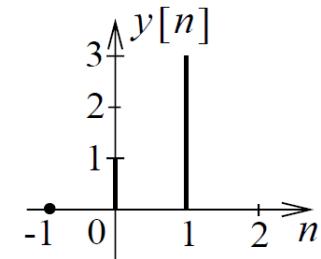
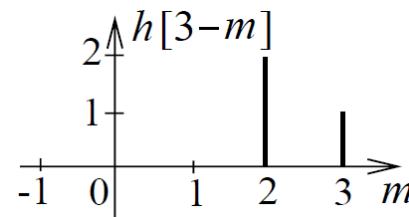
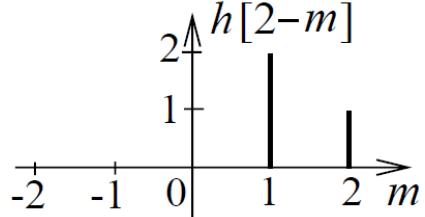
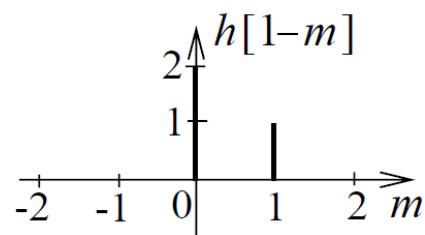
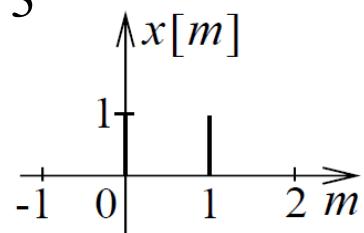


Figure 45: Solution for example on convolution sum

2.1 Discrete-Time and Continuous-Time LTI Systems Summary 7

□ Analysis of DT Systems

- Any DT LTI system can be uniquely defined by its impulse response, $h[n]$.
- The output of a DT LTI system is the convolution of the input signal and its impulse response.
- The DT convolution (or convolution sum) is defined as

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

- The graphical approach for evaluating the convolution sum.



*You have reached the end of 2.1a. Do reflect on your level of understanding.
Please proceed to 2.1b Continuous-Time LIT Systems.*



NANYANG
TECHNOLOGICAL
UNIVERSITY
SINGAPORE

IE2110

Signals and Systems Part 1

2.0 Linear Time-Invariant (LTI) Systems

with Instructor:
A/P Teh Kah Chan



Outline of Signals & Systems - Part 1

1. Signals and Systems
 - 1.1 Classification of Signals ✓
 - 1.2 Elementary and Singularity Signals ✓
 - 1.3 Operations on Signals ✓
 - 1.4 Properties of Systems ✓
2. Linear Time-Invariant (LTI) Systems
 - 2.1 Discrete-Time ✓ and **Continuous-Time LTI Systems** ↗
 - 2.2 Convolution
 - 2.3 LTI System Properties
 - 2.4 Correlation Functions

2.1b Continuous-Time LTI Systems

Example 18:

Sketch the waveform of $y(t) = x_1(t) * x_2(t)$ using the graphical approach for convolution integral.

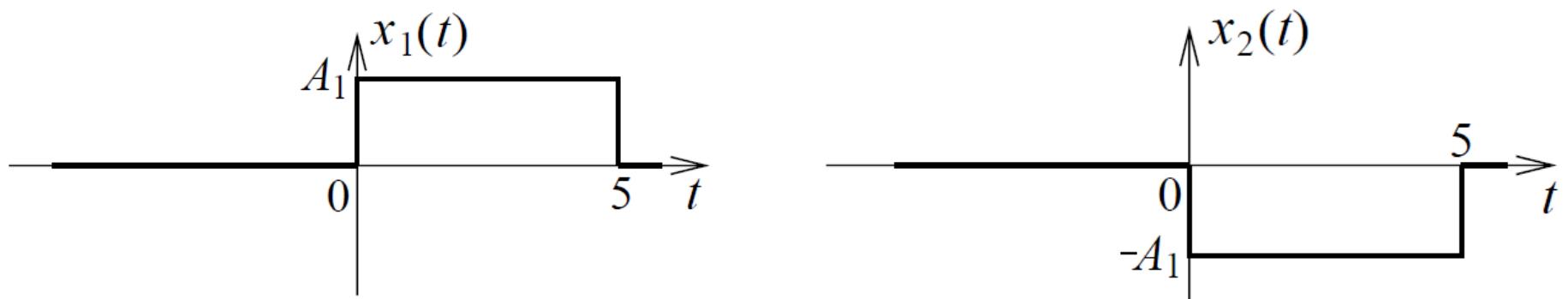
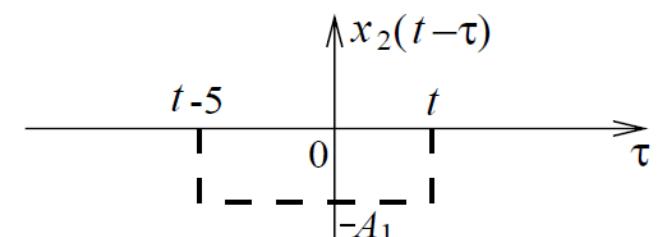
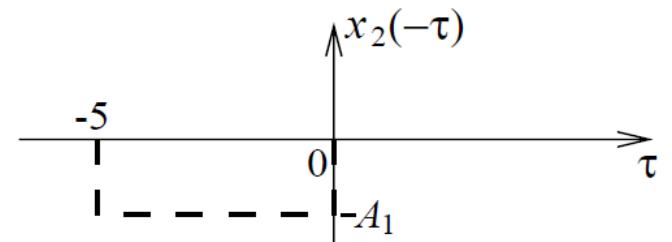
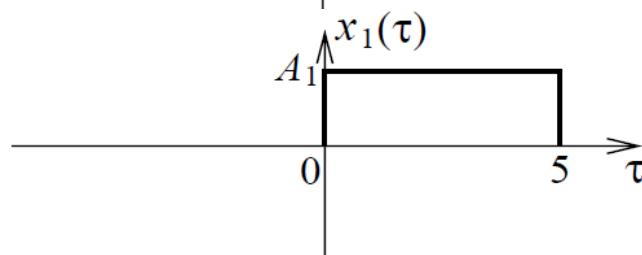
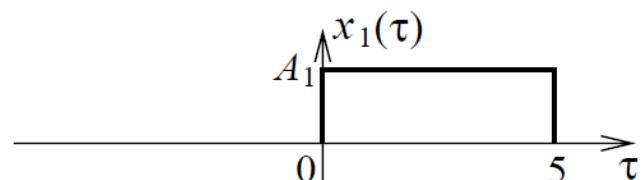


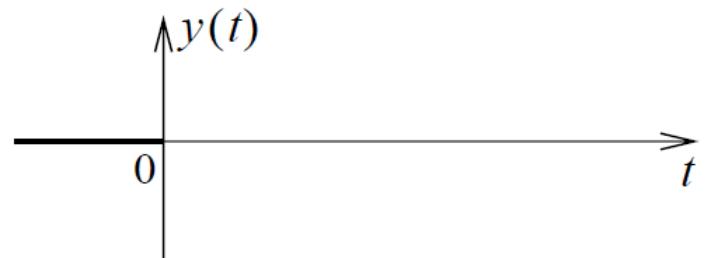
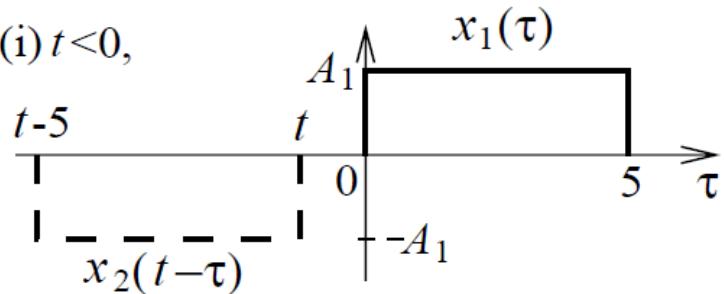
Figure 46: Example on convolution integral

2.1b Continuous-Time LTI Systems

$$y(t) = x(t) * x(t) = \int_{-\infty}^{\infty} x(\tau)x(t - \tau)d\tau$$

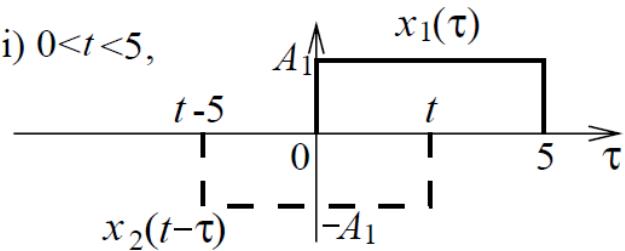


(i) $t < 0$,

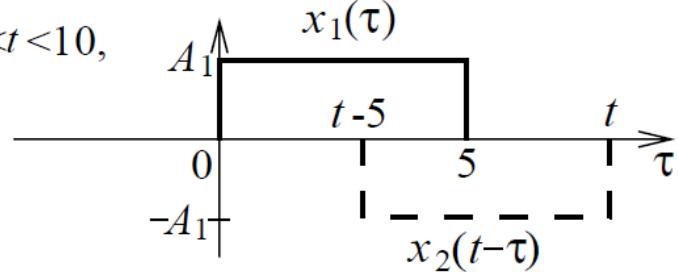


2.1b Continuous-Time LTI Systems

(ii) $0 < t < 5$,



(iii) $5 < t < 10$,



(iv) $t > 10$,

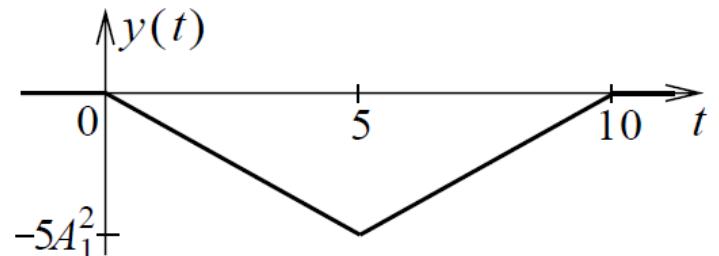
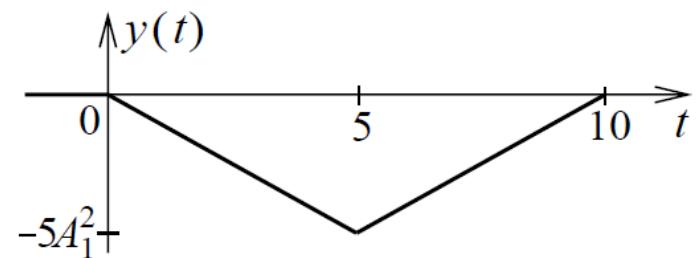
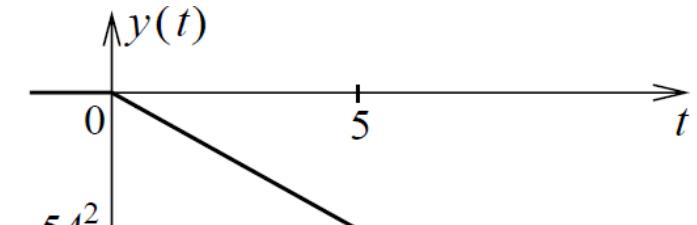
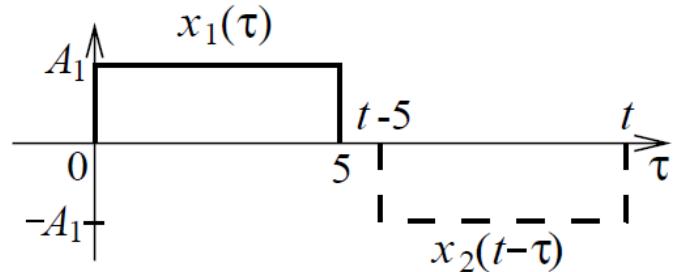


Figure 47: Solution for example on convolution integral

2.1 Discrete-Time and Continuous-Time LTI Systems

Summary 8

□ Analysis of CT Systems

- Any CT LTI system can be uniquely defined by its impulse response, $h(t)$.
- The output of a CT LTI system is the convolution of the input signal and its impulse response.
- The CT convolution (or convolution integral) is defined as

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$$

- The graphical approach for evaluating the convolution integral.



***You have reached the end of 2.1. Do reflect on your level of understanding.
Please proceed to 2.2 Convolution.***