#### **Problem 1**

Hits are happy as happy. False alarms are happy as sad.

```
Subject 1: d' = \Phi^{-1}(41/60) - \Phi^{-1}(41/60) = 0
Subject 2: d' = \Phi^{-1}(39/60) - \Phi^{-1}(42/60) < 0
Subject 3: d' = \Phi^{-1}(32/60) - \Phi^{-1}(32/60) = 0
```

Not a single subject had a d'>0 so none of the subjects could actually distinguish happy from sad faces.

- a) The animation did not work
- b) The researchers conclusion is not valid.

## **Problem 2**

a)

```
sensitivity d' = \Phi^{-1} (57/60) - \Phi^{-1} (35/60) = 1.43; bias = \Phi^{-1}(Correct rejections) = \Phi^{-1}(25/60) = -0.21
```

b)

A criterion smaller than d'/2 is a lax criterion meaning that the observer is more like to say yes than no. In this case the criterion is << d'/2 and the criterion is very lax. This could be due to the way the cat was trained. Reward was given only for hits and not for correct rejections.

c) based on signal detection theory assuming Gaussian noise the psychometric function is a cumulative Gaussian  $\Phi((x-x_0)/\sigma)$  where x is the sound level in db,  $x_0$  is the 50% threshold and  $\sigma$  is the standard deviation. Since d' is measured in standard deviations we have

```
d' = 1.43 \sigma = 5 dB \text{ so } \sigma = 5 db / 1.43 = 3.50 dB
```

The 50% threshold is the stimulus level corresponding to the criterion.

```
50% threshold = criterion = -0.21 \sigma = -0.21 x 3.50 dB = -0.73 dB
```

The negative threshold means that due to the cat's bias to say yes, it would actually say yes more than 50% for a sound level of 0 dB (no sound).

#### **Problem 3**

The solution to this problem is obtained in complete analogy to the signal detection exercise in the course.

The three level confidence ratings can be model using a signal detection model with five criteria dividing the line into 6 ordered response categories. For the leftmost criterion between no (high confidence) and no (medium confidence) the hit rate is (100-2)/100. For the next criterion the hit rate is (100-2-14)/100 and so forth. The corresponding false alarm rates are (100-2)/100 and (100-2-31)/57 respectively.

The receiver operating characteristics is a straight line in Gaussian coordinates so that

```
\Phi^{-1} (Hit rate) = (μ/σ)\Phi^{-1} (False alarm rate) + 1/σ
```

By fitting a straight line to the hit rates / false alarm rates for each of the five criteria we get  $\sigma$ =1.18 and  $\mu$ =1.19

## **Problem 4**

A Bayesian observer using the maximum a posteriori criterion will say yes if the posterior probability for a signal present is greater than the posterior probability for signal absent

```
L(c \text{ given S})P(S) > L(c \text{ given NS})P(NS)
```

where c is the criterion, L is the likelihood, S is signal, NS is no signal and P is the prior probability. According to the SDT model the likelihood for the internal representation value is Gaussian so that

$$\varphi(c,\mu_S,\sigma)P(S) > \varphi(c,\mu_{NS},\sigma)P(NS)$$

Where  $\Phi$  is the normal probability density function. Furthermore, note that P(S) = 1 – P(NS) so that

$$\varphi(c,\mu_S,\sigma)P(S) > \varphi(c,\mu_{NS},\sigma)(1-P(S)) = \varphi(c,\mu_{NS},\sigma) - \varphi(c,\mu_{NS},\sigma)P(S)$$

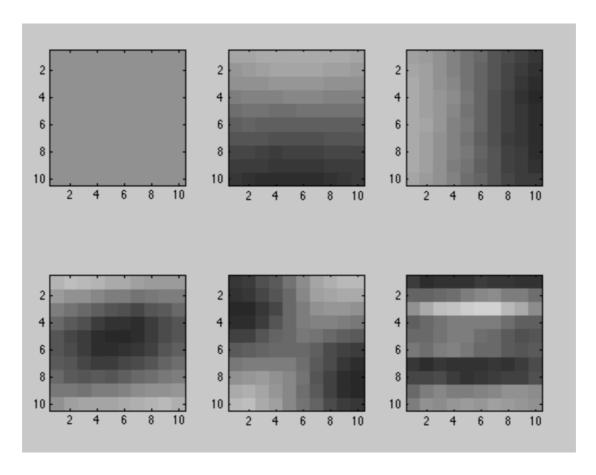
$$P(S)(\varphi(c,\mu_S,\sigma)+\varphi(c,\mu_{NS},\sigma))=\varphi(c,\mu_{NS},\sigma)$$

$$P(S) = \varphi(c,\mu_{NS},\sigma) / (\varphi(c,\mu_{S},\sigma) + \varphi(c,\mu_{NS},\sigma))$$

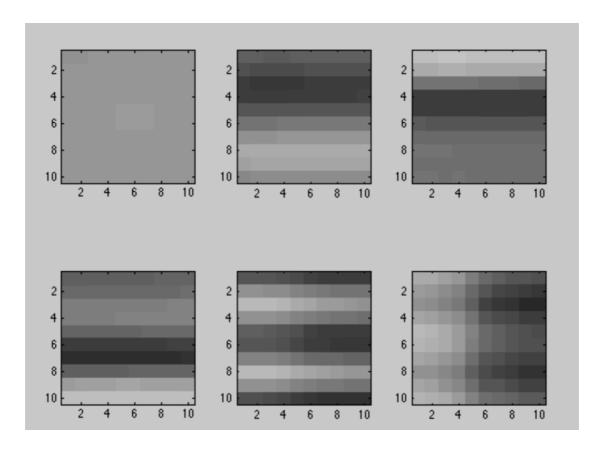
$$= \varphi(0.7,0,1) / (\varphi(0.7,1,1) + \varphi(0.7,0,1)) = 0.45$$

# Problem 5

The six first principal components for mona



the six first principal components for wood



- a) the first pc is nearly the same for the two images. The second is similar. The others differ
- b) the difference is due to the wood image containing mostly horizontal components.
- c) Neurons in the visual system are also sensitive to vertical lines reflecting the structure in the natural world.