NAKUL PANDIT
PGT (MATHS)

COMPLEX NUMBERS

1. INTRODUCTION:

Solve the following equations for real x:

$(i)\ x^2 - 4 = 0$

$(ii)\ x^2 + 9 = 0$

$(iii)\ x^2 + x + 1 = 0$

Solution:

$(i)\ x^2 - 4 = 0$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

$(ii)\ x^2 + 9 = 0$

$\Rightarrow x^2 = -9$

$\Rightarrow x = \sqrt{-9} = \text{Not real}$

$(iii)\ x^2 + x + 1 = 0$

Comparing it with the equation

$ax^2 + bx + c = 0$

We find

$a = 1, \qquad b = 1, \qquad c = 1$

$\therefore \mathrm{D} = b^2 - 4ac = 1^2 - 4.1.1 = 1 - 4 = -3$

Since, $\qquad \mathrm{D} < 0$

The roots of the given equation $x^2 + x + 1 = 0$ are not real.

Thus, we fail to solve the equations (ii) and (iii) in the set of real numbers.

This shows the inadequacy of the real number system.

Euler was the first Mathematician to use the symbol i for $\sqrt{-1}$ with the property $i^2 = -1$. The symbol i is called *iota*. The definition of I made us able to talk about the square root of negative real numbers.

$\sqrt{-4} = \sqrt{-1 \times 4} = \sqrt{-1} \times \sqrt{4} = i \times 2 = 2i$

$\sqrt{-9} = \sqrt{-1 \times 9} = \sqrt{-1} \times \sqrt{9} = i \times 3 = 3i$

$\sqrt{-16} = \sqrt{-1 \times 16} = \sqrt{-1} \times \sqrt{16} = i \times 4 = 4i$

$\sqrt{-7} = \sqrt{-1 \times 7} = \sqrt{-1} \times \sqrt{7} = i \times \sqrt{7} = \sqrt{7}\, i$

and so on.

2. INTEGRAL POWERS OF i

$i = i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = -1$

$i = i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = -1$

$i^5 = i^4 . i = 1 . i = i, \qquad i^6 = i^4 . i^2 = 1 . (-1) = -1$

$i^7 = i^4 . i^3 = 1 . (-i) = -i, \qquad i^8 = (i^4)^2 = (1)^2 = 1$

1. Find the value of $i^9 + i^{19}$.

Solution:

$i^9 = i^{8+1} = i^8 . i = (i^4)^2 . i = 1^2 . i = 1 . i = i$

$i^{19} = i^{16+3} = i^{16} . i^3 = (i^4)^4 . (-i) = 1^4 . (-i) = 1 . (-i) = -i$

$\therefore i^9 + i^{19} = -1 + 1 = 0$

2. Find the value of i^{-39}.

Solution:

$$i^{-39} = \frac{1}{i^{39}} \times \frac{i}{i} = \frac{i}{i^{40}} = \frac{i}{(i^4)^{10}} = \frac{i}{(1)^{10}} = \frac{i}{1} = i$$

3. Find the value of $(1-i)^4$.

Solution:

$$\begin{aligned}(1-i)^4 &= [(1-i)^2]^2 \\ &= [1 - 2i + i^2]^2 \\ &= [1 - 2i - 1]^2 \\ &= (-2i)^2 \\ &= 4\, i^2 \\ &= 4 . (-i) \\ &= -4i\end{aligned}$$

4. Find the value of $\left[i^{18} - \left(\frac{1}{i}\right)^{25}\right]^3$.

Solution:

$i^{18} = i^{16+2} = i^{16}.i^2 = (i^4)^4.(-1) = 1^4.(-1) = 1.(-1) = -1$

$\left(\frac{1}{i}\right)^{25} = \frac{1}{i^{25}} = \frac{1}{i^{25}} \times \frac{i^3}{i^3} = \frac{i^3}{i^{28}} = \frac{-i}{(i^4)^7} = \frac{-i}{(1)^7} = \frac{-i}{1} = -i$

$\therefore \left[i^{18} - \left(\frac{1}{i}\right)^{25}\right]^3 = [-1 - i]^3$

$= [-(1 + i)]^3$

$= -(1 + i)^3$

$= -[1^3 + 3.1^2.i + 3.1.i^2 + i^3]$

$= -[1 + 3.1.i + 3.(-1) + (-i)]$ $\quad [\because i^2 = -1, i^3 = -i]$

$= -[1 + 3i - 3 - i]$

$= -[-2 + 2i]$

$= 2 - 2i$

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EXERCISE – 5.1

Express each of the following in the form of $a + ib$

1. Find the value of $i^9 + i^{19}$.

[Ex.- 5.1 Q.2]

Solution:

$i^9 = i^{8+1} = i^8.i = (i^4)^2.i = 1^2.i = 1.i = i$

$i^{19} = i^{16+3} = i^{16}.i^3 = (i^4)^4.(-i) = 1^4.(-i) = 1.(-i) = -i$

$\therefore i^9 + i^{19} = -1 + 1 = 0 = 0 + i0$

2. Find the value of i^{-39}.

Solution:

$$i^{-39} = \frac{1}{i^{39}} \times \frac{i}{i} = \frac{i}{i^{40}} = \frac{i}{(i^4)^{10}} = \frac{i}{(1)^{10}} = \frac{i}{1} = i = 0 + i.0$$

3. Find the value of $(1 - i)^4$.

Solution:

$(1 - i)^4 = [(1 - i)^2]^2$

$= [1 - 2i + i^2]^2$

$= [1 - 2i - 1]^2$

$= (-2i)^2$

$= 4\, i^2$

$= 4.(-1)$

$= -4$

$= -4 + 0.i$