

$$\delta a = (\Delta a_{\text{mean}} / a_{\text{mean}}) \times 100\%$$

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Combination of Error.

↳ Error of sum or difference.

$$\text{If, } Z = A + B$$

then,

$$\Delta Z = \pm \Delta A + \Delta B$$

$$\text{or, } \Delta Z = \Delta A + \Delta B$$



If,

$$Z = A - B$$

then,

$$\text{absolute error} \Rightarrow Z = \Delta A + \Delta B.$$

• Error of Product.

let,

$$Z = A \times B.$$

Then max relative error in Z,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}.$$

• Error of Division.

If,

$$Z = \frac{A}{B}$$

Then, max relative error in Z,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}.$$

• Error of Power.

$$\text{if, } Z = A^p$$

then, maximum relative error in Z,

$$\frac{\Delta Z}{Z} = p \frac{\Delta A}{A}.$$

In general,

$$\text{if, } Z = \frac{A^p \times B^q}{C^r}$$

$$\Rightarrow \frac{\Delta Z}{Z} = p \frac{\Delta A}{A} + q \frac{\Delta B}{B} + r \frac{\Delta C}{C}$$

Q. If length of a rectangle is measured as $l = 20 \pm 0.2$ m and breadth = 10 ± 0.3 m. Find perimeter, area, with error limits.

$$\text{here, length} = l = 20 \pm 0.2 \text{ m}$$

$$\text{breadth} = b = 10 \pm 0.3 \text{ m.}$$

Therefore,

$$\text{Perimeter} = 2(l+b)$$

$$= 2(20+30)$$

$$= 2(50)$$

$$= 100\text{m}$$

& error in perimeter,

$$\Rightarrow \Delta p = \Delta l + \Delta b$$

$$= 0.2 + 0.3$$

$$= 0.5\text{ m.}$$

$$\therefore \text{Perimeter} = 100 \pm 0.5\text{ m.}$$

$$\text{Area} = l \times b$$

$$= 20 \times 10$$

$$= 200\text{ m}^2$$

&

$$\text{error in area} \Rightarrow \frac{\Delta a}{a} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$$

$$= \frac{0.2}{20} + \frac{0.3}{10}$$

$$= \frac{1}{100} + \frac{3}{100}$$

$$= \frac{4}{100}$$

$$= \frac{1}{25}$$

$$= 0.04$$

$$\therefore \text{Area} = 200 \pm 0.04\text{ m}^2.$$

- Q. If heat dissipated in a resistance can be determined from the relation, $H = I^2 R t$ Joule, if the maximum error in the measurement of current, resistance and time are 2%, 1% and 1% respectively, what would be the maximum error in dissipated heat?

$$H = I^2 R t$$

$$\Rightarrow \frac{\Delta H}{H} = 2 \cdot \frac{\Delta I}{I} + \frac{\Delta R}{R} + \frac{\Delta t}{t}$$

$$\Rightarrow \frac{\Delta H}{H} \times 100\% = 2 \left(\frac{\Delta I}{I} \times 100\% \right) + \left(\frac{\Delta R}{R} \times 100\% \right) + \left(\frac{\Delta t}{t} \times 100\% \right)$$

$$= 2(2\%) + 1\% + 1\%$$

$$= 4\% + 2\%$$

$$= 6\%$$

- Significant figures.

The reliable digits plus the first uncertain digits are known as significant figures.

Significant figures depends on the least count of the instrument.

• Rules.

1. All non zero digits are significant.
2. All zeros between two non zero digits are significant, no matter where the decimal point is.
3. If the number is less than 1, the zero(s) of the right of decimal point but to the left of the first non-zero digit are not significant.
4. The terminal or trailing zero(s) in a number without a decimal point are not significant.
5. Trailing zeros in a number with a decimal point are significant.

