06th Sept 2021.

Find $\frac{dy}{dx}$.

1. $\sin x^2$

$\Rightarrow \frac{dy}{dx}(\sin x^2) \times \frac{dy}{dx}(x^2)$

$\Rightarrow \cos x^2 \times 2x$

$\Rightarrow 2x \cos x^2$.

2. $y = \sin 2x$

$\Rightarrow \frac{dy}{dx} = \cos 2x \times \frac{dy}{dx}(2x)$

$\Rightarrow \cos 2x \times 2$

$\Rightarrow 2\cos 2x$

3. $y = \cos x^2$

$\Rightarrow \frac{dy}{dx} = -\sin x^2 \times \frac{dy}{dx}(x^2)$

$\Rightarrow -\sin x^2 \times 2x$

$\Rightarrow -2x \sin x^2$.

4. $y = (x^2-4)^8$

$\Rightarrow \frac{dy}{dx} = 8(x^2-4)^{8-1} \times \frac{d}{dx}(x^2-4)$

$= 8(x^2-4)^7 \times 2x$

$= 16x(x^2-4)^7$.

5. Find $\frac{d^2y}{dx^2}$

$y = 4x^5$

$\Rightarrow \frac{dy}{dx} = 4 \times 5x^4$

$= 20x^4$

Now,

differentiation with respect to x,

i.e.,

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(20x^4)$

$\Rightarrow \frac{d^2y}{dx^2} = 20 \times 4x^3$

$= 80x^3$

6. find $\frac{d^2y}{dx^2}$ — $y = 6x^{-2} + 5x^3$

$\frac{dy}{dx} = 6(-2)x^{-2-1} + 5(3)x^{3-1}$

$= -12x^{-3} + 15x^2$

$\hookrightarrow \frac{d^2y}{dx^2} = -12(-3)x^{-3-1} + 15(2)x^{2-1}$

$= +36x^{-4} + 30x^{2-1}$

$= 36x^{-4} + 30x^1$

$= 36x^{-4} + 30x.$

Q. The position of a particle with time t is,

$x = 5t^3 + 4t^2 - 3t.$

Find the velocity and accelaration of the particle at $t = 2s$.

$x = 5t^3 + 4t^2 - 3t.$

$\therefore$ velocity $= v = \frac{dx}{dt}$

$\Rightarrow v = \frac{dx}{dt}(5t^3 + 4t^2 - 3t)$

$= 15t^2 + 8t^{2-1} - 3t^{1-1}$

$= 15t^2 + 8t - 3.$

puting $t = 2$,

$v = 60 + 16 - 3$

$= 73$ m/s.

Now,

Accelaration $= a = \frac{dv}{dt}$

$\Rightarrow a = \frac{d}{dt}(15t^2 + 8t - 3)$

$= 30t + 8 - 0$

$= 30t + 8$

for, $t = 2$,

accelaration $= 60 + 8 = 68$ m/s^2.

Integration

$y = f(x)$

↳ $\frac{dy}{dx} = f'(x)$.

$\Rightarrow dy = f'(x) \times dx$

$\Rightarrow y = \int f'(x)\, dx$.

- **Indefinite integral.**

(i) $\int dx = x + c$.

(ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

(iii) $\int \frac{1}{x} dx = \log_e x + c$

(iv) $\int e^x dx = e^x + c$

(v) $\int \sin x\, dx = -\cos x + C$

(vi) $\int \cos x\, dx = \sin x + c$

- **Definite Integration (intigral).**

$$\int_a^b f'(x)\, dx = [F(x)]_a^b$$
$$= F(b) - F(a)$$

1. $\int 4x^2 dx$

$= 4 \int x^2 dx$

$= 4 \times \frac{x^{2+1}}{2+1}$

$= \frac{4}{3}x^3 + C$. Ans.

2. $\int_1^2 (3x-1)\, dx$

$= \int_1^2 3x - \int_1^2 dx$

$= 3\left[\frac{x^2}{2}\right]_1^2 - [x]_1^2$

$= \frac{3}{2}(2^2 - 1^2) - (2-1)$

$= \frac{3}{2}(4-1) - 1$

$= \frac{3}{2}(3) - 1$

$= \frac{9}{2} - 1$

$= \frac{9-2}{2}$

$= \frac{7}{2}$ Ans.

3. $\int_2^3 3x^2\,dx$

$= 3\int_2^3 x^2 \times \int_2^3 dx$

$= \cancel{3} \times \left[\frac{x^3}{\cancel{3}}\right]_2^3 \times [x]_2^3$

$= [x^3]_2^3 \times [x]_2^3$

$= (3^3 - 2^3) \times [3-2)$

$= (27-8) \times 1$

$= 19 \times 1$

$= 19$. Ans.