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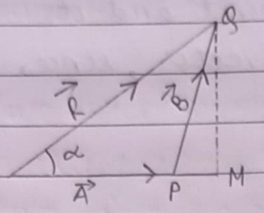
Date :

15th - Sept 2021.



direction

$\vec{R}$ makes an angle α with the direction of $\vec{A}$.

In ΔOMQ,

$$\tan\alpha = \frac{MQ}{OM}.$$

$$\Rightarrow \tan\alpha = \frac{B\sin\theta}{A+B\cos\theta}$$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{B\sin\theta}{A+B\cos\theta}\right).$$

Special cases.

for, $\theta = 0°$

$$R = \sqrt{A^2+B^2+2AB\cos\theta}$$
$$\Rightarrow R = \sqrt{A^2+B^2+2AB\cos 0°}$$
$$\Rightarrow R = \sqrt{A^2+B^2+2AB}$$
$$\Rightarrow R = \sqrt{(A+B)^2}$$
$$\Rightarrow R = A+B.$$

$$\alpha = \tan^{-1}\left(\frac{B\sin\theta}{A+B\cos\theta}\right)$$
$$= \tan^{-1}(0)$$
$$= 0.$$

for, $\theta = 90°$

$$R = \sqrt{A^2+B^2+2AB\cos 90°}$$
$$\Rightarrow R = \sqrt{A^2+B^2}$$

$$\alpha = \tan^{-1}\left(\frac{B\sin\theta}{A+B\cos\theta}\right)$$

$$\alpha = \tan^{-1}\left(\frac{B}{A}\right)$$

for, $\theta = 180°$

$$R = \sqrt{A^2+B^2+2AB\cos 180°}$$
$$\Rightarrow R = \sqrt{A^2+B^2-2AB}$$

$\Rightarrow R = \sqrt{(A-B)^2}$

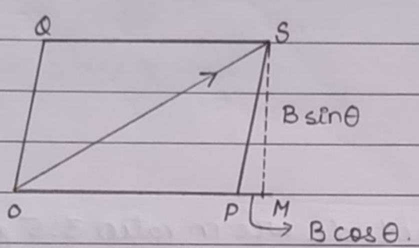
$\Rightarrow R = A - B.$

$$\alpha = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)$$

$$\Rightarrow \alpha = \tan^{-1}(0)$$

$$= 0.$$

- Parallelogram Law of Vector addition:

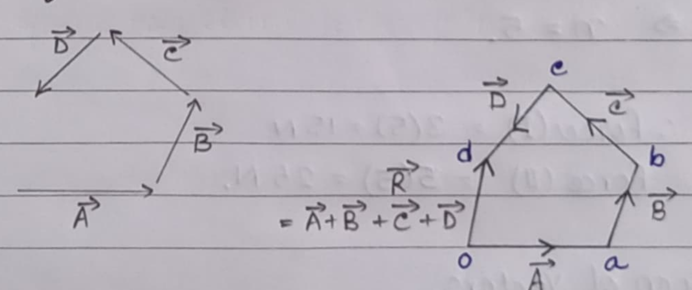


$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}.$$

$$\alpha = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right).$$

If two vectors are represented in magnitude and direction, by two adjacent sides of a parallelogram, drawn from a point, then their resultant is represented in magnitude and direction, by the diagonal for the parallelogram, drawn from the ~~diagram~~ same point.

- Polygon Law of Vector addition.



If more than two vectors can be representated in magnitude and direction, by the sides of its polygon, taken in same order, then the closing side of the polygon represent the resultane of this vectors, in magnitude and direction, in opposite order.

Q. Two forces of equal magnitude of 3N act on a body making angle θ. If the resultant forces of this two forces is also 3N, then find the value of θ.

$$3 = \sqrt{3^2+3^2+2.3.3\cos\theta}$$
$$\Rightarrow (3)^2 = 9+9+2.9\cos\theta$$
$$\Rightarrow 9-18 = 18\cos\theta$$
$$\Rightarrow 18\cos\theta = -9$$
$$\Rightarrow \cos\theta = \frac{-9}{18} = \frac{-1}{2}$$
$$\Rightarrow \theta = \cos^{-1}\left(\frac{-1}{2}\right).$$

Q. Two forces, whose magnitude are in ratio 3:5 give a resultant of 35 N. If the angle of inclination is 60°, calculate the magnitude of each force.

Let the two forces, whose magnitude are in ratio 3:5 be $3n$, $5n$.

∴ A/Q,

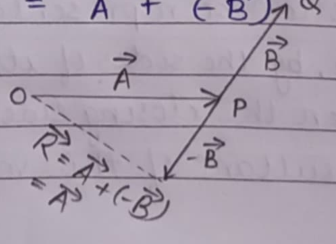
$$\vec{R} = \sqrt{A^2+B^2+2AB\cos\theta}$$
$$\Rightarrow 35 = \sqrt{(3n)^2+(5n)^2+2.3n5n.\cos 60^\circ}$$
$$\Rightarrow 35 = \sqrt{9n^2+25n^2+\cancel{2}.15n^2\times\frac{1}{\cancel{2}}}$$
$$\Rightarrow 35 = \sqrt{34n^2+15n^2}$$
$$\Rightarrow 35 = \sqrt{49n^2}$$
$$\Rightarrow 7n = 35$$
$$\Rightarrow n = 5.$$

∴ Force (I) = 3(5) = 15 N

Force (II) = 5(5) = 25 N.

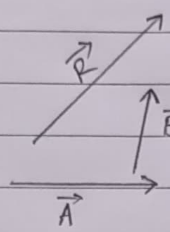
- Substraction of vectors.

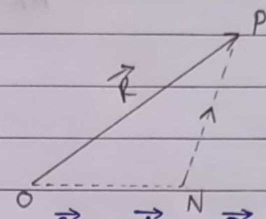
$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$



Resolution of vectors.

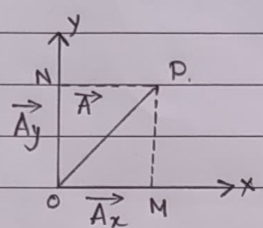
The process of splitting a vector into two or more vectors is called resolution of a vector. The resolved vectors are called compound of a given vector.





$\vec{R} = n\vec{A} + m\vec{B}$

Rectangular resolution:-



$$\vec{A} = \vec{A_x} + \vec{A_y}$$

$$\boxed{\vec{A} = A_x \hat{i} + A_y \hat{j}} \quad \text{———} \quad (I)$$

In ΔOMP,

$$\cos\theta = \frac{OM}{OP}$$
$$\Rightarrow OM = OP\cos\theta.$$
$$\Rightarrow \boxed{A_x = A\cos\theta} \quad \text{———} \quad (II)$$

&

$$\sin\theta = \frac{PM}{OP}$$
$$\Rightarrow PM = OP\sin\theta$$
$$\Rightarrow \boxed{A_y = A\sin\theta.} \quad \text{———} \quad \cancel{(3)} \ (III).$$

squaring and adding eq. (II) & (III),

$$\hookrightarrow A_x^2 + A_y^2 = A^2(\sin^2\theta + \cos^2\theta$$
$$\Rightarrow A_x^2 + A_y^2 = A^2(1)$$
$$\Rightarrow A^2 = A_x^2 + A_y^2$$
$$\Rightarrow \boxed{A = \sqrt{A_x^2 + A_y^2}} \quad \text{———} \quad (IV).$$

dividing eq. (3) by (2),

$$\frac{A_y}{A_x} = \tan\theta.$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \quad \text{—— (5).}$$