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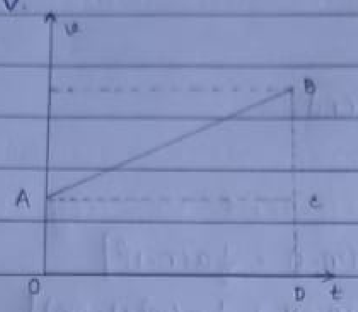
03/Sept/2021

EQUATION OF KINEMATICS.

Let an object moving along a straight line with intial velocity u and accelarationo a. It travels distance in time t and its final velocity is v.



(i) Here,

accelaration, a = slope of (v-t) graph

$\Rightarrow a = \frac{BC}{AC}$

$\Rightarrow a = \frac{v-u}{t}$

$\Rightarrow at = v-u$

$\Rightarrow v = at+u$

$\Rightarrow v = u+at$ ——— (i)

ii) Distance travelled by the object in time t, s = area under (v-t) graph.

$\Rightarrow S = \text{area}(OABD)$

$\Rightarrow S = \text{area}(OACD) + \text{area}(\Delta ABC)$

$\Rightarrow S = OA \times OD + \frac{1}{2} AC \times BC$

$\Rightarrow S = ut + \frac{1}{2} \times t \times (v-u)$

$v-u = at$

$\hookrightarrow S = ut + \frac{1}{2} t\,(at)$

$\Rightarrow S = ut + \frac{1}{2} at^2$ ——— (ii)

iii) S = area ~~of~~ under (v-t) graph

$\Rightarrow S = \text{area of trapezium } (OABD)$

$\Rightarrow S = \frac{1}{2}[OA + BD] \times OD$

$\Rightarrow S = \frac{1}{2}(v+u)\left(\frac{v-u}{a}\right)$

$$\Rightarrow s = \frac{v^2 - u^2}{2a}$$

$$\Rightarrow 2as = v^2 - u^2$$

$$\Rightarrow v^2 = 2as + u^2$$

$$\Rightarrow v^2 = u^2 + 2as \quad \text{——— (iii)}$$

↳ distance travelled in n^{th} second.

$$S_{nth} = S_n - S_{n-1}$$

$$\Rightarrow S_{nth} = (un + \tfrac{1}{2}an^2) - [u(n-1) + \tfrac{1}{2}a(n-1)^2]$$

$$\Rightarrow S_{nth} = (un + \tfrac{1}{2}an^2) - [un - u + \tfrac{1}{2}a(n^2+1-2n)]$$

$$\Rightarrow S_{nth} = \cancel{un} + \tfrac{1}{2}an^2 - \cancel{un} + u - \tfrac{1}{2}a(n^2+1-2n)$$

$$\Rightarrow S_{nth} = \tfrac{1}{2}\cancel{a}n^2 + u - \tfrac{1}{\cancel{2}}an^2 - a/2 + an$$

$$\Rightarrow S_{nth} = u + an - \frac{a}{2}$$

$$\Rightarrow S_{nth} = u + \frac{a}{2}(2n-1) \quad \text{——— (iv)}$$

Equation of object under gravity.

- For downward motion,

$$a = +g$$
$$v = u + gt$$
$$h = ut + \tfrac{1}{2}gt^2$$
$$v^2 = u^2 + 2gh$$

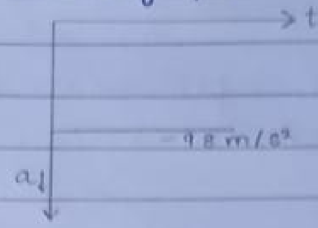
- For upward motion,

$$a = -g.$$
$$v = u - gt$$
$$h = ut - \tfrac{1}{2}gt^2$$
$$v^2 = u^2 - 2gh.$$

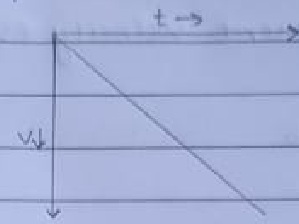
Graph of freely falling body,

when a body is falling from a height with only gravitational force acting on it, it is called free fall.

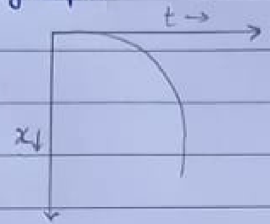
- accelaration time graph.



- velocity time graph,



- position time graph



Relative Velocity.

The relative velocity of an object A, with respect to object B. is the time rate of change of position of object B with respect to that of A.

Relative velocity of A w.r.t to B,

$$V_{AB} = V_A - V_B$$
$$V_{BA} = V_B - V_A.$$
when moving in same direction

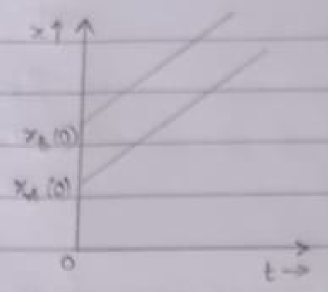
when objects are moving in opposite direction.

$$V_{AB} = V_A - (-V_B)$$
$$= V_A + V_B.$$

→ Relative velocity and Position Time graph.

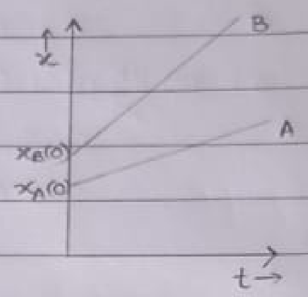
1. Objects moving with same velocity along same

direction.

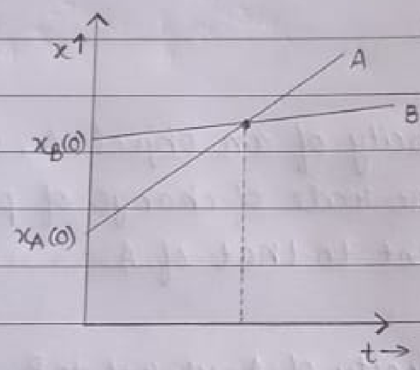


② objects moving with different velocity ~~$(V_B > V_A)$~~

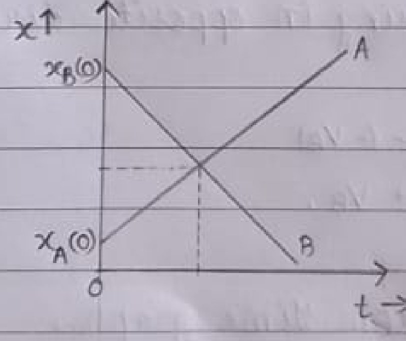
a) $(V_B > V_A)$



b) $(V_B < V_A)$



③ objects moving in opposite direction,



Differentiation.

$$y = f(x) \quad \text{---} \; ①$$

If, Δx be the small change in x,

then, the corresponding change is Δy.

$\therefore$ The average rate of change of y w.r.to x in $\frac{\Delta y}{\Delta x}$

$$\boxed{\therefore \; \frac{dy}{dx} = \lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta x} \right)}$$

- formula of differentiation.

① $\frac{d}{dx}(c) = 0$ $\qquad$ c = constant.

② $\frac{d}{dx}(x^n) = n x^{n-1}$

③ $\frac{d}{dx}(x) = 1$

④ $\frac{d}{dx}(cy) = c\frac{dy}{dx}$.

⑤ $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

⑥ $\frac{d}{dx}(u \times v) = u\frac{dv}{dx} + v\frac{du}{dx}$

⑦ $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} + u\frac{dv}{dx}}{v^2}$

⑧ $\frac{d}{dx}(\log_e x) = 1/x$

⑨ $\frac{d}{dx}(e^x) = e^x$

⑩ $\frac{d}{dx}(\sin x) = \cos x$

⑪ $\frac{d}{dx}(\cos x) = (-\sin x)$

⑫ $\cancel{\frac{d}{dx}}$ $y = f(z)$ & $z = f(x)$

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}.$$

Q Find $\frac{dy}{dx}$ –

① $y = x^5$

$\frac{dy}{dx} = 5x^{5-1}$

$= 5x^4$

② $y = 3x^{-2}$

$\frac{dy}{dx} = 3(-2)x^{-2-1}$

$= -6x^{-3}$

③ $y = ax^2 + bx + c$

$\frac{dy}{dx} = a\frac{dy}{dx}(x^2) + b(\frac{dy}{dx}(x) + 0$

$= a \times 2x + b$

$= 2ax + b.$

④ $y = x^{-3/4} + x^{1/4}$

$\frac{dy}{dx} = -\frac{3}{4}x^{-3/4-1} + \frac{1}{4}x^{1/4-1}$

$= -\frac{3}{4}x^{-7/4} + \frac{1}{4}x^{-3/4}.$