计算机视觉与模式识别

Computer Vision and Pattern Recognition

-- Geometric Vision

Reconstruction ; F Matrix estimation;
8-points algorithm; Robust F estimation and 3D from weak calibration

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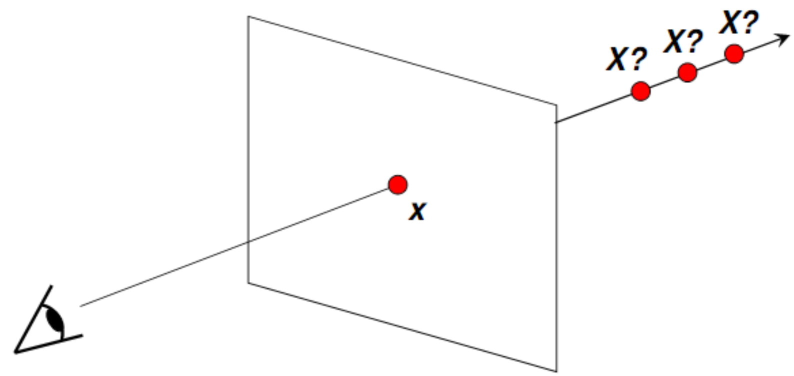
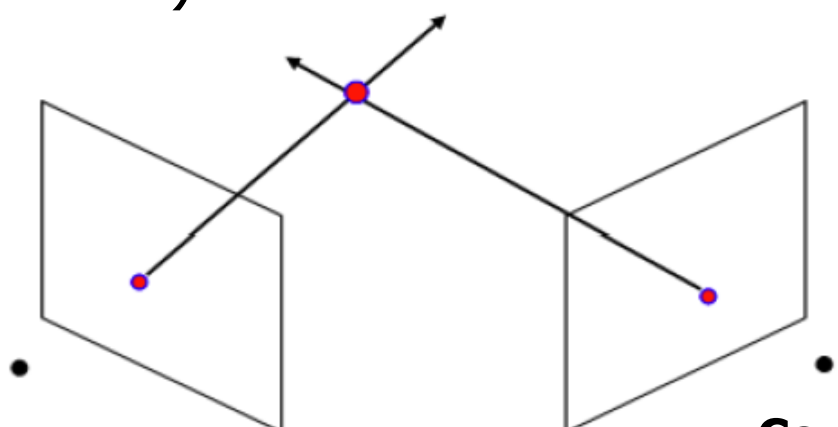
Recovery of 3D Structure

- **Recovery of structure from one image is inherently ambiguous**



- **3D structure from two-view (two images)**

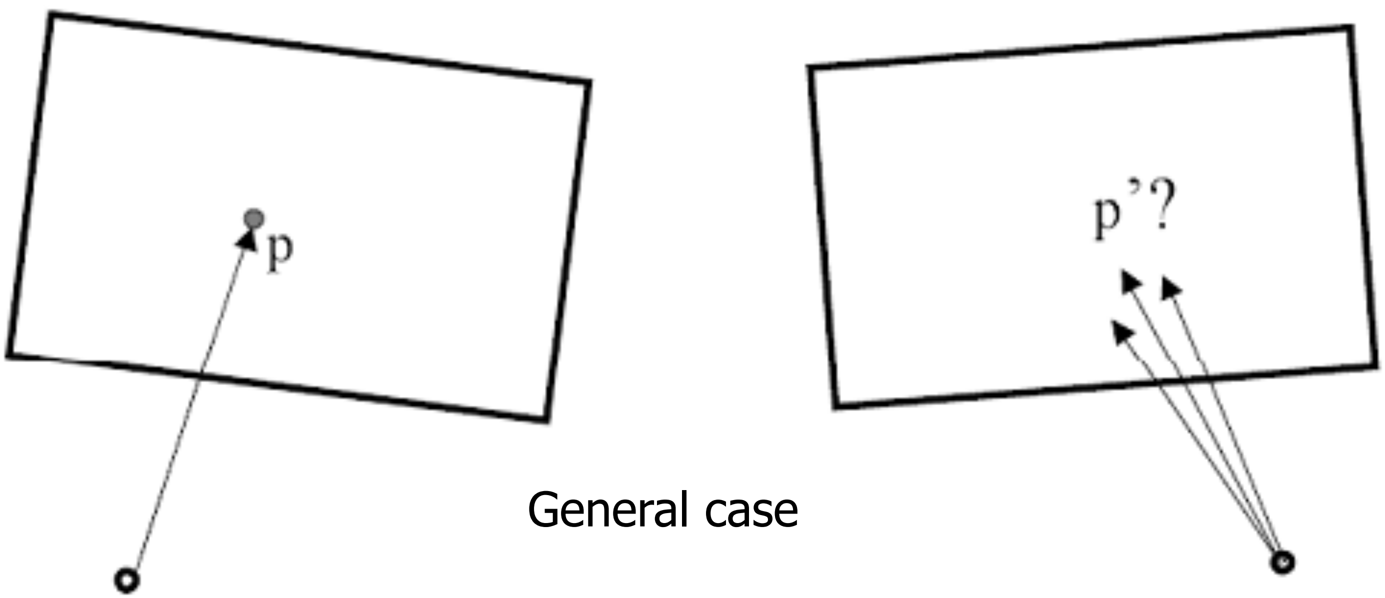
---- Reconstruction as intersection of two rays (**given correspondence**)



Correspondence problem?

Stereo Correspondence Constraints

- Given p in the left image, where can the corresponding point p' in the right image be?



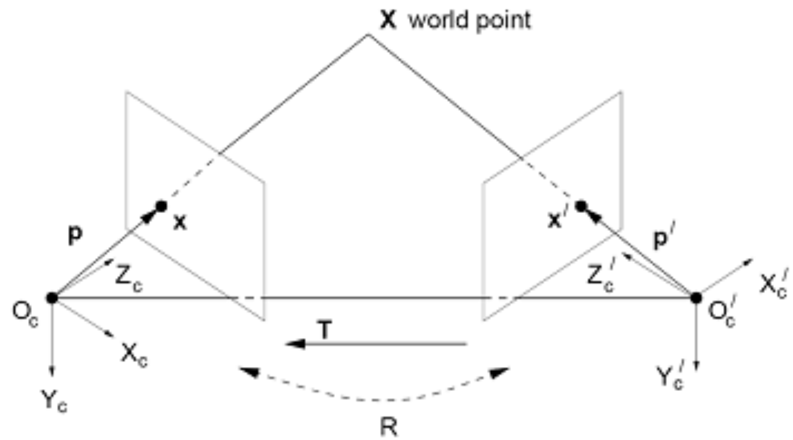
General case

- Correspondence (stereo matching): Given a point in just one image, how does it constrain the position of the corresponding point x' in another image?

Essential Matrix(calibrated cameras)

$$\boxed{\mathbf{X}' \cdot (\mathbf{T} \times \mathbf{R}\mathbf{X}) = 0}$$

$$\mathbf{X}' \cdot ([\mathbf{T}_\times]\mathbf{R}\mathbf{X}) = 0$$



Let $\mathbf{E} = [\mathbf{T}_\times]\mathbf{R}$

$$\mathbf{X}'^T\mathbf{E}\mathbf{X} = 0$$

- This holds for the rays p and p' that are parallel to the camera-centered position vectors X and X', so we have: $\boxed{\mathbf{p}'^T\mathbf{E}\mathbf{p} = 0}$
- $\mathbf{E}$ is called the essential matrix, which relates corresponding image points [Longuet-Higgins 1981]

Slide credit: Kristen Grauman

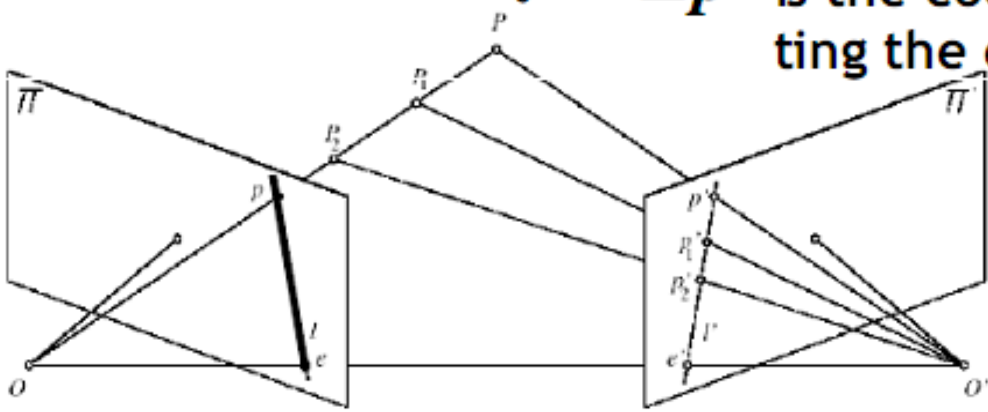
Essential Matrix and Epipolar Lines

$$\boxed{\mathbf{p}'^{\mathrm{T}}\mathbf{E}\mathbf{p} = 0}$$

Epipolar constraint: if we observe point p in one image, then its position p' in second image must satisfy this equation.

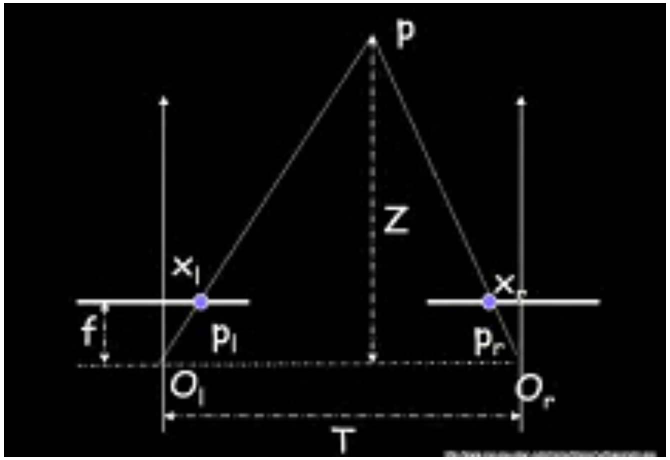
$l' = Ep$ is the coordinate vector representing the epipolar line for point p

(i.e., the line is given by: $l'^{\top}\mathbf{x} = 0$)



$l = E^T p'$ is the coordinate vector representing the epipolar line for point p'

Essential Matrix Example: Parallel Cameras



$$\mathbf{R} = \mathbf{I}$$

$$\mathbf{T} = [-d, 0, 0]^{\mathrm{T}}$$

$$\mathbf{E} = [\mathbf{T}_{\mathrm{x}}]\mathbf{R} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & d \\ 0 & -d & 0 \end{pmatrix}$$

$$\mathbf{p}'^{\mathrm{T}}\mathbf{E}\mathbf{p} = 0$$

$$\begin{bmatrix} x' & y' & f \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & d \\ 0 & -d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ f \end{bmatrix} = 0$$

$$\Leftrightarrow \begin{bmatrix} x' & y' & f \end{bmatrix} \begin{bmatrix} 0 \\ df \\ -dy \end{bmatrix} = 0$$

$$\boxed{\Leftrightarrow y = y'}$$

For the parallel cameras, image of any point must lie on same horizontal line in each image plane.

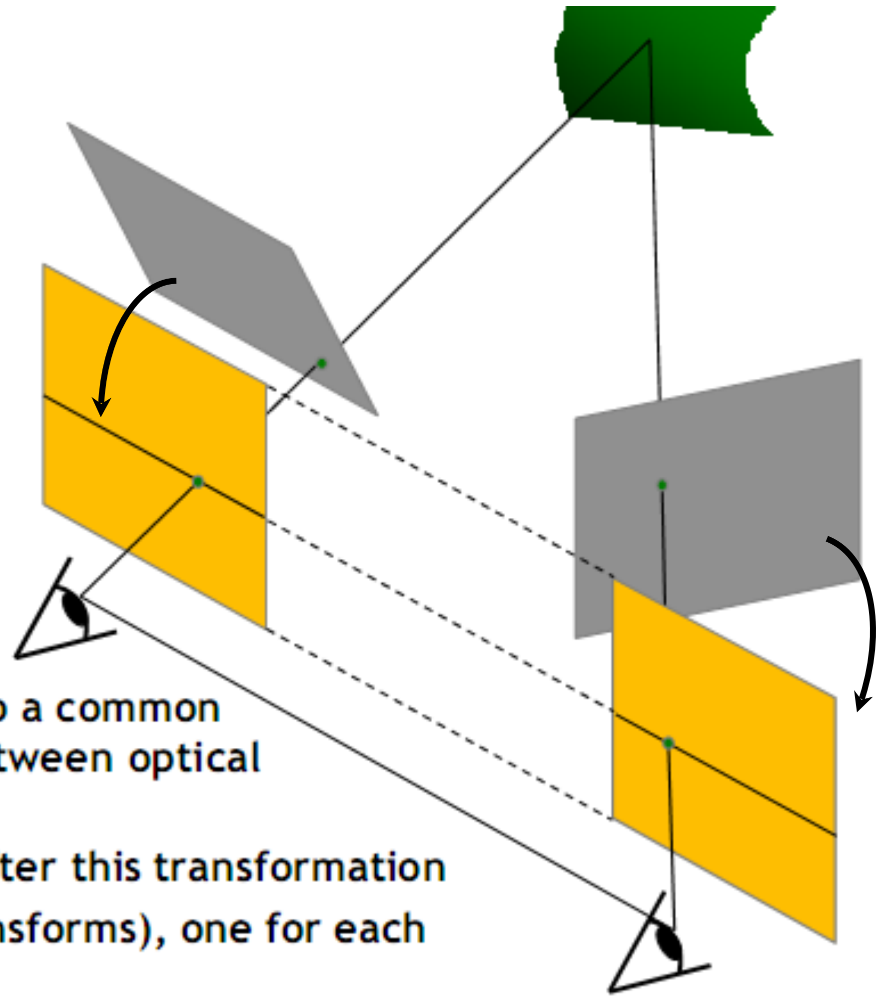
Image I(x,y) Disparity map D(x,y) Image I´(x´,y´)

$$(x', y') = (x + D(x,y),\ y)$$

- What about when cameras' optical axes are not parallel?

Stereo Image Rectification

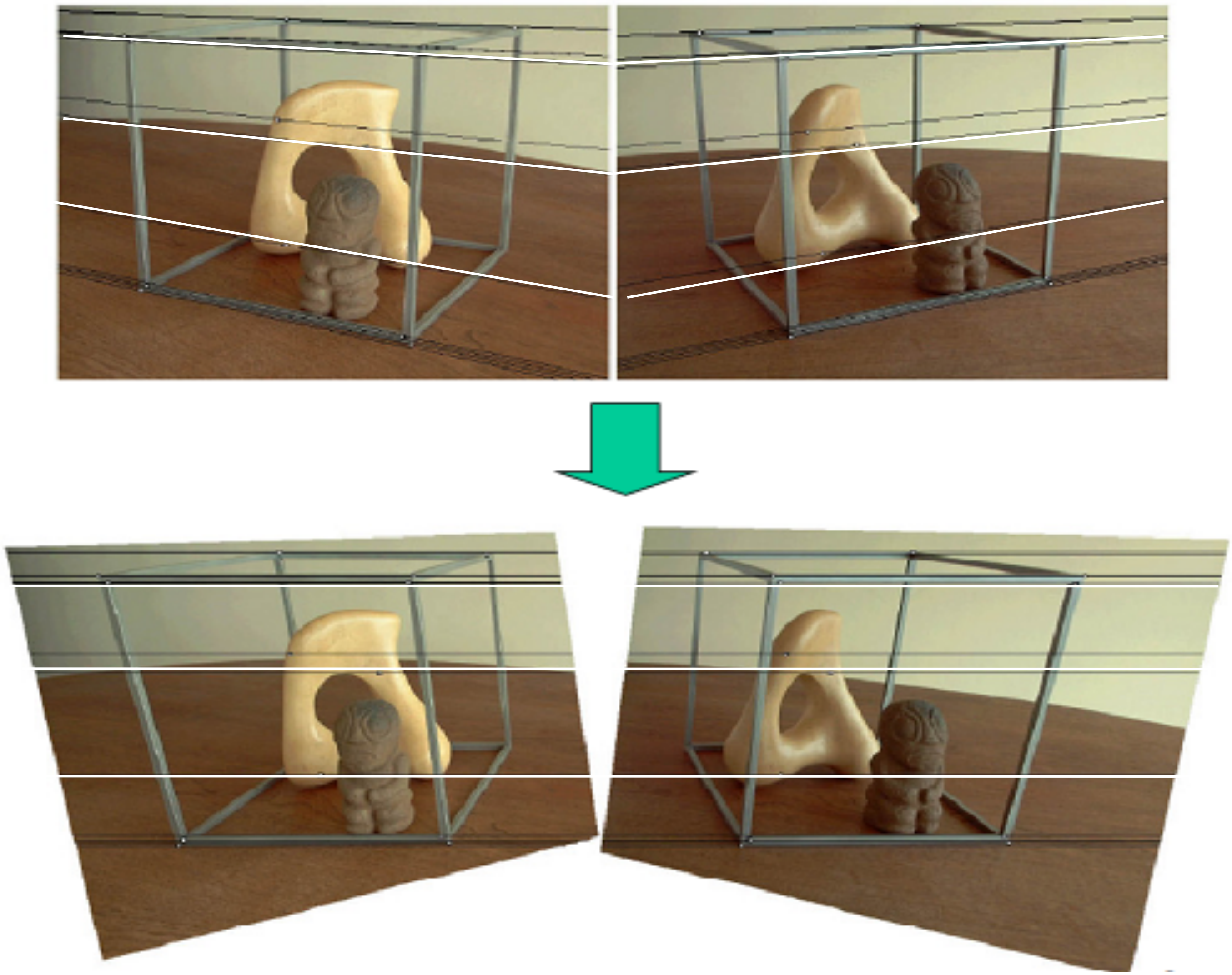
- In practice, it is convenient if image scanlines are the epipolar lines.



- Algorithm
  - Reproject image planes onto a common plane parallel to the line between optical centers
  - Pixel motion is horizontal after this transformation
  - Two homographies (3x3 transforms), one for each input image reprojection

Slide adapted from Li Zhang

Stereo Image Rectification: Example



Source: Alyosha Efros

Stereo Reconstruction

- Main Steps
  - Calibrate cameras
  - Rectify images
  - Compute disparity
  - Estimate depth

Slide credit: Kristen Grauman

Correspondence Search & Additional Constraints

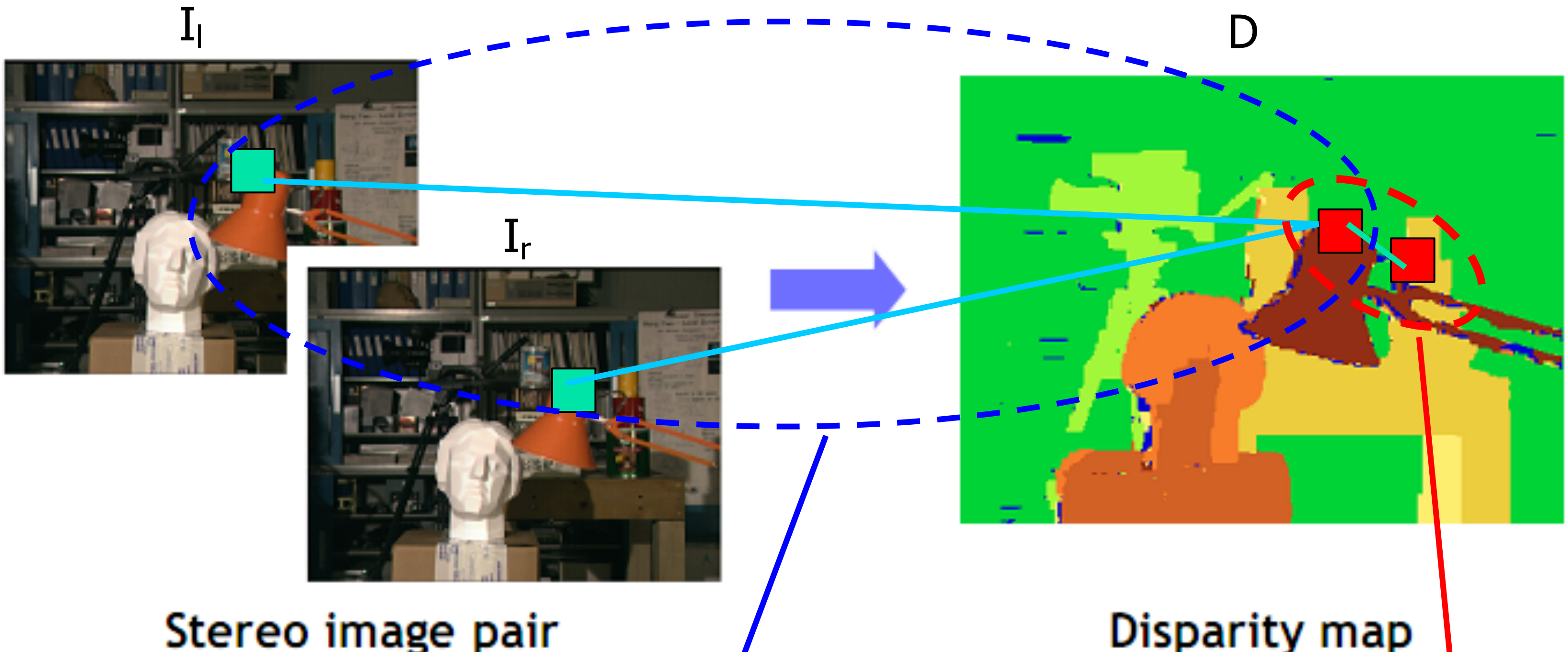
- Correspondence search on epipolar line

$$\mathbf{p}'\mathbf{E}\mathbf{p} = 0$$

- Additional correspondence constraints
  - Similarity
  - Uniqueness
  - Ordering
  - Disparity gradient

MRFs for disparity estimation

- **Stereo depth estimation**



$$E(D) = \sum_{\mathbf{p}} \boxed{C\left(I_l(\mathbf{p}), I_r(\mathbf{p}+D_{\mathbf{p}})\right)} + \sum_{(\mathbf{p},\mathbf{q})\in E} \boxed{S\left(D_{\mathbf{p}}, D_{\mathbf{q}}\right)}$$

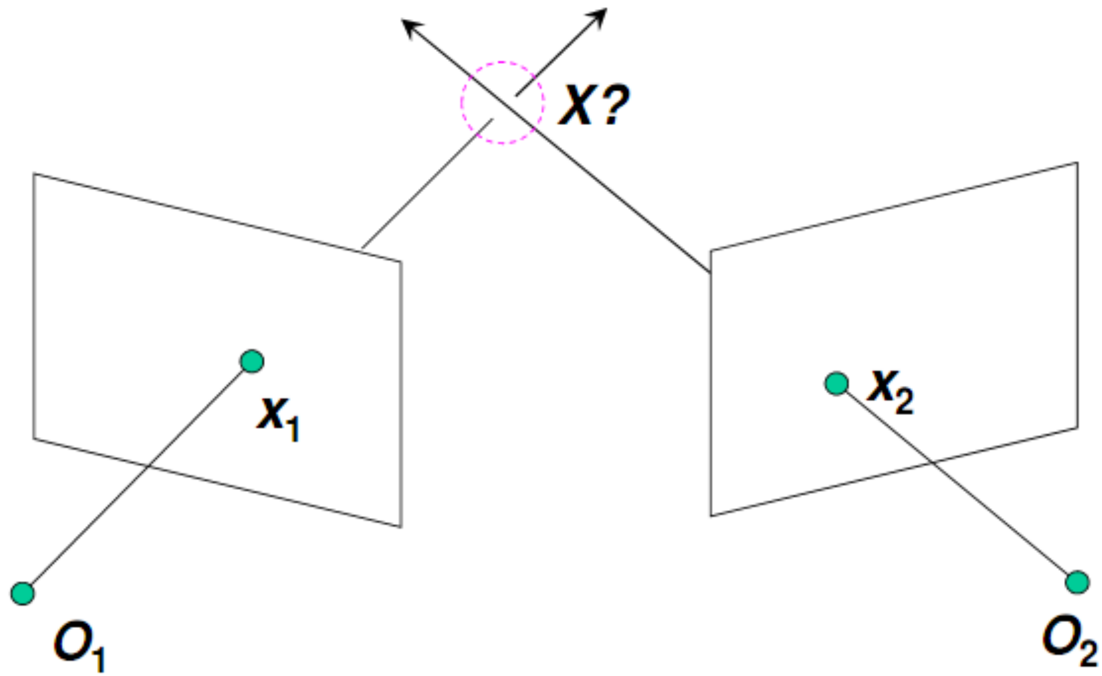
Content

- **3D reconstruction**
- Epipolar constrain (Uncalibrated cameras case)
- E/F Parameters estimation
- Robust estimation

3D reconstruction

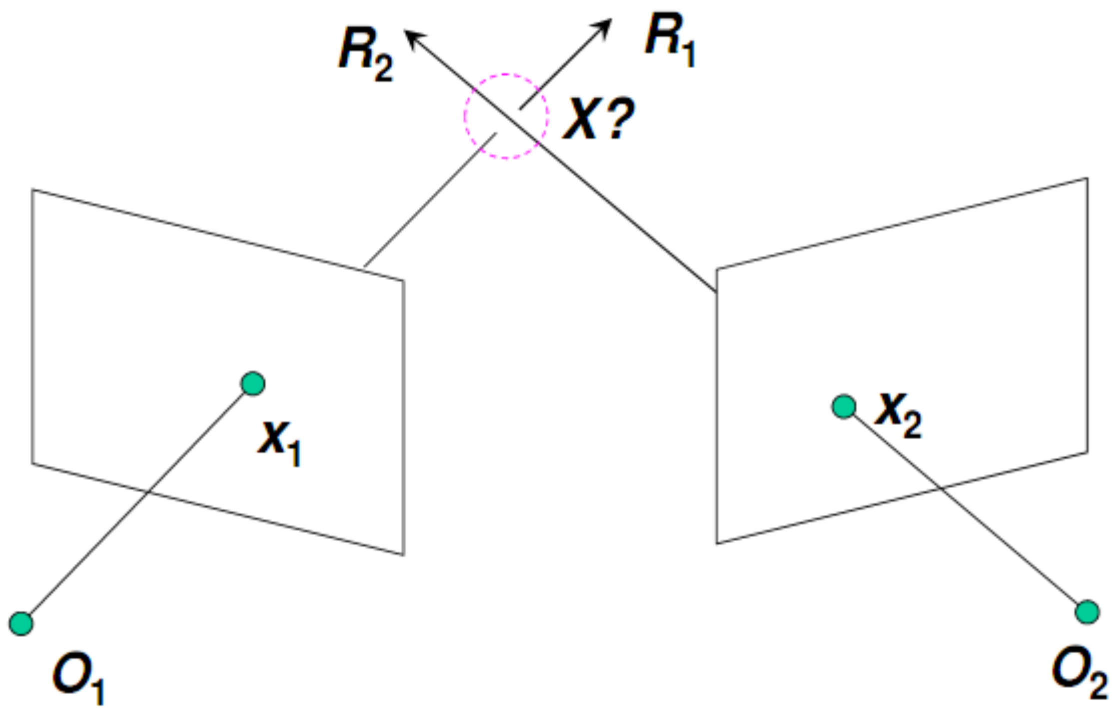
Revisiting Triangulation （3D）

- Given projections of a 3D point in two or more images (with known camera matrices), find the coordinates of the point



Revisiting Triangulation

- We want to intersect the two visual rays corresponding to x_1 and x_2, but because of noise and numerical errors, they will never meet exactly. How can this be done?



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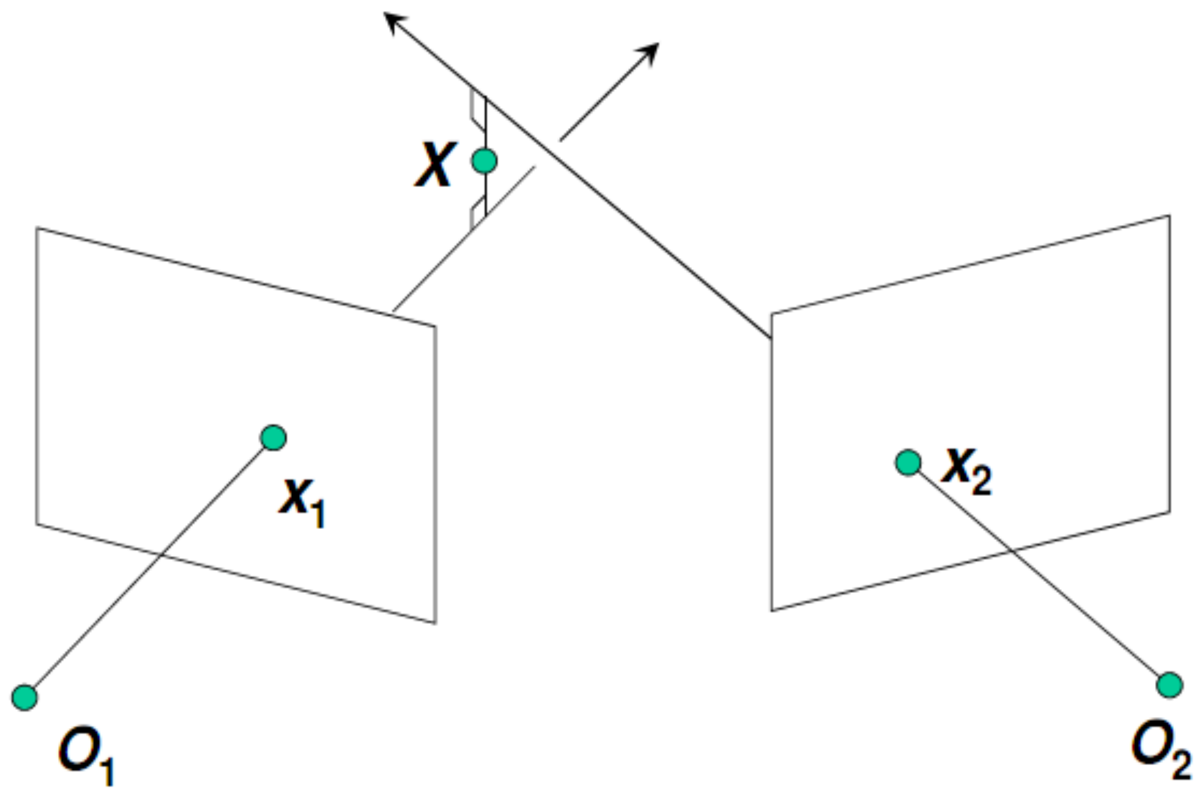
Triangulation: 1) Geometric Approach

- Find shortest segment connecting the two viewing rays and let X be the midpoint of that segment.



Triangulation: 2)Linear Algebraic Approach

$$\lambda_1 \mathrm{x}_1 = \mathrm{P}_1 \mathrm{X} \qquad \mathrm{x}_1 \times \mathrm{P}_1 \mathrm{X} = 0 \qquad [\mathrm{x}_{1\times}]\mathrm{P}_1 \boxed{\mathrm{X}} = 0$$

$$\lambda_2 \mathrm{x}_2 = \mathrm{P}_2 \mathrm{X} \qquad \mathrm{x}_2 \times \mathrm{P}_2 \mathrm{X} = 0 \qquad [\mathrm{x}_{2\times}]\mathrm{P}_2 \boxed{\mathrm{X}} = 0$$

Cross product as matrix multiplication:

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [\mathbf{a}_\times]\mathbf{b}$$

Slide credit: Svetlana Lazebnik

Triangulation: 2) Linear Algebraic Approach

$$\lambda_1 x_1 = P_1 X \qquad x_1 \times P_1 X = 0 \qquad [x_{1\times}]P_1 X = 0$$

$$\lambda_2 x_2 = P_2 X \qquad x_2 \times P_2 X = 0 \qquad [x_{2\times}]P_2 X = 0$$

Two independent equations each in terms of three unknown entries of X

$\Rightarrow$ Stack them and solve using SVD!

$$\boxed{\begin{aligned} &\mathbf{AX} = 0; \\ &\mathbf{A} = \begin{bmatrix} \mathbf{x}_{1\times}\mathbf{P}_1 \\ \mathbf{x}_{2\times}\mathbf{P}_2 \end{bmatrix} \end{aligned}}$$

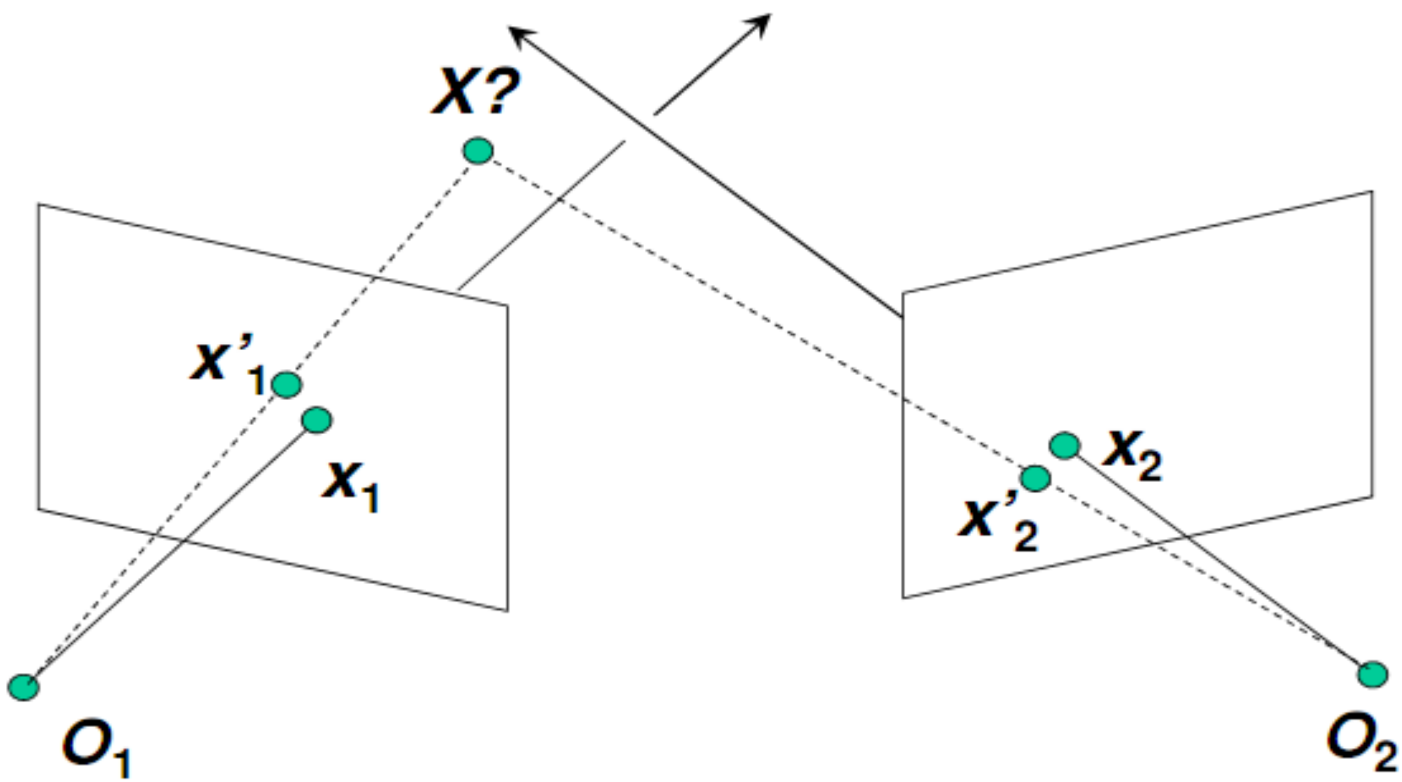
- This approach is often preferable to the geometric approach, since it nicely generalizes to multiple cameras.

Slide credit: Svetlana Lazebnik

Triangulation: 3) Nonlinear Approach

- **Find X that minimizes**

$$d^2(x_1, P_1X) + d^2(x_2, P_2X)$$



Triangulation: 3) Nonlinear Approach

- Find X that minimizes

$$d^2(x_1, P_1X) + d^2(x_2, P_2X)$$

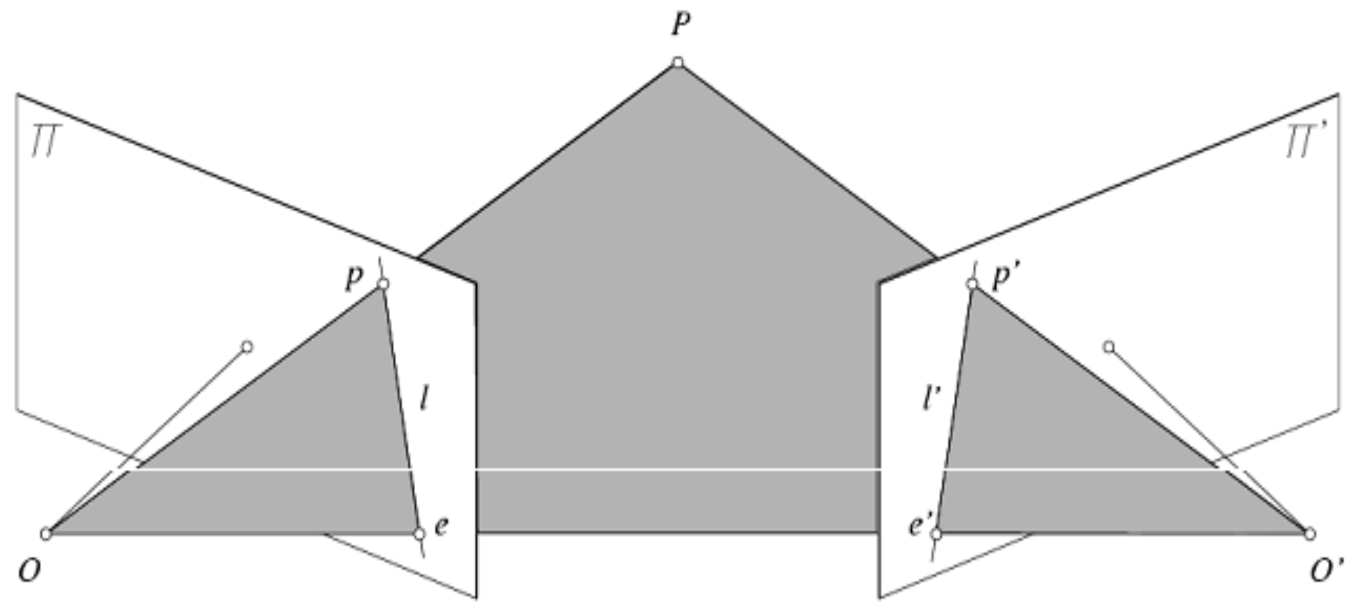
- This approach is the most accurate, but unlike the other two methods, it doesn't have a closed-form solution.

- Iterative algorithm
  - Initialize with linear estimate.
  - Optimize with $\boxed{\text{Gauss-Newton or Levenberg-Marquardt}}$ (see F&P sec. 3.1.2 or H&Z Appendix 6).

Content

- **3D reconstruction**
- Epipolar constrain (**Uncalibrated cameras case**)
- E/F Parameters estimation
- Robust estimation

Revisiting **Epipolar Geometry**



- Let's look again at the epipolar constraint
  - For the calibrated case (but in homogenous coordinates)
  - For the uncalibrated case

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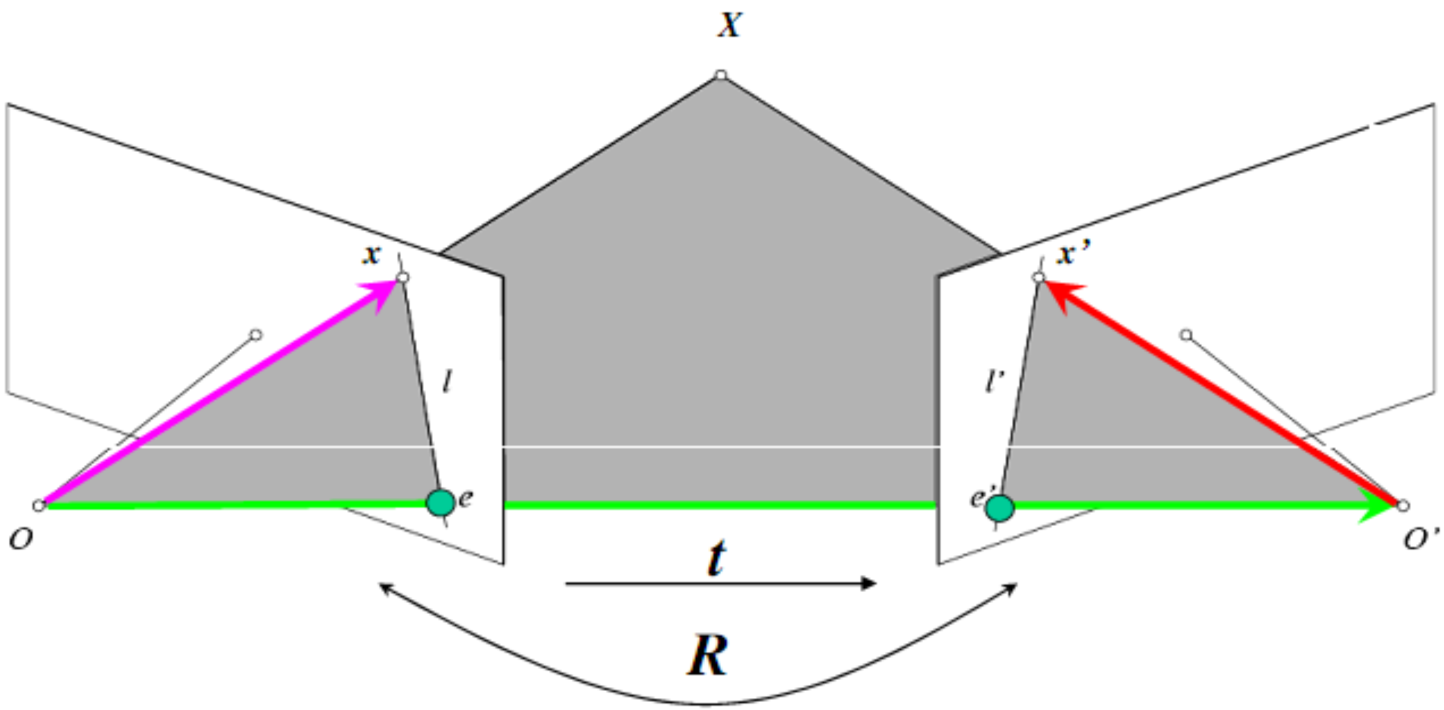
Epipolar Geometry: **Calibrated Case**

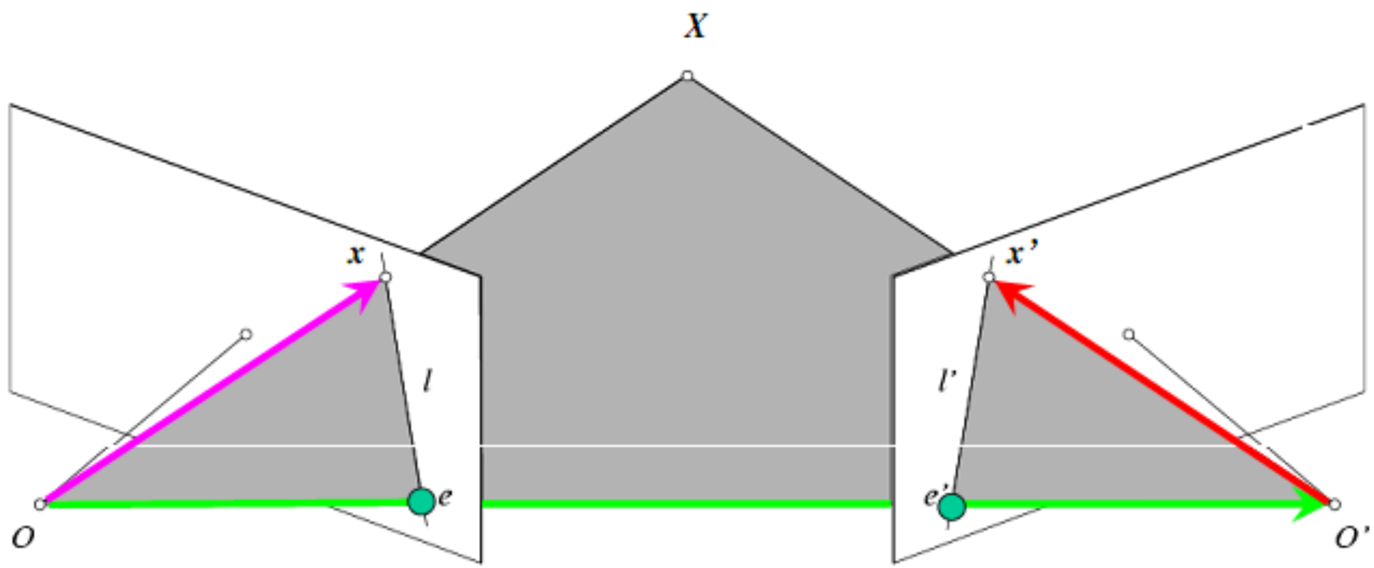


Camera matrix: $[\mathbf{I}|\mathbf{0}]$
$X = (u, v, w, 1)^T$
$x = (u, v, w)^T$

Camera matrix: $[R^T \mid -R^T t]$
Vector x' in second coord. system has coordinates Rx' in the first one.

The vectors x, t, and Rx' are coplanar

Epipolar Geometry: Calibrated Case



$$x \cdot [t \times (R x')] = 0 \quad \Rightarrow \quad x^T E x' = 0 \quad \text{with} \quad E = [t_\times] R$$

Essential Matrix (Longuet-Higgins, 1981)

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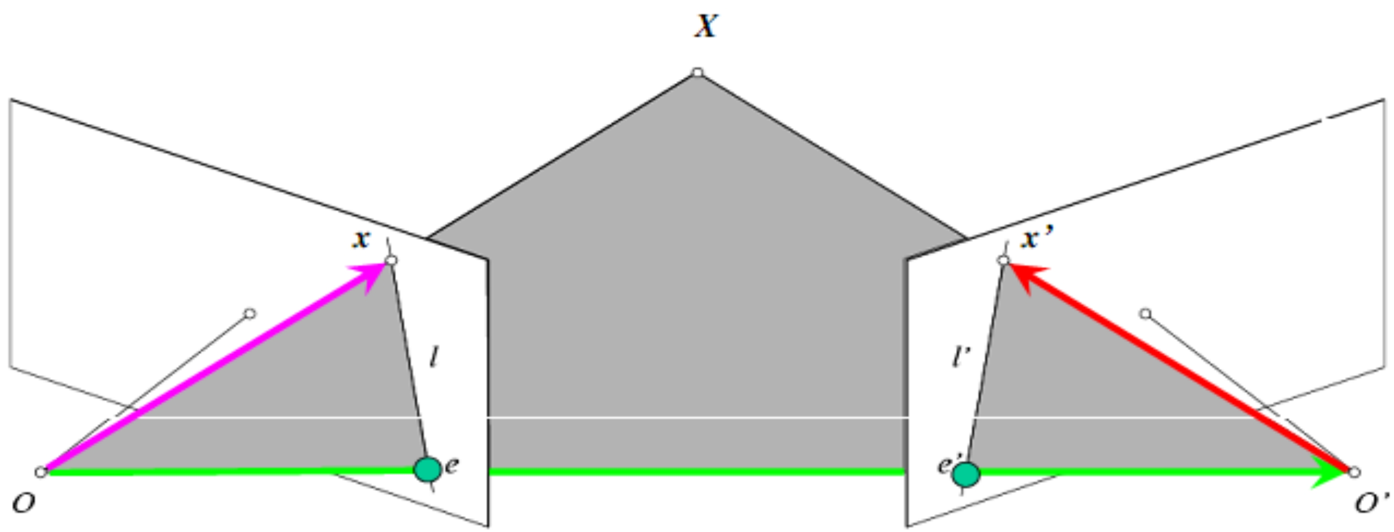
Epipolar Geometry: Calibrated Case



$$x \cdot [t \times (R x')] = 0 \quad \Rightarrow \quad x^T E x' = 0 \quad \text{with} \quad E = [t_\times]R$$

- $E x'$ is the epipolar line associated with x' ($l = E x'$)
- $E^T x$ is the epipolar line associated with x ($l' = E^T x$)
- $E e' = 0$ and $E^T e = 0$
- E is singular (rank two)
- E has five degrees of freedom (up to scale)

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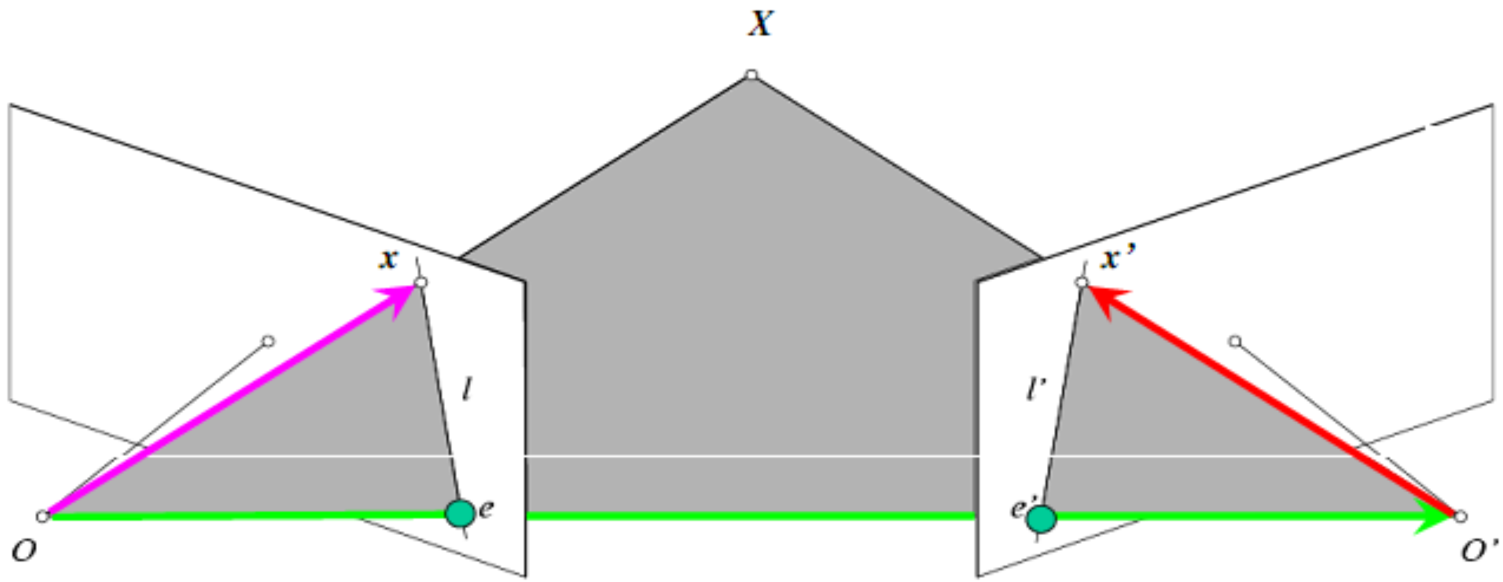
Epipolar Geometry: **Uncalibrated Case**



- The calibration matrices K and K' of the two cameras are unknown
- We can write the epipolar constraint in terms of *unknown* normalized coordinates:

$$\hat{x}^T E \hat{x}' = 0 \qquad x = K\hat{x}, \quad x' = K'\hat{x}'$$

? ?

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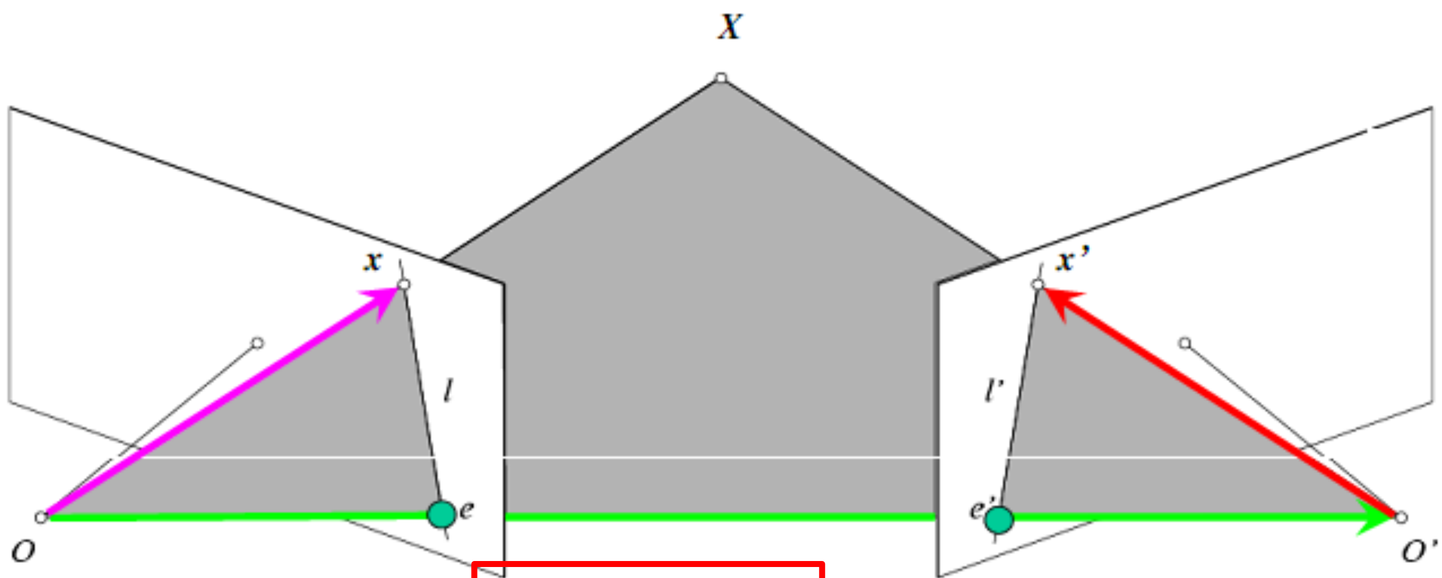
Epipolar Geometry: Uncalibrated Case



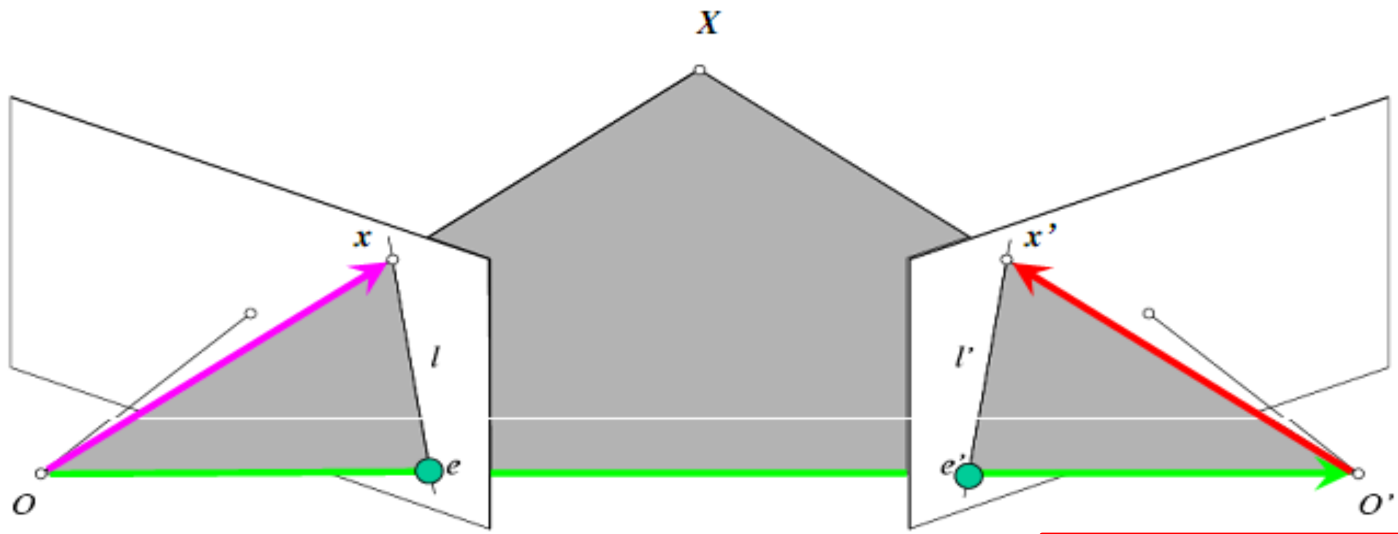
$$\hat{x}^T E \hat{x}' = 0 \quad \Rightarrow \quad \boxed{x^T F x' = 0} \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

$$x = K \hat{x}$$

$$x' = K' \hat{x}'$$

Fundamental Matrix
(Faugeras and Luong, 1992)

Epipolar Geometry: Uncalibrated Case



$$\hat{x}^T E \hat{x}' = 0 \quad \Rightarrow \quad x^T F x' = 0 \quad \text{with} \quad \boxed{F = K^{-T} E K'^{-1}}$$

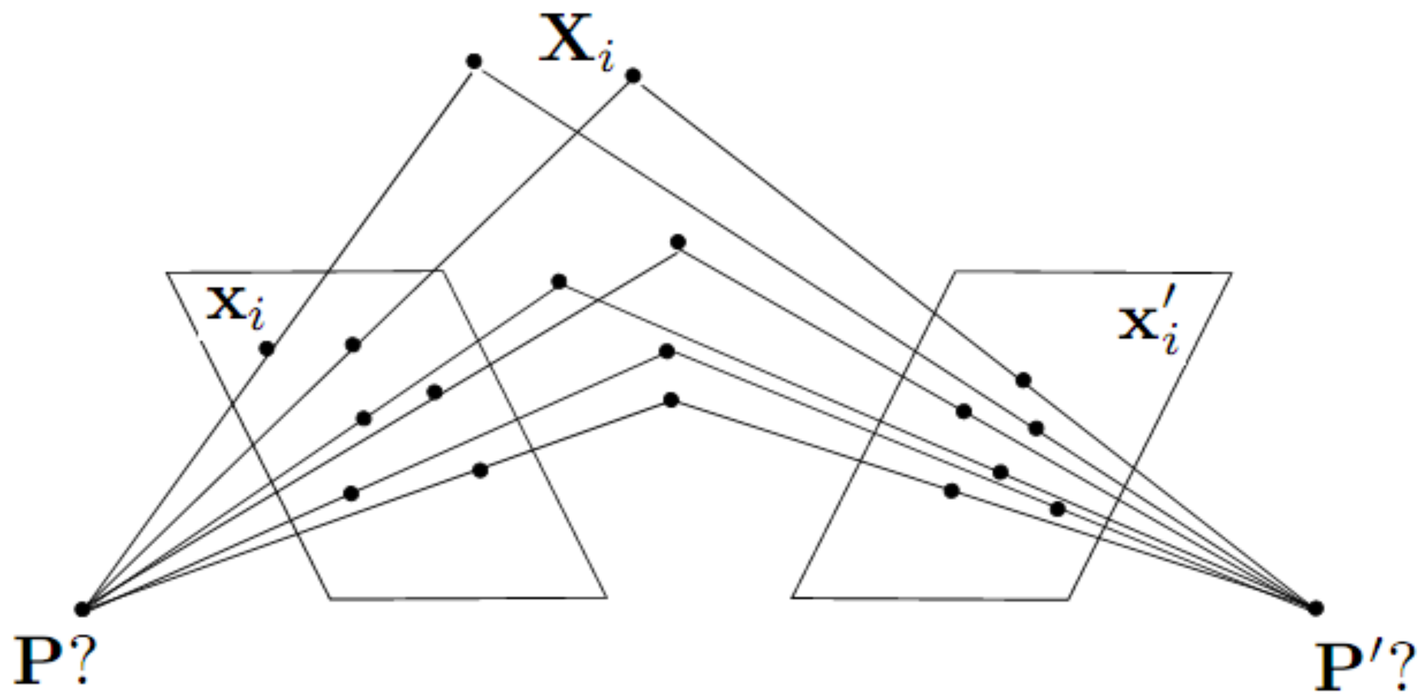
- *F* **x'** is the epipolar line associated with **x'** (*l* = *F* **x'**)
- F^T**x** is the epipolar line associated with **x** (*l'* = F^T**x**)
- *F* **e'** = 0 and F^T**e** = 0
- *F* is singular (rank two)
- *F* has seven degrees of freedom

Content

- **3D reconstruction**
- Epipolar constrain (Uncalibrated cameras case)
- E/F Parameters estimation
- Robust estimation

Estimating the Fundamental Matrix

- The Fundamental matrix defines the epipolar geometry between two uncalibrated cameras.
- How can we estimate F from an image pair?
  - We need correspondences…



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The Eight-Point Algorithm

$\boldsymbol{x} = (u, v, 1)^T, \quad \boldsymbol{x}' = (u', v', 1)^T$

$$(u, v, 1)\begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix}\begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \quad \Rightarrow \quad [u'u, u'v, u', uv', vv', v', u, v, 1]\begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

$$\begin{bmatrix} u_1'u_1 & u_1'v_1 & u_1' & u_1v_1' & v_1v_1' & v_1' & u_1 & v_1 & 1 \\ u_2'u_2 & u_2'v_2 & u_2' & u_2v_2' & v_2v_2' & v_2' & u_2 & v_2 & 1 \\ u_3'u_3 & u_3'v_3 & u_3' & u_3v_3' & v_3v_3' & v_3' & u_3 & v_3 & 1 \\ u_4'u_4 & u_4'v_4 & u_4' & u_4v_4' & v_4v_4' & v_4' & u_4 & v_4 & 1 \\ u_5'u_5 & u_5'v_5 & u_5' & u_5v_5' & v_5v_5' & v_5' & u_5 & v_5 & 1 \\ u_6'u_6 & u_6'v_6 & u_6' & u_6v_6' & v_6v_6' & v_6' & u_6 & v_6 & 1 \\ u_7'u_7 & u_7'v_7 & u_7' & u_7v_7' & v_7v_7' & v_7' & u_7 & v_7 & 1 \\ u_8'u_8 & u_8'v_8 & u_8' & u_8v_8' & v_8v_8' & v_8' & u_8 & v_8 & 1 \end{bmatrix}\begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{A}\mathbf{f} = 0$$

Solve using... SVD!

This minimizes:

$$\sum_{i=1}^{N}(x_i^T F x_i')^2$$

1981, H. C. Longuet-Higgins (Nature 293:133-135),

Slide credit: Svetlana Lazebnik

Properties of SVD

- Frobenius norm
  - Generalization of the Euclidean norm to matrices

$$\|A\|_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}|a_{ij}|^2} = \sqrt{\sum_{i=1}^{\min(m,n)} \sigma_i^2}$$

- Partial reconstruction property of SVD

$$A = UDV^T$$

  - Let σ_i $i=1,\ldots,N$ be the singular values of A.
  - Let $A_p = U_p D_p V_p^T$ be the reconstruction of A when we set $\sigma_{p+1}, \ldots, \sigma_N$ to zero.
  - Then $A_p = U_p D_p V_p^T$ is the best rank-p approximation of A in the sense of the Frobenius norm

(i.e. the best least-squares approximation).

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The Eight-Point Algorithm

- Problem with noisy data
  - The solution will usually not fulfill the constraint that F only has rank 2.

  $\Rightarrow$ *There will be no epipoles through which all epipolar lines pass!*

- Enforce the rank-2 constraint using SVD

$$F \overset{\text{SVD}}{=} UDV^T = U \begin{bmatrix} d_{11} & & \\ & d_{22} & \\ & & d_{33} \end{bmatrix} \begin{bmatrix} v_{11} & \cdots & v_{13} \\ \vdots & \ddots & \vdots \\ v_{31} & \cdots & v_{33} \end{bmatrix}^T$$

Set d_{33} to zero and reconstruct F

- As we have just seen, this provides the best least-squares approximation to the rank-2 solution.

1981, H. C. Longuet-Higgins (Nature 293:133-135),

Problem with the Eight-Point Algorithm

- In practice, this often looks as follows:

$$\begin{bmatrix} u_1'u_1 & u_1'v_1 & u_1' & u_1v_1' & v_1v_1' & v_1' & u_1 & v_1 & 1 \\ u_2'u_2 & u_2'v_2 & u_2' & u_2v_2' & v_2v_2' & v_2' & u_2 & v_2 & 1 \\ u_3'u_3 & u_3'v_3 & u_3' & u_3v_3' & v_3v_3' & v_3' & u_3 & v_3 & 1 \\ u_4'u_4 & u_4'v_4 & u_4' & u_4v_4' & v_4v_4' & v_4' & u_4 & v_4 & 1 \\ u_5'u_5 & u_5'v_5 & u_5' & u_5v_5' & v_5v_5' & v_5' & u_5 & v_5 & 1 \\ u_6'u_6 & u_6'v_6 & u_6' & u_6v_6' & v_6v_6' & v_6' & u_6 & v_6 & 1 \\ u_7'u_7 & u_7'v_7 & u_7' & u_7v_7' & v_7v_7' & v_7' & u_7 & v_7 & 1 \\ u_8'u_8 & u_8'v_8 & u_8' & u_8v_8' & v_8v_8' & v_8' & u_8 & v_8 & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Longuet Higgins 的**8**点算法，是一种线性算法**,** 对噪声太敏感**,**实用性不大。

Problem with the Eight-Point Algorithm

- In practice, this often looks as follows:

$$
\begin{bmatrix}
250906.36 & 183269.57 & 921.81 & 200931.10 & 146766.13 & 738.21 & 272.19 & 198.81 & 1 \\
2692.28 & 131633.03 & 176.27 & 6196.73 & 302975.59 & 405.71 & 15.27 & 746.79 & 1 \\
416374.23 & 871684.30 & 935.47 & 408110.89 & 854384.92 & 916.90 & 445.10 & 931.81 & 1 \\
191183.60 & 171759.40 & 410.27 & 416435.62 & 374125.90 & 893.65 & 465.99 & 418.65 & 1 \\
48988.86 & 30401.76 & 57.89 & 298604.57 & 185309.58 & 352.87 & 846.22 & 525.15 & 1 \\
164786.04 & 546559.67 & 813.17 & 1998.37 & 6628.15 & 9.86 & 202.65 & 672.14 & 1 \\
116407.01 & 2727.75 & 138.89 & 169941.27 & 3982.21 & 202.77 & 838.12 & 19.64 & 1 \\
135384.58 & 75411.13 & 198.72 & 411350.03 & 229127.78 & 603.79 & 681.28 & 379.48 & 1
\end{bmatrix}
\begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

$\Rightarrow$ Poor numerical conditioning

$\Rightarrow$ Can be fixed by rescaling the data

The Normalized Eight-Point Algorithm

1. Center the image data at the origin, and scale it so the mean squared distance between the origin and the data points is 2 pixels.
2. Use the eight-point algorithm to compute *F* from the normalized points.
3. Enforce the rank-2 constraint using SVD.

$$F \overset{\text{SVD}}{=} UDV^T = U\begin{bmatrix} d_{11} & & \\ & d_{22} & \\ & & d_{33} \end{bmatrix}\begin{bmatrix} v_{11} & \cdots & v_{13} \\ \vdots & \ddots & \vdots \\ v_{31} & \cdots & v_{33} \end{bmatrix}^T$$

Set d_{33} to zero and reconstruct F

4. Transform fundamental matrix back to original units: if T and T' are the normalizing transformations in the two images, than the fundamental matrix in original coordinates is $T^T F T'$.

[R. Hartley, 1995]

The Eight-Point Algorithm

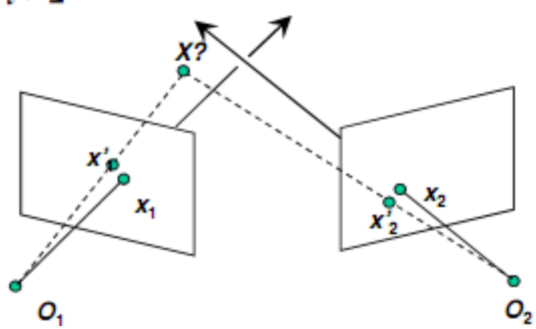
- **Meaning of error** $\sum_{i=1}^{N}(x_i^T F x_i')^2$:

  Sum of Euclidean distances between points x_i and epipolar lines Fx'_i (or points x'_i and epipolar lines $F^T x_i$), multiplied by a scale factor

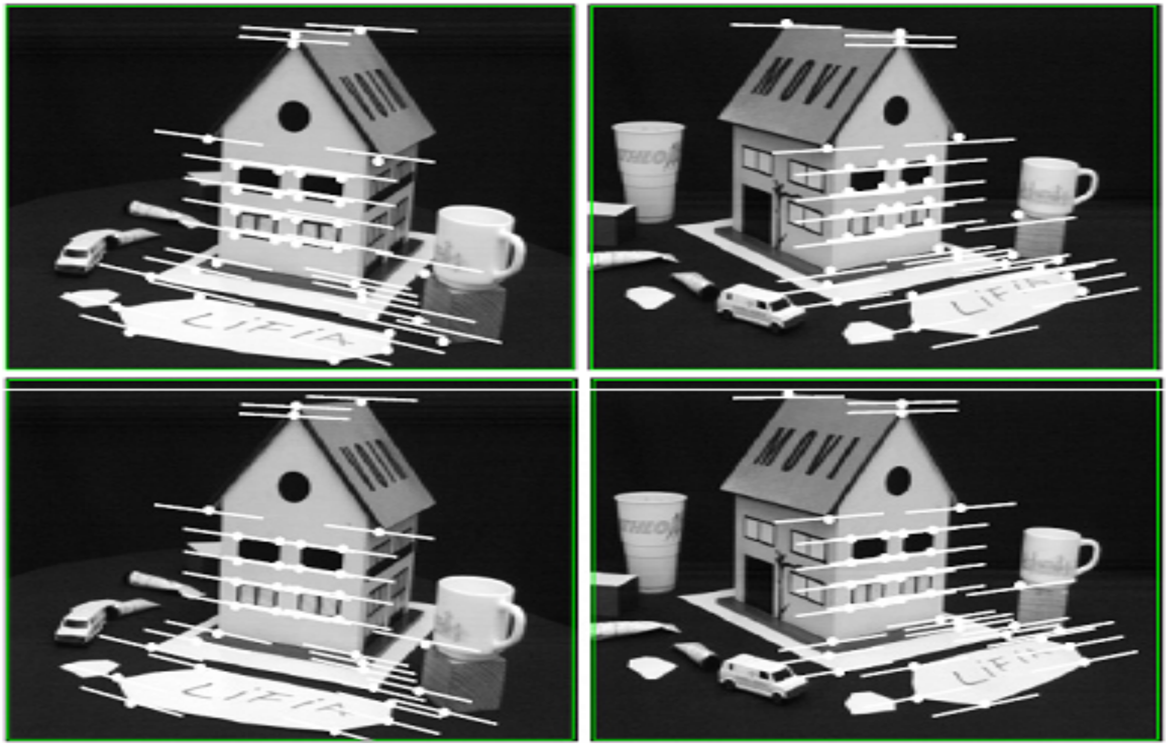
- **Nonlinear approach: minimize**

$$\sum_{i=1}^{N}\left[\mathrm{d}^2(x_i, F x_i') + \mathrm{d}^2(x_i', F^T x_i)\right]$$

  - **Similar to nonlinear minimization approach for triangulation.**
  - **Iterative approach (Gauss-Newton, Levenberg-Marquardt,…)**



Comparison of Estimation Algorithms



	8-point	Normalized 8-point	Nonlinear least squares
Av. Dist. 1	2.33 pixels	0.92 pixel	0.86 pixel
Av. Dist. 2	2.18 pixels	0.85 pixel	0.80 pixel

Slide credit: Svetlana Lazebnik

Using fewer than eight points

- F matrix (7 DoF)
- E matrix (5 DoF)

◆F有7个自由度，本质上7对对应点可以求F矩阵（结合1个约束），非线性解（多解）；

◆E有5个自由度， 本质上5对对应点可以求E矩阵（结合3个约束），非线性解（多解）；

3D Reconstruction with Weak Calibration

- Want to estimate world geometry without requiring calibrated cameras.
- Many applications:
  - Archival videos
  - Photos from multiple unrelated users
  - Dynamic camera system
- Main idea:
  - Estimate epipolar geometry from a (redundant) set of point correspondences between two uncalibrated cameras.

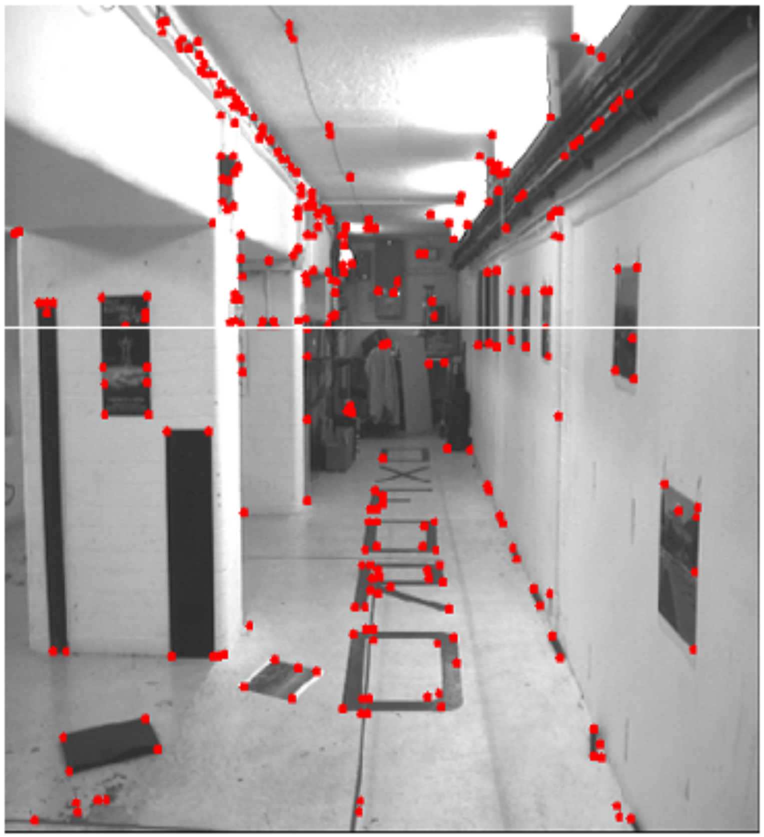
Stereo Pipeline with Weak Calibration

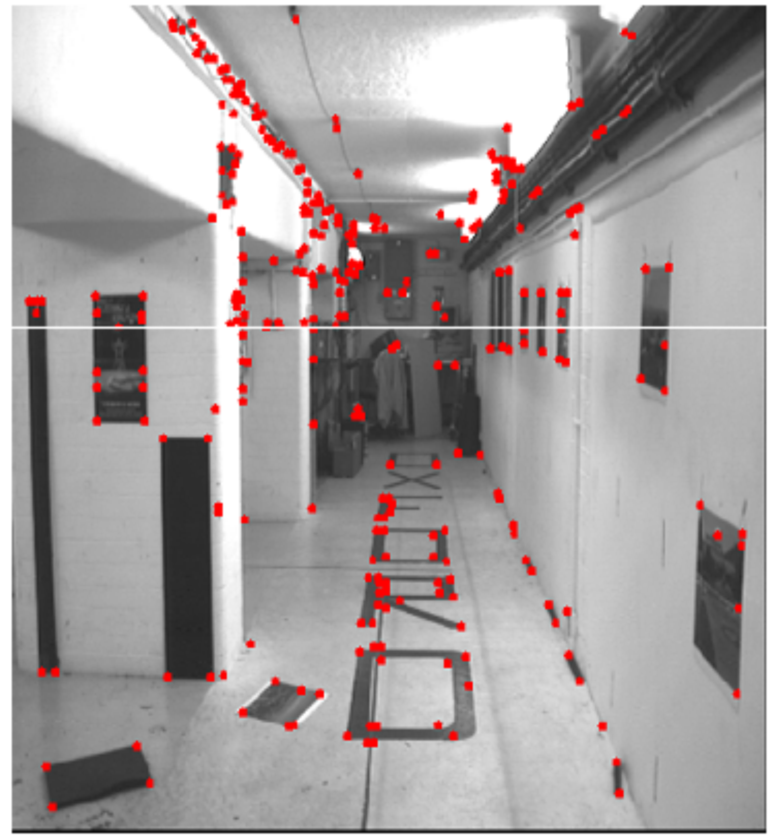
- So, where to start with uncalibrated cameras?
  - Need to find fundamental matrix F *and* the correspondences (pairs of points (u',v') ↔ (u,v)).

- Procedure
  1. Find interest points in both images
  2. Compute correspondences
  3. Compute epipolar geometry
  4. Refine

Stereo Pipeline with Weak Calibration

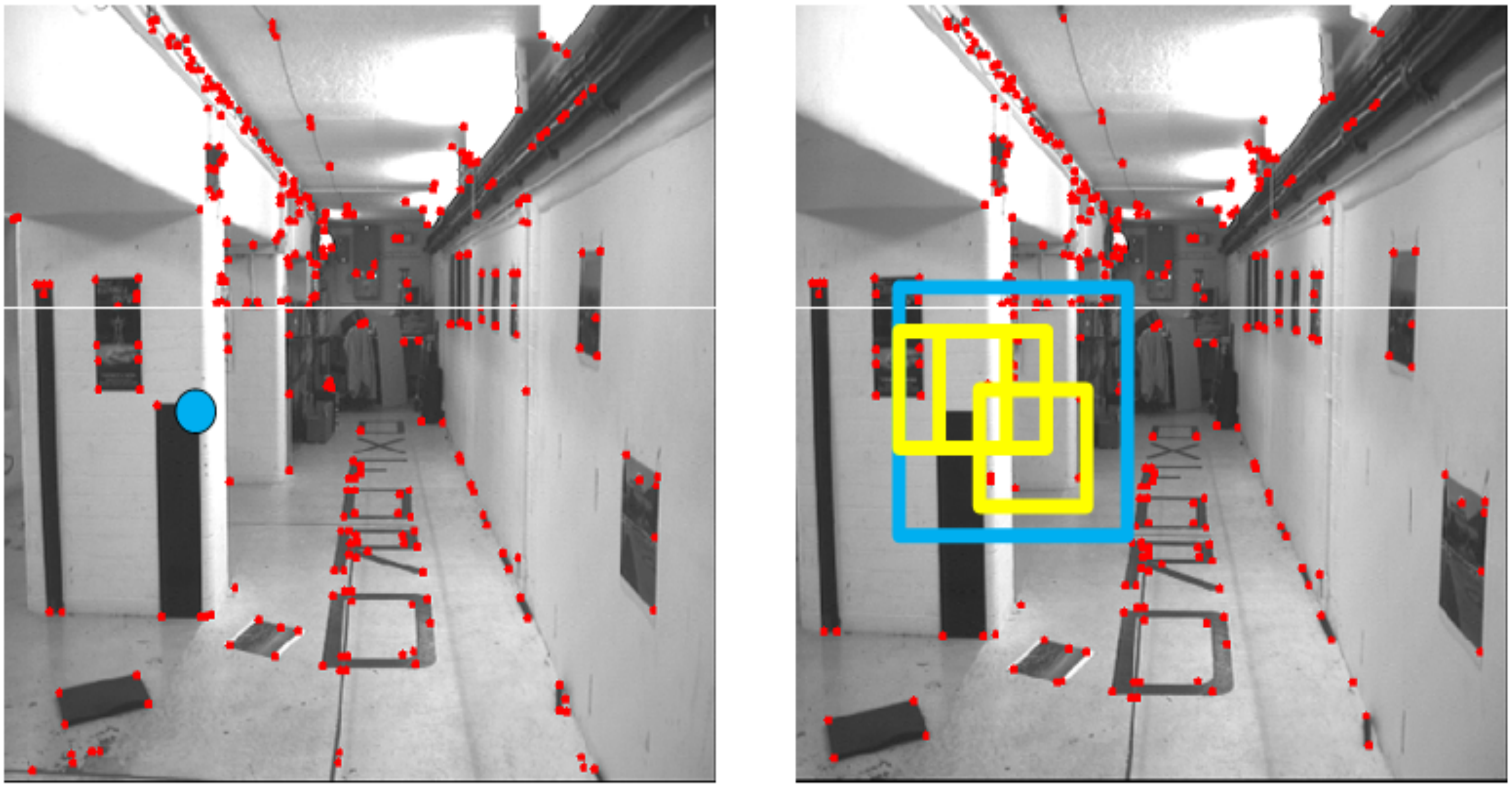
1. Find interest points (e.g. Harris corners)



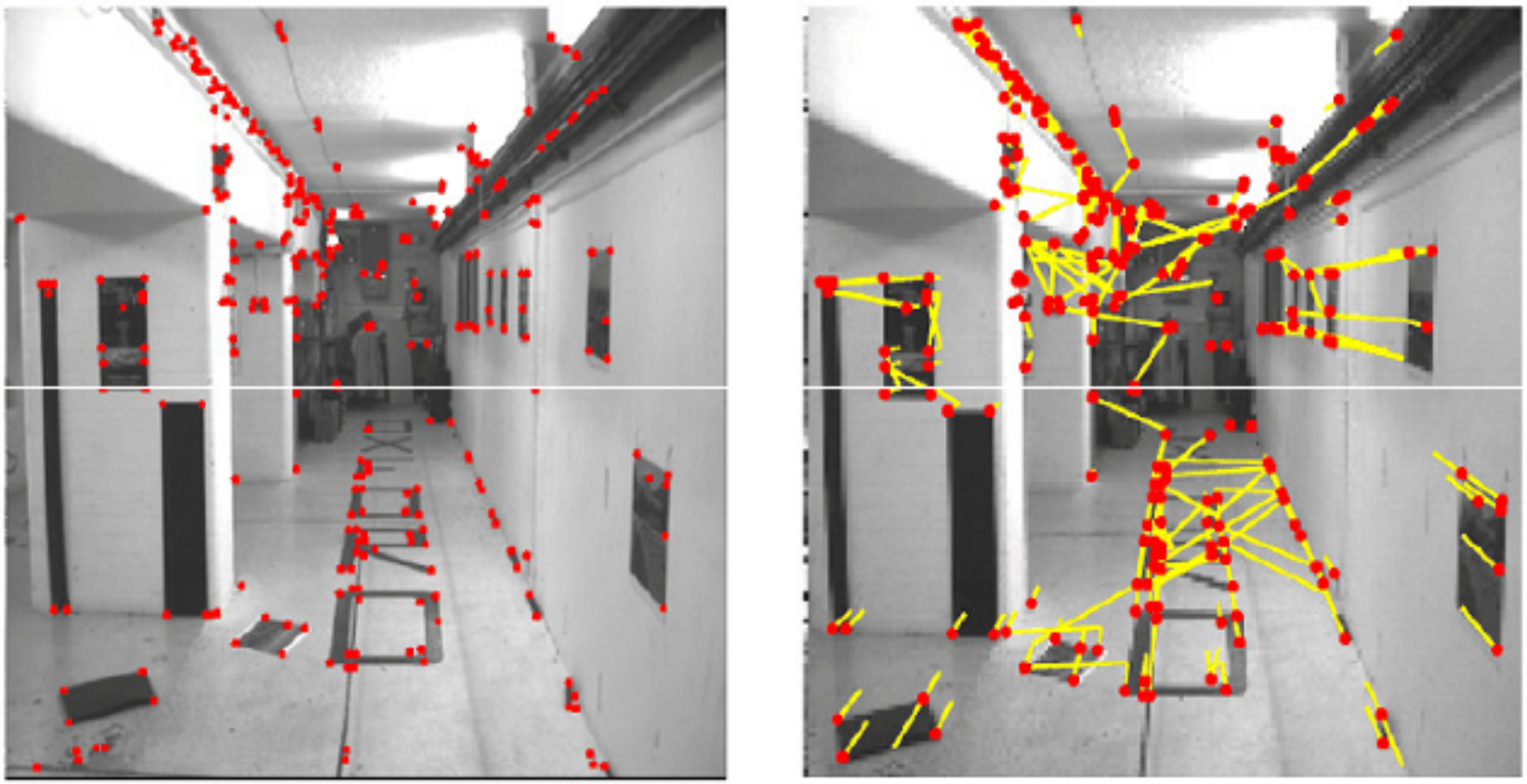


Stereo Pipeline with Weak Calibration

2. Match points using only proximity



Putative Matches based on Correlation Search



- Many wrong matches (10-50%), but enough to compute F

Content

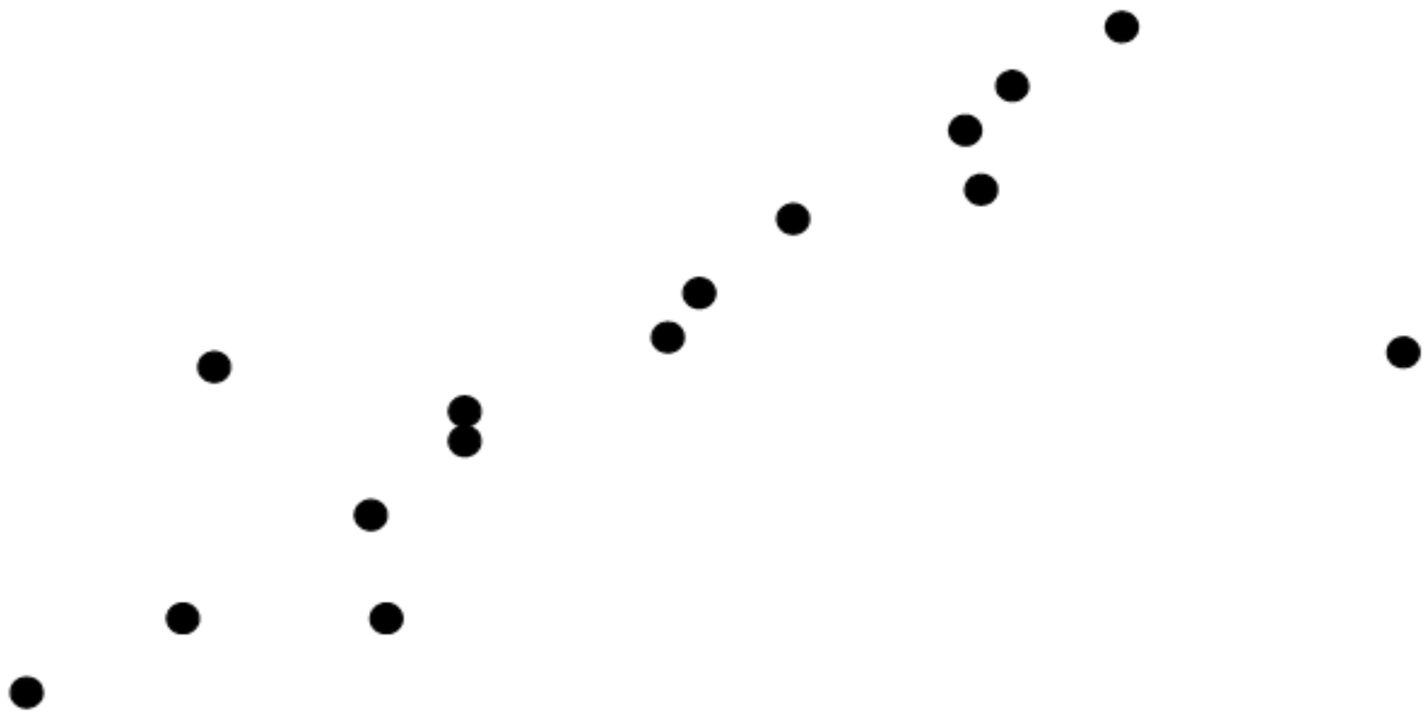
- 3D reconstruction
- Epipolar constrain (Uncalibrated cameras case)
- E/F Parameters estimation
- **Robust estimation (RANSAC)**

RANSAC for Robust Estimation of F

- Select random sample of correspondences
- Compute F using them
  - This determines epipolar constraint
- Evaluate amount of support - number of inliers within threshold distance of epipolar line
- Choose F with most support (#inliers)

RANSAC Line Fitting Example

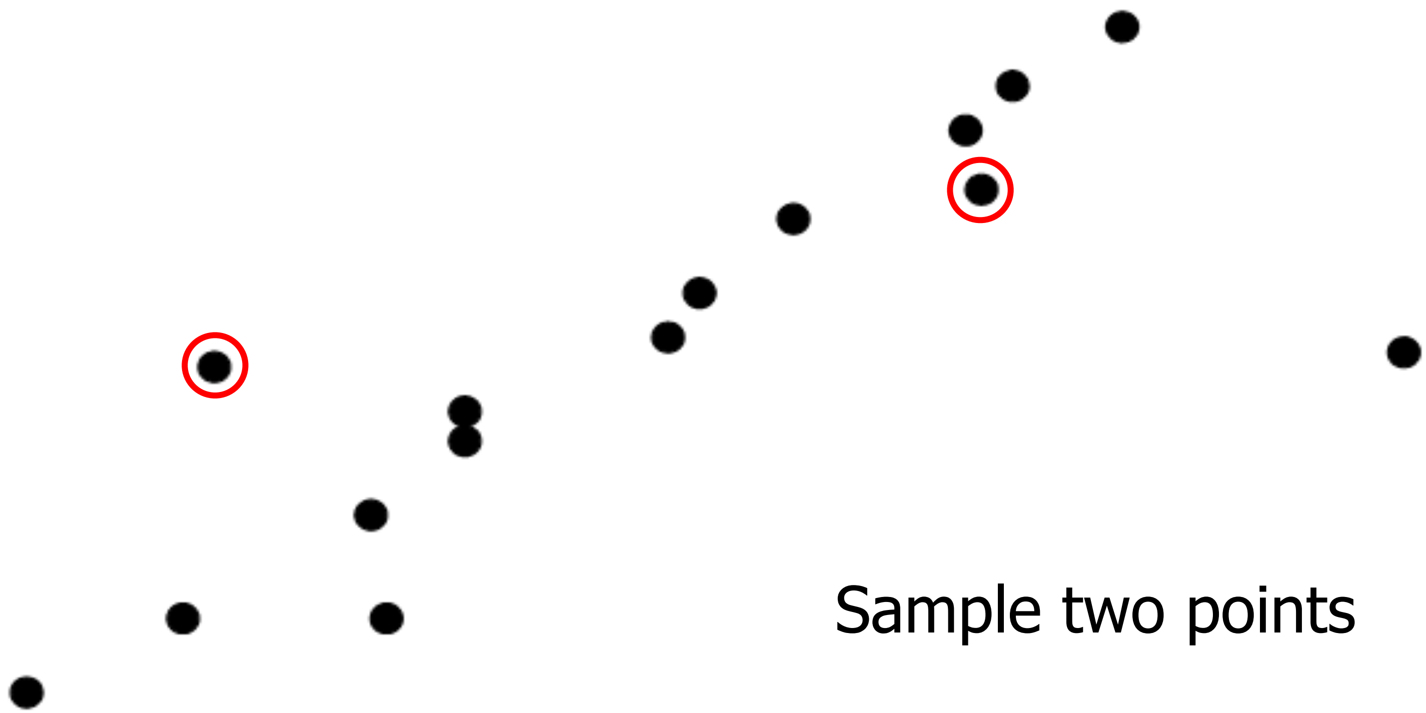
- Task: Estimate the best line



Slide credit: Jinxiang Chai

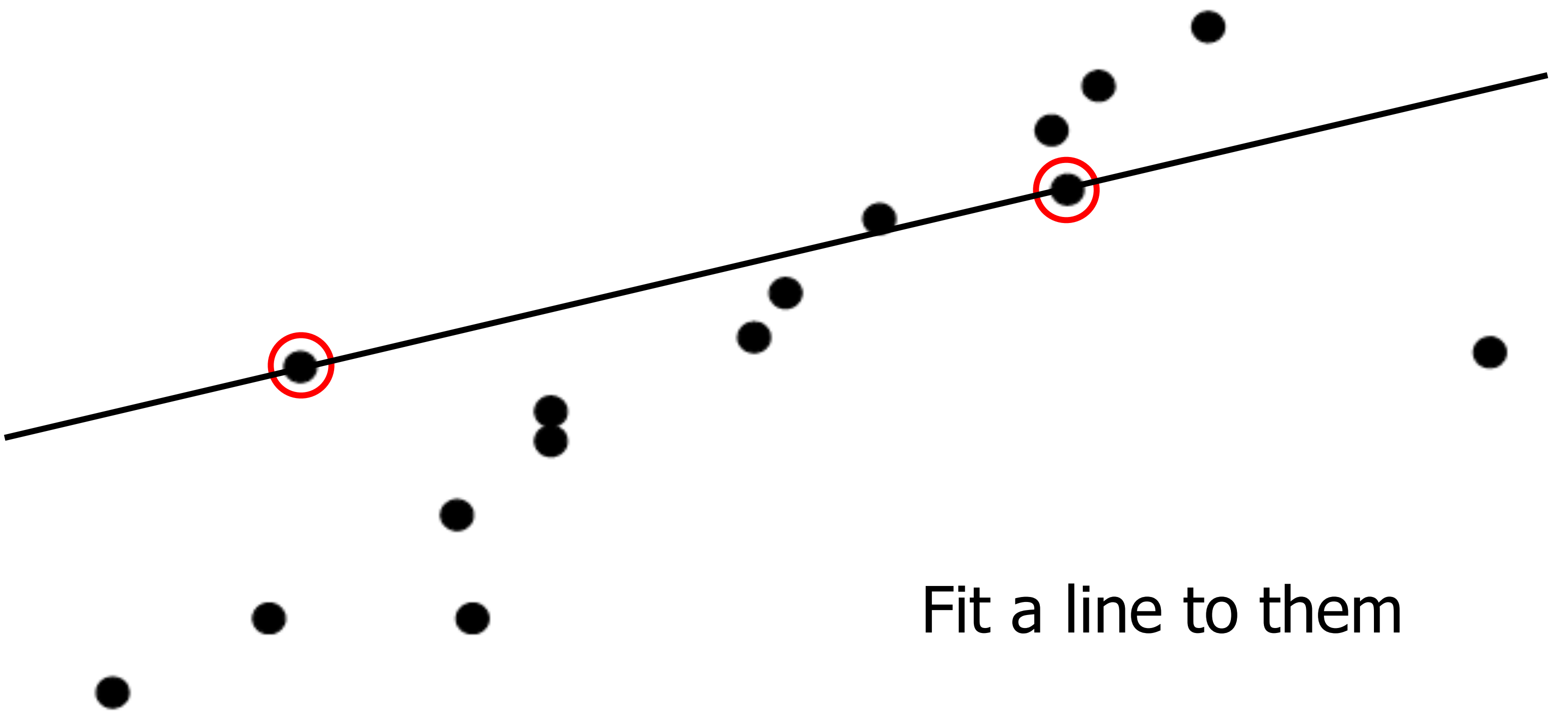
RANSAC Line Fitting Example

- **Task: Estimate the best line**



RANSAC Line Fitting Example

- **Task: Estimate the best line**

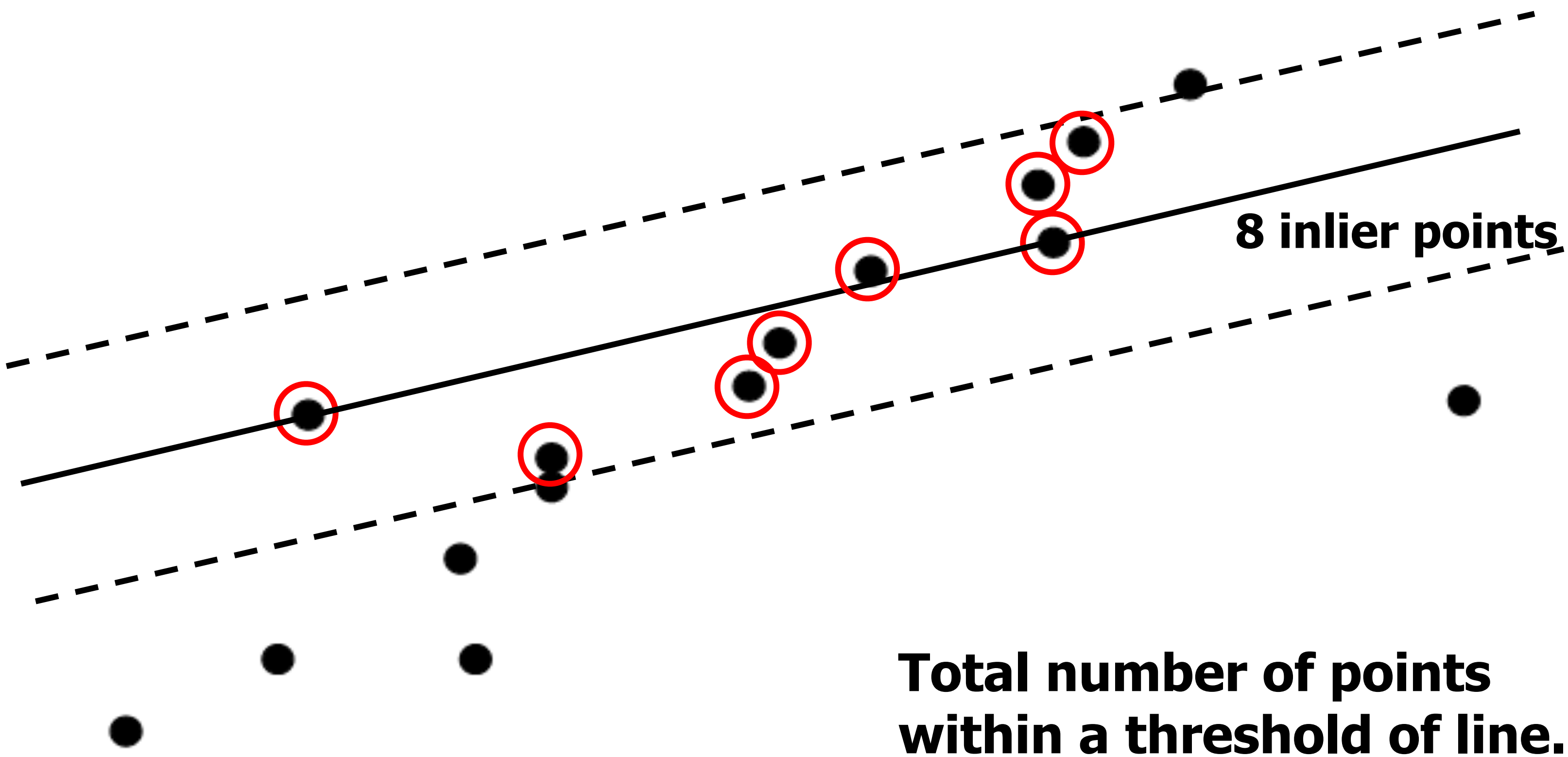


Fit a line to them

Slide credit: Jinxiang Chai

RANSAC Line Fitting Example

- Task: Estimate the best line

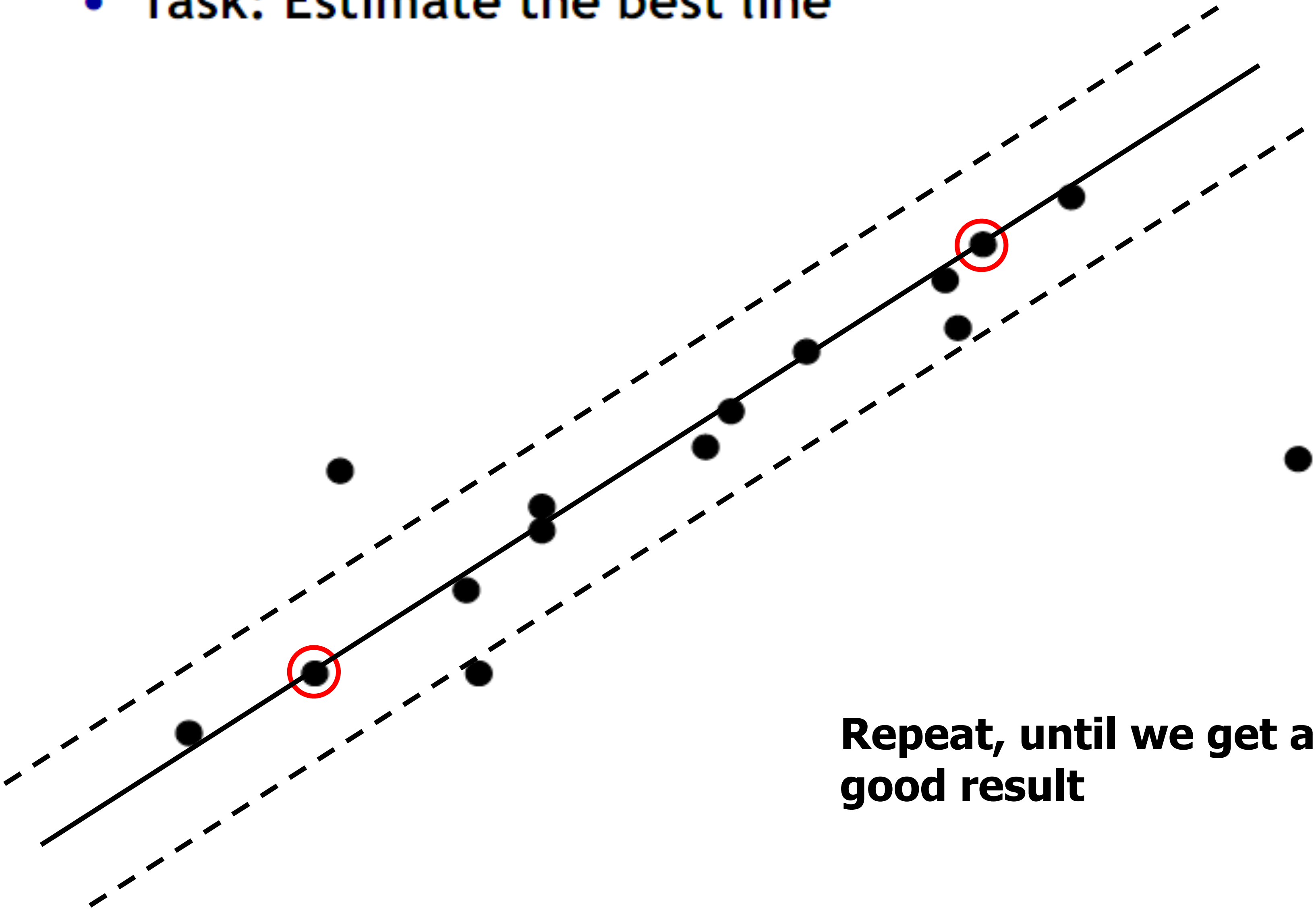


Total number of points within a threshold of line.

Slide credit: Jinxiang Chai

RANSAC Line Fitting Example

- Task: Estimate the best line



Slide credit: Jinxiang Chai

RANSAC: How many samples?

- How many samples are needed?
  - Suppose w is fraction of inliers (points from line).
  - n points needed to define hypothesis (2 for lines)
  - k samples chosen.
- Prob. that a single sample of n points is correct: w^n
- Prob. that all k samples fail is: $(1-w^n)^k$

$\Rightarrow$ Choose k high enough to keep this below desired failure rate.

Slide credit: David Lowe

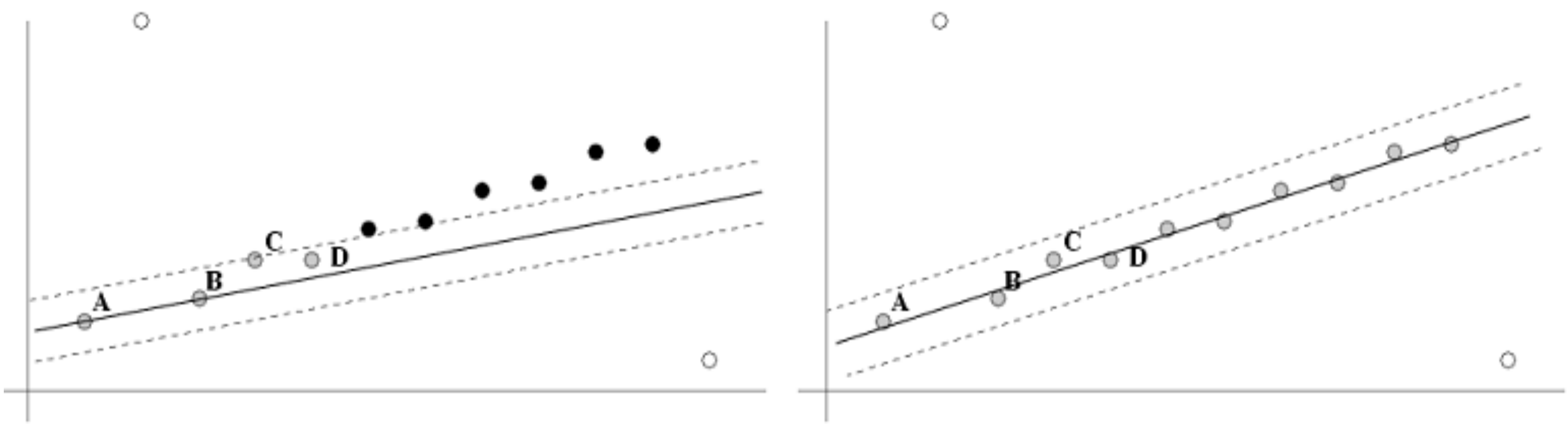
RANSAC: Computed k (p=0.99)

Sample size n	Proportion of outliers 5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

Slide credit: David Lowe

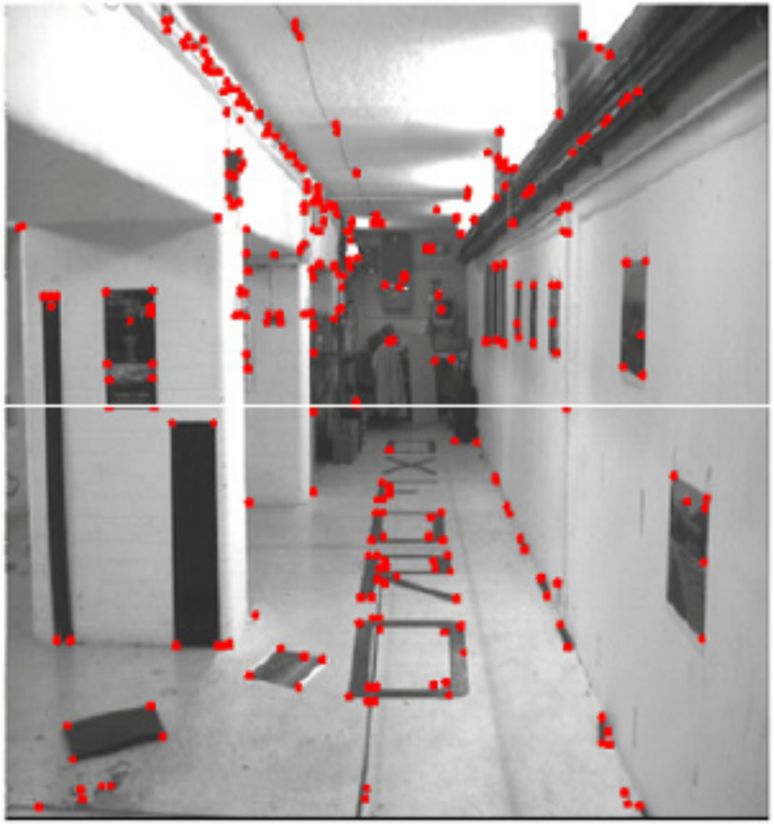
After RANSAC

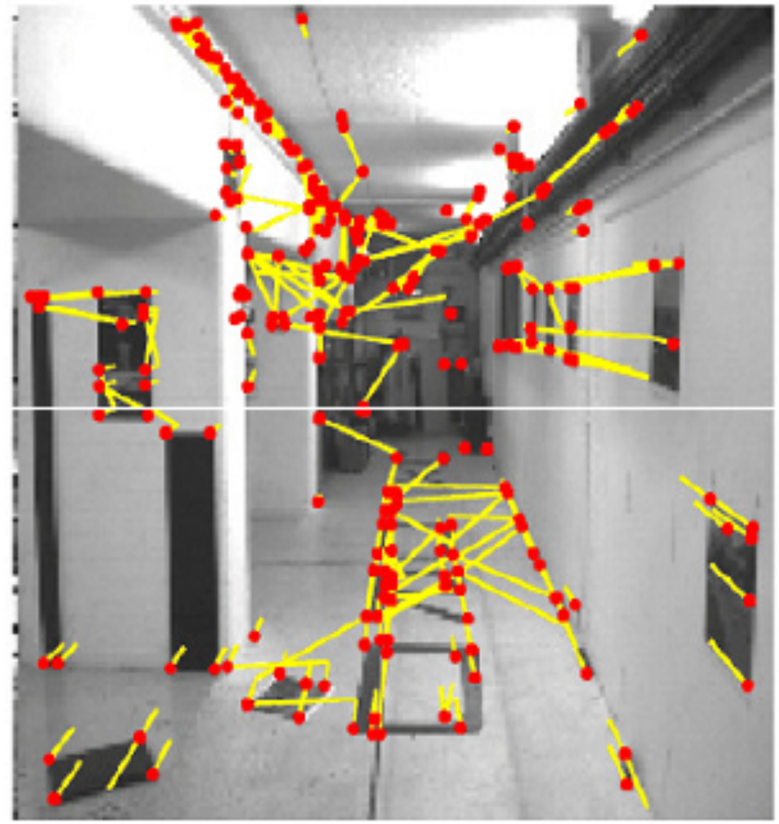
- RANSAC divides data into inliers and outliers and yields estimate computed from minimal set of inliers.
- Improve this initial estimate with estimation over all inliers (e.g. with standard least-squares minimization).
- But this may change inliers, so alternate fitting with re-classification as inlier/outlier.



Slide credit: David Lowe

Putative Matches based on Correlation Search

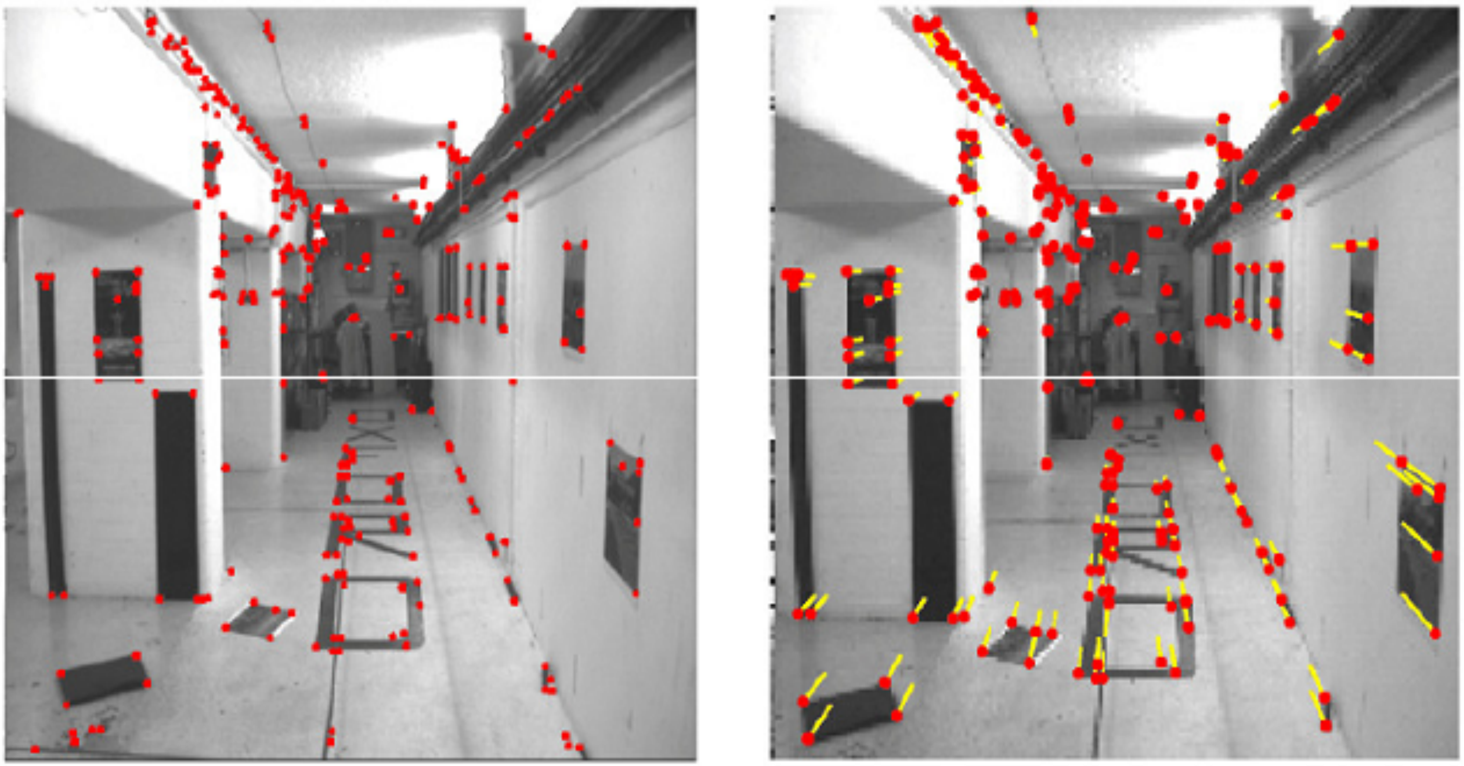




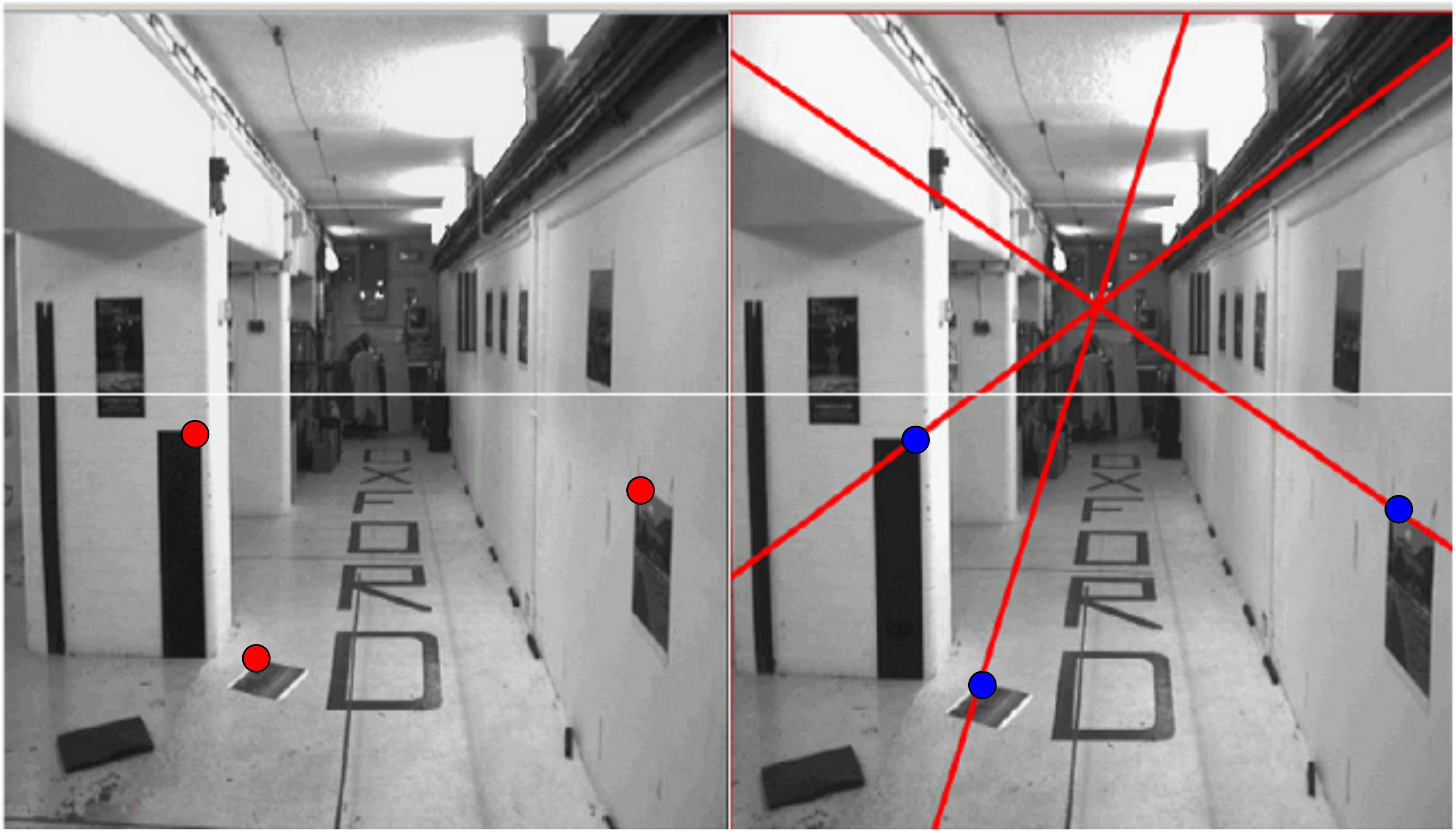
- Many wrong matches (10-50%), but enough to compute F

Pruned Matches

- Correspondences consistent with epipolar geometry



Resulting Epipolar Geometry



Summary

- (Robust) 3D reconstruction
  - ✓Geometric approach
  - ✓Linear Algebraic Approach
  - ✓Nonlinear Approach (LM iterative method)
- Epipolar constrain (Uncalibrated cameras case)
  - ✓Fundamental matrix (Longuet Higgins equation)
- E/F Parameters estimation
  - ✓8-points algorithm & Normal 8-points algorithm (SVD)
  - ✓ Nonlinear Approach (LM iterative method)
- Robust estimation & Reconstruction
  - ✓ RANSAC method for F matrix estimation

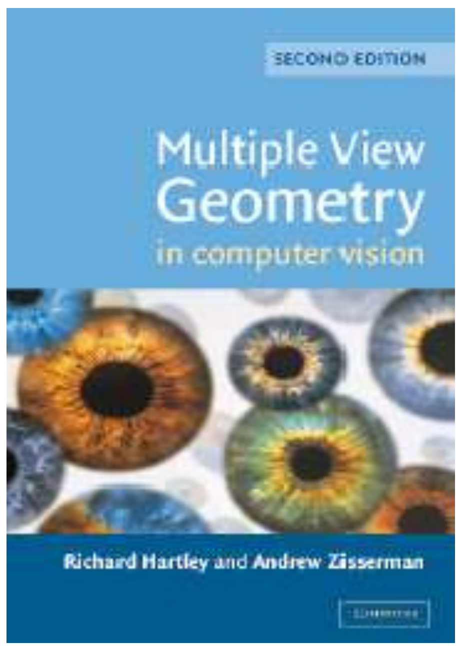
References and Further Reading

- Camera models and calibration algorithms：

R. Hartley, A. Zisserman, Multiple View Geometry in Computer Vision, 2nd Ed., Cambridge Univ. Press, 2004, Chapters 6-7, Chapter 9, 11.1-6.

- 马颂德，张正友，计算机视觉—计算理论与算法，科学出版社，2003.

Chapter 9.



Minimal Problems in Computer Vision

http://cmp.felk.cvut.cz/minimal/index.php

Minimal problems in Computer Vision

cmp.felk.cvut.cz/minimal/

» Home
» P3P problem
» P4P + unknown focal length
» P4P + focal + radial distortion
» 5-pt relative pose problem
» 6-pt focal length problem
» 6-pt calibrated-uncalibrated
» 4-pt 3-views relative pose problem
» 2-pt panorama stitching (1 focal)
» 3-pt panorama stitching (2 focals)
» 3-pt panorama stitching (1 focal+ radial distortion)
» 6-pt generalized camera problem
» 6-pt calibrated radial distortion
» 8-pt uncalibrated radial distortion
» 9-pt different distortion problem
» 3-view triangulation
» 9-pt catadioptric
» Automatic generator
» Grobener basis solver

Overview

Minimal problems in computer vision arise when computing geometrical models from image data.
They often lead to solving systems of algebraic equations.

This page provides links to publications, software, data, and evaluation of minimal problems.

NEW: ICCV 2009

Bujnak M., Kukelova Z., and Pajdla T. 3D reconstruction from image collections with a single known focal length. ICCV 2009, Kyoto, Japan, September 29 - October 2, 2009. [pdf]
CODE:
6-point relative pose problem for one calibrated and one up to focal length calibrated camera

Minimal problems:

3-point absolute pose problem (P3P)
4-point absolute pose problem with unknown focal length (P4Pf)
4-point absolute pose problem with unknown focal length and radial distortion (P4Pfr) **NEW**
5-point relative pose problem
6-point relative pose problem with unknown focal length
6-point relative pose problem for one calibrated and one up to focal length calibrated camera **NEW**
6-point generalized camera problem
4-point 3-view calibrated realtive pose problem **NEW**
2-point panorama stitching problem with one unknown focal lenth **NEW**
3-point panorama stitching problem with two different unknown focal lenths **NEW**
3-point panorama stitching problem with one unknown focal lenth and radial distortion **NEW**
6-point relative pose problem with radial distortion
8-point "uncalibrated" relative pose problem with radial distortion
9-point "uncalibrated" relative pose problem with different radial distortions
3-view triangulation
9-point catadioptric problem **NEW**