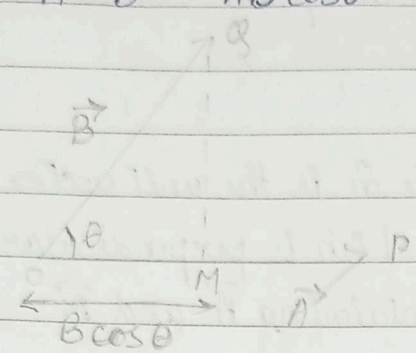
17/09/21

**Dot product:** The product of two vectors $\vec{A}$ and $\vec{B}$ is the product of magnitude of two vectors and the cosine of the angle between them.

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$




$QM \perp OP$

$$\vec{A} \cdot \vec{B} = A(B\cos\theta)$$
$$= A \times OM$$
$$= A \times (\text{Projection of } \vec{B} \text{ along } \vec{A})$$

∴ Dot product of two vectors is a scaler product.

**Properties of Dot product:**

1) $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
2) $\vec{A}(\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$
3) If $\theta = 0$,
   $\vec{A} \cdot \vec{B} = AB$

(a) If $\theta = 90^\circ$
   $\vec{A} \cdot \vec{B} = 0$.

(b) If $\theta = 180^\circ$
   $\vec{A} \cdot \vec{B} = -\vec{A}\vec{B}$

4) $\vec{A} \cdot \vec{A} = A^2$
5) $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$
6) $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{i}\hat{k} = 0$.

7) $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$
$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

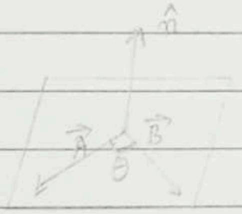
$$\therefore \vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

= Vector Product / Cross Product of two vectors:-

The vector product of two vectors is defined as a vector having a magnitude equal to the product of the magnitudes of the two vectors and the sine of the angle between them, and having the direction perpendicular to the plane containing the two vectors.

Thus,

$\vec{A}\times\vec{B} = AB\sin\theta\,(\hat{n})$, where, $\hat{n}$ is the unit vector along a direction which is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.




= Right Hand Thumb Rule:- If the fingers of the right hand are curled in such a way that the point along the direction of rotation from $\vec{A}$ to $\vec{B}$ through a small angle them, the thumb points the direction of cross product of the two vectors.

= Properties of Cross Product:

1) Product of two vectors is a vector quantity.
2) $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$

however,
$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$

3) $\vec{A}\times\vec{B} = AB\sin\theta$

(i) $\theta = 0°$ or $180°$
$|\vec{A}\times\vec{B}| = 0.$

(ii) $\theta = 90°$
$|\vec{A}\times\vec{B}| = AB.$

Date 17/09/21
Page

4) $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0.$

5) $\hat{i} \times \hat{j} = \hat{k}$ $\quad\quad \hat{j} \times \hat{i} = -\hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$ $\quad\quad \hat{k} \times \hat{j} = -\hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$ $\quad\quad \hat{i} \times \hat{k} = -\hat{j}$

6) If $\vec{A} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

and $\vec{B} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$

Then,

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_y b_z - a_z b_y)\hat{i} - (a_x b_z - a_z b_x)\hat{j} + (a_x b_y - a_y b_x)\hat{k}.$$