

FUNCTIONS

1. FUNCTION AS A RELATION:

“A function is a relation in which no two ordered pairs have the same first components.”

A function is denoted by f .

2. DOMAIN AND RANGE OF A FUNCTION:

The set of first components of the ordered pairs belonging to the function f is called domain of f and the set of second components is called the range of f .

Example:

Consider the relation R given by

$$R = \{(1, 2), (2, 3), (3, 4), (4, 5)\}$$

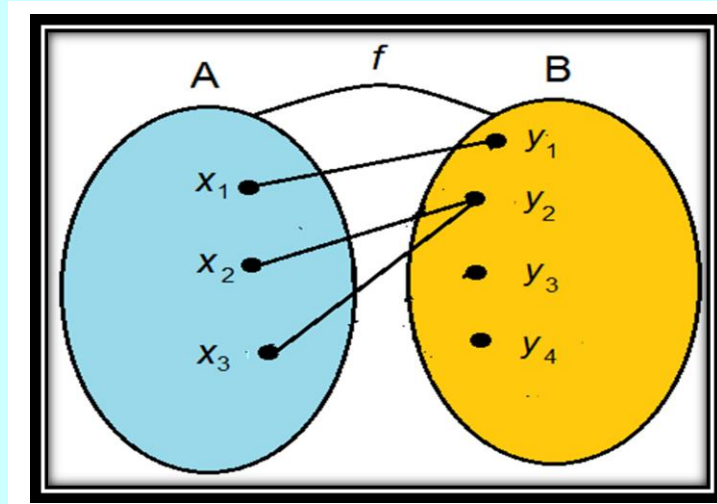
Since, no two ordered pairs have the same first components. So, R is a function.

$$\text{Domain} = \{a: (a, b) \in R\} = \{1, 2, 3, 4\}$$

$$\text{Range} = \{b: (a, b) \in R\} = \{2, 3, 4, 5\}$$

3. FUNCTION

Let A and B be any two non-empty sets. Then a rule or an association under which every element of A gets associated with a unique element of B , is called a function or mapping from set A to the set B .



Important Remarks:

- (i) Every element of the set A must get associated to a unique (single) element of the set B .
- (ii) Two or more elements of the set A may be associated with the same element of the set B .
- (iii) There may be some elements in the set B , which are not assigned to any element of the set A .
- (iv) A function may sometimes be represented by a formula.

4. DOMAIN AND CODOMAIN OF A FUNCTION

A function f defined from a set A to the set B is denoted by $f: A \rightarrow B$.

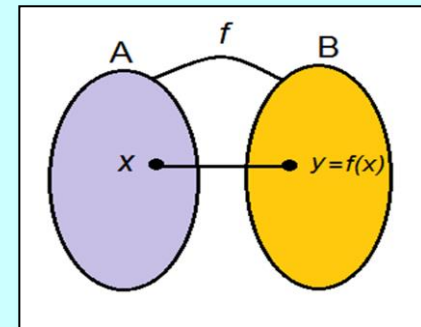
Here, the set A is called the domain and the set B is called Codomain.

5. IMAGE AND PRE-IMAGE OF A FUNCTION

If an element $x \in A$ is associated to an element $y \in B$ under the function f , then we say that y is the image of x under the function f and we write it as under

$$f(x) = y$$

Here, x is called the pre-image of y .



6. RANGE OF A FUNCTION

The 'set of images' of the elements of the domain set A under a function f is called range of the function f and is denoted by $f(A)$.

i. e. $\text{Range}(f) = f(A) = \{f(x) : x \in A\}$

Note:

The range of a function is always a subset of the codomain.

7. REAL VALUED FUNCTION and REAL FUNCTION

A function which has either \mathbb{R} or one of its subsets as its range is called **a real valued function**. Further, if its domain is also either \mathbb{R} or a subset of \mathbb{R} , it is called **a real function**.

8. SOME FUNCTIONS AND THEIR GRAPHS

(1) IDENTITY FUNCTION

A function $f: A \rightarrow A$ is called an identity function if each element of the set A is mapped onto itself.

$$i. e. f(x) = x, \quad \forall x \in A$$

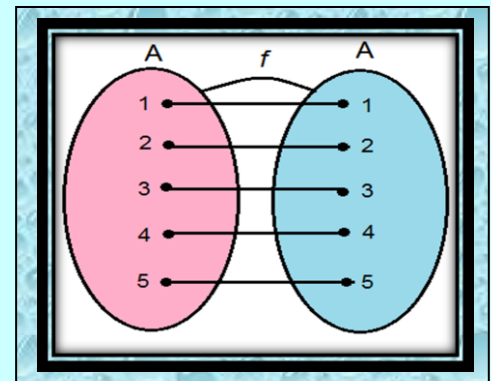
Domain = \mathbb{R} and Range = \mathbb{R}

Example:

If $A = \{1, 2, 3, 4, 5\}$ then,

$$\begin{aligned} f &= \{(x, x) : x \in A\} \\ &= \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\} \end{aligned}$$

Is an identity function on A .



Note: An identity function on the set A is denoted by I_A and is defined by

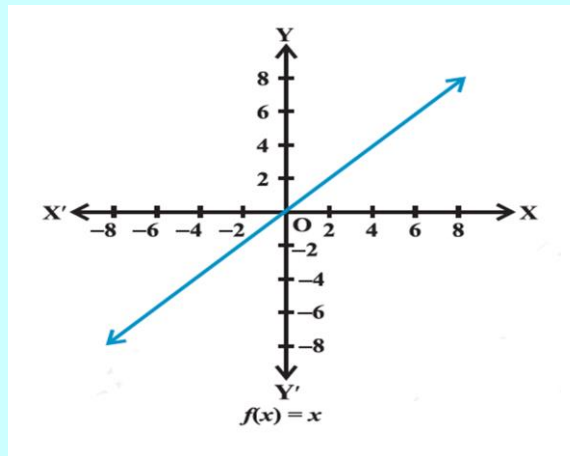
$$I_A(x) = x, \quad \forall x \in A$$

GRAPH OF THE IDENTITY FUNCTION

Let us obtain some points on the graph of identity function

$$y = f(x) = x$$

x	-3	-2	-1	0	1	2	3	4
$y = x$	-3	-2	-1	0	1	2	3	4



(2) CONSTANT FUNCTION

A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is called a constant function

i.e. $f(x) = c, \quad \forall x \in \mathbb{R}$

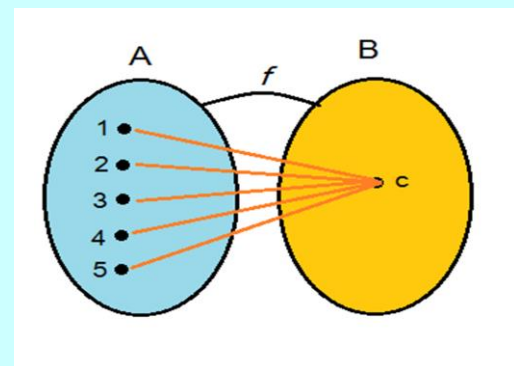
Domain = \mathbb{R} and Range = $\{c\}$

Example:

If $A = \mathbb{N}$ then, the function defined by

$$f(x) = x^0, \quad \forall x \in \mathbb{R}$$

is a constant function.



GRAPH OF THE CONSTANT FUNCTION

Let us obtain some points on a constant function

$$y = f(x) = 3$$

x	-3	-2	-1	0	1	2	3	4
$y = x$	3	3	3	3	3	3	3	3

