EXERCISE – 5.1

Express each of the following in the form of $a + ib$

1. Find the value of $i^9 + i^{19}$.

[Ex.- 5.1 Q.2]

Solution:

$i^9 = i^{8+1} = i^8 . i = (i^4)^2 . i = 1^2 . i = 1 . i = i$

$i^{19} = i^{16+3} = i^{16} . i^3 = (i^4)^4 . (-i) = 1^4 . (-i) = 1 . (-i) = -i$

$\therefore i^9 + i^{19} = -1 + 1 = 0 = 0 + i0$

2. Find the value of i^{-39}.

Solution:

$$i^{-39} = \frac{1}{i^{39}} \times \frac{i}{i} = \frac{i}{i^{40}} = \frac{i}{(i^4)^{10}} = \frac{i}{(1)^{10}} = \frac{i}{1} = i = 0 + i.0$$

3. Find the value of $(1 - i)^4$.

Solution:

$(1 - i)^4 = [(1 - i)^2]^2$

$= [1 - 2i + i^2]^2$

$= [1 - 2i - 1]^2$

$= (-2i)^2$

$= 4\, i^2$

$= 4.(-1)$

$= -\,4$

$= -4 + 0.i$

4. Simplify:

$$\left[\left(\frac{1}{3} + i\frac{7}{3}\right) + \left(4 + i\frac{1}{3}\right)\right] - \left(-\frac{4}{3} + i\right)$$

Solution:

$$\left[\left(\frac{1}{3} + i\frac{7}{3}\right) + \left(4 + i\frac{1}{3}\right)\right] - \left(-\frac{4}{3} + i\right)$$

$$= \left[\frac{1}{3} + i\frac{7}{3} + 4 + i\frac{1}{3}\right] + \frac{4}{3} - i$$

$$= \frac{1}{3} + i\frac{7}{3} + 4 + i\frac{1}{3} + \frac{4}{3} - i$$

$$= \left(\frac{1}{3} + 4 + \frac{4}{3}\right) + i\left(\frac{7}{3} + \frac{1}{3} - 1\right)$$

$$= \left(\frac{1 + 12 + 4}{3}\right) + i\left(\frac{7 + 1 - 3}{3}\right)$$

$$= \frac{17}{3} + i\frac{5}{3}$$

5. Simplify:

$$\frac{(3 + i\sqrt{5})(3 - i\sqrt{5})}{(\sqrt{3} + i\sqrt{2}) - (\sqrt{3} - i\sqrt{2})}$$

Solution:

$$\frac{(3 + i\sqrt{5})(3 - i\sqrt{5})}{(\sqrt{3} + i\sqrt{2}) - (\sqrt{3} - i\sqrt{2})}$$

$$= \frac{3^2 - i^2(\sqrt{5})^2}{\sqrt{3} + i\sqrt{2} - \sqrt{3} + i\sqrt{2}}$$

$$= \frac{9 - (-1).5}{i\sqrt{2} + i\sqrt{2}}$$

$$= \frac{9 + 5}{i\,2\sqrt{2}} = \frac{14}{2\sqrt{2}i}$$

$$= \frac{7}{\sqrt{2}i} \times \frac{\sqrt{2}i}{\sqrt{2}\,i}$$

$$= \frac{7\sqrt{2}i}{2\,i^2}$$

$$= \frac{7\sqrt{2}i}{2\,(-1)}$$

$$= 0 - i\,\frac{7\sqrt{2}}{2} \qquad \text{Ans.}$$

6. Simplify:

$$\left(-2 - \tfrac{1}{3}i\right)^3$$

Solution:

$$\left(-2-\frac{1}{3}i\right)^3$$

$$=-\left(2+\frac{1}{3}i\right)^3$$

$$=-\left[2^3+3.\,2^2.\frac{1}{3}i+3.2.\left(\frac{1}{3}i\right)^2+\left(\frac{1}{3}i\right)^3\right]$$

$$=-\left[8+3.4.\frac{1}{3}i+6.\frac{1}{9}i^2+\frac{1}{27}i^3\right] \qquad [\because i^2=-1, i^3=-i\,]$$

$$=-\left[8+4\,i-\frac{2}{3}-\frac{1}{27}i\right]$$

$$=-\left[\left(8-\frac{2}{3}\right)+i\left(4-\frac{1}{27}\right)\right]$$

$$=-\left[\frac{24-2}{3}+\frac{108-1}{27}i\right]$$

$$=-\left[\frac{22}{3}+\frac{107}{27}i\right]$$

$$=-\left(\frac{22}{3}\right)+\left(-\frac{107}{27}\right)i \quad \text{Ans}$$

7. Find the multiplicative inverse of $4-3i$.

Solution:

The multiplicative inverse of $z=4-3i$ is

$$z^{-1}=\frac{1}{z}=\frac{1}{4-3i}$$

$$=\frac{1}{4-3i}\times\frac{4+3i}{4+3i}$$

$$=\frac{4+3i}{4^2-(3i)^2}$$

$$=\frac{4+3i}{16-9i^2}$$

$$=\frac{4+3i}{16-9\,(-1)}$$

$$=\frac{4+3i}{16+9}$$

$$=\frac{4+3i}{25}$$

$$=\frac{4}{25}+i\,\frac{3}{25} \quad \text{Ans.}$$