Hw 1

1. In the regression problem, we use the error function $E(g|D) = \frac{1}{N}\sum_{i=1}^{N}[y_i - g(\mathbf{x}_i)]^2$ to sum up the squares of the differences between the actual value and the estimated value. This error function is not robust to outliers. What would be a better error function to implement robust regression. (Hint: You only need to find one error function and briefly explain why it is more robust to outliers than the squared error one.)

2. Assume a disease is seen in only one person out of every thousand. Assume also that we have a test that is effective in that if a person has the disease, there is a 95% chance that the test result will be positive; however, there is a one in a hundred chance that the test result will be positive on a healthy person. Assume that a new patient arrives and the test result is positive. What is the probability that the patient has the disease?

3. In a 2-class 1-D problem, the pdfs are the Gaussians $N(0, \sigma^2)$ and $N(1, \sigma^2)$ for the two classes, respectively. Show that the threshold x_0 minimizing the average error rate is $x_0 = \frac{1}{2} - \sigma^2 ln\frac{P(C_2)}{P(C_1)}$.

4. Consider a 3-class classification problem. Let $P(C_1) = P(C_2) = P(C_3) = \frac{1}{3}$. The class-conditional densities are multivariate normal densities with parameters:

$$\boldsymbol{\mu}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \boldsymbol{\mu}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \boldsymbol{\mu}_3 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\boldsymbol{\Sigma}_1 = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.7 \end{bmatrix}, \boldsymbol{\Sigma}_2 = \boldsymbol{\Sigma}_3 = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}$$

Classify the following points: (a) $\boldsymbol{x} = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix}$, (b) $\boldsymbol{x} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$.