# Learning Coursework: Task 2 report

April 9, 2020

## 2.3 - Polygon A classification

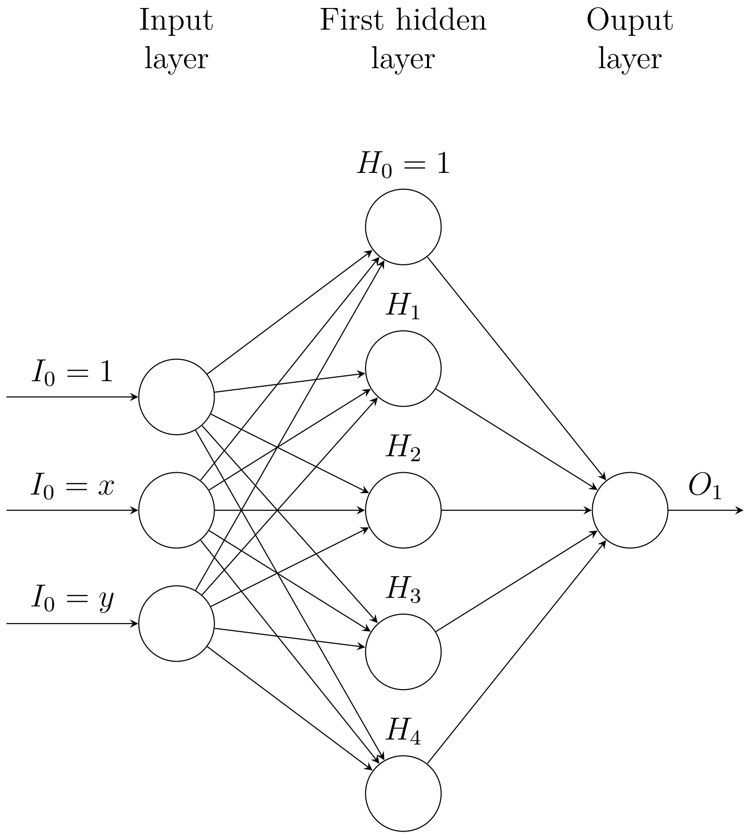


Figure 1: **Polygon A classification - neural network layout**

## Weights

1. The coordinates of the point we are classifying are in the form: $(x, y)$. Therefore, we need to put them in the input layer along with a one: $I_0 = 1, I_1 = x, I_2 = y$.

2. As for the first hidden layer, we need a neuron for each edge of the polygon. Starting from the upper-left edge (as seen on Figure 2) and going along the polygon in counter-clockwise order, each neuron decides if the input point is below/above the line that passes through the corresponding edge.

   (a) $\underline{H_1}$: **1** if below the line that passes through the first edge; otherwise **0**

   (b) $\underline{H_2}$: **1** if above the line that passes through the second edge; otherwise **0**

   (c) $\underline{H_3}$: **1** if above the line that passes through the third edge; otherwise **0**

   (d) $\underline{H_4}$: **1** if below the line that passes through the fourth edge; otherwise **0**

   For example, for $H_1$ we need to make sure that the value of the neuron corresponds to the fact that the following inequality holds (approximately): $y < 1.116 * x + 1.170$ as $y = 1.115 * x + 1.170$ is the equation of the line that passes through the upper-left edge. Now, we can transform this inequality into vector form and normalise it by dividing by the biggest absolute value (1.170 in this case):

$$\begin{bmatrix} 1 \\ x \\ y \end{bmatrix} \bullet \begin{bmatrix} 1.170 \\ 1.116 \\ -1 \end{bmatrix} > 0 \Leftrightarrow \begin{bmatrix} 1 \\ x \\ y \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 0.954 \\ -0.855 \end{bmatrix} > 0 \tag{1}$$

   Now, we can take the vector on the right as the weight vector for $H_1$. Once the step function is applied, this will ensure that the requirements for the output of $H_1$ set above are met. For $H_2, H_3, H_4$, we can proceed in an equivalent fashion.

3. Finally, the output layer weight are constructed so that the output is 1 if and only if $H_1 = H_2 = H_3 = H_4 = 1$. This can be achieved by taking $[-\frac{4}{5}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}]$ as the weight vector. If at least one of $H_1, H_2, H_3, H_4$ is equal to 0, then the following will hold (as the rest can only sum up to $\frac{3}{4}$ at maximum, which is smaller than $\frac{4}{5}$):

$$\begin{bmatrix} 1 \\ H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} \bullet \begin{bmatrix} -4/5 \\ 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix} < 0 \tag{2}$$

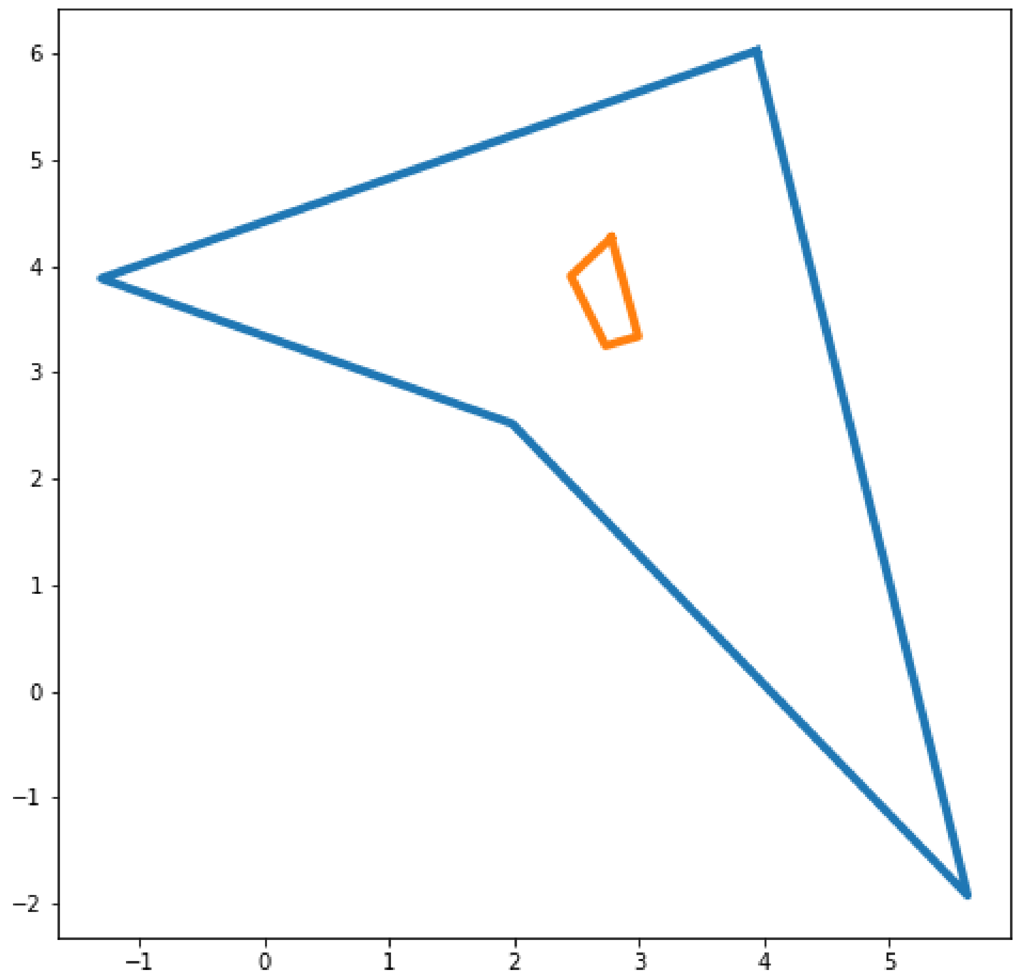


Figure 2: **Polygons A, B**; The A polygon is in orange, the B polygon is in blue.

## 2.10 - Different decision regions for sigmoidal/step functions