NAKUL PANDIT
PGT (MATHS)

TYPES OF FUNCTIONS

(3) POLYNOMIAL FUNCTION

A function $f: \mathrm{R} \to \mathrm{R}$ is said to be polynomial function if for each x in R,

$y = f(x) = a_0 + a_1x + a_2x^2 + \cdots . + a_nx^n$,

where n is a non-negative integer and $a_0,\ a_1,\ a_2, \ldots a_n \in \mathrm{R}$

Domain $=$ R and Range $=$ R

Examples:

(i) $f(x) = 3x^2 + 4x + 5$

(ii) $f(x) = x^3 - 2x^2 + \sqrt{3}x - 7$

(iii) $f(x) = \frac{2}{3}x^2 + \pi x - \frac{\sqrt{3}}{4}$

GRAPH OF A POLYNOMIAL FUNCTION

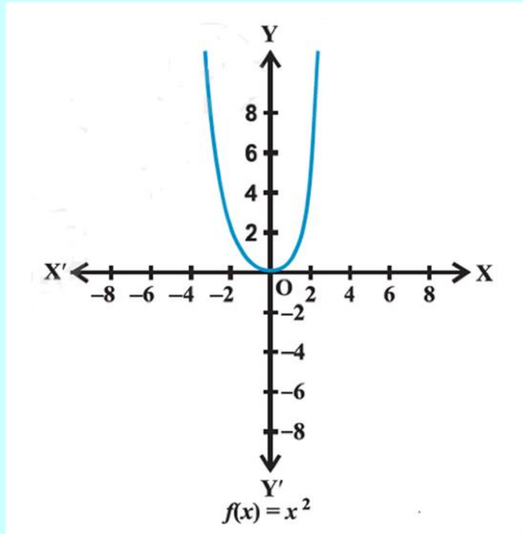
(1) Let us consider a polynomial function

$f(x) = x^2$

Let us obtain some points on a constant function

$y = f(x) = x^2$

x	-3	-2	-1	0	1	2	3	4
$y = x^2$	9	4	1	0	1	4	9	16



$f(x) = x^2$

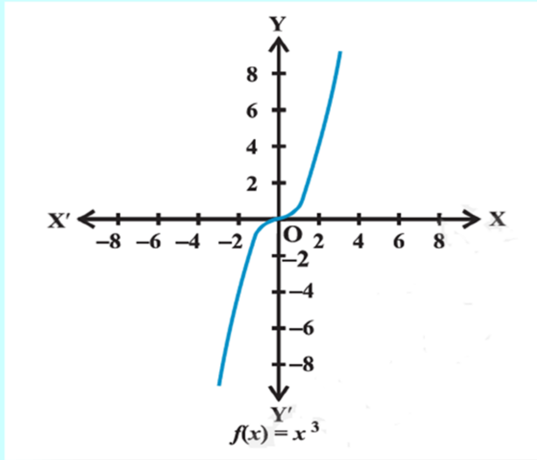
(2) Let us consider a polynomial function

$f(x) = x^3$

Let us obtain some points on a constant function

$y = f(x) = x^3$

x	-3	-2	-1	0	1	2	3	4
$y = x^3$	-27	-8	-1	0	1	8	27	64



$f(x) = x^3$

(4) RATIONAL FUNCTION

A rational function $f(x)$ is defined by

$$f(x) = \frac{p(x)}{q(x)}$$

where $p(x) and\ q(x)$ are polynomial functions and $q(x) \neq 0$

Examples:

$(i)\quad f(x) = \dfrac{3x^2 - 5x + 2}{x - 4}, x \neq 4$

$(ii)\quad f(x) = \dfrac{x^2 - 3x + 2}{4x^2 - x - 5}, x \neq -1$

$(iii)\quad f(x) = \dfrac{y^3 - 5y^2 + 2y + 7}{y^2 + y - 6}, x \neq 2$

GRAPH OF A RATIONAL FUNCTION

Let us consider a rational function

$f(x) = \dfrac{1}{x}, x \neq 0$

Clearly , Domain = $\mathrm{R} - \{0\}$

Let us obtain some points on a constant function

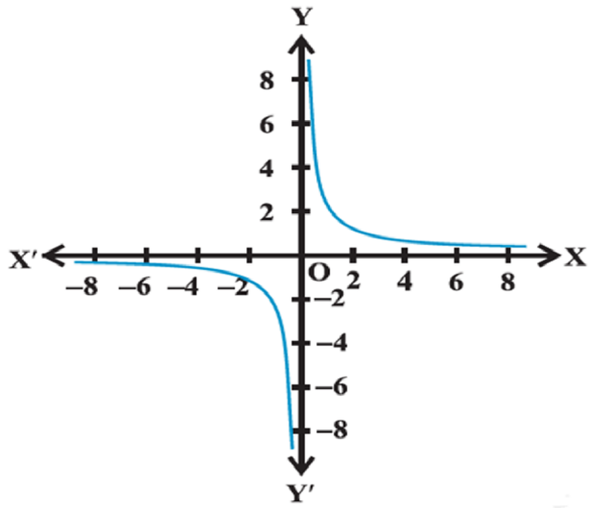
$$y = f(x) = \frac{1}{x}$$

Table-1

x	…	-5	-4	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	$-\frac{1}{5}$	….
$y = \frac{1}{x}$	…	$-\frac{1}{5}$	$-\frac{1}{4}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	-2	-3	-4	-5	…

Table-2

x	…	5	4	3	2	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	….
$y = \frac{1}{x}$	…	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3	4	5	…



$f(x) = \frac{1}{x}$

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VALUE OF A FUNCTION

The value of the function $f(x)$ obtained by putting $x = a$ is called the value of the function at $x = a$ and id denoted by $f(a)$.

Examples:

$$f(x) = \frac{x^2 - 3x + 4}{x - 2}$$

The value of the function

at $x = 0$ is $f(0) = \dfrac{0^2 - 3.0 + 4}{0 - 2} = \dfrac{4}{-2} = -2$

at $x = 1$ is $f(1) = \dfrac{1^2 - 3.1 + 4}{1 - 2} = \dfrac{2}{-1} = -2$

at $x = -1$ is $f(-1) = \dfrac{(-1)^2 - 3.(-1) + 4}{(-1) - 2} = \dfrac{8}{-3}$

at $x = 2$ is $f(0) = \dfrac{2^2 - 3.2 + 4}{2 - 2} = \dfrac{2}{0} = \text{Not real} = \infty$

Thus, it is clear that the function $f(x)$ is well defined at $x = 0, 1, -1$ but it is not defined at $x = 2$

∴The domain of the given function is $= \mathrm{R} - \{2\}$

Remarks: A function $f(x)$ is said to be undefined at a point $x = a$ if its value $f(a)$ takes a value of the type

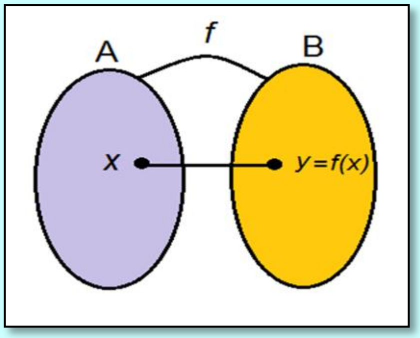
$\frac{1}{0}, \frac{0}{0}, 0^0, 1^\infty, \infty^0, \infty^\infty, \frac{\infty}{\infty}, \sqrt{-ve}$ etc.

DOMAIN AND RANGE OF A FUNCTION

Let $f: A \to B$ be a function defined from set A to set B.

Let y be the image of x under the function f

i.e. $y = f(x)$



Then, domain is the set of all those real values of x for which the value of the function $f(x)$ is well defined.

i. e. Domain= $\{x: x \in \mathrm{A} \text{ and } f(x) \in \mathrm{B}\}$

And the range is the set of all those values of y for which $y = f(x)$

i. e. Range= $\{y: y \in \mathrm{B} \text{ and } y = f(x)\}$