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Boolean Algebra.

In Boolean Algebra, logical statements are representated by symbols and such symbols are connected with logical connections to represent complex statements. The truth of logical statements is depicted by logical constants. true and false or by 1 and 0, respectively.

X	Y	Z = X.Y
0	0	0
0	1	0
1	0	0
1	1	1

X	Y	Z	Y+Z	XY	XZ	X(Y+Z)	XY+XZ
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	1	1	1
1	1	0	1	1	0	1	1
1	1	1	1	1	1	1	1

Logic gates.

Boolean Algebra operations are implemented with the help of logic gates. In other words, one can say that, logic gates are electronic circuits which operate on one or more input signals and produce an output signal.

a) AND Gate.

An "AND Gate" consists of two or more than two input signals and produces one output signal. The output signal is only true (=1) if all inputs are true.

X	Y	Z	Output.
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

b) OR Gate.

The 'OR' operator returns true, if either ~~are both~~ one or all of the inputs are true.

X	Y	Z	Output
0	0	0	0
0	0	1	1
0	1	0	1
1	0	0	1
0	1	1	1
~~0~~ 1	0	1	1
1	1	0	1
1	1	1	1

c) NOR Gate

The "NOT" operator is ~~one~~ a unary one represented by ('). It inverts the truth of the operand on which it operates. i.e., if A = true, then A' = false.

X	Y	Z	Output
0	0	0	1
0	0	1	0
0	1	0	0
1	0	0	0
0	1	1	0
1	0	1	0
1	1	0	0
1	1	1	0

X	X'
0	1
1	0

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a. NAND Gate.

The "NAND" gate is logically equivalent to an "AND Gate" with its output inverted. The output of a "NAND" Gate will be true if any or all inputs are false. On the other side output will be false if all inputs are true.

X	Y	Z	Output
0	0	0	1
0	0	1	~~0~~ 1
0	1	0	1
1	0	0	1
1	1	0	1
1	0	1	1
0	1	1	1
1	1	1	0

b. NOR Gate.

A "NOR" Gate is logically equivalent to an "OR" Gate with its output inverted. If give output (=0) if any or all inputs are (=1) and a true value (=1) if all the inputs a zero.

X	Y	Z	Output
0	0	0	1
0	0	1	0
0	1	0	0
1	0	0	0
1	0	1	0
0	1	1	0
1	1	0	0
1	1	1	0

c. XOR Gate (Exclusive OR Gate).

The output of an "Exclusive OR Gate" (also called "XOR" Gate) is true, if either, but not both of the ~~outputs~~ inputs are true. It may also be true if all the inputs are true.

X	Y	Z	Output
0	0	0	0
0	0	1	1
1	0	0	1
0	1	0	1
1	1	0	0
0	1	1	0
1	0	1	0
1	1	1	1

Basic Laws of Boolean Algebra:

1. Properties of 0. $0+X=X$ $\quad 0.X=0.$
2. Properties of 1 $1+X=1$ $\quad 1.X=X.$
3. Indempotence Law $X+X=X$, $\quad X.X=X.$
4. Involution. $\overline{\overline{X}}=X$,
5. complementary Law $X+\overline{X}=1$, $\quad X.\overline{X}=0.$
6. Commutative Law $X+Y=Y+X$ $\quad X.Y=Y.X$
7. Associative Law $X+(Y+Z)=(X+Y)+Z$ $\quad X(YZ)=(XY)Z$
8. Distributive Law $X(Y+Z)=XY+XZ$, $\quad X+YZ=(X+Y)(X+Z).$
9. Absorption Law $X+XY=X$, $\quad X(X+Y)=X.$
10. Other $X+\overline{X}Y=X+Y.$
   (3rd distributive Law)

1. de Morgan's first Law.

It states that,

$$\overline{X+Y}=\overline{X}.\overline{Y}.$$

let,

$P=X+Y.$

according to complementary laws.

$$P+\overline{P}=1 \quad \& \quad P.\overline{P}=0.$$

if, $\overline{X+Y}=\overline{X}.\overline{Y}$, then.

$(X+Y)+\overline{X}\,\overline{Y}$ must be equal to 1.

$(X+Y).\overline{X}\,\overline{Y}$ must be equal to 0.

$$\begin{aligned}(X+Y)+\overline{X}\,\overline{Y} &= ((X+Y)+\overline{X})((X+Y)+\overline{Y}).\\ &= (X+\overline{X}+Y).(X+Y+\overline{Y})\\ &= (1+Y).(X+1)\\ &= 1.1\\ &= 1.\end{aligned}$$

also,

$$\begin{aligned}(X+Y).\overline{X}\,\overline{Y} = \overline{X}\,\overline{Y}.(X+Y) &= \overline{X}\,\overline{Y}X+\overline{X}\,\overline{Y}Y\\ &= X\overline{X}\,\overline{Y}+\overline{X}Y\overline{Y}\\ &= 0.\overline{Y}+\overline{X}.0\\ &= 0+0=0.\end{aligned}$$

Thus,

$$\overline{X+Y} = \bar{X}\,\bar{Y}.$$

• de Morgan's second law.

$$\overline{X.Y} = \bar{X} + \bar{Y}.$$

If, XY's complement is $\bar{X} + \bar{Y}$ then it must be true that,

(a) $XY + (\bar{X} + \bar{Y}) = 1$ and (b) $XY(\bar{X} + \bar{Y}) = 0$.

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$XY + (\bar{X} + \bar{Y})$

$\Rightarrow (\bar{X} + \bar{Y}) + XY$

$\Rightarrow (\bar{X} + \bar{Y} + X).(\bar{X}\bar{Y} + Y)$

$\Rightarrow (\bar{X} + X + \bar{Y})(\bar{X} + Y + \bar{Y})$

$\Rightarrow$ $\cancel{Ex}$ $(1 + \bar{Y})(\bar{X} + 1)$

$\Rightarrow 1.1$

$\Rightarrow 1.$

II

$XY.(\bar{X} + \bar{Y})$

$= XY\bar{X} + XY\bar{Y}$

$= X\bar{X}Y + XY\bar{Y}$

$= 0.Y + X.0$

$= 0 + 0$

$=$ RHS.

$\therefore \overline{XY} = \bar{X} + \bar{Y}$. Proved