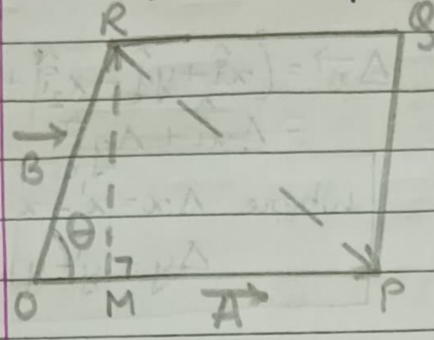
PAGE NO.:
DATE: / /

20.9.21

## Area of Parallelogram




$RM = \vec{B} \sin\theta$.

Area of Parallelogram $= OP \times RM$

$= \vec{A} B \sin\theta$

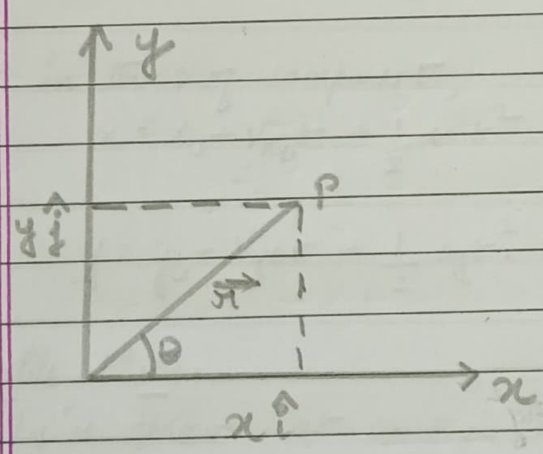
$= |\vec{A} \times \vec{B}|$

Area of triangle OPR =

$\therefore Ar(OPR) = \frac{1}{2} |\vec{A} \times \vec{B}|$.

## Motion in two dimension

### Position




The P is the position vector.

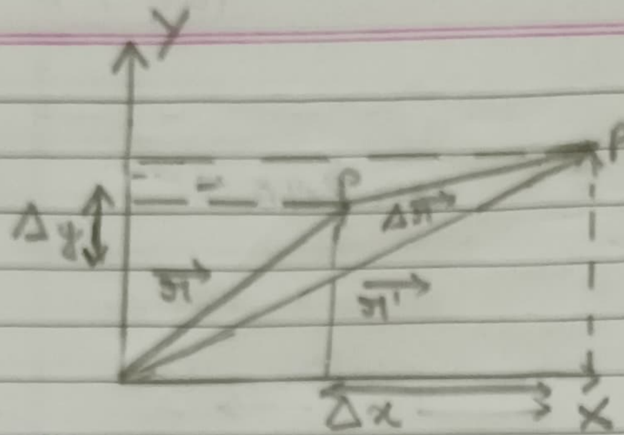
$\therefore \vec{r} = x\hat{i} + y\hat{j} \longrightarrow (1)$

$|\vec{r}| = \sqrt{x^2 + y^2}$ [Magnitude of a vector]

$\therefore \theta = \tan^{-1} = \frac{y}{x}$

Let the particle move from ~~P to P'~~ P to P'.

PAGE NO.:
DATE: / /




$$\Delta\vec{r} = (x\hat{i} + y\hat{j} - x_2\hat{i} + y\hat{j})$$
$$= \Delta x\hat{i} + \Delta y\hat{j}$$

$$\boxed{\text{where } \Delta x = x' - x \text{ and, } \Delta y = y' - y.}$$

$$\vec{V}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x\hat{i} + \Delta y\hat{j}}{\Delta t}$$
$$= \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}$$
$$= \Delta x\hat{i} + \Delta y\hat{j}$$

Magnitude of $\vec{v}$

$$\therefore |\vec{v}| = \sqrt{vx^2 + vy^2}$$

direction,

$$\theta = \tan^{-1}\frac{v_y}{v_x}$$

~~Acceleration~~ Accelaration

$$\vec{A} = \frac{\Delta v}{\Delta t} \Rightarrow A_x\hat{i} + A_y\hat{j}$$

$\therefore$ Magnitude $\Rightarrow |\vec{A}| = \sqrt{(A_x^2 + A_y^2}$

direction $\Rightarrow \tan\theta = \frac{a_y}{a_x}$

$$\therefore \theta = \tan^{-1}\left\{\cancel{A_y}\left(\frac{A_y}{A_x}\right)\right.$$

PAGE NO. :
DATE : / /

Motion in a plane with constant acceleration

$$\vec{a} = \frac{\vec{v} - \vec{v_0}}{t - 0} \Rightarrow \frac{\vec{v} - \vec{v_0}}{t}$$

$$\vec{v} = \vec{v_0} + \vec{a}t$$

Now, in terms of ~~companions~~, components,

$$\boxed{\begin{aligned} V_x &= V_{0x} + a_x t \\ V_y &= V_{0y} + a_y t \end{aligned}}$$

Now,

$$\vec{r} - \vec{r_0} = \left(\frac{\vec{v} - \vec{v_0}}{2}\right)t$$

$$= \left(\frac{\vec{v_0} + \vec{a}t + \vec{v_0}}{2}\right)t$$

$$\vec{r} = \vec{r_0} + \vec{v_0}t + \frac{1}{2}\vec{a}t^2$$

Now, in terms of components,

$$x = x_0 + V_{x0}t + \frac{1}{2}a_x t^2$$

$$y = y_0 + V_{y0}t + \frac{1}{2}a_y t^2$$

Projectile Motion

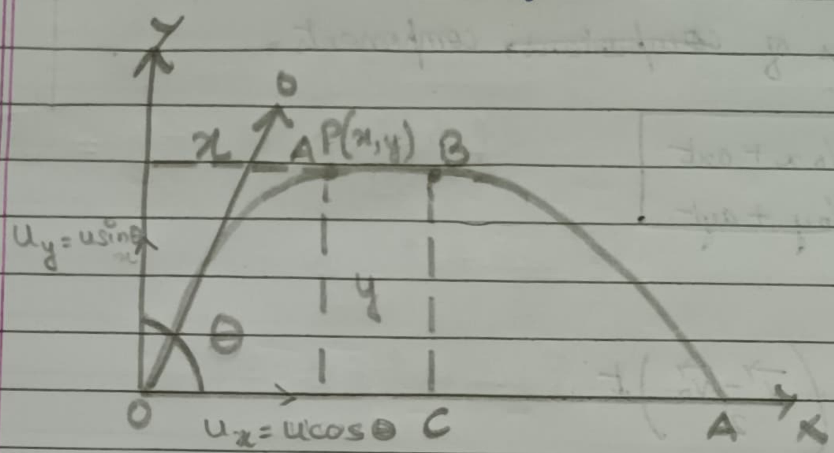
A body is thrown at an angle $\theta$ with a certain velocity and it move with the influance of gravity then it is called Projectile Motion.

Types of Projectile Motion

- Obilique Projectile Motion
- Horizontal Projectile Motion
- Projectile motion and an inclined.

PAGE NO. :
DATE : / /

**OBLIQUE PROJECTILE :-** When a body is thrown upward in a direction other than vertical and actual upon force of gravity only, then it moves in a curved path. This motion is called projectile motion.




At time 't', P(x,y) is position of the particle. It means the body has covered horizontal distance x and vertical distance 'y' in time 't'.

**HORIZONTAL MOTION :-** The body will cover horizontal distance x with velocity $u\cos\theta$.

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$x = u\cos\theta \times t \quad [a_x = 0]$$

$$\Rightarrow t = \frac{x}{u\cos\theta} \longrightarrow (1)$$

**VERTICAL MOTION :-** The body will cover vertical ~~tot~~ distance with acceleration due to gravity

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$y = u\sin\theta \times t - \frac{1}{2} g t^2 \longrightarrow (2)$$

PAGE NO.:
DATE: / /

From eq (1) putting the value of t

$$y = \cancel{u}\sin\theta \times \frac{x}{\cancel{u}\cos\theta} - \frac{1}{2} g \left(\frac{x}{\cos\theta}\right)^2$$

$$\boxed{y = x\tan\theta - \frac{1}{2} g \frac{x^2}{u^2\cos^2\theta}} \longrightarrow (3)$$

Eq. (3) is the equation of a trajector of a projectile which is parabollic in nature.

| TIME OF FLIGHT :- | Second Method |
|---|---|
| Let T is time of Flight.<br>$y = u\sin\theta \times t - \frac{1}{2}gt^2$.<br>$\Rightarrow 0 = u\sin\theta \times T - \frac{1}{2}gt^2$.<br>$\Rightarrow \frac{1}{2}gt^2 = u\sin\theta \times T$<br>$\Rightarrow T = \frac{2u\sin\theta}{g} \longrightarrow (1)$ | Time to reach max. height (t).<br>$\cancel{v_y} = v_y = u_y - gt$<br>$\Rightarrow 0 = u\sin\theta - gt$<br>$\Rightarrow t = \frac{u\sin\theta}{g} \longrightarrow (1)$<br>Time of flight<br>$T = 2t = \frac{2u\sin\theta}{g} \longrightarrow (2)$ |

Time of ascent = Time of decent $= \frac{T}{2} = \frac{u\sin\theta}{g}$.

HORIZONTAL RANGE :-

$$R = u_x \times T$$

$$R = u\cos\theta \times \frac{2u\sin\theta}{g}$$

$$R = \frac{u^2(2\sin\theta \cdot \cos\theta)}{g}$$

$(\because \sin 2\theta = 2\sin\theta\cos\theta)$.

$$R = \frac{u^2\sin 2\theta}{g} \longrightarrow (2)$$

For max. horizontal range,

$$\sin 2\theta = 1$$

$$\Rightarrow \sin 2\theta = 90^\circ$$

$$\Rightarrow 2\theta = 90^\circ$$

$$\Rightarrow \theta = 45^\circ.$$

PAGE NO. :

DATE : / /

Max. Horizontal range, $R_{max} = \frac{u^2}{g} \longrightarrow$ (3)

Maximum height reached :-

$v_y^2 = u_y^2 + 2as$ ($v_y = 0$ at max height).

$\Rightarrow 0 = (u\sin\theta)^2 - 2gH$

$\Rightarrow 2gH = u^2\sin^2\theta$

$\Rightarrow \boxed{H = \frac{u^2\sin^2\theta}{2g}}$