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PGT (MATHS)

CARTESIAN PRODUCT

1. ORDERED PAIR:

Let A and B be two sets.
If $a \in \mathrm{A}, b \in \mathrm{B}$ then (a, b) is called an ordered pair whose first component is *a* and the second component is *b*.

In the ordered pair (a, b), the order in which the elements a and b appear in the bracket is important

Thus,

(i) $(a, b) \neq (b, a)$, if $a \neq b$

(ii) $(a, b) = (c, d)$, if and only if $a = c$ and $b = d$

Example-1

If $(x + 1,\ y - 2) = (3, 1))$, find the values of x and y.

Solution:

Given,

$(x + 1, y - 2) = (3, 1)$

$\Leftrightarrow x + 1 = 3$ and $y - 2 = 1$

$\Leftrightarrow x = 2$ and $y = 3$ $\quad$ Ans.

2. CARTESIAN PRODUCT OF TWO SETS:

The set of all ordered pairs (a, b) of elements $a \in \mathrm{A}, b \in \mathrm{B}$ is called the Cartesian Product of sets A and B and is denoted by $\mathrm{A} \times \mathrm{B}$.

Thus, $\mathrm{A} \times \mathrm{B} = \{(a, b): a \in \mathrm{A}, b \in \mathrm{B}\}$

Example:

If $\mathrm{A} = \{1, 2\}$ and $\mathrm{B} = \{1, 2, 3\}$ then

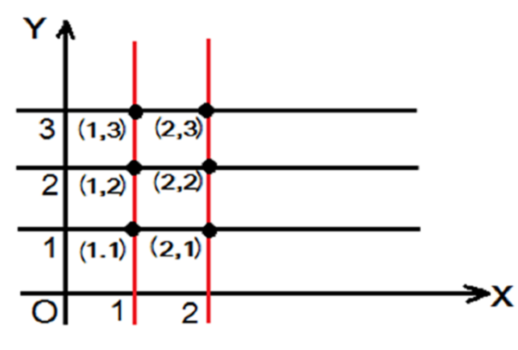
$\mathrm{A} \times \mathrm{B} = \{1, 2\} \times \{1, 2, 3\}$

$\mathrm{A} \times \mathrm{B} = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\}$

And

$\mathrm{B} \times \mathrm{A} = \{1, 2, 3\} \times \{1, 2\}$

$\mathrm{B} \times \mathrm{A} = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 1), (3, 2)\}$



Graphical Representation of A X B

REMARKS:

(i) $\mathrm{A} \times \mathrm{B} \neq \mathrm{B} \times \mathrm{A}$, If $\mathrm{A} \neq \mathrm{B}$

(ii) $\mathrm{A} \times \mathrm{B} = \emptyset$, if $\mathrm{A} = \emptyset$ or $\mathrm{B} = \emptyset$

(iii) $n(\mathrm{A} \times \mathrm{B}) = n(\mathrm{A}) \times n(\mathrm{B})$

NOTE:

$\mathrm{A} \times \mathrm{B} \times \mathrm{C} = \{(a, b, c): a \in \mathrm{A}, b \in \mathrm{B}, \mathrm{c} \in \mathrm{C}\}$

Here, (a, b, c) is called ordered triplet.

Example-2:

If $A = \{1, 2\}$, find $A \times A \times A$

Answer:

$A \times A = \{1, 2\} \times \{1, 2\}$

$= \{(1, 1), (1, 2), , (2, 1), (2, 2)\}$

$\therefore A \times A \times A = A \times (A \times A)$

$= \{1, 2\} \times \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

$= \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2)\}$

Note:

$n(A \times B \times C) = n(A) \times n(B) \times n(C)$

Example-3

Let $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$. Find

(i) $A \times (B \cap C)$ (ii) $(A \times B) \cap (A \times C)$

(iii) $A \times (B \cup C)$ (iv) $(A \times B) \cup (A \times C)$

Solution:

(i) $(B \cap C) = \{3, 4\} \cap \{4, 5, 6\} = \{4\}$

$\therefore A \times (B \cap C) = \{1, 2, 3\} \times \{4\} = \{(1, 4), (2, 4), (3, 4)\}$.

(ii) $(A \times B) = \{1, 2, 3\} \times \{3, 4\} = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$

$(A \times C) = \{1, 2, 3\} \times \{4, 5, 6\} = \{(1,4), (1,5), (1,6), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6)\}$

$\therefore (A \times B) \cap (A \times C) = \{(1, 4), (2, 4), (3, 4)\}$.

(iii) $(B \cup C) = \{3, 4\} \cup \{4, 5, 6\} = \{3, 4, 5, 6\}$

$\therefore A \times (B \cup C)$

$= \{1, 2, 3\} \times \{3, 4, 5, 6\}$

$= \{(1,3), (1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6)\}$.

(iv)

$(A \times B) \cup (A \times C)$ [Using $(A \times B)$ and $(A \times C)$ from (ii)]

$= \{(1,3), (1,4), (2,3), (2,4), (3,3), (3,4)\} \cup \{(1,4), (1,5), (1,6), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6)\}$

$= \{(1,3), (1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6)\}$.

Example - 4

Let A and B be two sets such that $(A) = 3$ and $n(B) = 2$. If $(x, 1), (y, 2), (z, 1)$ are in $A \times B$, find A and B, where x, y and z are distinct elements.

Solution:

Since, $(x, 1), (y, 2), (z, 1) \in (\mathrm{A} \times \mathrm{B})$

$\therefore \mathrm{A} = \{a: (a, b) \in (\mathrm{A} \times \mathrm{B})\} = \{x, y, z\}$

and $\mathrm{B} = \{b: (a, b) \in (\mathrm{A} \times \mathrm{B})\} = \{1, 2\}$

Example - 5

The Cartesian product $\mathrm{A} \times \mathrm{A}$ has 9 elements among which are found $(-1, 0)$ and $(0, 1)$. Find the set A and the remaining elements of $\mathrm{A} \times \mathrm{A}$.

Solution:

Two elements out of 9 elements of $\mathrm{A} \times \mathrm{A}$ are $(-1, 0)$ and $(0, 1)$.

Here, $a \in \mathrm{A},\ b \in \mathrm{A} \quad \forall\ (a, b)\ \in (\mathrm{A} \times \mathrm{A})$

$\therefore \mathrm{A} = \{-1, 0, 1\}$

Now,

$\mathrm{A} \times \mathrm{A} = \{-1, 0, 1\} \times \{-1, 0, 1\}$

$\qquad = \{(-1, -1), (-1, 0), (-1, 1), (0, -1), (0, 0), (0, 1)(1, -1), (1, 0), (1, 1)\}$

Thus, the remaining elements of $\mathrm{A} \times \mathrm{A}$ are

$(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0)$ and $(1, 1)$