111061702 常安彥

Pattern Recognition ex2

1. Bernoulli samples (Dataset generation was not truly like p)

```
>> ex2_1
(a) True p: 0.3
Estimate: 0.29
(b) True p: 0.5
Estimate: 0.501
```

2. Exponential Density

(a) Derivation

$$P(x|\theta)=\begin{cases}\theta e^{-\theta x}, & x\geq 0\\ 0, & x<0\end{cases}$$

$$L(\theta)=P(x_1|\theta)\cdot P(x_2|\theta)\cdots P(x_n|\theta)$$

$$L(p)=\sum_{i=1}^{n}\ln(p|\theta)$$

$$=\sum_{i=1}^{n}[\ln\theta-\theta x_i]$$

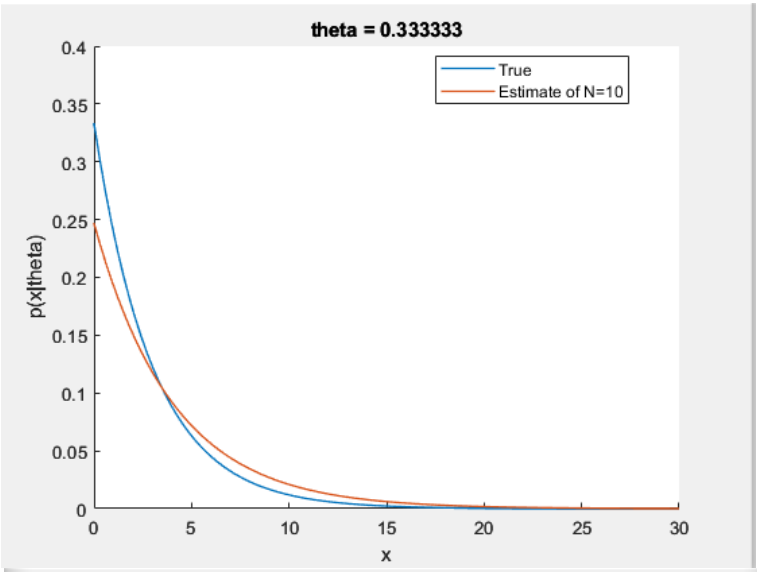
$$\frac{\partial}{\partial\theta}L(\theta)=\sum_{i=1}^{n}\left[-x_i+\frac{1}{\theta}\right]=0$$

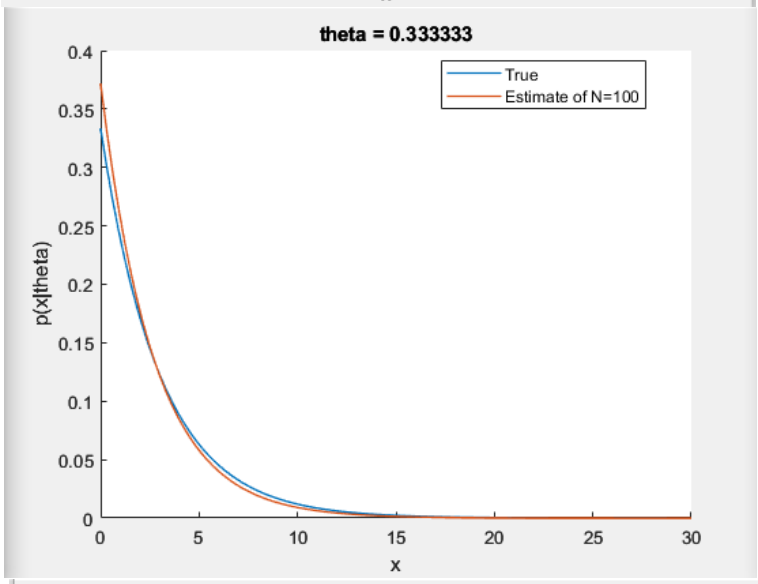
$$\Rightarrow \hat{\theta}_{ML}=\frac{n}{\sum_{i=1}^{n}x_i}\ \#$$

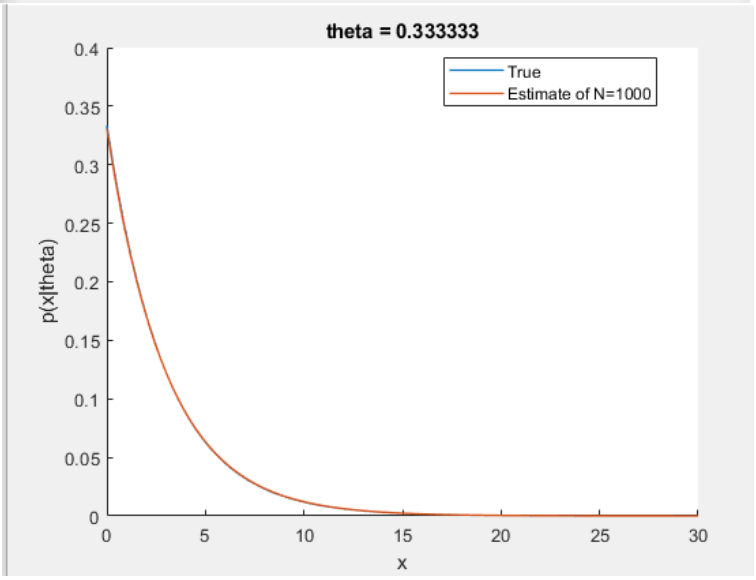
(b)

```
>> ex2_2
When theta = 0.333333
  Estimate theta of N = 10: 0.246745
  Estimate theta of N = 100: 0.371896
  Estimate theta of N = 1000: 0.330757
When theta = 0.5
  Estimate theta of N = 10: 0.384346
  Estimate theta of N = 100: 0.464675
  Estimate theta of N = 1000: 0.509666
When theta = 1
  Estimate theta of N = 10: 0.924147
  Estimate theta of N = 100: 1.0722
  Estimate theta of N = 1000: 0.924359
```

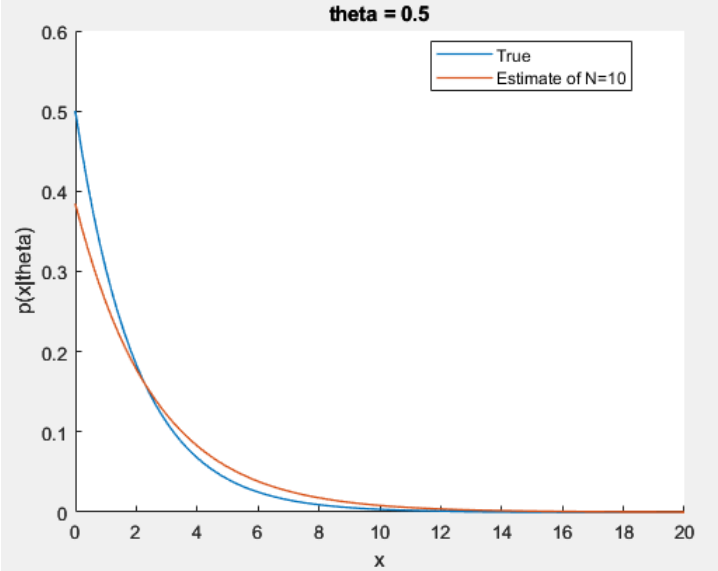
Theta = 1/3

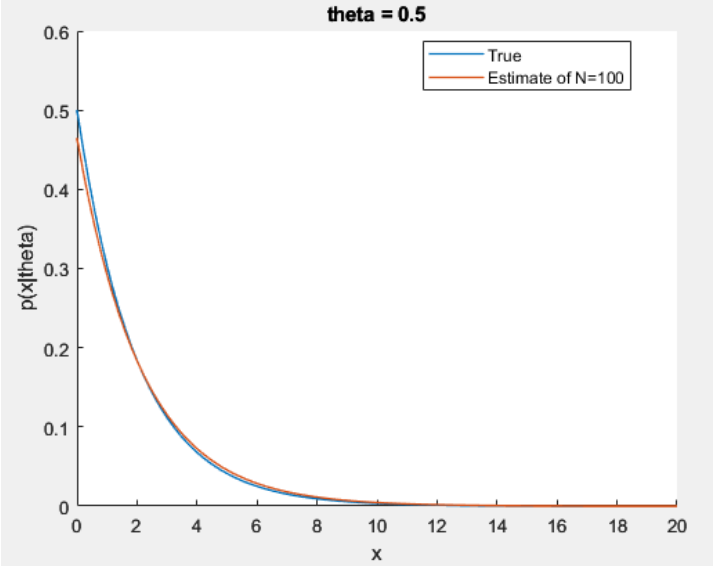

theta = 0.333333
True
Estimate of N=10
p(x|theta)
x
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0
5
10
15
20
25
30

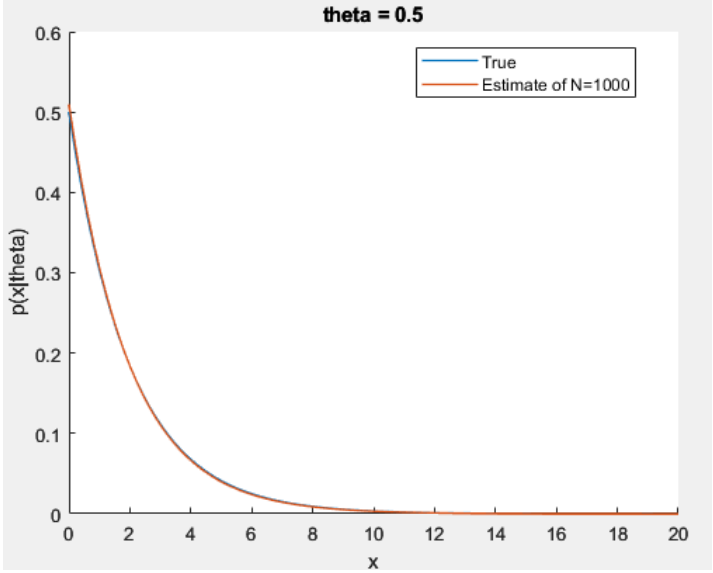

theta = 0.333333
True
Estimate of N=100
p(x|theta)
x
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0
5
10
15
20
25
30


theta = 0.333333
True
Estimate of N=1000
p(x|theta)
x
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0
5
10
15
20
25
30

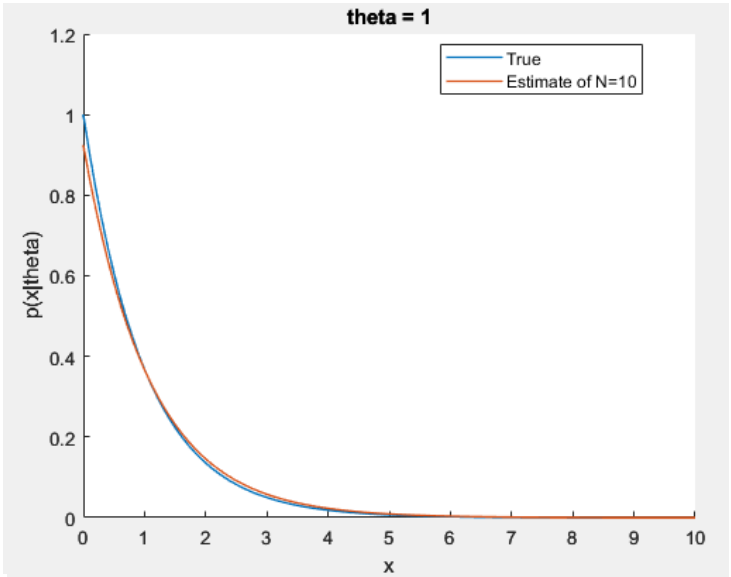
Theta = 1/2

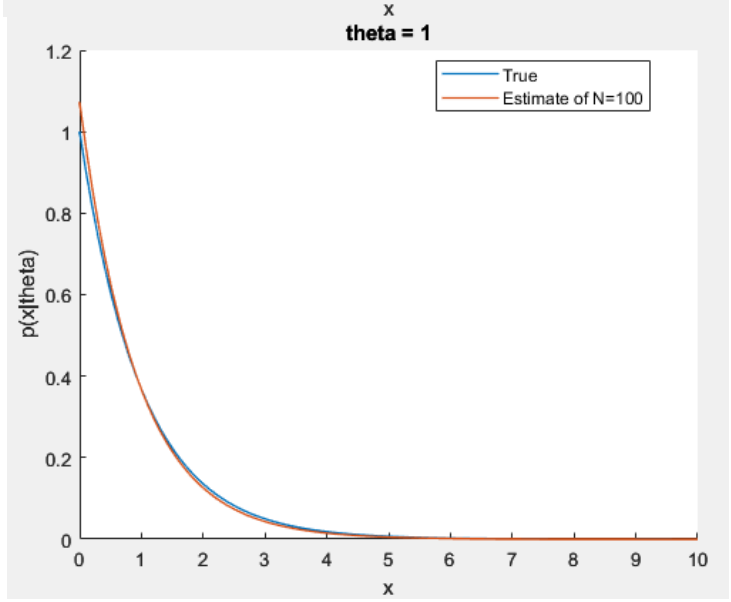

theta = 0.5
True
Estimate of N=10
p(x|theta)
x

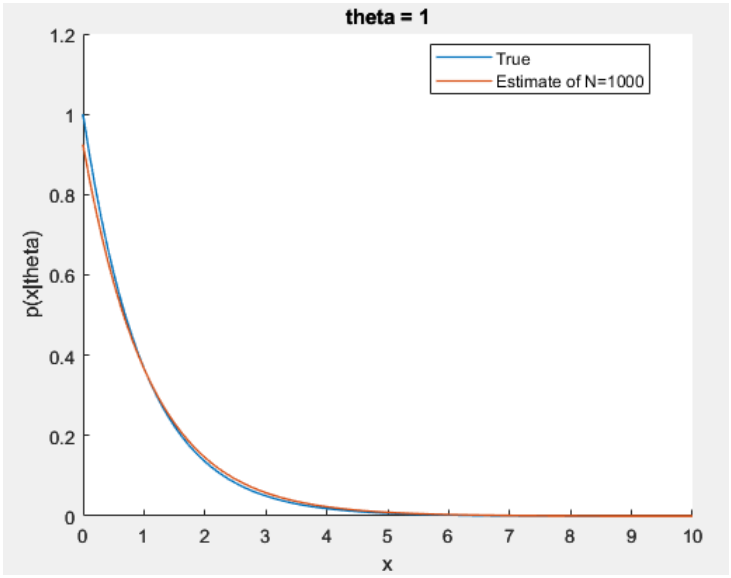

theta = 0.5
True
Estimate of N=100
p(x|theta)
x


theta = 0.5
True
Estimate of N=1000
p(x|theta)
x

Theta = 1

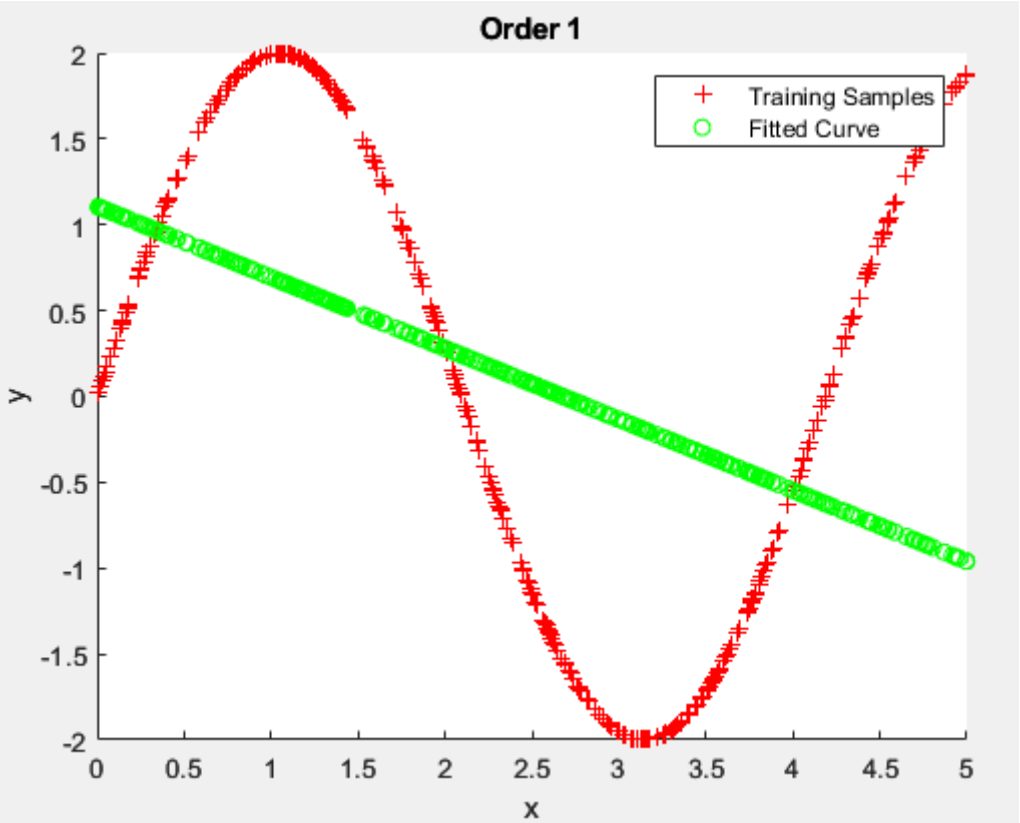

theta = 1
True
Estimate of N=10
p(x|theta)
x


theta = 1
True
Estimate of N=100
p(x|theta)
x

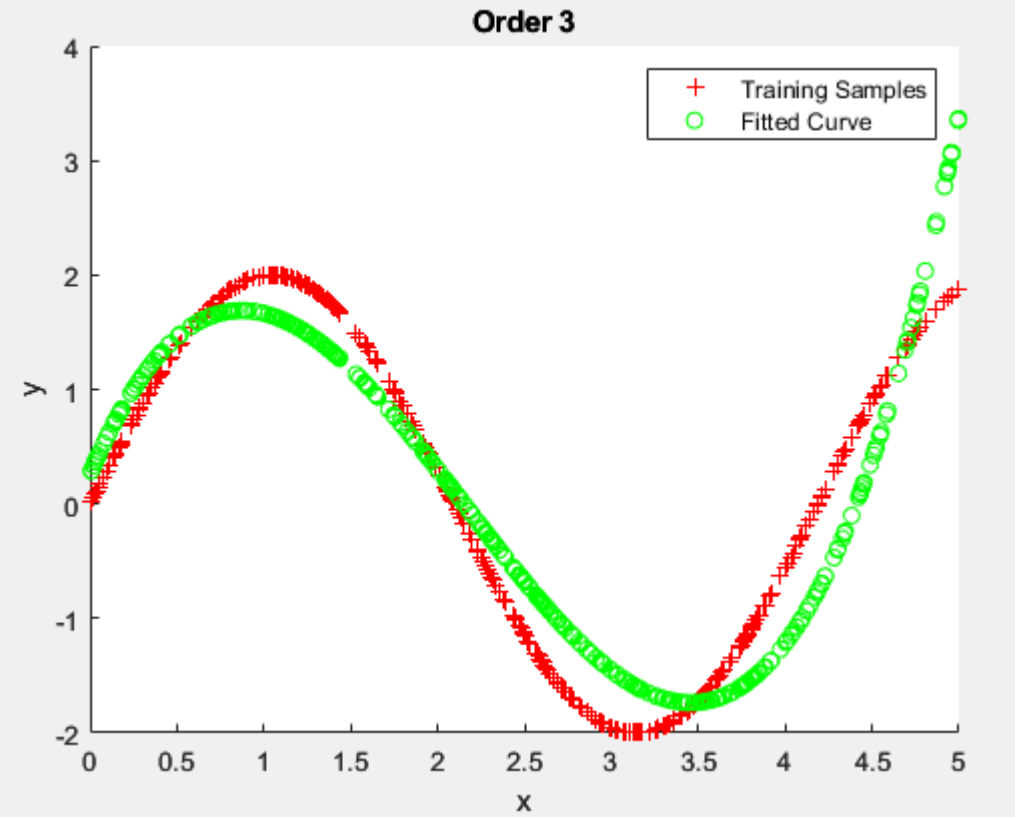

theta = 1
True
Estimate of N=1000
p(x|theta)
x

3. Polynomial Regression

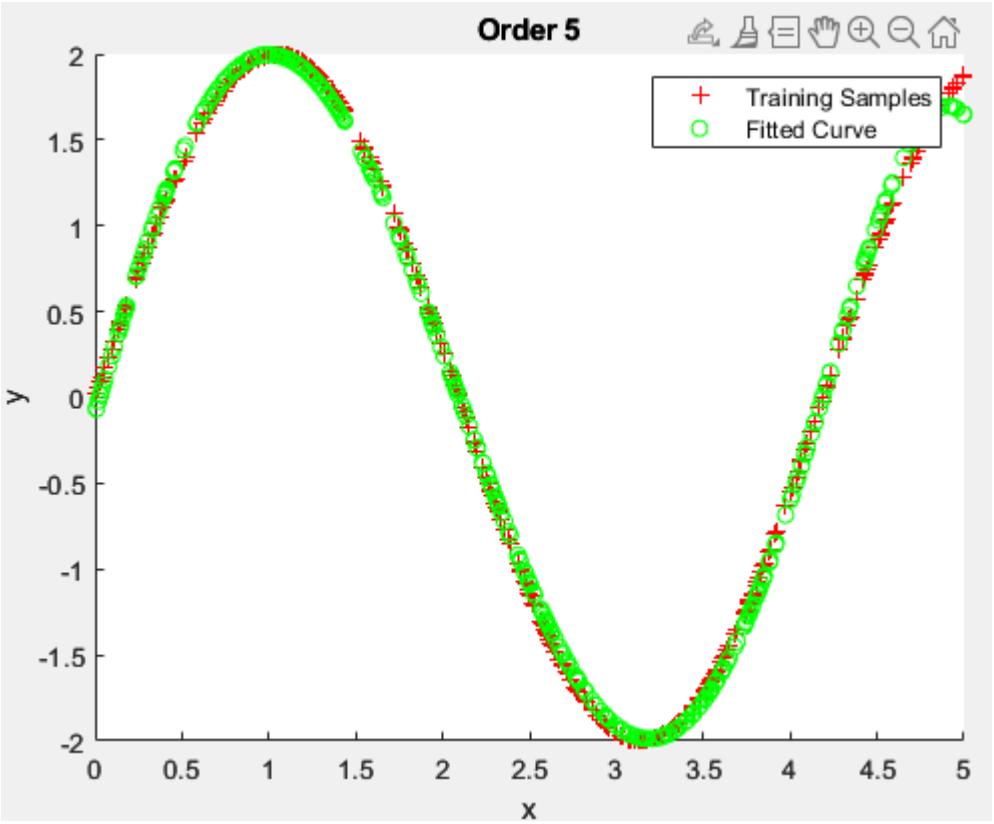
k = 1:



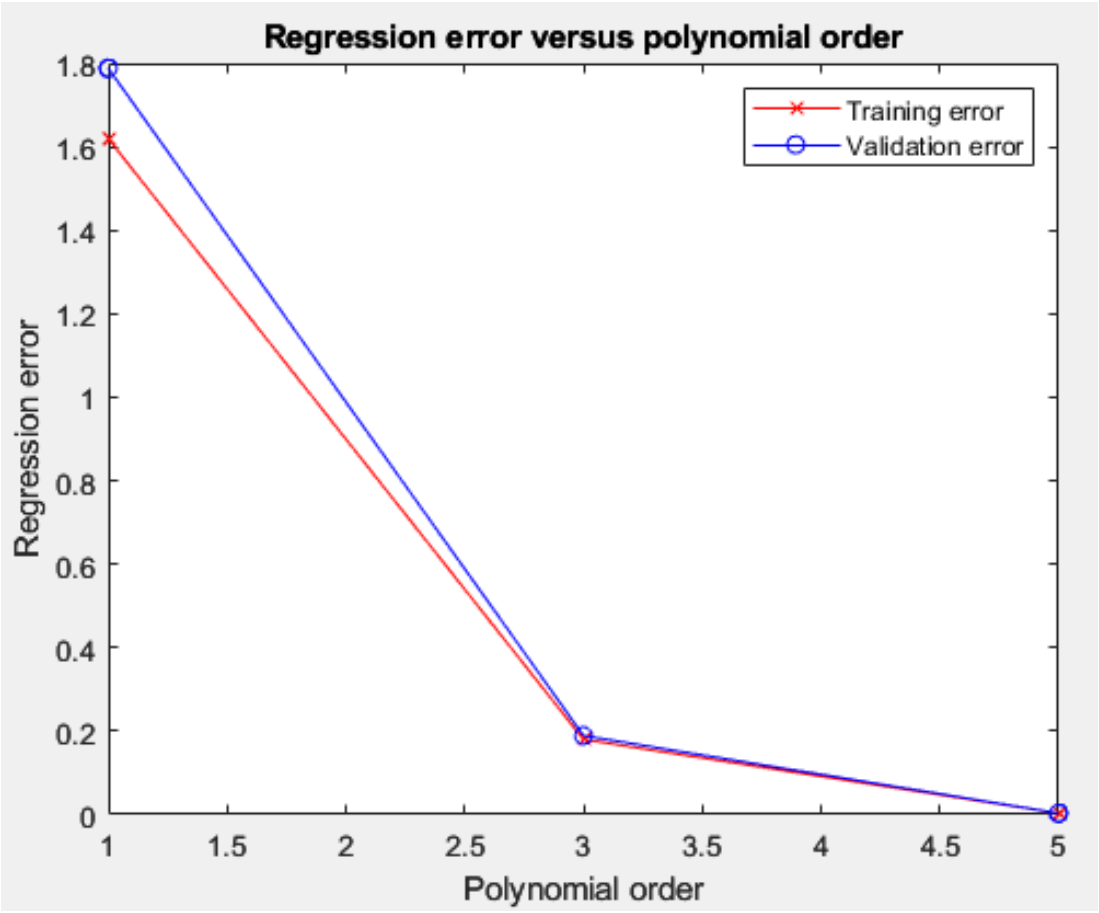

k = 3:



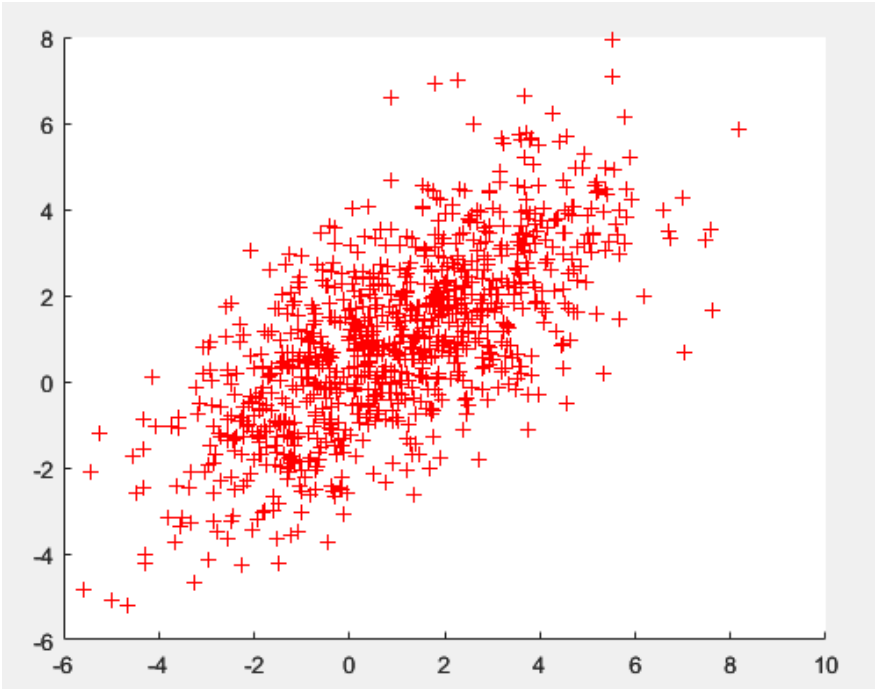
k = 5:




(c) Regression error vs. polynomial order

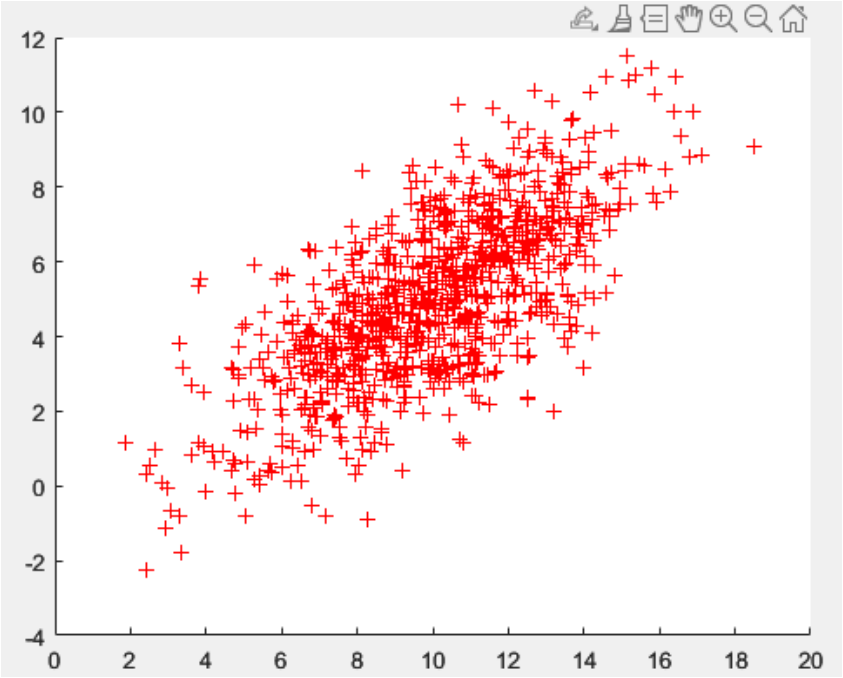


4. (a) dataset




```
>> ex2_4
(a)
True Mu:[1 1]
Unbiased Estimate of Mean mu_hat_ML:[0.969197 0.972917]
True sigma: [5 3; 3 4]
biased Estimate of Variance sigma_hat_ML: [5.30528 3.19283; 3.19283 4.14724]
```

(b) dataset




```
(b)
True Mu:[10 5]
Unbiased Estimate of Mean mu_hat_ML:[9.97453 4.98774]
True sigma: [7 4; 4 5]
biased Estimate of Variance sigma_hat_ML: [7.12853 4.11376; 4.11376 4.82365]
```