Computer experiment 2

1. Write the code that generates the Bernoulli samples $X = \{x_1, \dots, x_N\}, x_i \in \{0,1\}$ with (a) $p = 0.3, N = 1000$, and (b) $p = 0.5, N = 1000$; and the code that calculates the estimate $\hat{p}_{ML}$ from the sample X.

2. Let x have an exponential density $p(x|\theta) = \begin{cases} \theta e^{-\theta x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$
   (a) Derive the maximum-likelihood estimate $\hat{\theta}_{ML}$ for θ.
   (b) Write the code that generate $X = \{x_1, \dots, x_N\}$ according to $p(x|\theta)$ with $\theta = \frac{1}{3}, \frac{1}{2}$, and 1 . Plot $p(x|\theta)$ and $p(x|\hat{\theta}_{ML})$ for different Ns

3. Write the code that generates the samples $X = \{(x_i, y_i), i = 1, \dots, n\}$ multiple times by $y = 2\sin\left(\frac{3}{2}x\right) + \varepsilon,\ \ \varepsilon \sim N(0,1)$. Divide your samples into two as training and validation sets.
   (a) Use a polynomial in x of order k $(k = 1, 3, 5)$ to fit the training data. Refer to Fig 4.5 to show your training samples and the fitted curves.
   (b) Compute error on the validation set. Refer to Fig. 4.7(b) to show the regression error vs. polynomial order.

4. Write the code that generates the 2-D normal samples $X = \{\mathbf{x}_1, \dots, \mathbf{x}_N\}, \mathbf{x}_i \in R^2$ with (a) $\boldsymbol{\mu} = [1,1]^T, \boldsymbol{\Sigma} = \begin{bmatrix} 5 & 3 \\ 3 & 4 \end{bmatrix}, N = 1000$, and (b) $\boldsymbol{\mu} = [10,5]^T, \boldsymbol{\Sigma} = \begin{bmatrix} 7 & 4 \\ 4 & 5 \end{bmatrix}, N = 1000$; and the code that calculates the estimates $\hat{\boldsymbol{\mu}}_{ML}, \hat{\boldsymbol{\Sigma}}_{ML}$ from the sample X.