NAKUL PANDIT
PGT (MATHS)

OPERATIONS ON SETS

1. UNION OF SETS

Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and all the elements of B, the common elements being taken only once.
The union of A and B is symbolically denoted by AUB and is read as "A union B"

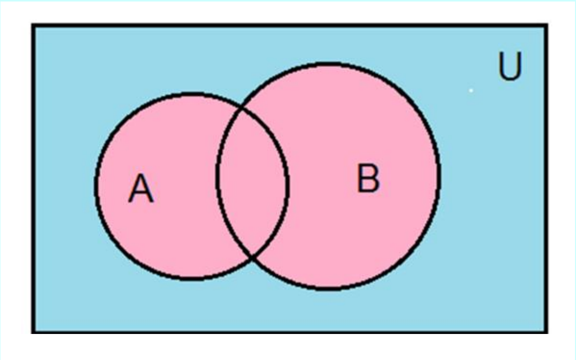
Thus,

$$\text{AUB} = \{x : x \in \text{A or } x \in \text{B}\}$$

Note:

1. $x \in (\text{AUB}) \Rightarrow x \in \text{A or } x \in \text{B}$
2. $x \notin (\text{AUB}) \Rightarrow x \notin \text{A and } x \notin \text{B}$

Venn Diagram of AUB



The shaded region in pink coloured represents AUB

Example-1

Let $\text{A} = \{2, 3, 4, 6, 8\}$ and $\text{B} = \{1, 2, 3, 4, 5\}$

Then, $\text{A} \cup \text{B} = \{1, 2, 3, 4, 5, 6, 8\}$

Example-2

Let $\text{A} = \{1, 2, 3\}$ and $\text{B} = \{1, 2, 3, 4, 5\}$

It is clear that $\text{A} \subset \text{B}$

Then, $\text{A} \cup \text{B} = \{1, 2, 3, 4, 5\} = \text{B}$

Conclusion:

If $\text{A} \subset \text{B}$, then $\text{A} \cup \text{B} = \text{B} = (\text{superset})$

2. PROPERTIES INVOLVING THE OPERATION UNION

(i) **Commutative Law**:

$$\text{A} \cup \text{B} = \text{B} \cup \text{A}$$

(ii) **Associative Law**:

$(A \cup B) \cup C = A \cup (B \cup C)$

(iii) **Idempotent Law**:

$A \cup A = A$

(iv) **Law of $\emptyset$:**

$A \cup \emptyset = A$

(v) **Law of U:**

$A \cup U = U$

(vi) $A \subset (A \cup B)$ and $B \subset (A \cup B)$

(vi) If $A \subset B$, then $A \cup B = B =$ superset

3. INTERSECTION OF SETS

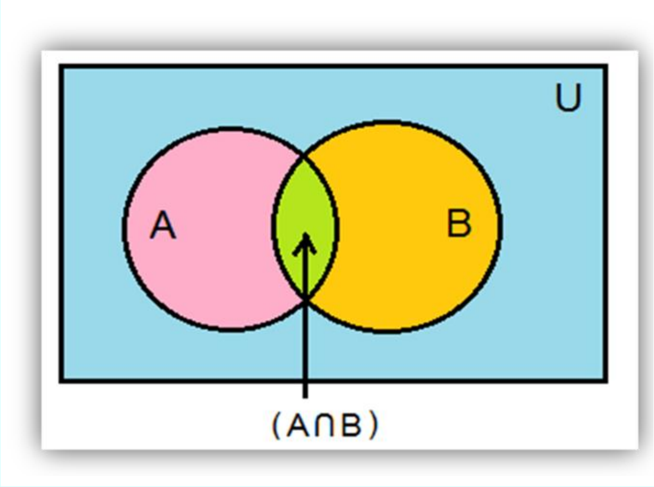
Let A and B be any two sets. The intersection of A and B is the set which consists of all common elements of the sets A and B.
The intersection of A and B is symbolically denoted by A ∩ B and is read as "A intersection B"
Thus,

$$\boxed{A \cap B = \{x : x \in A \text{ and } x \in B\}}$$

Note:

1. $x \in (A \cap B) \Rightarrow x \in A$ and $x \in B$
2. $x \notin (A \cap B) \Rightarrow x \notin A$ or $x \notin B$

Venn Diagram of A∩B



The shaded region in green coloured represents $A \cap B$.

Example-1

Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6, 8\}$

Then, $A \cap B = \{ 2, 4\}$

Example-2

Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$

It is clear that $A \subset B$

Then, $A \cap B = \{1, 2, 3\} = A$

Conclusion:

If $A \subset B$, then $A \cap B = A =$ subset

2. PROPERTIES INVOLVING THE OPERATION INTERSECTION

(i) **Commutative Law**:

$$A \cap B = B \cap A$$

(ii) **Associative Law**:

$$(A \cap B) \cap C = A \cap (B \cap C)$$

(iiI) **Distributive Law**:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

(iii) **Idempotent Law**:

$$A \cap A = A$$

(iv) **Law of $\emptyset$:**

$$A \cap \emptyset = \emptyset$$

(v) **Law of U:**

$$A \cap U = A$$

(vi) $(A \cap B) \subset A$ and $(A \cap B) \subset B$

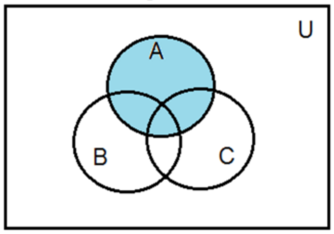
(vi) If $A \subset B$, then $A \cap B = A =$ subset

Q. Using Venn Diagrams, show that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

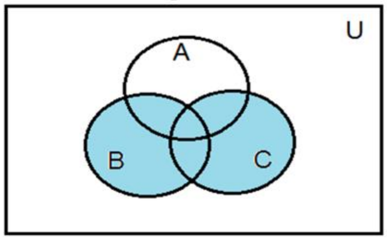
Proof:

Diagram-1



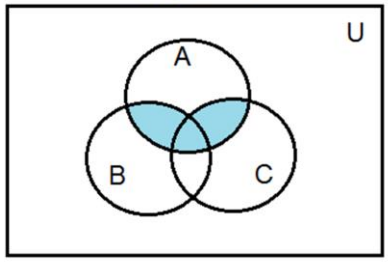
The shaded region is A

Diagram- 2



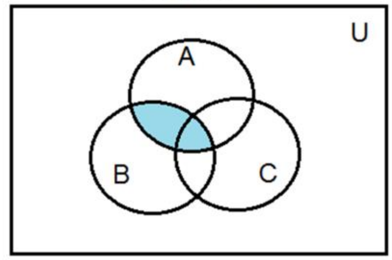
The shaded region is B U C

Diagram-3



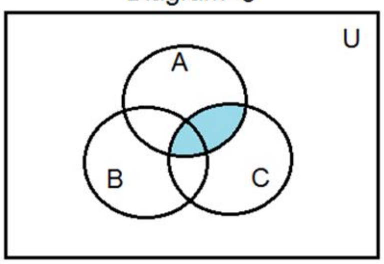
The shaded region is $A \cap (B \cup C)$

Diagram- 4



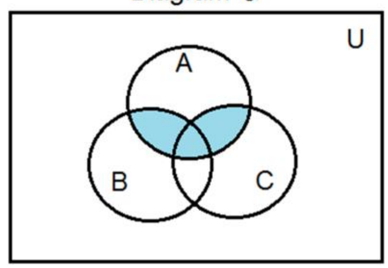
The shaded region is $A \cap B$

Diagram- 5



The shaded region is $A \cap C$

Diagram-6

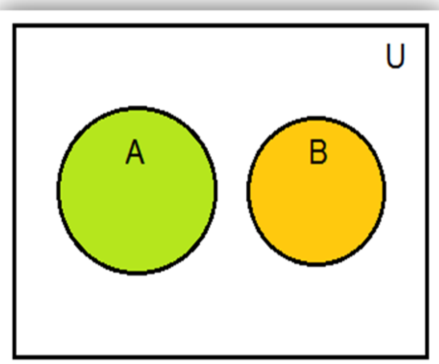


The shaded region is $(A \cap B) \cup (A \cap C)$

Since, venn diagrams numbers 3 and 6 are exactly the same, so it is verified that

$$\mathrm{A} \cap (\mathrm{B} \cup \mathrm{C}) = (\mathrm{A} \cap \mathrm{B}) \cup (\mathrm{A} \cap \mathrm{C})$$

4. DISJOINT SETS



A and B are disjoint sets

Two sets A and B are said to be disjoint if $\mathrm{A} \cap \mathrm{B} = \emptyset$.

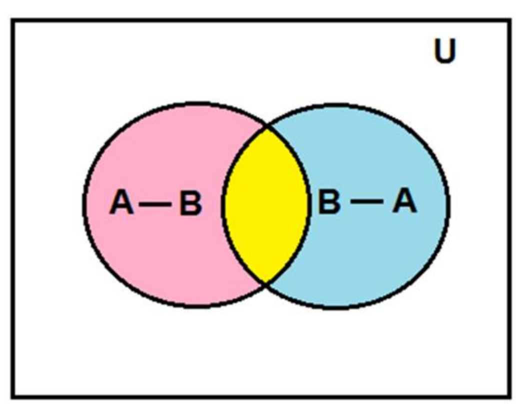
Example:

Let $\mathrm{A} = \{a, e, i, o, u\}$ and $\mathrm{B} = \{c, d, f, g, h\}$

Since, $\mathrm{A} \cap \mathrm{B} = \emptyset$

So, A and B are disjoint sets,

5. DIFFERENCE OF TWO SETS



The difference of two sets A – B is the set of elements which belong to A but not to B.

Example-1

Let $\mathrm{A} = \{a, b, c, d\}$ and $\mathrm{B} = \{a, b, c, g, h\}$, then

$\mathrm{A} - \mathrm{B} = \{a, b, c, d\} - \{a, b, c, g, h\} = \{d\}$ and

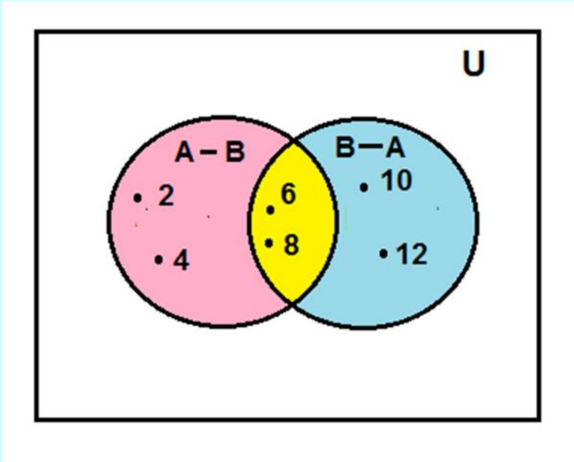
$\mathrm{B} - \mathrm{A} = \{a, b, c, g, h\} - \{a, b, c, d\} = \{g, h\}$

Example -2

Let $\mathrm{A} = \{2, 4, 6, 8\}$ and $\mathrm{B} = \{6, 8, 10, 12\}$, then

$\mathrm{A} - \mathrm{B} = \{2, 4, 6, 8\} - \{6, 8, 10, 12\} = \{2, 4\}$ and

$\mathrm{B} - \mathrm{A} = \{6, 8, 10, 12\} - \{2, 4, 6, 8\} = \{10, 12\}$



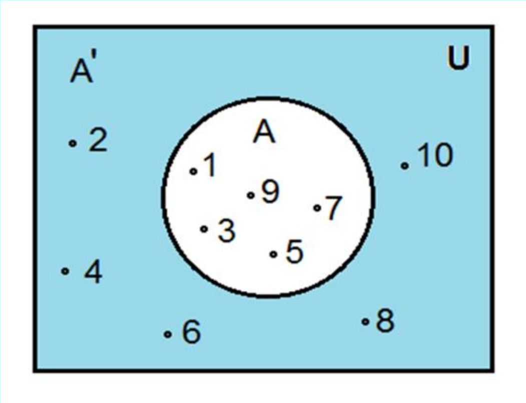
6. COMPLEMENT OF A SET

Let U be the universal set and A a subset of U. Then the complement of A is the set of all elements Of U which are not the elements of A. Symbolically, we write A' or A^c to denote the complement of A with respect to U.

Thus,

$A' = \{x : x \in \mathrm{U} \text{ and } x \notin \mathrm{A}\}$

$\quad = \mathrm{U} - \mathrm{A}$



Example - 3

Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

and A = {1, 3, 5, 7, 9}. Find A′.

Solution:

$\mathrm{A}' = \mathrm{U} - \mathrm{A}$

$\quad = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 3, 5, 7, 9\}$

$\quad = \{2, 4, 6, 8, 10\}$

7. DE-MORGAN'S LAWS

Let A and B be any two sets. Then,

(I) $(\mathrm{A} \cup \mathrm{B})' = \mathrm{A}' \cap \mathrm{B}'$

(II) $(\mathrm{A} \cap \mathrm{B})' = \mathrm{A}' \cup \mathrm{B}'$

These two results are known as De-Morgan's Laws.

SOME PROPERTIES OF COMPLEMENT SETS

1. Complement laws:

$(i)\ \mathrm{A} \cup \mathrm{A}' = \mathrm{U}$ $\qquad (ii)\ \mathrm{A} \cap \mathrm{A}' = \emptyset$

2. Law of double complementation: $(\mathrm{A}')' = \mathrm{A}$

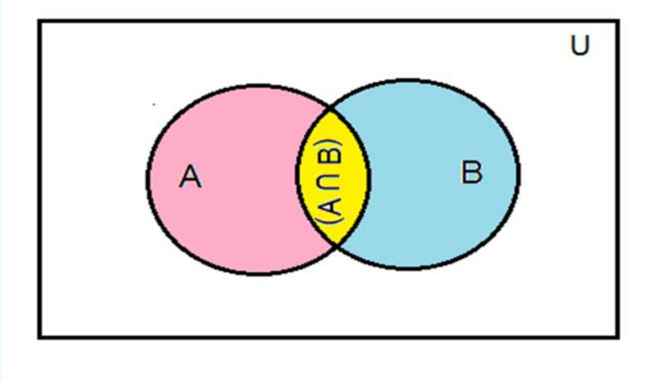
3. Laws of empty set: $\emptyset' = \mathrm{U}$

4. Laws of universal et: $\mathrm{U}' = \emptyset$

NAKUL PANDIT
PGT (MATHS)

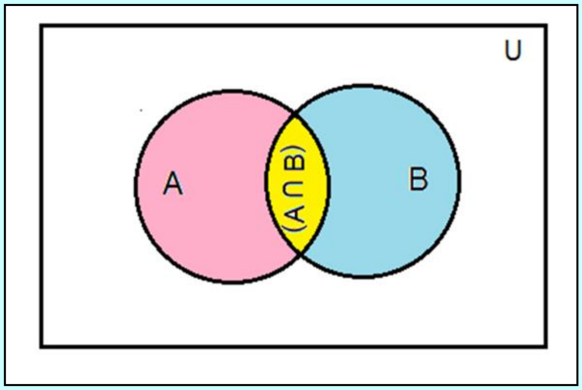
PRACTICAL PROBLEMS ON UNION AND INTERSECTION OF TWO SETS

In this Section, we will go through some practical problems related to our daily life.
We will derive an useful formula.



From Venn diagram, it is obvious that

$$n(\mathrm{AUB}) = n(\mathrm{A}) + n(\mathrm{B}) - n(\mathrm{A} \cap \mathrm{B})$$

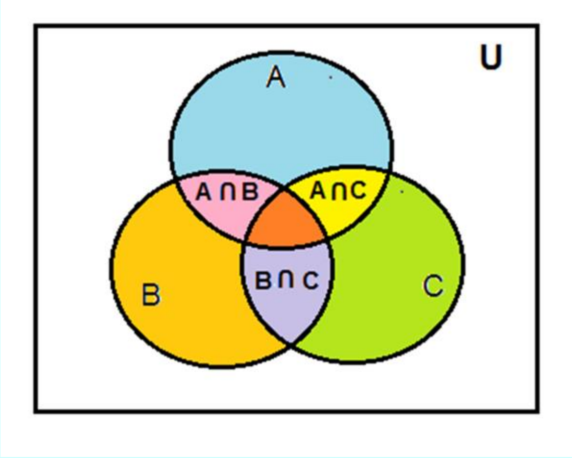


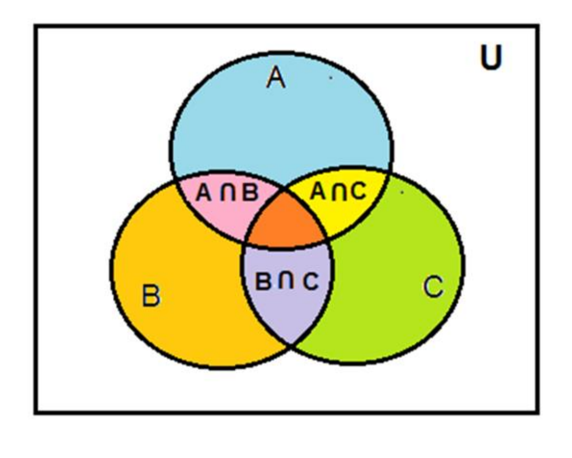
From above diagram, it is clear that,

1. $n(\mathrm{A\ only}) = n(\mathrm{A}) - n(\mathrm{A} \cap \mathrm{B})$
2. $n(\mathrm{B\ only}) = n(\mathrm{B}) - n(\mathrm{A} \cap \mathrm{B})$

A similar result involving three set is given below

$$n(\mathrm{AUBUC}) = n(\mathrm{A}) + n(\mathrm{B}) + n(\mathrm{C}) - n(\mathrm{A} \cap \mathrm{B}) - n(\mathrm{B} \cap \mathrm{C}) - n(\mathrm{C} \cap \mathrm{A}) + n(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C})$$





From above diagram, it is clear that,

1. $n(\text{A only}) = n(\text{A}) - n(\text{A} \cap \text{B}) - n(\text{A} \cap \text{C}) + n(\text{A} \cap \text{B} \cap \text{C})$
2. $n(\text{B only}) = n(\text{B}) - n(\text{A} \cap \text{B}) - n(\text{B} \cap \text{C}) + n(\text{A} \cap \text{B} \cap \text{C})$
3. $n(\text{C only}) = n(\text{C}) - n(\text{B} \cap \text{C}) - n(\text{A} \cap \text{C}) + n(\text{A} \cap \text{B} \cap \text{C})$

EXERCISE-1.6

Q.1 If X and Y are two sets such that $X \cup Y$ has 18 elements; X has 8 elements and Y has 15 elements; how many elements does $X \cap Y$ have ? [Ex.-1.6 Q.2]

Solution:

Given:

$n(X) = 8, n(Y) = 15$ and $n(X \cup Y) = 18$

To find: $n(X \cap Y)$

We know that

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$\Rightarrow 18 = 8 + 15 - n(X \cap Y)$

$\Rightarrow 18 = 23 - n(X \cap Y)$

$\Rightarrow n(X \cap Y) = 23 - 18$

$\Rightarrow n(X \cap Y) = 5$ Ans.

Q.2 In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages ?

[Ex.-1.6 Q.8]

Solution:

Let

X = The set of people who speak French

Y = The set of people who speak Spanish

∴ As per question

$n(X) = 50, \quad n(Y) = 20$ and $n(X \cap Y) = 10$

To find: $n(X \cup Y)$

We know that

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$= 50 + 20 - 10$

$= 70 - 10$

$= 60$

∴ 60 people speak at least one of these two languages.

Q.3 In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket ? How many like tennis ?

[Ex.-1.6 Q.7]

Solution:

Let

C = The set of people who like cricket

T $=$ The set of people who like tennis

$\therefore$ As per question

$n(\mathrm{C} \cup \mathrm{T}) = 65.$ $n(\mathrm{C}) = 40,$ and $n(\mathrm{C} \cap \mathrm{T}) = 10$

To find: (i) How many like tennis only and not cricket ?

(ii) How many like tennis?

We know that

$n(\mathrm{C} \cup \mathrm{T}) = n(\mathrm{C}) + n(\mathrm{T}) - n(\mathrm{C} \cap \mathrm{T})$

$\Rightarrow 65 = 40 + n(\mathrm{T}) - 10$

$\Rightarrow 65 = 30 + n(\mathrm{T})$

$\Rightarrow n(\mathrm{T}) = 65 - 30$

$\Rightarrow n(\mathrm{T}) = 35$

$\therefore$ 35 people like tennis.

Next,

The number of people who like tennis only and not cricket

$= n(\mathrm{T}) - n(\mathrm{C} \cap \mathrm{T})$

$= 35 - 10$

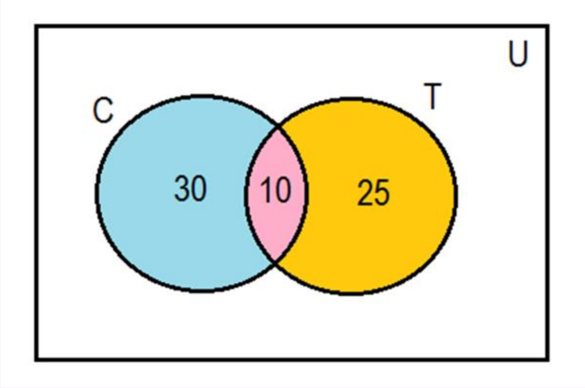
$= 25$

Thus,

(i) 25 people like tennis only and not cricket.

(ii) 35 people like tennis.

Understanding of solution through Venn diagram



Q.4 In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find:

(i) the number of people who read at least one of the newspapers.

(ii) the number of people who read exactly one newspaper. [Ex.-Miscl. Q.15]

Solution:

As per question,

$n(\mathrm{U}) = 60, \quad n(\mathrm{H}) = 25, \quad n(\mathrm{T}) = 26, \quad n(\mathrm{I}) = 26,$

$n(\mathrm{H} \cap \mathrm{T}) = 11,\ n(\mathrm{T} \cap \mathrm{I}) = 8,\ n(\mathrm{H} \cap \mathrm{I}) = 9$ and $n(\mathrm{H} \cap \mathrm{T} \cap \mathrm{I}) = 3$

Now, we know that

(i) $n(\mathrm{H} \cup \mathrm{T} \cup \mathrm{I}) = n(\mathrm{H}) + n(\mathrm{T}) + n(\mathrm{I}) - n(\mathrm{H} \cap \mathrm{T}) - n(\mathrm{H} \cap \mathrm{I}) - n(\mathrm{T} \cap \mathrm{I}) + n(\mathrm{H} \cap \mathrm{T} \cap \mathrm{I})$

$= 25 + 26 + 26 - 11 - 9 - 8 + 3$

$= 77 - 28 + 3$

$= 80 - 28$

$= 52$

(ii) $n(\mathrm{H\ only}) = n(\mathrm{H}) - n(\mathrm{H} \cap \mathrm{T}) - n(\mathrm{H} \cap \mathrm{I}) + n(\mathrm{H} \cap \mathrm{T} \cap \mathrm{I})$

$= 25 - 11 - 9 + 3$

$= 28 - 20$

$= 8$

$n(\mathrm{T\ only}) = n(\mathrm{T}) - n(\mathrm{H} \cap \mathrm{T}) - n(\mathrm{T} \cap \mathrm{I}) + n(\mathrm{H} \cap \mathrm{T} \cap \mathrm{I})$

$= 26 - 11 - 8 + 3$

$= 29 - 19$

$= 10$

$n(\mathrm{I\ only}) = n(\mathrm{I}) - n(\mathrm{H} \cap \mathrm{I}) - n(\mathrm{T} \cap \mathrm{I}) + n(\mathrm{H} \cap \mathrm{T} \cap \mathrm{I})$

$= 26 - 9 - 8 + 3$

$= 29 - 17$

$= 12$

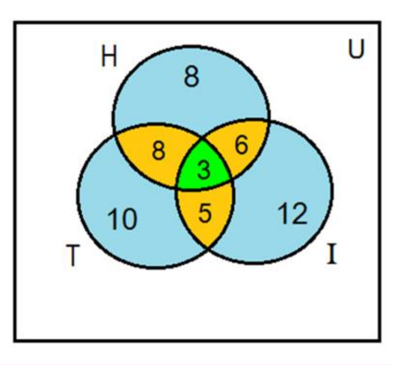
$\therefore$ The number of people who read exactly one newspaper

$= 8 + 10 + 12$

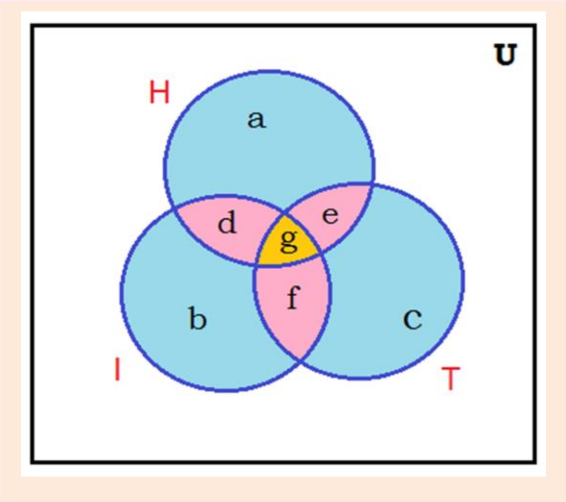
$= 30$

(i) The number of people who read at least one of the newspapers = 52

(ii) The number of people who read exactly one newspaper = 30



ALTERNATE SOLUTION



As per question

$n(\mathrm{H}) = 25 \Rightarrow a + d + e + \mathrm{g} = 25 \ldots\ldots\ldots\ldots (1)$

$n(\mathrm{I}) = 26 \Rightarrow b + d + \mathrm{f} + \mathrm{g} = 26 \ldots\ldots\ldots\ldots (2)$

$n(\mathrm{T}) = 26 \Rightarrow c + \mathrm{f} + e + \mathrm{g} = 26 \ldots\ldots\ldots\ldots (3)$

$n(\mathrm{H} \cap \mathrm{T}) = 11 \Rightarrow\ e + \mathrm{g} = 11 \ldots\ldots\ldots\ldots (4)$

$n(\mathrm{H} \cap \mathrm{I}) = 9 \Rightarrow\ \mathrm{d} + \mathrm{g} = 9 \ldots\ldots\ldots\ldots (5)$

$n(\mathrm{T} \cap \mathrm{I}) = 8 \Rightarrow \mathrm{f} + \mathrm{g} = 8 \ldots\ldots\ldots\ldots (6)$

$n(\mathrm{H} \cap \mathrm{T} \cap \mathrm{I}) = 3 \Rightarrow \mathrm{g} = 3 \ldots\ldots\ldots\ldots (7)$

Putt g = 3 in equations (4), (5), (6), we get

$e = 8,\ d = 6$ and $\mathrm{f} = 5$

Putting these values in equations (1), (2) and (3),

$a = 8,\ b = 12,\ c = 10$

(i) The number of people who read at least one of the newspapers

$= a + b + c + d + e + f + g$

$= 8 + 12 + 10 + 6 + 8 + 5 + 3$

$= 52$

(ii) The number of people who read exactly one newspaper

$= a + b + c$

$= 8 + 12 + 10$

$= 30$

MISCELLANEOUS EXERCISE on Ch-1

Q.1 Show that $A \cap B = A \cap C$ need not imply $B = C$. [Miscl. Ex. Q.10]

Solution:

Let

$A = \{1, 2\}, B = \{1, 3\}$ and $C = \{1, 4\}$

Now,

$A \cap B = \{1, 2\}, \cap \{1, 3\} = \{1\}$

$A \cap C = \{1, 2\}, \cap \{1, 4\} = \{1\}$

Clearly,

$A \cap B = A \cap C$

But, $B \neq C$

Hence, proved.

Q. 2 Find sets A, B and C such that $A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets and $A \cap B \cap C = \emptyset$. [Miscl. Ex. Q.12]

Solution:

Let

$A = \{1, 2\}, B = \{2, 3\}$ and $C = \{1, 3\}$

Now,

$A \cap B = \{1, 2\} \cap \{2, 3\} = \{2\}$

$B \cap C = \{2, 3\} \cap \{1, 3\} = \{3\}$

$A \cap C = \{1, 2\} \cap \{1, 3\} = \{1\}$

Clearly,

$A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets

But, $A \cap B = B \cap C = A \cap C = \emptyset$

Q. 3 Let A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that $B = C$. [Miscl. Ex. Q.3]

Solution:

We have given

$A \cup B = A \cup C \ldots\ldots\ldots\ldots\ldots (1)$

$A \cap B = A \cap C \ldots\ldots\ldots\ldots\ldots (2)$

From (1),

$A \cup B = A \cup C$

$\Rightarrow B \cap (A \cup B) = B \cap (A \cup C)$ [Taking intersection of both sides with B]

$\Rightarrow (B \cap A) \cup (B \cap B) = (B \cap A) \cup (B \cap C)$ [Using Left Distributive Law]

$\Rightarrow (A \cap B) \cup B = (A \cap B) \cup (B \cap C)$ [Using commutative law]

$\Rightarrow B = (A \cap B) \cup (B \cap C)$ .... (3) [$\because (A \cap B) \subset B$]

Again from (1),

$A \cup B = A \cup C$

$\Rightarrow C \cap (A \cup B) = C \cap (A \cup C)$ [Taking intersection of both sides with C]

$\Rightarrow (C \cap A) \cup (C \cap B) = (C \cap A) \cup (C \cap C)$ [Using Left Distributive Law]

$\Rightarrow (A \cap C) \cup (B \cap C) = (A \cap C) \cup C$ [Using commutative law]

$\Rightarrow (A \cap B) \cup (B \cap C) = (A \cap C) \cup C$ [Using (2)]

$\Rightarrow (A \cap B) \cup (B \cap C) = C$ .... (4) [$\because (A \cap C) \subset C$]

Finally from (3) and (4), we conclude that

$B = C$

Hence proved

Q. 4 Let A and B be sets. If $A \cap X = B \cap X = \emptyset$ and $A \cup X = B \cup X$ for some set X, show that $A = B$. [Miscl. Ex. Q.11]

Solution:

We have given

$A \cup X = B \cup X$ … … … … … (1)

$A \cap X = B \cap X = \emptyset$ … … … … … (2)

From (1),

$A \cup X = B \cup X$

$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$ [Taking intersection of both sides with A]

$\Rightarrow (A \cap A) \cup (A \cap X) = (A \cap B) \cup (A \cap X)$ [Using Left Distributive Law]

$\Rightarrow A \cup \emptyset = (A \cap B) \cup \emptyset$ [Using (2)]

$\Rightarrow A = A \cap B$ .... (3)

Again from (1),

$A \cup X = B \cup X$

$\Rightarrow B \cap (A \cup X) = B \cap (B \cup X)$ [Taking intersection of both sides with B]

$\Rightarrow (B \cap A) \cup (B \cap X) = (B \cap B) \cup (B \cap X)$ [Using Left Distributive Law]

$\Rightarrow (A \cap B) \cup \emptyset = B \cup \emptyset$ [Using (2)]

$\Rightarrow A \cap B = B$ .... (3)

Finally from (3) and (4), we conclude that

$A = B$

Hence proved

Q. 5 Show that if $A \subset B$, then show that $C - B \subset C - A$.

[Miscl. Ex. Q.5]

Proof:

Given,

$A \subset B \ldots \ldots \ldots \ldots \ldots (1)$

Let x be an arbitrary element $C - B$

$\therefore x \in (C - B) \Rightarrow x \in C \text{ and } x \notin B$

$\Rightarrow x \in C \text{ and } x \notin A \quad [\because A \subset B, \text{by } (1)]$

$\Rightarrow x \in (C - A)$

Thus, $x \in (C - B) \Rightarrow x \in (C - A)$

$\therefore (C - B) \subset C - A$ Proved.

Q. 6 Assume that $P(A) = P(B)$. Show that $A = B$. [Miscl. Ex. Q.6]

Proof:

Given,

$P(A) = P(B) \ldots \ldots \ldots \ldots \ldots (1)$

Let x be an arbitrary element A

$\therefore x \in A \Rightarrow \{x\} \subset A$

$\Rightarrow \{x\} \in P(A)$

$\Rightarrow \{x\} \in P(B) \quad [\text{Using } (1)]$

$\Rightarrow \{x\} \subset B$

$\Rightarrow x \in B$

Thus, $x \in A \Rightarrow x \in B$

$\therefore A \subset B$ …………… (2)

Next, let y be an arbitrary element B

$\therefore y \in B \Rightarrow \{y\} \subset B$

$\Rightarrow \{y\} \in P(B)$

$\Rightarrow \{y\} \in P(A) \quad [\text{Using } (1)]$

$\Rightarrow \{y\} \subset A$

$\Rightarrow y \in A$

Thus, $y \in B \Rightarrow y \in A$

$\therefore B \subset A$ …………… (3)

Thus, from (2) and (3), we conclude that,

$A = B$

Hence, proved.

Q. 7 Is it true that for any sets A and B, $P(A) \cup P(B) = P(A \cup B)$?
Justify your answer. [Miscl. Ex. Q.7]

Solution:

Let

$A = \{a\}$ and $B = \{b\}$

$\therefore$ $P(A) = \{\emptyset, \{a\}\}$ and $P(B) = \{\emptyset, \{b\}\}$

So, $P(A) \cup P(B) = \{\emptyset, \{a\}\} \cup \{\emptyset, \{b\}\} = \{\emptyset, \{a\}, \{b\}\}$

Next, $A \cup B = \{a\} \cup \{b\} = \{a, b\}$

$\therefore$ $P(A \cup B) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

Now, it is clear that

$P(A) \cup P(B) \neq P(A \cup B)$