NAKUL PANDIT
PGT (MATHS)

COMPLEX NUMBERS

1. INTRODUCTION:

Solve the following equations for real x:

$(i)\ x^2 - 4 = 0$

$(ii)\ x^2 + 9 = 0$

$(iii)\ x^2 + x + 1 = 0$

Solution:

$(i)\ x^2 - 4 = 0$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

$(ii)\ x^2 + 9 = 0$

$\Rightarrow x^2 = -9$

$\Rightarrow x = \sqrt{-9} = \text{Not real}$

$(iii)\ x^2 + x + 1 = 0$

Comparing it with the equation

$ax^2 + bx + c = 0$

We find

$a = 1, \quad b = 1, \quad c = 1$

$\therefore \text{D} = b^2 - 4ac = 1^2 - 4.1.1 = 1 - 4 = -3$

Since, $\quad \text{D} < 0$

The roots of the given equation $x^2 + x + 1 = 0$ are not real.

Thus, we fail to solve the equations (ii) and (iii) in the set of real numbers.

This shows the inadequacy of the real number system.

Euler was the first Mathematician to use the symbol i for $\sqrt{-1}$ with the property $i^2 = -1$. The symbol i is called *iota*. The definition of I made us able to talk about the square root of negative real numbers.

$\sqrt{-4} = \sqrt{-1 \times 4} = \sqrt{-1} \times \sqrt{4} = i \times 2 = 2i$

$\sqrt{-9} = \sqrt{-1 \times 9} = \sqrt{-1} \times \sqrt{9} = i \times 3 = 3i$

$\sqrt{-16} = \sqrt{-1 \times 16} = \sqrt{-1} \times \sqrt{16} = i \times 4 = 4i$

$\sqrt{-7} = \sqrt{-1 \times 7} = \sqrt{-1} \times \sqrt{7} = i \times \sqrt{7} = \sqrt{7}\, i$

and so on.

2. INTEGRAL POWERS OF i

$i = i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1$

$$i = i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1$$

$$i^5 = i^4.i = 1.i = i, \qquad i^6 = i^4.i^2 = 1.(-1) = -1$$

$$i^7 = i^4.i^3 = 1.(-i) = -i, \qquad i^8 = (i^4)^2 = (1)^2 = 1$$

1. Find the value of $i^9 + i^{19}$.

Solution:

$i^9 = i^{8+1} = i^8.i = (i^4)^2.i = 1^2.i = 1.i = i$

$i^{19} = i^{16+3} = i^{16}.i^3 = (i^4)^4.(-i) = 1^4.(-i) = 1.(-i) = -i$

$\therefore i^9 + i^{19} = i - i = 0$

2. Find the value of i^{-39}.

Solution:

$$i^{-39} = \frac{1}{i^{39}} \times \frac{i}{i} = \frac{i}{i^{40}} = \frac{i}{(i^4)^{10}} = \frac{i}{(1)^{10}} = \frac{i}{1} = i$$

3. Find the value of $(1 - i)^4$.

Solution:

$$\begin{aligned}(1 - i)^4 &= [(1 - i)^2]^2 \\ &= [1 - 2i + i^2]^2 \\ &= [1 - 2i - 1]^2 \\ &= (-2i)^2 \\ &= 4\, i^2 \\ &= 4.(-i) \\ &= -4i\end{aligned}$$

4. Find the value of $\left[i^{18}-\left(\frac{1}{i}\right)^{25}\right]^3$.

Solution:

$i^{18}=i^{16+2}=i^{16}.i^2=(i^4)^4.(-1)=1^4.(-1)=1.(-1)=-1$

$\left(\frac{1}{i}\right)^{25}=\frac{1}{i^{25}}=\frac{1}{i^{25}}\times\frac{i^3}{i^3}=\frac{i^3}{i^{28}}=\frac{-i}{(i^4)^7}=\frac{-i}{(1)^7}=\frac{-i}{1}=-i$

$\therefore\left[i^{18}-\left(\frac{1}{i}\right)^{25}\right]^3=[-1-i]^3$

$=[-(1+i)]^3$

$=-(1+i)^3$

$=-[1^3+3.1^2.i+3.1.i^2+i^3]$

$=-[1+3.1.i+3.(-1)+(-i)]$ $\quad[\because i^2=-1, i^3=-i]$

$=-[1+3i-3-i]$

$=-[-2+2i]$

$=2-2i$

3. COMPLEX NUMBERS

If a and b are any two real numbers, then the symbol $a+ib$ is called a complex number. The se of complex number is denoted by C.

Thus,

$C=\{(a+ib): a, b\in R \text{ and } i=\sqrt{-1}\}$

Examples:

$2+3i,\quad -4+5i,\ 3-7i,\quad -4-5i,\ 0+3i,\ 6+0i,\ 0+0i,\ \sqrt{3}-4i$ etc.

Note: (i) A complex number is usually denoted by the letter z.

(ii) $a+ib=a+bi$

4. REAL AND IMAGINARY PARTS OF A COMPLEX NUMBER

Let $z=a+ib$ be a complex number.

Then, a is called real part of z and is written as Re(z) and b is called imaginary part of z and is written as Im(z).

Thus,

$\therefore \text{Re}(z)=a$ and $\text{Im}(z)=b$

Examples:

Complex Number (z)	Re (z)	Im (z)
$2+3i$	2	3
$-2+3i$	-2	3
$2-3i$	2	-3
$-2-3i$	-2	-3
$-5i=0-5i$	0	-5
$3=3+0i$	3	0
$0=0+0i$	0	0

5. PURELY REAL AND PURELY IMAGINARY COMPLEX NUMBERS

A complex number $\mathrm{z}=a+ib$ is said be purely real if $\mathrm{Im(z)}=0$

Examples:

$2=2+0i,\ -4=-4+0i,\ 3=3+0i,\ -2=-2+0i$ etc.

A complex number $\mathrm{z}=a+ib$ is said be purely imaginary if $\mathrm{Re(z)}=0$

Examples:

$0+2i,\ 0-3i,\ 0-5i,\ 0-6i,\ 0+7i,\ 0+9i$ etc.

6. EQUALITY OF COMLEX NUMBERS

Two complex numbers are said to be equal if their real parts and imaginary parts are separately equal.

Thus,

$$a+ib=c+id \Leftrightarrow a=c\,, b=d$$

Example:

If $4x+\ i(3x-y)=3+i\,(-\,6)$, where x and y are real numbers, then find the values of x and y.

Solution:

Given,

$4x+\ i(3x-y)=3+i\,(-\,6)$

$\Rightarrow\ 4x=3,\ 3x-y=-6$

$\Rightarrow\ x=\frac{3}{4},\ \ y=3x+6$

$\Rightarrow\ x=\frac{3}{4},\ \ y=3.\frac{3}{4}+6$

$\Rightarrow\ x=\frac{3}{4},\ \ y=\frac{9}{4}+6$

$\Rightarrow\ x=\frac{3}{4},\ \ y=\frac{33}{4}$