

Parsing

Sujit Kumar Chakrabarti

IITB

Recursive Descent Parsing

Recursive Descent Parsing

- Appropriate for manual implementation
- Top-down parsing
- Starts with the root
- Prepares the parse tree in pre-order depth first sequence
- Finds the *leftmost derivation* for a string
- $LL(k)$ Grammars

Leftmost and Rightmost Derivation

Grammar:

$$E \rightarrow E + E \mid (E) \mid \text{id}$$

Example:

$$1 + (2 + 3) \rightarrow \text{id}_1 + \text{id}_2 + \text{Id}_3$$

Leftmost Derivation

Rightmost Derivation

E

Leftmost and Rightmost Derivation

Grammar:

$$E \rightarrow E + E \mid (E) \mid \text{id}$$

Example:

$$1 + (2 + 3) \rightarrow \text{id}_1 + \text{id}_2 + \text{Id}_3$$

Leftmost Derivation

Rightmost Derivation

$$E \Rightarrow E + E$$

Leftmost and Rightmost Derivation

Grammar:

$$E \rightarrow E + E \mid (E) \mid \text{id}$$

Example:

$$1 + (2 + 3) \rightarrow \text{id}_1 + \text{id}_2 + \text{Id}_3$$

Leftmost Derivation

Rightmost Derivation

$$\begin{aligned} E &\Rightarrow E + E \\ &\Rightarrow \text{id}_1 + E \end{aligned}$$

Leftmost and Rightmost Derivation

Grammar:

$$E \rightarrow E + E \mid (E) \mid \text{id}$$

Example:

$$1 + (2 + 3) \rightarrow \text{id}_1 + \text{id}_2 + \text{Id}_3$$

Leftmost Derivation

Rightmost Derivation

$$\begin{aligned} E &\Rightarrow E + E \\ &\Rightarrow \text{id}_1 + E \\ &\Rightarrow \text{id}_1 + (E) \end{aligned}$$

Leftmost and Rightmost Derivation

Grammar:

$$E \rightarrow E + E \mid (E) \mid \text{id}$$

Example:

$$1 + (2 + 3) \rightarrow \text{id}_1 + \text{id}_2 + \text{Id}_3$$

Leftmost Derivation

Rightmost Derivation

$$\begin{aligned}
 E &\Rightarrow E + E \\
 &\Rightarrow \text{id}_1 + E \\
 &\Rightarrow \text{id}_1 + (E) \\
 &\Rightarrow \text{id}_1 + (E + E)
 \end{aligned}$$

Leftmost and Rightmost Derivation

Grammar:

$$E \rightarrow E + E \mid (E) \mid \text{id}$$

Example:

$$1 + (2 + 3) \rightarrow \text{id}_1 + \text{id}_2 + \text{Id}_3$$

Leftmost Derivation

Rightmost Derivation

$$\begin{aligned}
 E &\Rightarrow E + E \\
 &\Rightarrow \text{id}_1 + E \\
 &\Rightarrow \text{id}_1 + (E) \\
 &\Rightarrow \text{id}_1 + (E + E) \\
 &\Rightarrow \text{id}_1 + (\text{id}_2 + E)
 \end{aligned}$$

Leftmost and Rightmost Derivation

Grammar:

$$E \rightarrow E + E \mid (E) \mid \text{id}$$

Example:

$$1 + (2 + 3) \rightarrow \text{id}_1 + \text{id}_2 + \text{id}_3$$

Leftmost Derivation

$$\begin{aligned} E &\Rightarrow E + E \\ &\Rightarrow \text{id}_1 + E \\ &\Rightarrow \text{id}_1 + (E) \\ &\Rightarrow \text{id}_1 + (E + E) \\ &\Rightarrow \text{id}_1 + (\text{id}_2 + E) \\ &\Rightarrow \text{id}_1 + (\text{id}_2 + \text{id}_3) \end{aligned}$$

Rightmost Derivation

$$E$$

Leftmost and Rightmost Derivation

Grammar:

$$E \rightarrow E + E \mid (E) \mid \text{id}$$

Example:

$$1 + (2 + 3) \rightarrow \text{id}_1 + \text{id}_2 + \text{id}_3$$

Leftmost Derivation

$$\begin{aligned} E &\Rightarrow E + E \\ &\Rightarrow \text{id}_1 + E \\ &\Rightarrow \text{id}_1 + (E) \\ &\Rightarrow \text{id}_1 + (E + E) \\ &\Rightarrow \text{id}_1 + (\text{id}_2 + E) \\ &\Rightarrow \text{id}_1 + (\text{id}_2 + \text{id}_3) \end{aligned}$$

Rightmost Derivation

$$E \Rightarrow E + E$$

Leftmost and Rightmost Derivation

Grammar:

$$E \rightarrow E + E \mid (E) \mid \text{id}$$

Example:

$$1 + (2 + 3) \rightarrow \text{id}_1 + \text{id}_2 + \text{id}_3$$

Leftmost Derivation

$$\begin{aligned}
 E &\Rightarrow E + E \\
 &\Rightarrow \text{id}_1 + E \\
 &\Rightarrow \text{id}_1 + (E) \\
 &\Rightarrow \text{id}_1 + (E + E) \\
 &\Rightarrow \text{id}_1 + (\text{id}_2 + E) \\
 &\Rightarrow \text{id}_1 + (\text{id}_2 + \text{id}_3)
 \end{aligned}$$

Rightmost Derivation

$$\begin{aligned}
 E &\Rightarrow E + E \\
 &\Rightarrow E + (E)
 \end{aligned}$$

Leftmost and Rightmost Derivation

Grammar:

$$E \rightarrow E + E \mid (E) \mid \text{id}$$

Example:

$$1 + (2 + 3) \rightarrow \text{id}_1 + \text{id}_2 + \text{id}_3$$

Leftmost Derivation

$$\begin{aligned} E &\Rightarrow E + E \\ &\Rightarrow \text{id}_1 + E \\ &\Rightarrow \text{id}_1 + (E) \\ &\Rightarrow \text{id}_1 + (E + E) \\ &\Rightarrow \text{id}_1 + (\text{id}_2 + E) \\ &\Rightarrow \text{id}_1 + (\text{id}_2 + \text{id}_3) \end{aligned}$$

Rightmost Derivation

$$\begin{aligned} E &\Rightarrow E + E \\ &\Rightarrow E + (E) \\ &\Rightarrow E + (E + E) \end{aligned}$$

Leftmost and Rightmost Derivation

Grammar:

$$E \rightarrow E + E \mid (E) \mid \text{id}$$

Example:

$$1 + (2 + 3) \rightarrow \text{id}_1 + \text{id}_2 + \text{id}_3$$

Leftmost Derivation

$$\begin{aligned} E &\Rightarrow E + E \\ &\Rightarrow \text{id}_1 + E \\ &\Rightarrow \text{id}_1 + (E) \\ &\Rightarrow \text{id}_1 + (E + E) \\ &\Rightarrow \text{id}_1 + (\text{id}_2 + E) \\ &\Rightarrow \text{id}_1 + (\text{id}_2 + \text{id}_3) \end{aligned}$$

Rightmost Derivation

$$\begin{aligned} E &\Rightarrow E + E \\ &\Rightarrow E + (E) \\ &\Rightarrow E + (E + E) \\ &\Rightarrow E + (E + \text{id}_3) \end{aligned}$$

Leftmost and Rightmost Derivation

Grammar:

$$E \rightarrow E + E \mid (E) \mid \text{id}$$

Example:

$$1 + (2 + 3) \rightarrow \text{id}_1 + \text{id}_2 + \text{id}_3$$

Leftmost Derivation

$$\begin{aligned} E &\Rightarrow E + E \\ &\Rightarrow \text{id}_1 + E \\ &\Rightarrow \text{id}_1 + (E) \\ &\Rightarrow \text{id}_1 + (E + E) \\ &\Rightarrow \text{id}_1 + (\text{id}_2 + E) \\ &\Rightarrow \text{id}_1 + (\text{id}_2 + \text{id}_3) \end{aligned}$$

Rightmost Derivation

$$\begin{aligned} E &\Rightarrow E + E \\ &\Rightarrow E + (E) \\ &\Rightarrow E + (E + E) \\ &\Rightarrow E + (E + \text{id}_3) \\ &\Rightarrow E + (\text{id}_2 + \text{id}_3) \end{aligned}$$

Leftmost and Rightmost Derivation

Grammar:

$$E \rightarrow E + E \mid (E) \mid \text{id}$$

Example:

$$1 + (2 + 3) \rightarrow \text{id}_1 + \text{id}_2 + \text{id}_3$$

Leftmost Derivation

$$\begin{aligned} E &\Rightarrow E + E \\ &\Rightarrow \text{id}_1 + E \\ &\Rightarrow \text{id}_1 + (E) \\ &\Rightarrow \text{id}_1 + (E + E) \\ &\Rightarrow \text{id}_1 + (\text{id}_2 + E) \\ &\Rightarrow \text{id}_1 + (\text{id}_2 + \text{id}_3) \end{aligned}$$

Rightmost Derivation

$$\begin{aligned} E &\Rightarrow E + E \\ &\Rightarrow E + (E) \\ &\Rightarrow E + (E + E) \\ &\Rightarrow E + (E + \text{id}_3) \\ &\Rightarrow E + (\text{id}_2 + \text{id}_3) \\ &\Rightarrow \text{id}_1 + (\text{id}_2 + \text{id}_3) \end{aligned}$$

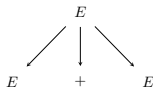
Leftmost and Rightmost Derivation

Formation of Parse Tree with Leftmost Derivation

E

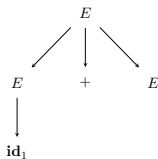
Leftmost and Rightmost Derivation

Formation of Parse Tree with Leftmost Derivation



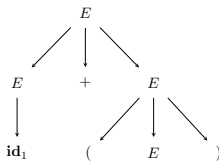
Leftmost and Rightmost Derivation

Formation of Parse Tree with Leftmost Derivation



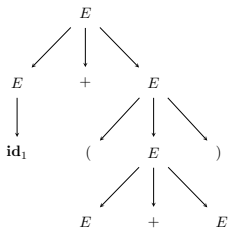
Leftmost and Rightmost Derivation

Formation of Parse Tree with Leftmost Derivation



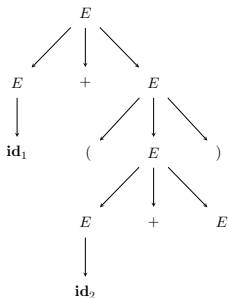
Leftmost and Rightmost Derivation

Formation of Parse Tree with Leftmost Derivation



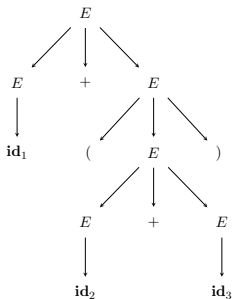
Leftmost and Rightmost Derivation

Formation of Parse Tree with Leftmost Derivation



Leftmost and Rightmost Derivation

Formation of Parse Tree with Leftmost Derivation



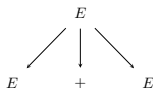
Leftmost and Rightmost Derivation

Formation of Parse Tree with Rightmost Derivation

E

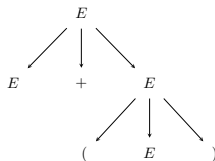
Leftmost and Rightmost Derivation

Formation of Parse Tree with Rightmost Derivation



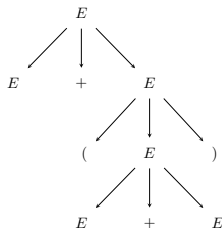
Leftmost and Rightmost Derivation

Formation of Parse Tree with Rightmost Derivation



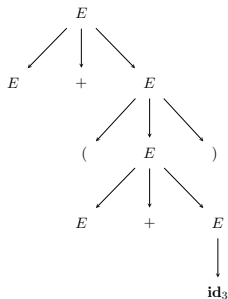
Leftmost and Rightmost Derivation

Formation of Parse Tree with Rightmost Derivation



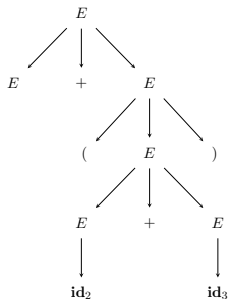
Leftmost and Rightmost Derivation

Formation of Parse Tree with Rightmost Derivation



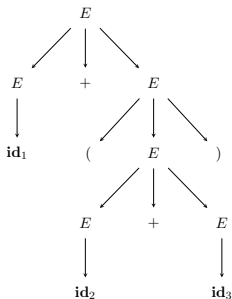
Leftmost and Rightmost Derivation

Formation of Parse Tree with Rightmost Derivation



Leftmost and Rightmost Derivation

Formation of Parse Tree with Rightmost Derivation



Recursive Descent Parsing

With backtracking – Example

Grammar:

$$S \rightarrow c A d$$
$$A \rightarrow a b$$
$$A \rightarrow a$$

Input string:

$$w = \text{"cad"}$$

Recursive Descent Parsing

With backtracking – Example

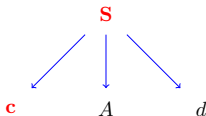
Grammar:

$$S \rightarrow c A d$$
$$A \rightarrow a b$$
$$A \rightarrow a$$

Input string:

$$w = \text{"cad"}$$

Step 1:



Recursive Descent Parsing

With backtracking – Example

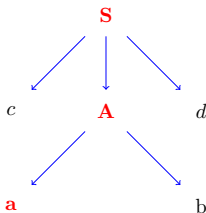
Grammar:

$$S \rightarrow c A d$$
$$A \rightarrow a b$$
$$A \rightarrow a$$

Input string:

$$w = \text{"c} \color{red}{a} d \text{"}$$

Step 2:



Recursive Descent Parsing

With backtracking – Example

Grammar:

$S \rightarrow c A d$

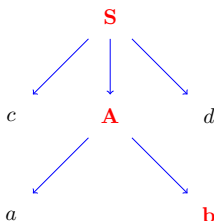
$A \rightarrow a b$

$A \rightarrow a$

Input string:

$w = \text{"cad"}^{\mathbf{d}}$

Step 3:



Failure

Recursive Descent Parsing

With backtracking – Example

Grammar:

$S \rightarrow c A d$

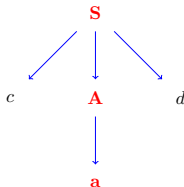
$A \rightarrow a b$

$A \rightarrow a$

Input string:

$w = \text{"c} \color{red}{a} \text{d"}$

Step 4:



Backtracking

Recursive Descent Parsing

With backtracking – Example

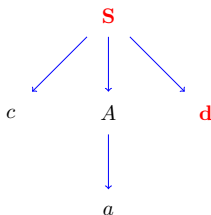
Grammar:

$$S \rightarrow c A d$$
$$A \rightarrow a b$$
$$A \rightarrow a$$

Input string:

$$w = \text{"ca} \mathbf{d} \text{"}$$

Step 5:



Success

Recursive Descent Parsing

With backtracking

```
procedure  $S(pos)$ 
  if MATCH( $pos, 'c'$ ) then
    if  $A_1(pos + 1)$  then
      if match( $pos + 3, 'd'$ ) then
        return true
      else
        return false
    else if ( $A_2(pos + 1)$ ) then
      if MATCH( $(pos + 2, 'd')$ ) then
        return true
    else
      return false
```

Recursive Descent Parsing

With backtracking

```
procedure  $A_1(pos)$   
  if MATCH( $pos$ , 'a') and MATCH( $pos + 1$ , 'b') then  
    return true  
  else  
    return false
```

```
procedure  $A_2(pos)$   
  if MATCH( $pos$ , 'a') then  
    return true  
  else  
    return false
```

Recursive Descent Parsing

With backtracking

- Powerful algorithm
- Backtracking
- Back and forth movement of input pointer
- May lead to inefficiency and complexity

Recursive Descent Parsing

Predictive Parsing – Example

Grammar:

$$E \rightarrow \text{num } E'$$

$$E' \rightarrow + E E' \mid \epsilon$$

Example:

$$1 + 2 + 3 \rightarrow \text{num}_1 + \text{num}_2 + \text{num}_3$$

Recursive Descent Parsing

Predictive Parsing – Example

Grammar:

$$E \rightarrow \text{num } E'$$

$$E' \rightarrow + E E' \mid \epsilon$$

Example:

$\text{num}_1 + \text{num}_2 + \text{num}_3$

E

Recursive Descent Parsing

Predictive Parsing – Example

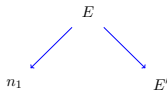
Grammar:

$$E \rightarrow \text{num } E'$$

$$E' \rightarrow + E E' \mid \epsilon$$

Example:

num₁ + **num**₂ + **num**₃



Recursive Descent Parsing

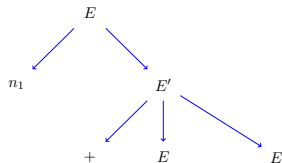
Predictive Parsing – Example

Grammar:

$$E \rightarrow \text{num } E'$$
$$E' \rightarrow + E E' \mid \epsilon$$

Example:

num₁ + **num**₂ + **num**₃



Recursive Descent Parsing

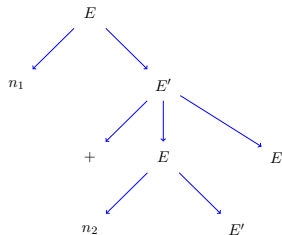
Predictive Parsing – Example

Grammar:

$$E \rightarrow \text{num } E'$$
$$E' \rightarrow + E E' \mid \epsilon$$

Example:

num₁ + **num**₂ + **num**₃



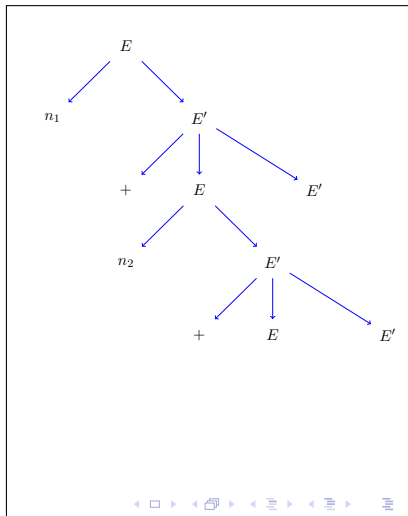
Recursive Descent Parsing

Predictive Parsing – Example

Grammar:

$$E \rightarrow \text{num } E'$$
$$E' \rightarrow + E E' \mid \epsilon$$

Example:

$$\text{num}_1 + \text{num}_2 + \text{num}_3$$


Recursive Descent Parsing

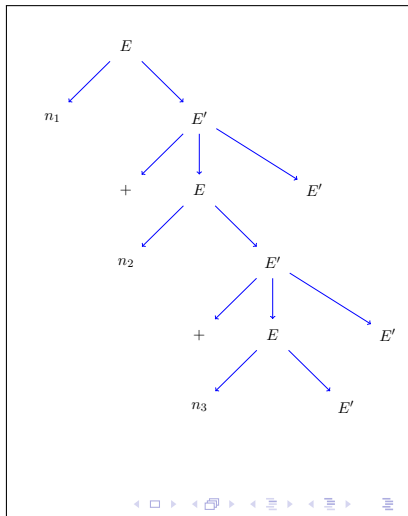
Predictive Parsing – Example

Grammar:

$$E \rightarrow \text{num } E'$$
$$E' \rightarrow + E E' \mid \epsilon$$

Example:

num₁ + num₂ + num₃



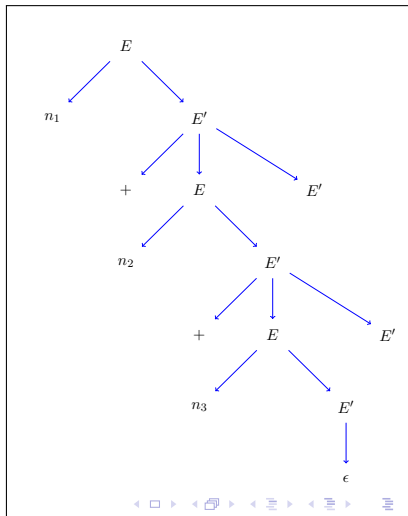
Recursive Descent Parsing

Predictive Parsing – Example

Grammar:

$$E \rightarrow \text{num } E'$$
$$E' \rightarrow + E E' \mid \epsilon$$

Example:

$$\text{num}_1 + \text{num}_2 + \text{num}_3$$


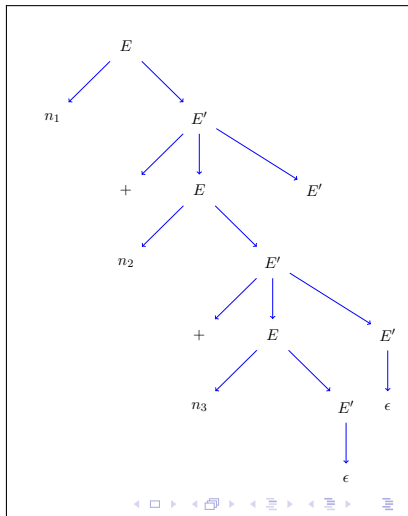
Recursive Descent Parsing

Predictive Parsing – Example

Grammar:

$$E \rightarrow \text{num } E'$$
$$E' \rightarrow + E E' \mid \epsilon$$

Example:

$$\text{num}_1 + \text{num}_2 + \text{num}_3$$


Recursive Descent Parsing

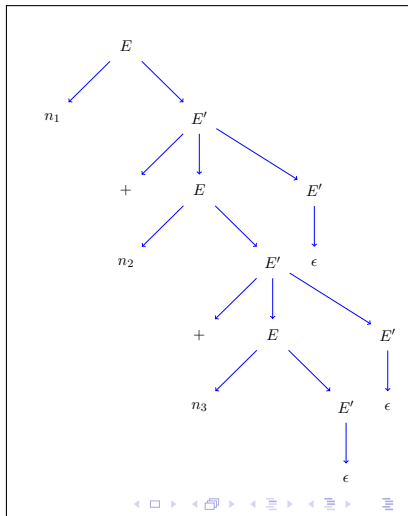
Predictive Parsing – Example

Grammar:

$$E \rightarrow \text{num } E'$$

$$E' \rightarrow + E E' \mid \epsilon$$

Example:

$$\text{num}_1 + \text{num}_2 + \text{num}_3$$


Recursive Descent Parsing

Algorithm

Grammar:

$$E \rightarrow \mathbf{num} E'$$

$$E' \rightarrow + E E' \mid \epsilon$$

Recursive Descent Parsing

Algorithm

Grammar:

$$E \rightarrow \text{num } E'$$
$$E' \rightarrow + E E' \mid \epsilon$$

```
procedure  $E$ 
  return MATCH(num) and  $E'$ 

procedure  $E'$ 
  return  $E'_1$  or  $E'_2$ 

procedure  $E'_1$ 
  return MATCH(+) and  $E$  and  $E'$ 

procedure  $E'_2$ 
  return true
```

Recursive Descent Parsing

Algorithm – Activity

Grammar:

$expr \rightarrow \mathbf{num} \mid term$

$term \rightarrow factor \mid factor + term$

$factor \rightarrow \mathbf{num} \mid \mathbf{num} * factor$

Recursive Descent Parsing

Left Recursion

Grammar:

$$E \rightarrow E + T \mid \text{num}$$

Recursive Descent Parsing

Left Recursion

Grammar:

$$E \rightarrow E + T \mid \text{num}$$

```
procedure E
  return E and MATCH(+) and MATCH(num)
```

```
procedure T
  return  $E'_1$  or  $E'_2$ 
```

Recursive Descent Parsing

Left Recursion

Grammar:

$$E \rightarrow E + T \mid \mathbf{num}$$

Recursive Descent Parsing

Left Recursion

Grammar:

$$E \rightarrow E + T \mid \text{num}$$

 \Rightarrow

Modified Grammar:

$$\begin{array}{lcl} E & \rightarrow & \text{num } E' \\ E' & \rightarrow & + E E' \mid \epsilon \end{array}$$

Recursive Descent Parsing

Left Recursion

Grammar:

$$E \rightarrow E + T \mid \mathbf{num}$$

 \Rightarrow

Modified Grammar:

$$\begin{array}{lcl} E & \rightarrow & \mathbf{num} E' \\ E' & \rightarrow & + E E' \mid \epsilon \end{array}$$

- Algorithm available for removing left recursion
- Self-study