6.4

$E(\hat{\theta}) = \theta$

$\hat{\theta}_2 = \dfrac{\Sigma(x_i - \bar{x})^2}{n-1}$

$\hat{\theta}_1 = \dfrac{\Sigma(x_i - \bar{x})^2}{n}$ $E(x_i) = \mu$ $V(x_i) = \sigma^2 = E(x_i^2) - \mu^2$

$E(\bar{x}) = \mu$ $V(\bar{x}) = \dfrac{\sigma^2}{n} = E(\bar{x}^2)$

$E(\hat{\theta}_1) \neq \sigma^2$

$E(\hat{\theta}_2) = \sigma^2$

$$E(\hat{\theta}_1) = E\left[\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}\right]$$

$$= \frac{1}{n} E\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n}(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \frac{n-1}{n}\sigma^2$$

$$E(\hat{\theta}_2) = E\left[\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}\right]$$

$$= \frac{1}{n-1} E\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n-1}(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \sigma^2$$