Diamonds

02450 Introduction to Machine Learning & Data Mining

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Contribution Table

Task	Oriade	Pietro
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Question 1	x	
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Question 1

Give an Overview and Introduction to the Dataset

The International Diamond Grading System, created by The Gemological Institute of America,focuses on the 4C's as a means to categorise diamonds. The categories are explained on the Diamonds Search Engine, which contains information about how loose diamonds are graded.

The Diamonds dataset used in this report is based on the Diamonds Search Engine [1].

The 4C's include :

- carat
- cut
- colour and
- clarity.

The Diamonds dataset was obtained from the TidyVerse package developed in R by Hadley Wickham and other contributors [4].

The Diamonds dataset records 53,940 rows of diamonds and registers 10 attributes , including the 4C's.

Below, there is a brief overview about the variables in the dataset.

The length (**X**) , width (**Y**) and depth (**Z**) of each diamond has been measured. The length ranges from 0 mm to 10.74 mm. The width ranges from 0 mm to 58.9 mm and the depth ranges from 0 to 31.8 mm.

The carat of a diamond represents the weight of a diamond. The carat ranges from 0.2 to 5.01 carats, where 1 carat is equal to 200 milligrams (1/5 th of a gram).

The quality of the cut is categorised into fair, good, very good, premium and ideal. The cut is an important feature because it determines the sparkle and brilliance observed due to light refraction, which potentially has an influence on the price.

The colour of a diamond is classified from D (best) to J (lowest). The diamonds in group D are colourless. In the international diamond grading system, colourless diamonds have the highest the grade. Diamonds that are classified as J, often have a yellow tinge. One interesting factor is the colour of the diamond, as many diamonds inside engagement rings are G to J in colour. This colour is offset by a gold or silver band [7].

Clarity is categorised from IF to I1. IF is the best class. Here, F means Flawless - without visible blemishes. Diamonds that contain inclusions that are visible to the naked eye are given the worst class of I1.
The last 3 variables or attributes of the dataset include the price, the total depth percentage and the table.

The price of diamonds in US dollars ranges from $326 to $18,823 US dollars. In the market, the price of diamonds may be based on the carat weight.

The depth (total depth percentage) is a continuous variable which is the total depth, from top to bottom of each diamond, divided by the mean length and width.

The table is a continuous variable which is a measurement of the width of the top of a diamond in relation to the widest point.

What is the Overall Statement of Interest ?

The idea is to analyse the dataset in order to determine relationships between variables. More specifically, it may be interesting to discover the relationship between two attributes: i.e. the relationship between:

- The length and width of a colourless diamond
- The colour and the table of the diamond
- The carat and price of a colourless diamond
- The carat and the table of a colourless diamond
- The carat and the length of a colourless diamond

- The carat and the width of a colourless diamond
- The carat and the depth of a colourless diamond
- The price corresponding to the cut of a diamond

How to Transform Data

Ideally the price could be determined in Danish Kroner, Pounds Sterling or Euros, rather the US dollars. It may also be a good idea to transform carats into milligrams to represent the weight of the diamond. The attributes that are recorded in millimetres could also be converted to centimetres.

What is the Conclusion of previous Analysis?

In his R publication, Jon Ong has performed a brief analysis of this dataset looking at the relationship between Cut vs. Colour, Price vs. Cut, and Price vs. Clarity. The results show that diamonds with a Premium Cut have the highest carat [5].

Poonam Rao has also performed a brief analysis of the diamonds dataset where she performs data visualisation using scatterplots , histograms and box-plots. She explains that diamonds that are categorised in the Ideal group with regard to cut, often have a low weight [6] .

What do you aim to learn from the data using Classification and Regression?

A regression problem analyses the relationship between a dependent variable and an independent variable. The Regression can be linear regression or multiple linear regression.

It is possible to analyse the relationship between diamond length (**X**) and diamond width (**Y**), the carat and the depth(**Z**) in order to understand the relationship between these variables. In the regression problem, we will predict the carat based on the length, width and depth.

It is possible to look at the relationship between carat and price for 1 specific colour, cut and clarity of diamond in our regression problem. One could observe the carat of the diamonds that are priced between $10,000 to $20,000 USD.

A classification problem requires a dataset to be classified into two or more categories. One idea is to choose the category of cut and determine if the diamond is very good, premium cut or ideal cut? This can be done using the diamond length (**X**) and diamond width (**Y**), the carat and the depth(**Z**).

Another idea is to choose the category of colour and determine if the diamond is D or J. This can be also be done using the diamond length (**X**) and diamond width (**Y**), the carat and the depth(z).

It is also possible to choose the category of clarity and determine if the diamond is IF , VS1 or I1 . This can be done using the diamond length (**X**) and diamond width (**Y**), the carat and the depth(**Z**).

Question 2

Describe if the attributes are discrete/continuous, Nominal/Ordinal/Interval/Ratio

The table below shows the classification of the attributes. The cut, colour, and clarity are ordinal factor variables. In the table below, Cont. means continuous and Categ. means Categorical.

Attribute Table

Attribute	Carat	Cut	Colour	Clarity.	Depth	Table	Price	Length (X)	Width(Y)	Depth (Z)
Type	Cont.	Categ.	Categ.	Categ.	Cont.	Cont.	Discrete	Cont.	Cont.	Cont.
Type	Ratio	Ordinal	Ordinal	Ordinal	Ratio	Ratio	Interval	Ratio	Ratio	Ratio

There are no missing values or NAs in any column of the dataset. The data is tidy from the TidyVerse. In the Premium cut group and Ideal cut group of diamonds, there are some diamonds that have an outlying width, which corresponds to the VS1 ad SI2 clarity groups.

Include basic summary statistics of the attributes, including correlation

There are many diamonds that have a carat between 0.2 and 1, with the average carat being 0.7.
Therefore it may be interesting to narrow in on the dataset and only look at diamonds that are between 1 and 5 carat, and exclude all diamonds below 1 carat. The average length of a diamond (**X**) is 5.731 mm. It may be interesting to look at diamonds that are between 6 and 10 millimetres long.
The average price of a diamond in this dataset is $3,932.8 USD. There is a lower quantile of $950 dollars and upper quantile of $5,324.5 USD. It may be interesting to narrow down the price range and focus on diamonds that are between $326 - $5000 USD. However $5000 - $15,000 USD or $10,000 - £$20,000 USD may also be suitable ranges to focus on. Many of the diamonds have the average table of approximately 61.7. Diamonds have an average depth of 3.5. It may be interesting to explore diamonds that have a depth (**Z**) between 5 and 10. It may be interesting to choose diamonds with a specific clarity, IF, SI2, VS1 and narrow down on the cut to look at only 1 type of cut. There are 21,000 Ideal cut diamonds, which makes this the most common category. One could choose the very good (12,082) , premium (13,791) or ideal cut diamonds to focus on.

Summary Statistics Table

Variable	No. Obs.	Mean	Variance	Standard Dev	Lower Quart	Median	Upper quart
Carat.	53,940	0.797	0.2247	0.474	0.4	0.7	1.04
Cut.	53,940						
Colour	53,940						
Clarity	53,940						
Total Depth %	53,940	61.749	2.052	1.433	61	61.8	62.5
Table	53,940	57.457	4.992	2.234	56	57.0	59.0
Price	53,940	3932.80	15915629	3989.44	950	2401	5324.5
Length(X)	53,940	5.731	1.258	1.122	4.71	5.70	6.54
Width (Y)	53,940	5.734	1.305	1.142	4.72	5.71	6.54
Depth(Z)	53,940	3.538	0.498	0.706	2.91	3.53	4.04

Correlation Table

Correlation	Value
X & Y	0.9747
Y & Z	0.9520
X & Z	0.9707
carat & X	0.9750

- There is a positive correlation between Length(**X**) and Width(**Y**).
- There is a positive correlation between Width(**Y**) and Depth(**Z**).
- There is a positive correlation between Length(**X**) and Depth(**Z**).
- There is a positive correlation between carat and Length(**X**)

3) Data visualization

Representation of the data set

The data set is composed by different type of data as discussed above. In order to manage the ordinal attributes (*cut*, *color* and *clarity*), they can be converted into ordinal numbers (from 1 to the upper level number) according to their ranking.

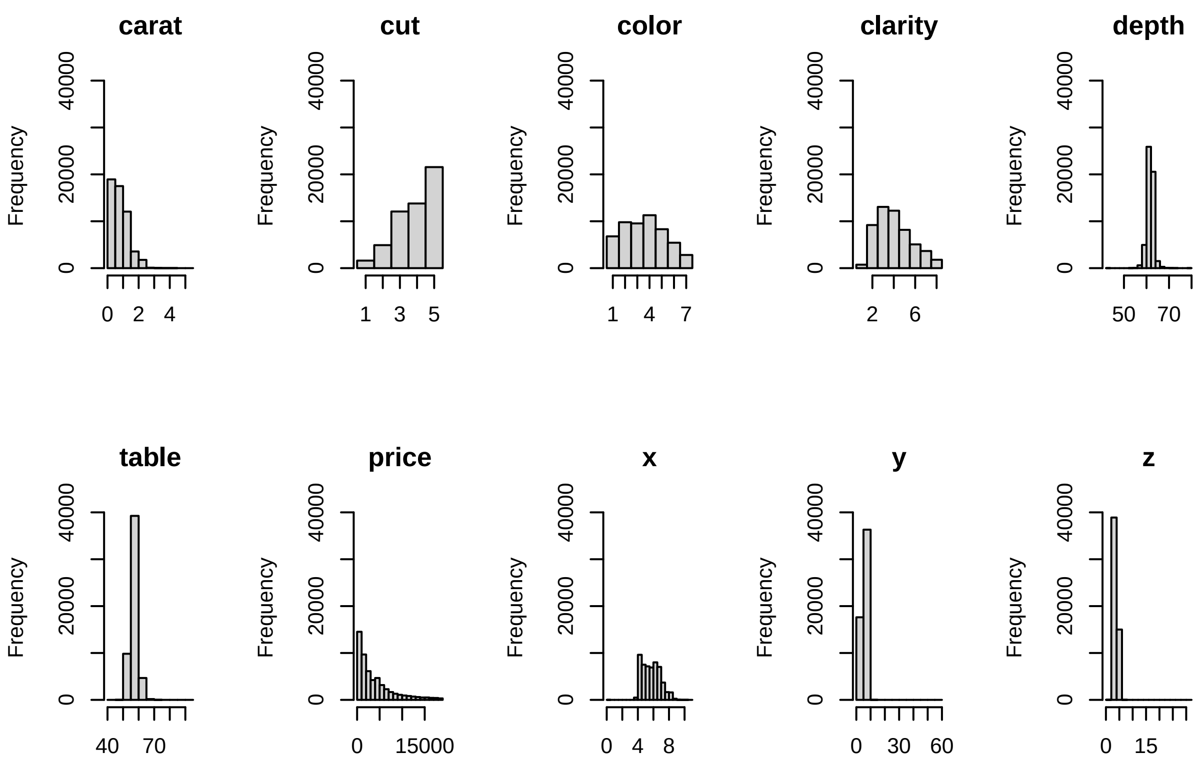
After the conversion, the data set appears as below:

Table 5: Head of the diamonds dataset arranged for the PCA

carat	cut	color	clarity	depth	table	price	x	y	z
0.23	5	2	2	61.5	55	326	3.95	3.98	2.43
0.21	4	2	3	59.8	61	326	3.89	3.84	2.31
0.23	2	2	5	56.9	65	327	4.05	4.07	2.31
0.29	4	6	4	62.4	58	334	4.20	4.23	2.63
0.31	2	7	2	63.3	58	335	4.34	4.35	2.75
0.24	3	7	6	62.8	57	336	3.94	3.96	2.48

Issues with outliers in the data

To have a general idea of the distribution of the values assumed by the attributes, a list of histograms can be displayed:



Some of the histograms seem to be affected by outliers in the data. It can be seen by looking at the horizontal axis of each graph and by noticing that bins do not cover all the axis values but only a limited range. This is very evident in the attributes *table* and in the dimensions *y* and *z*. This attributes contain very extreme values that wrongly set the axis limits. In order to detect them, the data set can be sorted from the smallest value of a specific attribute to the largest value of the same attribute. By doing so, it can be selected the 99.99 percentile, which corresponds to the highest observations of the data set sorted by one of the attributes.

If the values assumed by the attribute are not physical or are very different from the other values, the specific observation can be discarded.

The 99.99^{th} percentile of the data set sorted by the *table* shows the following data:

```
##       carat cut color clarity depth table price    x    y    z
## 24933  2.01   1     3       3  58.6    95 13387 8.32 8.31 4.87
## 50774  0.81   1     3       2  68.8    79  2301 5.26 5.20 3.58
## 51343  0.79   1     4       3  65.3    76  2362 5.52 5.13 3.35
```

The first row has a value of *table* very far from the other two rows. It is very unlikely that, considering a data set of 53940 diamonds, there is a diamond with the *table* 16 percentage points bigger than the second-biggest *table* diamond. For this reason, the row nr. 24933 of the data set will be canceled.

As for the attribute y (width), it can be followed the same procedure by sorting the data set by the width and looking at the 99.99^{th} percentile:

```
##       carat cut color clarity depth table price     x     y    z
## 24068  2.00   4     5       2  58.9    57 12210  8.09 58.90 8.06
## 25999  4.01   4     6       1  61.0    61 15223 10.14 10.10 6.17
## 27416  5.01   1     7       1  65.5    59 18018 10.74 10.54 6.98
## 27631  4.50   1     7       1  65.8    58 18531 10.23 10.16 6.72
## 49190  0.51   5     2       5  61.8    55  2075  5.15 31.80 5.12
```

It can be seen that the first and the last rows have very larger values of y with respect to the the other three rows belonging to the 99.99^{th} percentile. It is very unlikely that, considering such a big number of diamonds, there are two diamonds with dimensions so different from the others. So the two rows containing the two outliers will be canceled

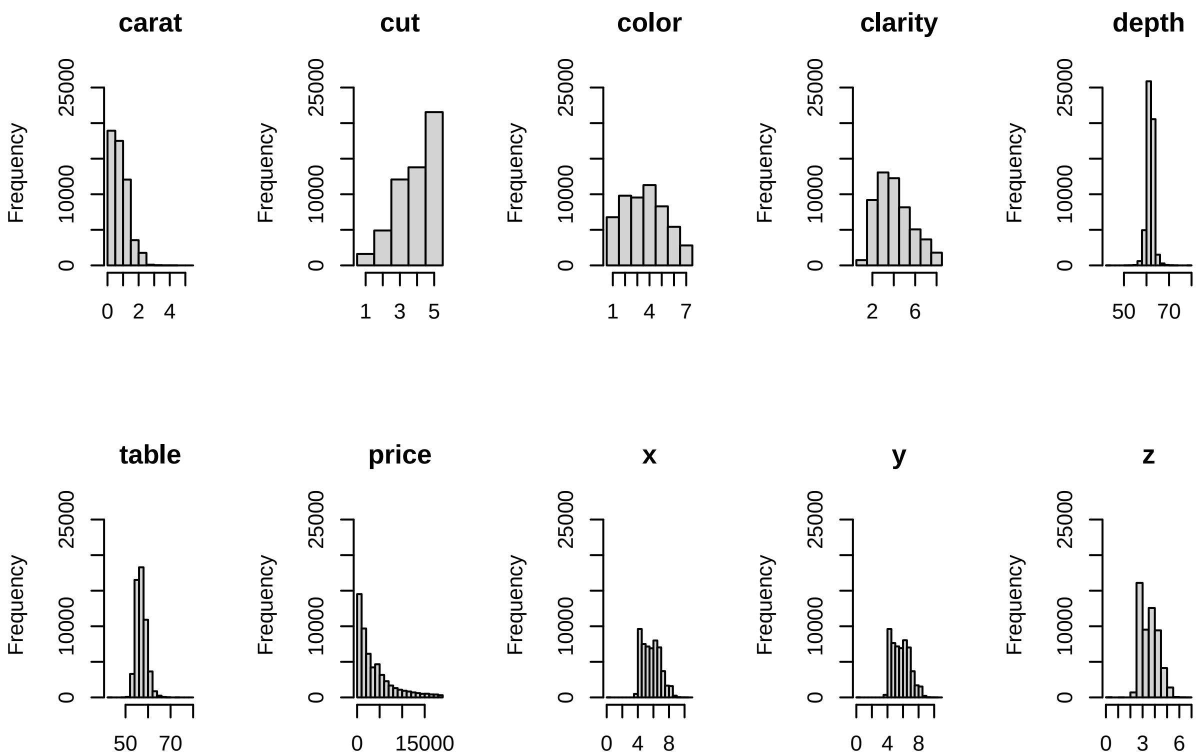
Finally, sorting the data set by the z-dimension, the 99.99^{th} percentile is composed by the following records:

```
##       carat cut color clarity depth table price     x     y     z
## 23645  3.65   1     5       1  67.1  53.0 11668  9.53  9.48  6.38
## 24068  2.00   4     5       2  58.9  57.0 12210  8.09 58.90  8.06
## 27131  4.13   1     5       1  64.8  61.0 17329 10.00  9.85  6.43
## 27416  5.01   1     7       1  65.5  59.0 18018 10.74 10.54  6.98
## 27631  4.50   1     7       1  65.8  58.0 18531 10.23 10.16  6.72
## 48411  0.51   3     2       5  61.8  54.7  1970  5.12  5.15 31.80
```

The last record has a value of z very far from the other ones, so it can be considered as an outlier and so it will be removed from the data set.

Possible distributions followed by the data

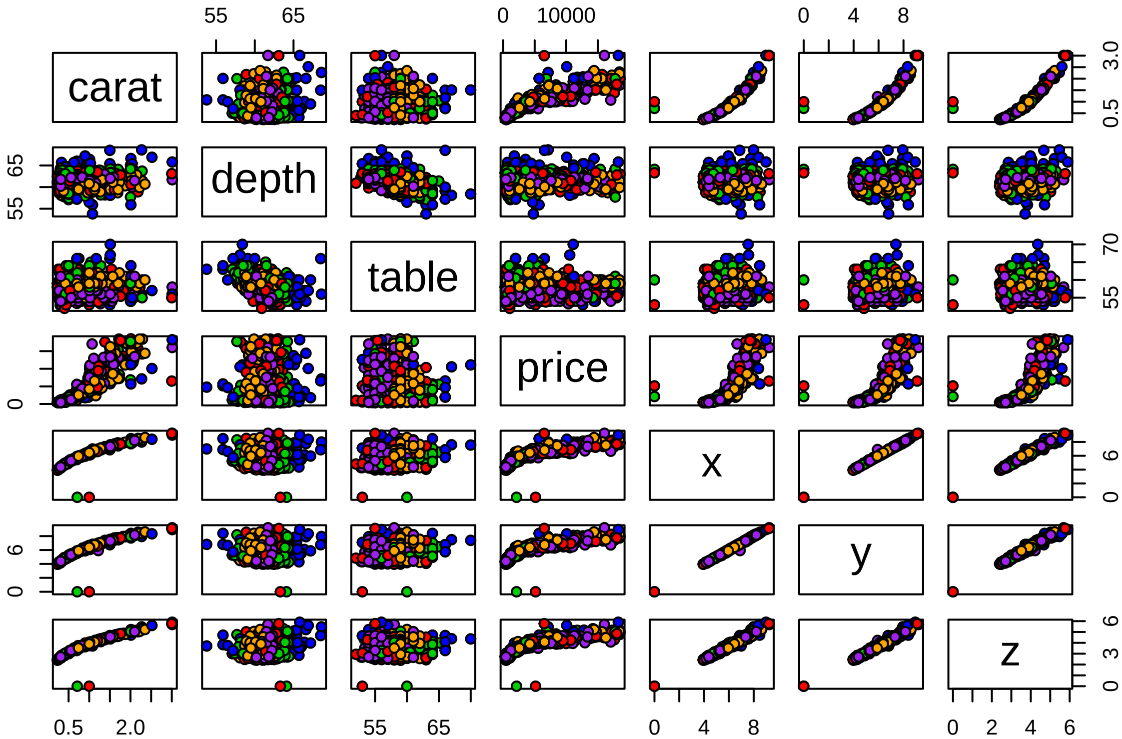
Now that the data set is free from outliers, the histograms of the attributes will be more meaningful about the possible distribution the data follow for each variable:



From the graphs, some considerations about the distribution of each attribute can be listed: - attributes *depth* and *table* appear to be normally distributed since they follow quite well the gaussian bell curve and they are almost symmetric; - attributes *carat* and *price* seem to follow an exponential distribution, since each bin is smaller than the previous one and larger than the following one; - attributes *clarity*, *x*, *y* and *z* appear to follow a log-normal distribution, since they follow a asymmetric bell curve with the sample-mode smaller than the mean.

Correlation of variables

Correlation between variables can be studied by producing the matrix of scatter plots of each combination of two attributes against each other. Scatter plots are not so meaningful with ordinal data (*cut*, *color* and *clarity*), so they are removed from the data set and points of the plots are colored according to the attribute *cut* in the attempt of finding some pattern of the data. The matrix is shown below:



The plots confirm the very high correlation between the variables *carat*, *x*, *y* and *z*, as already computed in the Table 3. In addition, also the *price* appears to be slightly correlated with the variables *carat*, *x*, *y* and *z*. The other variables do not show any specific pattern and also the *cut* does not seem to distinguish any cluster of the data.

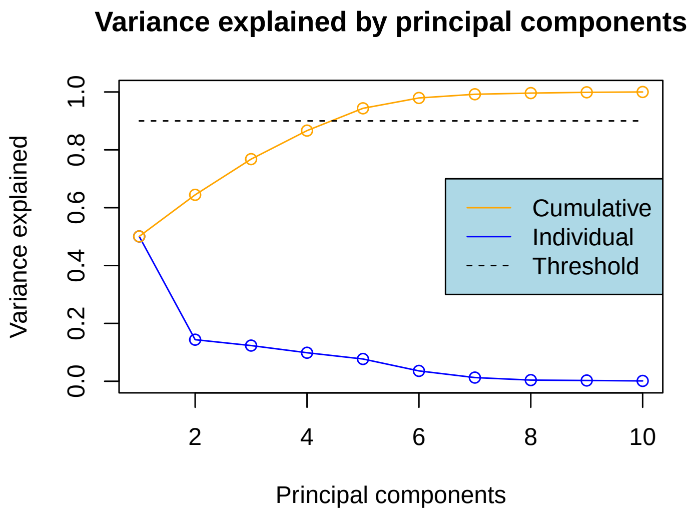
Principal component analysis

The aim of the Principal Component Analysis ("PCA" from now on), is to simplify the problem dimension by reducing the number of variables which explains the behavior of the diamonds' price.

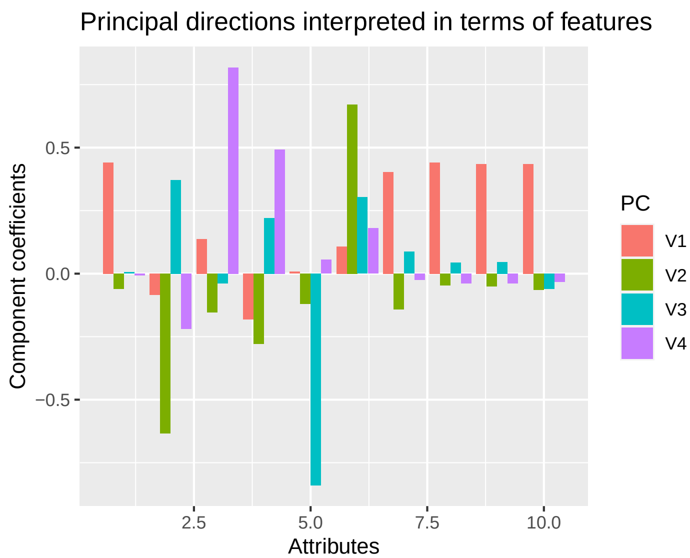
Below the standard deviations of the variables are shown:

```
##    carat      cut    color clarity    depth    table   price        x        y        z
##     0.47     1.12     1.70     1.65     1.43     2.23 3989.44     1.12     1.14     0.71
```

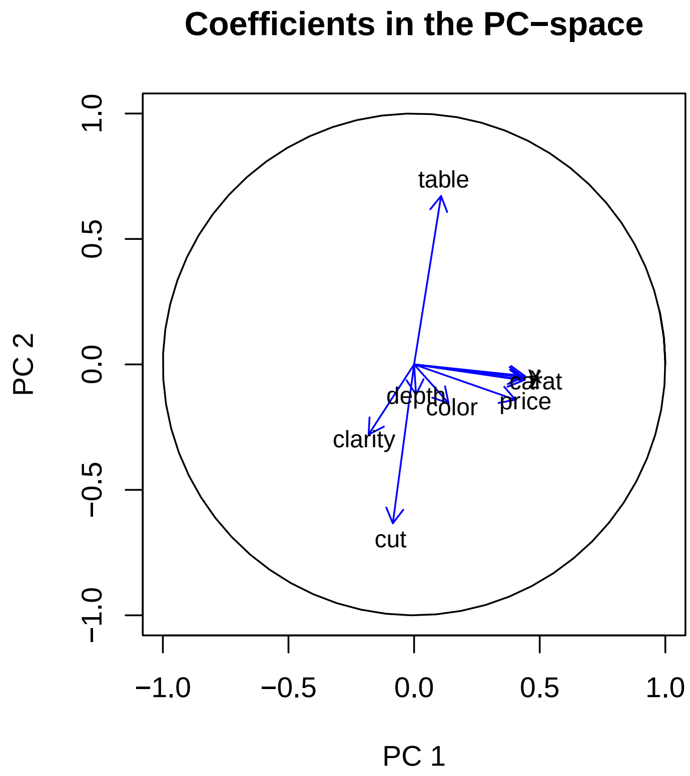
Since standard deviation of *price* is some orders of magnitude larger than the others, the dataset has to be standardized by subtracting the mean and dividing by the standard deviation of the whole set of observations of that variable.



The new standardized dataset can be now used to perform the Singular Value Decomposition (SVD), which gives rise to three different matrices: U, Σ and V^T. By the extraction of the diagonal of the matrix Σ, it can be seen how much variance is explained by each principal component. The cumulative explained variance should reach the percentage of 90% in order to describe properly the main features of the dataset.
The figure on the left shows that the first 4 principal components explain little less than the 90% of variance



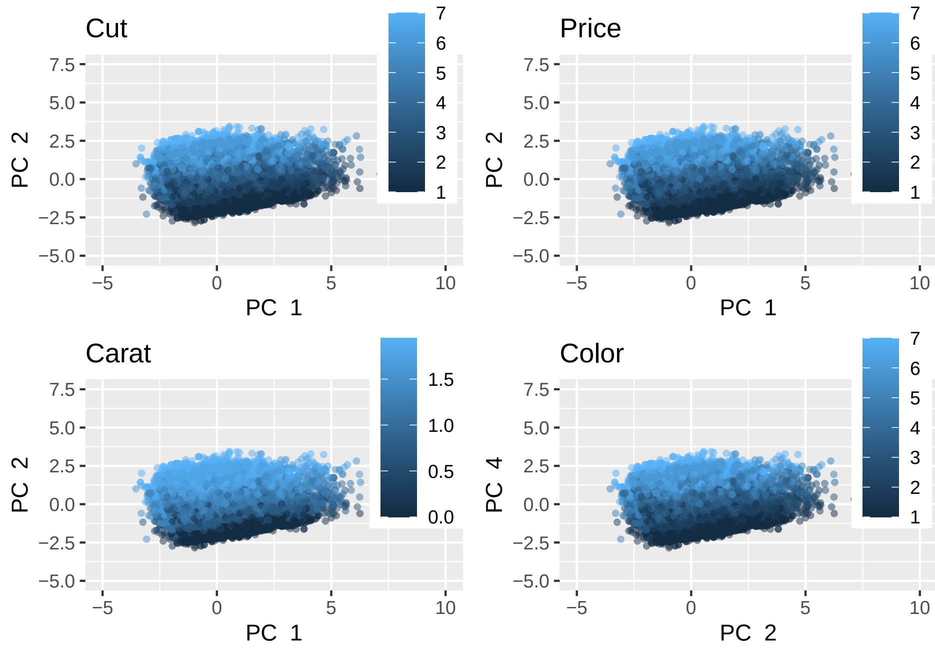
The matrix V^T contains the ten decadimensional vectors defining the principal components (PCs). Focusing on the first four PCs, they contains the weights associated to each of original component. The figure on the left shows the valuea of the weights. It can be noticed that the first PC mainly describes the dimensional quantities of the diamonds (*carat*, size in x, y, z) and the *price*. It seems that these five characteristics alone explain half of the variability of the diamonds. As for the second PC, it focuses more on the quality characteristics (*cut*, *color*, *clarity*) and on the *table*.



Unfortunately, it is impossible to represent data in a four-dimensional space, so data are projected onto the bi-dimensional space defined by the first two PCs. Coefficients can be projected in this space as well, showing the directions followed by original attributes. It can clearly be seen that the first PC is eastward oriented, so that it collects very well the eastward attributes. Probably, the second PC is southward directed so that it collects the southward attributes (with the plus sign) and the northward *table* attribute (with the minus sign)

Data can be projected in all the possible bi-dimensional spaces defined by each combination of two of the PCs. By projecting them, some insteresting features can be observed, as shown by the four graphs below.

- Variable *cut* can assume five possible levels, so it is the easiest one to be visualized. The graph shows that *cut* varies mostly in the direction of the second PC, so the *cut* of diamonds strongly depends on their quality features (the better are *color* and *clarity*, the better will be the cut), and is less affected by the size (x, y and z) and the *carat* (weight);
- variable *price* is strongly asymmethrical towards low values, and it is very evident also in the graph where cold blue is predominant with respect to light blue. *Price* strongly varies with the first PC, so it depends on quantity factors of diamonds (the bigger and heavier is the diamond, the more expensive will be) more than on quality ones;
- variable *carat* is very concentrated between 0 and 1 and widespread beyond 1. So, in order to have a better visualization, the graph below represent the logarithm of the *carat*. It can be seen that it is correlated to the dimensional quantities explained by the first PC (the more expensive and bigger is the diamond, the heavier will be) more than quality ones;
- variable *color* has a better representation if data are projected onto the plan defined by the second and forth PCs. It can be seen that *color* varies with the forth PC, so it is correlated with *clarity*, *cut* and *table*.



4) Results: what data have shown

Exam Problems

Question 1 | Spring 2019 question 2 Which of the following statements are true about the types of the attributes in the Urban Traffic dataset?

Answer **B**: the attribute x_1 is ordinal because it can be said that time-interval number 5 is greater than the time-interval number 4 since it is later in the day but it makes no sense to sum and subtract them

Question 2 | Spring 2019 question 2 Consider again the Urban Traffic dataset from table 1 and in particular the 14 and 18th observation . Which of the following statements about the p-norm distance dp (.,.) is correct ?

Answer **A**: The Maximum Norm Distance is 7.0

Question 4 | Spring 2019 question 2 Which one of the following statements is true?

Answer **D**: PC2 has a negative coefficient multiplying x_1, so if an observation has a low value of x_1 it means that the PC2 decreases very little for the contribution of x_1. On the other hand, PC2 has positive coefficients multiplying x_2, x_3 and x_5 so if an observation has a high value of them, it means that the PC2 increases a lot thanks to their the contribution

References

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Appendix