

# Diamonds

02450 Introduction to Machine Learning & Data Mining

Oriade Simpson (s172084)

Pietro Lombardo (s231756)

From: 2023-09-05 To: 2023-10-03



## Contribution Table

| Task        | Oriade  | Pietro  |
|-------------|---------|---------|
| Student ID  | s172084 | s231756 |
| Question 1  | x       |         |
| Question 2  | x       |         |
| Question 3  |         | x       |
| Question 4  |         | x       |
| Exam Prob 1 |         | x       |
| Exam Prob 2 | x       |         |
| Exam Prob 3 | x       |         |
| Exam Prob 4 |         | x       |
| Exam Prob 5 |         | x       |
| Exam Prob 6 | x       |         |

# Contents

|                                                             |           |
|-------------------------------------------------------------|-----------|
| <b>Contribution Table</b>                                   | <b>1</b>  |
| <b>Question 1</b>                                           | <b>3</b>  |
| Introduction to the Dataset . . . . .                       | 3         |
| Statement of Interest . . . . .                             | 3         |
| Data Transformation . . . . .                               | 4         |
| Conclusion of previous Analysis . . . . .                   | 4         |
| Learning Aims using Classification and Regression . . . . . | 4         |
| <b>Question 2</b>                                           | <b>4</b>  |
| Description of Attributes . . . . .                         | 4         |
| Attribute Table . . . . .                                   | 5         |
| Summary Statistics Table . . . . .                          | 5         |
| Correlation Table . . . . .                                 | 6         |
| <b>Question 3</b>                                           | <b>6</b>  |
| Representation of the data set . . . . .                    | 6         |
| Issues with outliers in the data . . . . .                  | 6         |
| Possible distributions followed by the data . . . . .       | 7         |
| Correlation of variables . . . . .                          | 8         |
| Principal component analysis . . . . .                      | 9         |
| <b>4) Results: what data have shown</b>                     | <b>12</b> |
| <b>Exam Problems</b>                                        | <b>13</b> |
| <b>References</b>                                           | <b>14</b> |
| <b>Appendix</b>                                             | <b>15</b> |



# Question 1

## Introduction to the Dataset

The International Diamond Grading System, created by The Gemological Institute of America, focuses on the 4C's as a means to categorise diamonds. The categories are explained on the Diamonds Search Engine, which contains information about how loose diamonds are graded.

The Diamonds dataset used in this report is based on the Diamonds Search Engine [1].

The 4C's include :

- carat
- cut
- colour and
- clarity.

The Diamonds dataset was obtained from the TidyVerse package developed in R by Hadley Wickham and other contributors [4].

The Diamonds dataset records 53,940 rows of diamonds and registers 10 attributes , including the 4C's.

Below, there is a brief overview about the variables in the dataset.

The length (**X**) , width (**Y**) and depth (**Z**) of each diamond has been measured. The length ranges from 0 mm to 10.74 mm. The width ranges from 0 mm to 58.9 mm and the depth ranges from 0 to 31.8 mm.

The carat of a diamond represents the weight of a diamond. The carat ranges from 0.2 to 5.01 carats, where 1 carat is equal to 200 milligrams (1/5 th of a gram).

The quality of the cut is categorised into fair, good, very good, premium and ideal. The cut is an important feature because it determines the sparkle and brilliance observed due to light refraction, which potentially has an influence on the price.

The colour of a diamond is classified from D (best) to J (lowest). The diamonds in group D are colourless. In the international diamond grading system, colourless diamonds have the highest the grade. Diamonds that are classified as J, often have a yellow tinge. One interesting factor is the colour of the diamond, as many diamonds inside engagement rings are G to J in colour. This colour is offset by a gold or silver band [7].

Clarity is categorised from IF to I1. IF is the best class. Here, F means Flawless - without visible blemishes.

Diamonds that contain inclusions that are visible to the naked eye are given the worst class of I1.

The last 3 variables or attributes of the dataset include the price, the total depth percentage and the table.

The price of diamonds in US dollars ranges from \$326 to \$18,823 US dollars. In the market, the price of diamonds may be based on the carat weight.

The depth (total depth percentage) is a continuous variable which is the total depth, from top to bottom of each diamond, divided by the mean length and width.

The table is a continuous variable which is a measurement of the width of the top of a diamond in relation to the widest point.

## Statement of Interest

The idea is to analyse the dataset in order to determine relationships between variables. More specifically, it may be interesting to discover the relationship between two attributes: i.e. the relationship between:

- The length and width of a colourless diamond

- The colour and the table of the diamond
- The carat and price of a colourless diamond
- The carat and the table of a colourless diamond
- The carat and the length of a colourless diamond
- The carat and the width of a colourless diamond
- The carat and the depth of a colourless diamond
- The price corresponding to the cut of a diamond

### **Data Transformation**

Ideally the price could be determined in Danish Kroner, Pounds Sterling or Euros, rather the US dollars. It may also be a good idea to transform carats into milligrams to represent the weight of the diamond. The attributes that are recorded in millimetres could also be converted to centimetres.

### **Conclusion of previous Analysis**

In his R publication, Jon Ong has performed a brief analysis of this dataset looking at the relationship between Cut vs. Colour, Price vs. Cut, and Price vs. Clarity. The results show that diamonds with a Premium Cut have the highest carat [5].

Poonam Rao has also performed a brief analysis of the diamonds dataset where she performs data visualisation using scatterplots, histograms and box-plots. She explains that diamonds that are categorised in the Ideal group with regard to cut, often have a low weight [6].

### **Learning Aims using Classification and Regression**

A regression problem analyses the relationship between a dependent variable and an independent variable. The Regression can be linear regression or multiple linear regression.

It is possible to analyse the relationship between diamond length (**X**) and diamond width (**Y**), the carat and the depth(**Z**) in order to understand the relationship between these variables. In the regression problem, we will predict the carat based on the length, width and depth.

It is possible to look at the relationship between carat and price for 1 specific colour, cut and clarity of diamond in our regression problem. One could observe the carat of the diamonds that are priced between \$10,000 to \$20,000 USD.

A classification problem requires a dataset to be classified into two or more categories. One idea is to choose the category of cut and determine if the diamond is very good, premium cut or ideal cut. This can be done using the diamond length (**X**) and diamond width (**Y**), the carat and the depth(**Z**).

Another idea is to choose the category of colour and determine if the diamond is D or J. This can be also be done using the diamond length (**X**) and diamond width (**Y**), the carat and the depth(**Z**).

It is also possible to choose the category of clarity and determine if the diamond is IF, VS1 or I1. This can be done using the diamond length (**X**) and diamond width (**Y**), the carat and the depth(**Z**).

## **Question 2**

### **Description of Attributes**

**Describe if the attributes are discrete/continuous, Nominal/Ordinal/Interval/Ratio**

The table below shows the classification of the attributes. The cut, colour, and clarity are ordinal factor variables. In the table below, Cont. means continuous and Categ. means Categorical.

## Attribute Table

| Attribute | Carat | Cut     | Colour  | Clarity. | Depth | Table | Price    | Length<br>(X) | Width(Y) | Depth<br>(Z) |
|-----------|-------|---------|---------|----------|-------|-------|----------|---------------|----------|--------------|
| Type      | Cont. | Categ.  | Categ.  | Categ.   | Cont. | Cont. | Discrete | Cont.         | Cont.    | Cont.        |
| Type      | Ratio | Ordinal | Ordinal | Ordinal  | Ratio | Ratio | Interval | Ratio         | Ratio    | Ratio        |

There are no missing values or NAs in any column of the dataset. The data is tidy from the TidyVerse. In the Premium cut group and Ideal cut group of diamonds, there are some diamonds that have an outlying width, which corresponds to the VS1 and SI2 clarity groups.

### Include basic summary statistics of the attributes, including correlation

There are many diamonds that have a carat between 0.2 and 1, with the average carat being 0.7. Therefore it may be interesting to narrow in on the dataset and only look at diamonds that are between 1 and 5 carat, and exclude all diamonds below 1 carat. The average length of a diamond (**X**) is 5.731 mm. It may be interesting to look at diamonds that are between 6 and 10 millimetres long.

The average price of a diamond in this dataset is \$3,932.8 USD. There is a lower quantile of \$950 dollars and upper quantile of \$5,324.5 USD. It may be interesting to narrow down the price range and focus on diamonds that are between \$326 - \$5000 USD. However \$5000 - \$15,000 USD or \$10,000 - £\$20,000 USD may also be suitable ranges to focus on. Many of the diamonds have the average table of approximately 61.7. Diamonds have an average depth of 3.5. It may be interesting to explore diamonds that have a depth (**Z**) between 5 and 10. It may be interesting to choose diamonds with a specific clarity, IF, SI2, VS1 and narrow down on the cut to look at only 1 type of cut. There are 21,000 Ideal cut diamonds, which makes this the most common category. One could choose the very good (12,082) , premium ( 13,791) or ideal cut diamonds to focus on.

## Summary Statistics Table

| Variable         | No.Obs.<br>$N$ | Mean $\hat{\mu}$ | Variance | Standard<br>Dev $\hat{\sigma}$ | Lower<br>Quart | Median | Upper<br>quart |
|------------------|----------------|------------------|----------|--------------------------------|----------------|--------|----------------|
| Carat.           | 53,940         | 0.797            | 0.2247   | 0.474                          | 0.4            | 0.7    | 1.04           |
| Cut.             | 53,940         |                  |          |                                |                |        |                |
| Colour           | 53,940         |                  |          |                                |                |        |                |
| Clarity          | 53,940         |                  |          |                                |                |        |                |
| Total Depth<br>% | 53,940         | 61.749           | 2.052    | 1.433                          | 61             | 61.8   | 62.5           |
| Table            | 53,940         | 57.457           | 4.992    | 2.234                          | 56             | 57.0   | 59.0           |
| Price            | 53,940         | 3932.80          | 15915629 | 3989.44                        | 950            | 2401   | 5324.5         |
| Length(X)        | 53,940         | 5.731            | 1.258    | 1.122                          | 4.71           | 5.70   | 6.54           |
| Width (Y)        | 53,940         | 5.734            | 1.305    | 1.142                          | 4.72           | 5.71   | 6.54           |
| Depth(Z)         | 53,940         | 3.538            | 0.498    | 0.706                          | 2.91           | 3.53   | 4.04           |

## Correlation Table

From the Correlation Table it can be seen that:

- There is a positive correlation between Length(**X**) and Width(**Y**).
- There is a positive correlation between Width(**Y**) and Depth(**Z**).
- There is a positive correlation between Length(**X**) and Depth(**Z**).
- There is a positive correlation between carat and Length(**X**)

| Correlation | Value  |
|-------------|--------|
| X & Y       | 0.9747 |
| Y & Z       | 0.9520 |
| X & Z       | 0.9707 |
| carat & X   | 0.9750 |

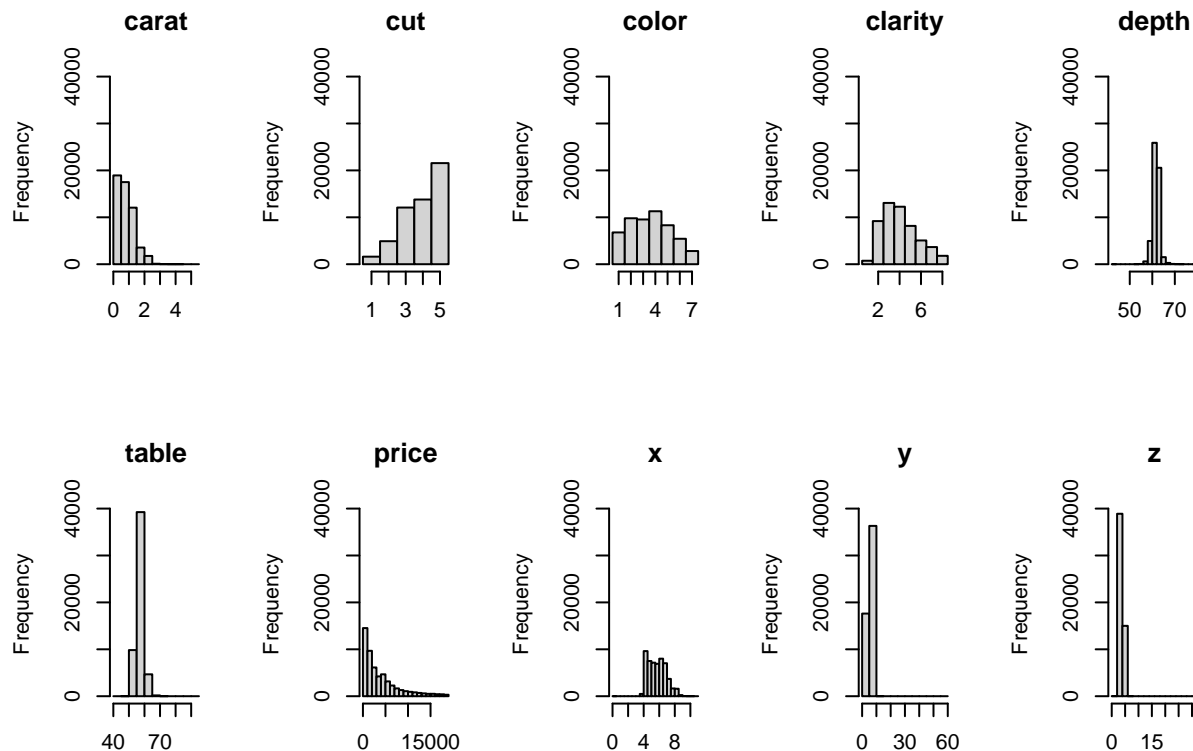
## Question 3

### Representation of the data set

The data set is composed by different type of data as discussed above. In order to manage the ordinal attributes (*cut*, *color* and *clarity*), they can be converted into ordinal numbers (from 1 to the upper level number) according to their ranking.

### Issues with outliers in the data

To have a general idea of the distribution of the values assumed by the attributes, a list of histograms can be displayed:



Some of the histograms seem to be affected by outliers in the data. It can be seen by looking at the horizontal axis of each graph and by noticing that bins do not cover all the axis values but only a limited

range. This is very evident in the attributes *table* and in the dimensions *y* and *z*. This attributes contain very extreme values that wrongly set the axis limits. In order to detect them, the data set can be sorted from the smallest value of a specific attribute to the largest value of the same attribute. By doing so, it can be selected the 99.99 percentile, which corresponds to the highest observations of the data set sorted by one of the attributes. If the values assumed by the attribute are not physical or are very different from the other values, the specific observation can be discarded.

The 99.99<sup>th</sup> percentile of the data set sorted by the *table* shows the following data:

| ##       | carat | cut | color | clarity | depth | table | price | x    | y    | z    |
|----------|-------|-----|-------|---------|-------|-------|-------|------|------|------|
| ## 24933 | 2.01  | 1   | 3     | 3       | 58.6  | 95    | 13387 | 8.32 | 8.31 | 4.87 |
| ## 50774 | 0.81  | 1   | 3     | 2       | 68.8  | 79    | 2301  | 5.26 | 5.20 | 3.58 |
| ## 51343 | 0.79  | 1   | 4     | 3       | 65.3  | 76    | 2362  | 5.52 | 5.13 | 3.35 |

The first row has a value of *table* very far from the other two rows. It is very unlikely that, considering a data set of 53940 diamonds, there is a diamond with the *table* 16 percentage points bigger than the second-biggest *table* diamond. For this reason, the row nr. 24933 of the data set will be canceled.

As for the attribute *y* (width), it can be followed the same procedure by sorting the data set by the width and looking at the 99.99<sup>th</sup> percentile:

| ##       | carat | cut | color | clarity | depth | table | price | x     | y     | z    |
|----------|-------|-----|-------|---------|-------|-------|-------|-------|-------|------|
| ## 24068 | 2.00  | 4   | 5     | 2       | 58.9  | 57    | 12210 | 8.09  | 58.90 | 8.06 |
| ## 27416 | 5.01  | 1   | 7     | 1       | 65.5  | 59    | 18018 | 10.74 | 10.54 | 6.98 |
| ## 27631 | 4.50  | 1   | 7     | 1       | 65.8  | 58    | 18531 | 10.23 | 10.16 | 6.72 |
| ## 49190 | 0.51  | 5   | 2     | 5       | 61.8  | 55    | 2075  | 5.15  | 31.80 | 5.12 |

It can be seen that the first and the last rows have very larger values of *y* with respect to the the other two rows belonging to the 99.99<sup>th</sup> percentile. It is very unlikely that, considering such a big number of diamonds, there are two diamonds with dimensions so different from the others. So the two rows containing the two outliers will be canceled

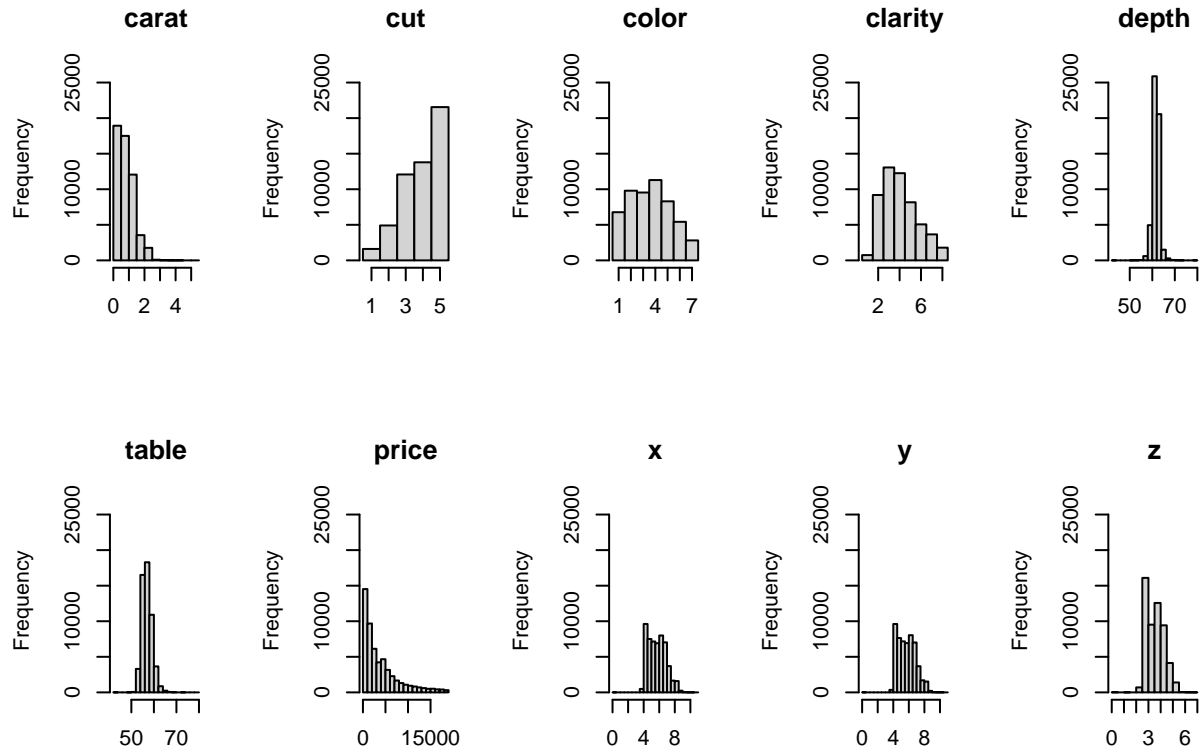
Finally, sorting the data set by the *z*-dimension, the 99.99<sup>th</sup> percentile is composed by the following records:

| ##       | carat | cut | color | clarity | depth | table | price | x     | y     | z     |
|----------|-------|-----|-------|---------|-------|-------|-------|-------|-------|-------|
| ## 24068 | 2.00  | 4   | 5     | 2       | 58.9  | 57.0  | 12210 | 8.09  | 58.90 | 8.06  |
| ## 27416 | 5.01  | 1   | 7     | 1       | 65.5  | 59.0  | 18018 | 10.74 | 10.54 | 6.98  |
| ## 48411 | 0.51  | 3   | 2     | 5       | 61.8  | 54.7  | 1970  | 5.12  | 5.15  | 31.80 |

The last record has a value of *z* very far from the other ones, so it can be considered as an outlier and so it will be removed from the data set.

### Possible distributions followed by the data

Now that the data set is free from outliers, the histograms of the attributes will be more meaningful about the possible distribution the data follow for each variable:

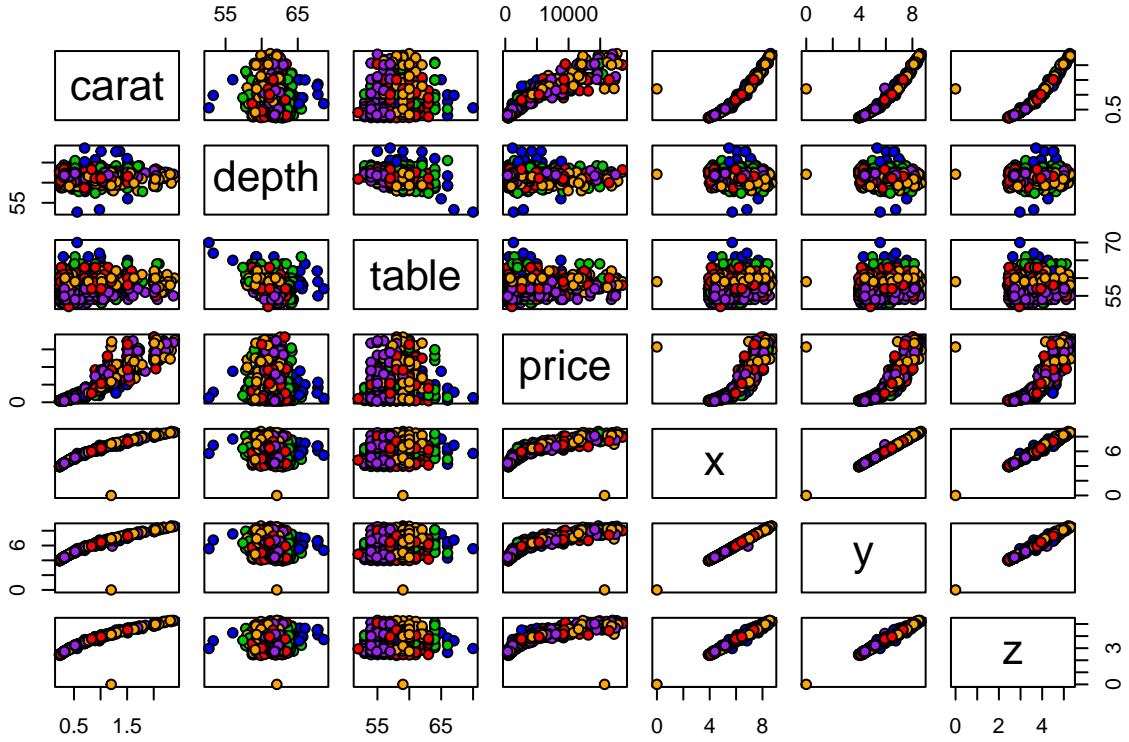


From the graphs, some considerations about the distribution of each attribute can be listed: - attributes *depth* and *table* appear to be normally distributed since they follow quite well the gaussian bell curve and they are almost symmetric; - attributes *carat* and *price* seem to follow an exponential distribution, since each bin is smaller than the previous one and larger than the following one; - attributes *clarity*, *x*, *y* and *z* appear to follow a log-normal distribution, since they follow a asymmetric bell curve with the sample-mode smaller than the mean.

### Correlation of variables

Correlation between variables can be studied by producing the matrix of scatter plots of each combination of two attributes against each other. Scatter plots are not so meaningful with ordinal data (*cut*, *color* and *clarity*), so they are removed from the data set and points of the plots are colored according to the attribute *cut* in the attempt of finding some pattern of the data. The matrix is shown below:





The plots confirm the very high correlation between the variables *carat*, *x*, *y* and *z*, as already computed in the Table 3. In addition, also the *price* appears to be slightly correlated with the variables *carat*, *x*, *y* and *z*.

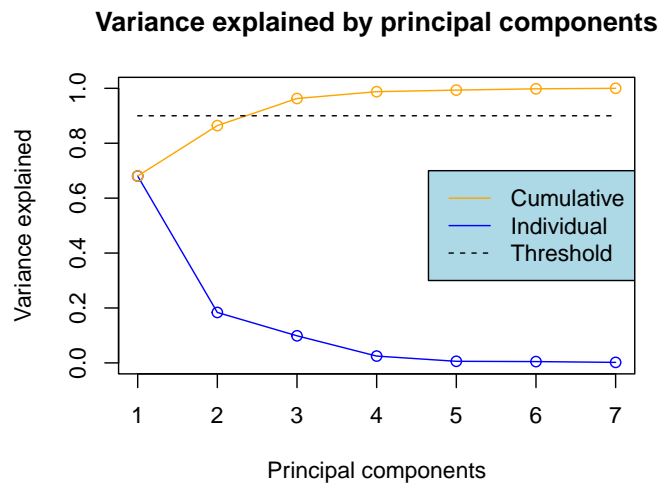
The other variables do not show any specific pattern and also the *cut* does not seem to distinguish any cluster of the data.

## Principal component analysis

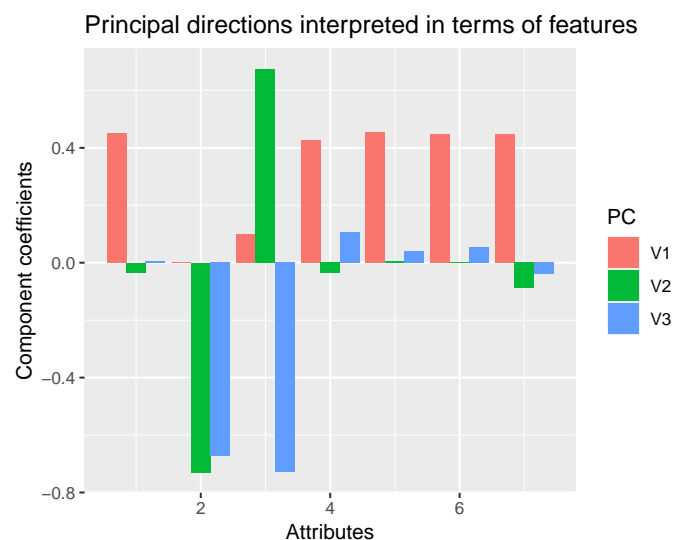
The aim of the Principal Component Analysis (“PCA” from now on), is to simplify the problem dimension by reducing the number of interval or ratio variables which explains the behavior of diamonds. So, for the PCA, the categorical attributes *cut*, *color* and *clarity* are removed from the data set.

Looking at the “Summary Statistics Table” (page 5), standard deviation of *price* is some orders of magnitude larger than the others, so the dataset has to be standardized by subtracting the mean and dividing by the standard deviation of the whole set of observations of that variable.

```
## [1] 0.680559258 0.183695440 0.098687323 0.024821905 0.005758174 0.004706656
## [7] 0.001771245
```



The new standardized dataset can be now used to perform the Singular Value Decomposition (SVD), which gives rise to three different matrices:  $U$ ,  $\Sigma$  and  $V^T$ . By the extraction of the diagonal of the matrix  $\Sigma$ , it can be seen how much variance is explained by each principal component. The cumulative explained variance should reach the percentage of 90% in order to describe properly the main features of the dataset. The figure on the left shows that the first 3 principal components explain the 96% of variance, so the problem can be reduced from a seven-dimensional to a three-dimensional one.

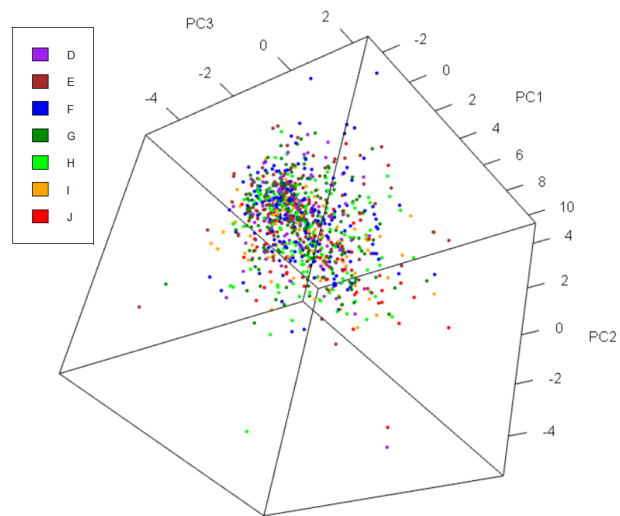
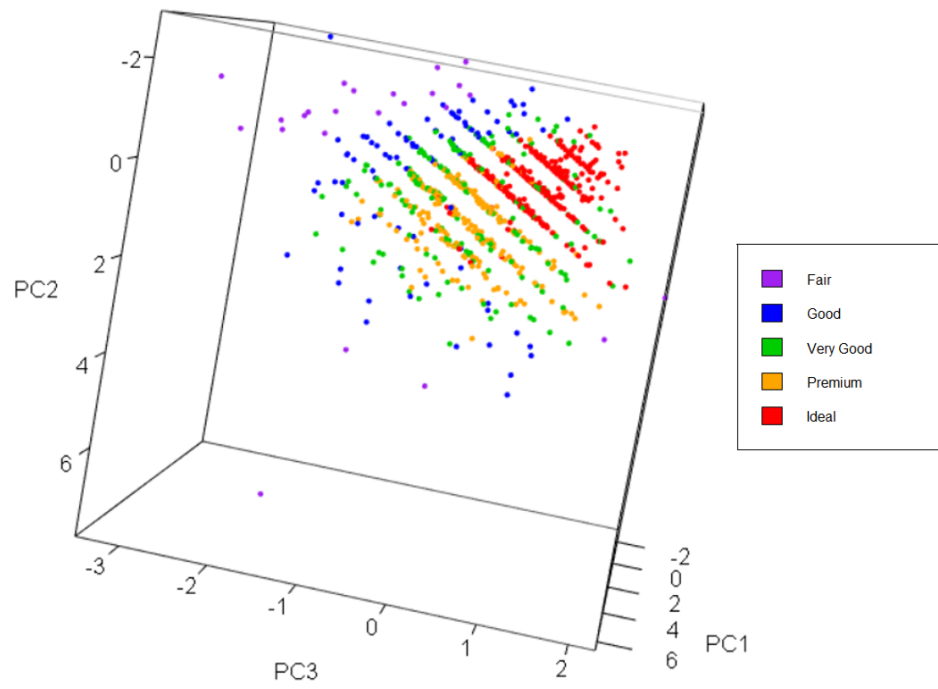


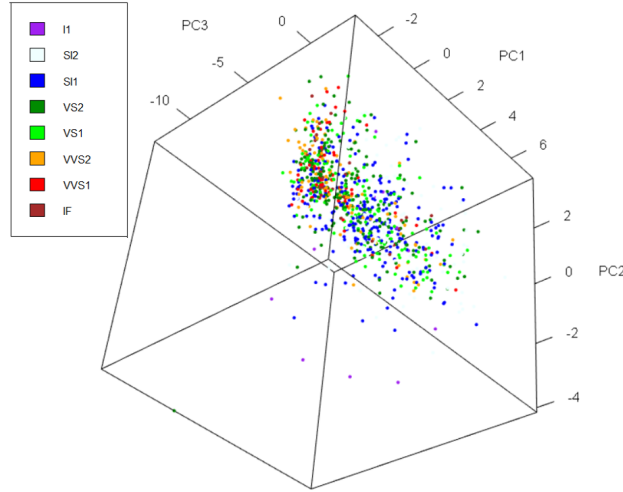
The matrix  $V^T$  contains the seven seven-dimensional vectors defining the principal components (PCs). Focusing on the first three PCs, they contain the weights associated to each of original component. The figure on the left shows the values of the weights. It can be noticed that the first PC mainly describes the dimensional quantities of the diamonds (*carat*, size in  $x$ ,  $y$ ,  $z$ ) and the *price*. It seems that these five characteristics alone explain two-thirds of the variability of the diamonds. As for the second PC, it focuses more on the cut characteristics (*depth* and *table*). The third PC mainly describes the cut characteristics as well, but considering them as contributing to the same direction.

Data can be projected in the three-dimensional space defined by the PCs. By projecting them, some interesting features can be observed, as shown by the 3D graph below.

In the top-figure below, data are colored by the attribute *cut* and, for visualization purposes, only a sample of 1000 data is plotted. It can be seen that data are layered following sloped planes. In addition, they are clustered according to the *cut*: borders of different classes are not so clear, in particular “very good” and “premium” diamonds are mixed together. “Ideal” diamonds are concentrated in a very visible cluster, as for “fair” diamonds that are very far from the rest of the diamonds. As it can be expected, the *cut* of diamonds is mainly explained by the second and third PCs, that are related to depth and table of diamonds.

In the bottom-figures below, data are colored by *color* (left) and *clarity* (right) and, for visualization purposes, only a sample of 1000 data is plotted. No pattern and clusters can be seen from the figures, so it seems that *color* and *clarity* are more difficult to be predicted than *cut*, also because there are more different levels that these attributes can assume.





#### 4) Results: what data have shown

The data set appears to be complete since each diamond is described by all the attributes and there are not missing values. This is a good thing because there is no lack of information. It is a good data set also because observations are coherent one to each other in the sense that there are very few extreme values that are very far from the other data.

By looking at the collection of diamonds, data show that small diamonds are more numerous than big ones (as it can be expected) and that the characteristics connected to the rarity of the diamond (so, *carat* and *price*) seems to be distributed exponentially, meaning that diamonds having low *carat* and low *price* are more numerous than the most expensive and heavy ones. In addition, a strong correlation between the dimensions of diamonds and their weight can be found, as it can be expected (bigger diamonds are also heavier). It is interesting the fact that the *price* is positively correlated to the dimensions and the *carat*, meaning that the bigger and heavier diamonds are also more expensive. All these connections reflect to the fact that the first PC (combining all these characteristics) alone explains the 68% of the variability of non-categorical attributes. As for the *table* and *depth*, they appear to be normally distributed, and so less subjected to governing rules. Indeed, they seem to be uncorrelated to the other attributes and they are described mainly by the second and third PCs. Nevertheless, they are fundamental in describing the *cut* behavior, since it is clear that data projected in the 2nd-3rd PCs plane and colored by the *cut* are somehow clustered. Unfortunately, the same is not valid for the other two categorical attributes (*color* and *clarity*) that seem to be not connected to any of the other attributes.

In conclusion, regression problems involving diamonds dimensions, *carat* and *price* appear to be feasible but it seems to be hard to make a regression of one of those attributes starting from *depth* and *table*. As for classification problems, *cut* appears to be easily classified starting from *depth* and *table*, conversely *color* and *clarity* seem to be hard to be predicted.

## Exam Problems

**Question 1** | Spring 2019 question 2 Which of the following statements are true about the types of the attributes in the Urban Traffic dataset?

Answer **B**: the attribute  $x_1$  is ordinal because it can be said that time-interval number 5 is greater than the time-interval number 4 since it is later in the day but it makes no sense to sum and subtract them

**Question 2** | Spring 2019 question 2 Consider again the Urban Traffic dataset from table 1 and in particular the 14 and 18th observation . Which of the following statements about the p-norm distance  $d_p$  (.,.) is correct ?

Answer **A**: The Maximum Norm Distance is 7.0

**Question 4** | Spring 2019 question 2 Which one of the following statements is true?

Answer **D**: PC2 has a negative coefficient multiplying  $x_1$ , so if an observation has a low value of  $x_1$  it means that the PC2 decreases very little for the contribution of  $x_1$ . On the other hand, PC2 has positive coefficients multiplying  $x_2$ ,  $x_3$  and  $x_5$  so if an observation has a high value of them, it means that the PC2 increases a lot thanks to their the contribution

**Question 6** | Spring 2019 question 2 Which is the probability an observation had  $\hat{x}_2 = 0$  given light congestion?

Answer **B**: according to the Sum Rule we know that  $p(x_2|y) = \sum_{i=0}^1 p(x_2, x_7 = i|y)$ , so a new Probability Table can be compiled:

| $p(x_2  y)$ | y=1  | y=2  | y=3  | y=4  |
|-------------|------|------|------|------|
| $x_2 = 0$   | 0.84 | 0.84 | 0.73 | 0.77 |
| $x_2 = 1$   | 0.16 | 0.16 | 0.27 | 0.23 |

## References

- 1) Grading Loose Diamonds, Diamonds Search Engine, “<https://www.diamondse.info/diamonds-grading.asp>”, Accessed 22 September 2023
- 2) Gemological Institute of America, Learn How a Diamond is Graded, (2009), YouTube “<https://www.youtube.com/watch?v=oqhaty0ny0g>”
- 3) Diamond Prices Index, “<https://www.diamondse.info/diamonds-price-index.asp>” Accessed 14 September 2023
- 4) Wickham H, Averick M, Bryan J, Chang W, McGowan LD, François R, Golemund G, Hayes A, Henry L, Hester J, Kuhn M, Pedersen TL, Miller E, Bache SM, Müller K, Ooms J, Robinson D, Seidel DP, Spinu V, Takahashi K, Vaughan D, Wilke C, Woo K, Yutani H (2019). “Welcome to the tidyverse.” *Journal of Open Source Software*, 4(43), 1686. doi:10.21105/joss.01686
- 5) Jon Ong, Diamond Data Analysis , (2018), “<https://rpubs.com/gokusun327/diamonddatatest>” , Accessed 22 September 2023
- 6) Poonam Rao , Exploratory Data Analytics, (2020), “<https://medium.com/swlh/exploratory-data-analysis-21bbf3887e28>”
- 7) Paul Gian, Beyond 4C’s - Real Insights to Buying Diamonds , “<https://beyond4cs.com/color/>”, Accessed 22 September 2023

## Appendix