NAKUL PANDIT
PGT (MATHS)

SUBSETS

1. SUBSETS

If A and B are two sets such that every element of set A is also an element of B, we say that A is a subset of B and we write $A \subset B$.

i.e. $A \subset B \text{ if } a \in A \Rightarrow a \in B$

Remarks:

1. *If* $A \subset B$ *and* $A \neq B$ *, then A is called a proper subset of B and B is called superset of A.*
2. *If* $A \subset B$ *and* $= B$ *, then A is called an improper subset of B.*
3. *If* $A \subset B$ *and* $B \subset A$*, then* $A = B$.
4. *If A is not a subset of B, then we write,* $A \not\subset B$.
5. *Every set A is a subset of itself i.e.* $A \subset A$
6. *The empty set is* $\emptyset$ *a subset of every set.*

Examples:

(i) $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5\}$

Here, A is a proper subset of B as $A \subset B$ *and* $A \neq B$. *Also B is the super set of A.*

(ii) $A = \{1, 2, 3\}$, $B = \{2, 3, 1\}$

Here, A is an improper subset of B as $A \subset B$ *and* $A = B$

(iii) $A = \{1, 2, 3\}$, $B = \{1, 2, 4, 5\}$

Here, A is not a subset of B i.e. $A \not\subset B$ *as* $3 \in A$ *but* $3 \notin B$.

NUMBER OF SUBSETS

Set	No. of Elements	Subsets	No. of subsets	
$\emptyset$	0	$\emptyset$	1	$1 = 2^0$
$\{a\}$	1	$\emptyset, \{a\}$	2	$2 = 2^1$
$\{a, b\}$	2	$\emptyset, \{a\}, \{b\}, \{a, b\}$	4	$4 = 2^2$
$\{a, b, c\}$	3	$\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}$	8	$8 = 2^3$
$\{a, b, c, d\}$	4	$\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$ $\{a, b, c, d\}$	16	$16 = 2^4$

Conclusion: *The number of subsets of the set containing* n *elements is* 2^n

The number of subsets of the set containing n elements is 2^n

Example-1

Write all possible subsets of the set $\{1, 2, 3\}$. How many of these are proper sub sets ?

Solution:

Let $A = \{1, 2, 3\}$

The possible subsets of the given set A are

$\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{3, 1\}, \{1, 2, 3\}$

Since, $\{1, 2, 3\}$ *is an improper set of A.*

∴ The total number of proper subsets of A is 7.

2. POWER SET

The collection of all possible subsets of a set A is called the power set of A.

It is denoted by P (A). In P (A), every element is a set.

Example-2

Write the power set of the set $A = \{a, b, c\}$.

Solution:

Given $A = \{1, 2, 3\}$

The power set of A is

$\mathrm{P(A)} = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}\}$

Example-3

How many elements has P (A), if $A = \emptyset$ *?*

Solution:

Given $A = \emptyset$

∴ The subset of A is $\emptyset$

So, the power set of A is

$P(A) = \{\emptyset\}$

Thus, the number of elements in P (A) is 1.

3. UNIVERSAL SET

If all the sets under consideration are subsets of a fixed set U, then this fixed set U is called the Universal Set.

Example-4

If $A = \{1, 2, 3, 4\}, B = \{2, 3, 5\}$ *and* $C = \{6, 7\}$, *then* $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ *is the universal set.*

Example-5

If

$A = \{x : x \text{ is a student of class IX of KV bkp (Army)}\}$,

$B = \{x : x \text{ is a student of class X of KV bkp (Army)}\}$,

$C = \{x : x \text{ is a student of class XI of KV bkp (Army)}\}$ *and*

$D = \{x : x \text{ is a student of class XII of KV bkp (Army)}\}$,

Then,

$U = \{x : x \text{ is a student of KV bkp (Army)}\}$ *is the universal set.*

Remarks:

1. *A set may behave like an element for another set. For example*

   $A = \{1, \{2, 3\}, 4\}$

   Here, $\{2, 3\}$ *is an element of the set A.*
2. *If* $a \in A$, *then* $\{a\} \subset A$
3. *If* $a \in A, b \in A, b \in A$ *then* $\{a, b, c\} \subset A$
4. *If* $\emptyset \notin A$, *but* $\emptyset \subset A$
5. *If* $A = \{\emptyset, 1, \{2, 3\}, 4\}$, *then* $\emptyset \in A$ *and* $\{\emptyset\} \subset A$

4. INTERVALS

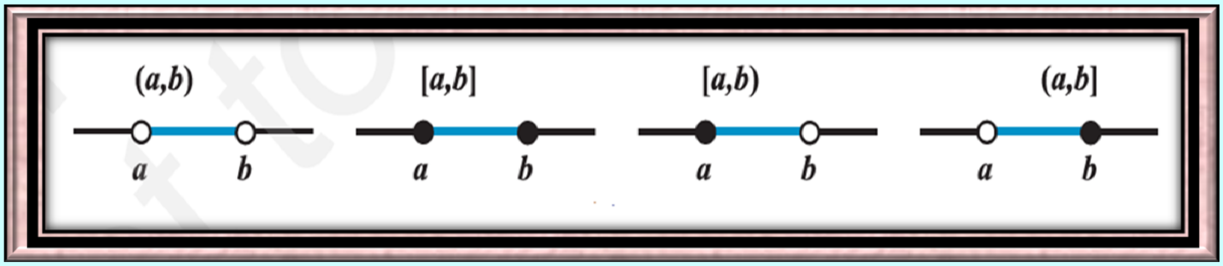
Let $a, b \in R$ *such that* $a < b$. *Then we define*

(i) **Open Interval**: *The set of real numbers* $\{x : a < x < b\}$ *is called an open interval and is denoted by* (a, b).

(ii) **Closed Interval**: *The set of real numbers* $\{x : a \le x \le b\}$ *is called a closed interval and is denoted by* $[a, b]$

(iii) **Left Half Open Interval**: *The set of real numbers* $\{x : a < x \le b\}$ *is called a closed interval and is denoted by* $(a, b]$

(iii) **Right Half Open Interval**: *The set of real numbers* $\{x : a \le x < b\}$ *is called a closed interval and is denoted by* $[a, b)$



Examples:

(i) $(-3,5) = \{x: x \in \mathrm{R}, -3 < x < 5\}$ *is an open interval from* -3 *to* 5, *which contains all real numbers between* -3 *and* 5.

(ii) $[-3,5] = \{x: x \in \mathrm{R}, -3 \leq x \leq 5\}$ *is a closed interval from* -3 *to* 5, *which contains all real numbers between* -3 *and* 5 *including -3 and 5.*

(iii) $(-3,5] = \{x: x \in \mathrm{R}, -3 < x \leq 5\}$ *is a left half open interval from* -3 *to* 5, *which contains all real numbers between* -3 *and* 5 *including 5 and excluding* -3.

(iv) $[-3,5) = \{x: x \in \mathrm{R}, -3 \leq x < 5\}$ *is a right half open interval from* -3 *to* 5, *which contains all real numbers between* -3 *and* 5 *including* -3 *and excluding 5.*

Remarks:

1. $(-\infty, \infty) =$ *The set of all real numbers.*

2. $(0, \infty) =$ *The set of all positive real numbers.*

3. $[\,0, \infty) =$ *The set of all non-negative real numbers.*

4. $(-\infty, 0) =$ *The set of all negative real numbers.*

Example-6

Write the following as intervals:

(a) $\{x: x \in \mathrm{R},\ 0 < x < 5\}$ (b) $\{x: x \in R, -2 < x \leq 7\}$

(c) $\{x: x \in R,\ 2 \leq x < 9\}$ (d) $\{x: x \in R, -5 \leq x \leq 0\}$

Solution:

(a) $\{x: x \in \mathrm{R},\ 0 < x < 5\} = (0,5)$

(b) $\{x: x \in \mathrm{R}, -2 < x \leq 7\} = (-2,7]$

(c) $\{x: x \in \mathrm{R},\ 2 \leq x < 9\} = [2,9)$

(d) $\{x: x \in \mathrm{R}, -5 \leq x \leq 0\} = [-5,0]$

Example-7

Write the following as intervals in set-builder form:

(a) $[-1,4]$ (b) $[3,11)$ (d) $(-2,8)$ (d) $(-4,0]$

Solution:

(a) $[-1,4] = \{x: x \in \mathrm{R}, -1 \leq x \leq 4\}$

(b) $[3,11) = \{x: x \in \mathrm{R},\ 3 \leq x < 11\}$

(c) $(-2,8) = \{x: x \in \mathrm{R}, -2 < x < 8\}$

(d) $(-4,0] = \{x: x \in \mathrm{R}, -4 < x \leq 0\}$

NAKUL PANDIT
PGT (MATHS)

VENN DIAGRAMS

1. VENN DIAGRAMS

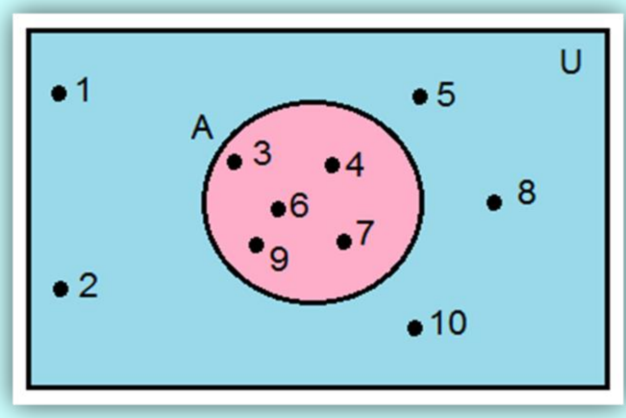
Most of the relationships between sets can be represented by means of diagrams, which are known as *venn diagrams*. Venn diagrams are named after the English logician John Venn (1834-1883).

These diagrams consist of rectangles and closed curves usually circles.
The universal set is represented usually by a rectangle and its subsets by circles.
In venn diagrams, the elements of the sets are written in their respective circles.

Examples:

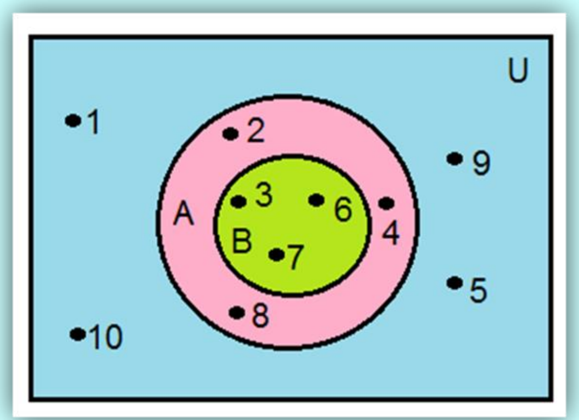
1. Represent the following sets using venn diagrams

   $U = \{1, 2, 3, \dots, 10\}$ and $A = \{3, 4, 6, 7, 9\}$



2. Represent the following sets using venn diagrams

   $U = \{1, 2, 3, \dots, 10\}$, $A = \{2, 3, 4, 6, 7, 8\}$ and $B = \{3, 6, 7\}$



NAKUL PANDIT
PGT (MATHS)

OPERATIONS ON SETS

1. UNION OF SETS

Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and all the elements of B, the common elements being taken only once.
The union of A and B is symbolically denoted by AUB and is read as "A union B"

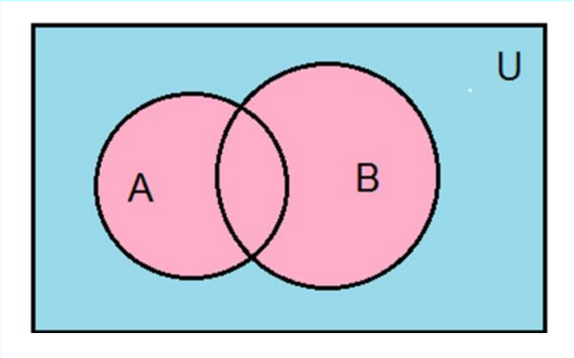
Thus,

$$\text{AUB} = \{x : x \in \text{A or } x \in \text{B}\}$$

Note:

1. $x \in (\text{AUB}) \Rightarrow x \in \text{A or } x \in \text{B}$
2. $x \notin (\text{AUB}) \Rightarrow x \notin \text{A and } x \notin \text{B}$

Venn Diagram of AUB



The shaded region in pink coloured represents AUB

Example-1

Let $\text{A} = \{2, 3, 4, 6, 8\}$ and $\text{B} = \{1, 2, 3, 4, 5\}$

Then, $\text{A} \cup \text{B} = \{1, 2, 3, 4, 5, 6, 8\}$

Example-2

Let $\text{A} = \{1, 2, 3\}$ and $\text{B} = \{1, 2, 3, 4, 5\}$

It is clear that $\text{A} \subset \text{B}$

Then, $\text{A} \cup \text{B} = \{1, 2, 3, 4, 5\} = \text{B}$

Conclusion:

If $\text{A} \subset \text{B}$, then $\text{A} \cup \text{B} = \text{B} = \text{(superset)}$