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Date : 13 Sept 2021.

Dimension.

Dimension of a physical quantity are the powers or the exponents to which the fundamental quantities are raised to represent that quantity.

In mechanics there are three base quantities, mass, length and time. It is represented as:

Mass = [M]

Length = [L]

Time = [T]

- dimension of velocity,

1. $V = \frac{S}{T}$

$\Rightarrow [V] = \frac{[L]}{[T]}$

$= [LT^{-1}]$.

2. $a = \frac{v}{t}$

$[a] = \frac{[LT^{-1}]}{[T]}$

$= [LT^{-2}]$

3. $F = ma$

$[F] = [M][LT^{-2}]$

$= [MLT^{-2}]$.

dimension of G,

$F = \frac{G m_1 m_2}{r^2}$

$\Rightarrow \frac{Fr^2}{m_1 \times m_2} = G$

$\Rightarrow [G] = \frac{Fr^2}{m_1 m_2}$

$= \frac{[MLT^{-2}][L]^2}{[M][M]}$

$= [M^{-1}L^3T^{-2}]$.

Principle of Homogenity.

It states that each term on both side of physical relation, ~~are~~ ~~be~~ must be same.

Application.

1. To check the correctness of a relation, the dimensions must be equal on both sides,

$$v^2 = u^2 + 2as.$$

LHS.

$[v^2] = [LT^{-1}]^2$

$= [L^2T^{-2}]$

RHS

$[u^2] + [2as] = [LT^{-1}]^2$ & $[2as]$,

$= L^2T^{-2}$ $= 2[LT^{-2}][L]$

$= L^2T^{-2}$

Thus,

dimension on each time on both sides is same.

2. $F = ax^2 + bt^3$, find dimension of a and b,

$$[F] = [ax^2]$$

$$[a] = \frac{[F]}{[x^2]}$$

$$\Rightarrow [a] = \frac{[MLT^{-2}]}{[L^2]}$$

$$\Rightarrow [a] = [ML^{-1}T^{-2}].$$

again,

$$[bt] = [MLT^{-2}]$$

$$[b] = \frac{[MLT^{-2}]}{[T]}$$

$$= [MLT^{-1}]$$

Q. $E = \frac{at^2}{b+x}$, find the dimension of a & b.

from principle of homogenity,

$$[b] = [x]$$
$$= [L].$$

$$\therefore \frac{[at^2]}{[x]} = [E]$$

$$\Rightarrow a = \frac{[E][L]}{[eT^2]}$$

$$= \frac{[ML^2T^{-2}][L]}{[T^2]}$$

$$= [ML^3T^{-4}]$$

- To deduce the relation among physical quantities.

Q. The centripetal force, F depends upon mass (m), velocity (v), and radius (r). Deride the formula of F using matter of dimension.

$$F \propto m^a v^b r^c$$
$$F = k m^a v^b r^c \quad \text{—— ①}$$

$$\Rightarrow [MLT^{-2}] = [M]^a [LT^{-1}]^b [L]^c$$
$$\Rightarrow [M^1L^1T^{-2}] = [M^a L^{b+c} T^{-b}]$$

$\therefore a = 1$ $\quad +b = +2$ $\quad b+c = 1$

$\Rightarrow b = 2$ $\quad \Rightarrow 2+c = 1$

$\Rightarrow c = 1-2$.

$= -1$.

~~$\therefore f \propto m^a b^b$.~~

Therefore,

$$F = km^1 v^2 r^{-1}$$
$$\Rightarrow F = k \frac{mv^2}{r}.$$

$$K = 1$$
$$\Rightarrow F = \frac{mv^2}{r}$$