### **Problem 1. Hamming distance**

(Time Limit: 3 seconds)

#### **Problem Description**

The hamming distance between two positive integers is the number of different bits in their binary encodings. For example, the binary encodings of 60 and 35 are 111100 and 100011, respectively. Then, the hamming distance between 60 and 35, denoted by d(60,35), is five since there are 5 different bits.

In this problem, you are given a set S of positive integers, and your job is to find a maximum cardinality subset S' of S such that d(x,y) > 4 for all x,y in S'.

#### **Input Format**

The first line of the input contains an integer indicating the number of test cases. For each test case, the first line contains an integer n indicating the number of integers in S. In the next n lines, each line has a positive integer. There are at most 80 integers and each is a 32-bit positive integer.

#### **Output Format**

Output in one line the maximum cardinality of subset of S for each test case.

| Sample Input: | Sample Output: |
|---------------|----------------|
| 1             | 2              |
| 3             |                |
| 60            |                |
| 35            |                |
| 8             |                |

## **Problem 2. Representatives**

(Time Limit: 2 seconds)

#### **Problem Description**

The PTC company decides to send some members abroad for a training course. Because of the limited budget, only k members can be chosen as the representatives. To choose the k members, the company list all combinations of k members in a list, and randomly selects a number i. The i-th group in the list will be the representatives. Assume that the members are numbered from i to i0, each group is presented as an increasing sequence of i1 numbers, and the list of all combinations is sorted in lexicographic order. Given the numbers i1, i2, and i3, find the group of representatives.

### **Technical Specification**

- $1 \le n \le 2000$
- $1 \le i \le 10^6$
- $1 \le k \le n$

#### **Input Format**

The first line of the input file contains an integer indicating the number of test cases, which is at most 10. Each test case is a line, consisting of 3 numbers, n, k, and i. Two numbers are separated by a space.

#### **Output Format**

For each test case, please output a line which is the i-th group, i.e. an increasing sequence of k numbers. Two consecutive numbers are separated by a space.

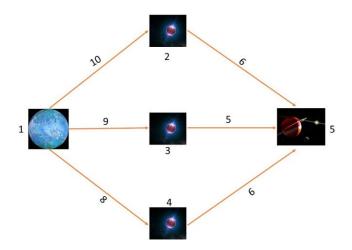
| Sample Input: | Sample Output: |
|---------------|----------------|
| 3             | 2 4 5          |
| 5 3 9         | 1 2 3 4 5 6 7  |
| 10 7 1        | 1 4            |
| 4 2 3         |                |

### **Problem 3. Interstellar Travel**

(Time Limit: 1 second)

#### **Problem Description**

In the distant future when interstellar travel is realized, an inter-star transportation network is constructed. The network is comprised of a lot of *one-way* hyperspace routes between stars. Traveling in such a route imposes a fixed cost which is proportional to the transmission speed of the route. A traveler with limited budget is planning a trip from a star s to another star t. Because of the time dilation effect of high-speed transmission, the traveler's clock will slow down. The faster the relative velocity, the greater the time dilation is. Just for the sake of curiosity, the traveler would like to experience the greatest time dilation as possible as his budget allows. In effect, he wants to find the path from s to t that goes through the route with the highest cost subject to the constraint that the total cost of the path is not beyond his budget limit. For example, let the transportation network be as follows:



Then, for s = 1, t = 5 and the budget limit is 15, there are three paths from 1 to 5

Path 1:  $1 \rightarrow 2 \rightarrow 5$ , total cost = 16(>15)

Path 2:  $1 \rightarrow 3 \rightarrow 5$ , total cost =  $14 \le 15$ , 9 is the maximum cost

Path 3:  $1 \rightarrow 4 \rightarrow 5$ , total cost =  $14 \le 15$ , 8 is the maximum cost

So the maximum cost for a route of all of the paths having total cost  $\leq 15$  is 9.

#### **Input Format**

The first line of the input file contains an integer  $T(T \le 50)$  which denotes the total number of test cases. The description of each test case is given below:

Each case starts with five integers  $N(2 \le N \le 10000$ , for the number of stars),  $M(1 \le M \le 100000$ , for the number of routes),  $s(1 \le s \le N)$ ,  $t(1 \le t \le N)$  and  $p(1 \le p \le 10^6)$ , for the budget limit). Then there are M lines each containing three integers u, v and c, where u and v are star numbers such that there is a hyperspace route from u to  $v(1 \le u, v \le N, u \ne v)$  and  $c(0 \le c \le 10^5)$  is the cost needed for that route.

#### **Output Format**

For each test case produce one line of output containing r, which is the maximum cost needed for a route in a path from s to t whose total cost is within the limit p, or '-1' if there is no such path.

| Sample Input: | Sample Output: |
|---------------|----------------|
| 2             | 9              |
| 5 6 1 5 15    | -1             |
| 1 2 10        |                |
| 2 5 6         |                |
| 1 3 9         |                |
| 3 5 5         |                |
| 1 4 8         |                |
| 4 5 6         |                |
| 2 1 1 2 10    |                |
| 1 2 20        |                |

### **Problem 4. Recovering a Tree**

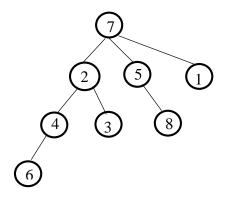
(Time Limit: 6 seconds)

#### **Problem Description**

Let T be a rooted tree of n nodes, which are labeled with 1,2,...,n. The *Euler tour* of T is the sequence of nodes we obtain if we write down the label of each node each time it is visited during a depth-first search. For example, in Figure 1, the Euler tour is (7,2,4,6,4,2,3,2,7,5,8,5,7,1,7). Note that the Euler tour has length 2n-1, because we start at the root and subsequently output a node each time we traverse an edge and we traverse each of the edges twice, once in each direction.

For each node v of T, let  $T_v$  be the subtree rooted at v and h(v) denote the height of  $T_v$ . For example, in Figure 1, (h(1), h(2), h(3), ..., h(8)) = (0,2,0,1,1,0,3,0).

In this problem, the Euler tour of a tree T is given. Please write a problem to compute the total height of all subtrees  $T_v$ , where v is a node of T. For example, given (7,2,4,6,4,2,3,2,7,5,8,5,7,1,7) as the input, which is the Euler tour of the tree in Figure 1, the output is 7.



**Figure 1.** A tree rooted T.

#### **Technical Specification**

- There are at most 10 test cases.
- The size, n, of T is an integer between 1 and  $10^6$ .

### **Input Format**

The first line of the input file contains an integer  $t \le 10$  indicating the number of test cases. Each test case contains 2 lines. The first line contains an integer n, where  $1 \le n \le 10^6$ , indicating the size of T. Next, the second line gives the Euler tour of T.

### **Output Format**

For each test case, output the total height of all subtrees  $T_{\nu}$  in one line.

| Sample Input: |       |     |     | Sample Output: |   |   |   |   |    |   |   |    |
|---------------|-------|-----|-----|----------------|---|---|---|---|----|---|---|----|
| 3             |       |     |     |                |   |   |   |   |    |   |   | 7  |
| 8             |       |     |     |                |   |   |   |   |    |   |   | 0  |
| 7 2 4         | 6 4 2 | 3 2 | 7 5 | 8 5            | 7 | 1 | 7 |   |    |   |   | 10 |
| 1             |       |     |     |                |   |   |   |   |    |   |   |    |
| 1             |       |     |     |                |   |   |   |   |    |   |   |    |
| 10            |       |     |     |                |   |   |   |   |    |   |   |    |
| 4 1 5         | 6 5 1 | 4 2 | 7 2 | 8 9            | 8 | 2 | 4 | 3 | 10 | 3 | 4 |    |

### **Problem 5. Palindromic Numbers**

(Time Limit: 1 second)

#### **Problem Description**

To celebrate the 1001<sup>st</sup> anniversary of the Palin Kingdom, King Drome decided to play games about palindromic numbers with his lovely people. A palindromic number is an integer which is a palindrome when written in base-10 without leading zeroes (e.g., 0, 1, 2, 12321, and 123321). At the beginning of this year, he assigned a distinct palindromic number to each person. There will be several games in the next few months, and they all run independently. The rules of a game are as follows:

- 1. The royal family announces an interval [l, r]. Only people whose numbers lie in the interval are invited.
- 2. Two invited people can form a team, and the score of the team is the difference between their numbers.
- 3. The teams with the smallest score are the winners, and they will be awarded 246642 Palin dollars.

All people want to win the game, and they wonder how small the winners' score can be for each game. As you are the only mathematician and computer scientist in the kingdom, they all seek your help.

#### **Input Format**

The first line contains an integer T indicating the number of games. Each of the following T lines contains two integers, l and r, indicating the interval announced by the royal family.

#### **Output Format**

For each game, output -1 if the number of invited people is less than 2. Otherwise, output the smallest possible score.

# **Technical Specification**

- $1 \le T \le 10^5$
- $0 \le l \le r \le 10^9$

| Sample Input: | Sample Output: |
|---------------|----------------|
| 3             | 1              |
| 6 10          | -1             |
| 87 98         | 100            |
| 12320 12422   |                |