02417: Time Series Analysis
Rune Grønborg Junker
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DTU Compute
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Assignment 2: ARMA Processes and Seasonal Processes

This assignment has a more theoretical focus compared to the rest of the assignments. You will study how the choice of coefficients (of the operator polynomials in ARMA processes) affects the structure of the process, through simulated data and empirical autocorrelation functions. Be aware that the model parametrizations in the assignment and in your favorite software package may not be identical. Sometimes AR-coefficients are formulated with positive signs on the right-hand side of the equation and sometimes on the left-hand side.

Question 2.1: Stability Let the process $\{X_t\}$ be given by

$$X_t + \phi_1 X_{t-1} + \phi_2 X_{t-2} = \varepsilon_t$$

where ε_t is a white noise process with finite moments.

Investigate analytically for which values of ϕ_2 the process is stationary when $\phi_1 = -\frac{1}{3}$.

In addition it should be investigated for which values of ϕ_2 the autocorrelation function shows damping harmonic oscillations (Complex roots). Still for $\phi_1 = -\frac{1}{3}$.

Question 2.2: Predicting consumer price index In finance there is great interest in predicting future values for a large range of indices including the consumer price index. An institution wants to predict the Consumer Price Index for All Urban Consumers from FRED, Federal Reserve Bank of St. Louis.

Based on historical data the following model has been identified:

$$(1 - 0.9B + 0.3B^2)(1 - 0.6B^4)(\log(Y_t) - \mu) = \varepsilon_t$$

where ε_t is a white-noise process with zero mean and variance σ_ε^2. Based on 77 observations, it is found that $\sigma_\varepsilon^2 = 0.01^2$. Furthermore, μ was estimated to 5.5. The table below shows the last eight observations of Y_t:

t	2015Q2	2015Q3	2015Q4	2016Q1	2016Q2	2016Q3	2016Q4	2017Q1
CPI	237.9	236.5	238.1	241	241.4	241.4	243.8	245.0

Predict the values of Y_t corresponding to t = 2017Q2 and 2017Q3, together with 95% prediction intervals for the predictions.

Question 2.3: Random walk Let the process Y be given by

$$Y_t = \exp(2) + \sum_{i=1}^{t} \varepsilon_i$$

where ε_t is a normally distributed white noise process with mean zero and variance σ_ε^2.

1. Find the mean value, variance and covariance functions of the process.

2. Is the process Y stationary? why?/why not?
3. Simulate 10 realizations of the process Y_t. Save the realizations and plot. Also plot their estimated autocorrelation functions (Preferably in one plot). Comment on the graphs, and use them to argue for your answer in step 2.

Question 2.4: Simulating seasonal processes A process Y_t is said to follow a multiplicative $(p,d,q)\times(P,D,Q)_s$ seasonal model if

$$\phi(B)\Phi(B^s)\nabla^d\nabla_s^D Y_t = \theta(B)\Theta(B^s)\varepsilon_t$$

where (ε_t) is a white noise process, and $\phi(B)$ and $\theta(B)$ are polynomials of order p and q, respectively. Furthermore, $\Phi(B^s)$ and $\Theta(B^s)$ are polynomials in B^s. All according to definition 5.22 in the textbook.

Simulate the following models (where monthly data are assumed). Plot the simulations and the associated autocorrelation functions (ACF and PACF).

1. A $(1,0,0)\times(0,0,0)_{12}$ model with the parameter $\phi_1=-0.8$.
2. A $(0,0,0)\times(1,0,0)_{12}$ model with the parameter $\Phi_1=0.8$.
3. A $(1,0,0)\times(0,0,1)_{12}$ model with the parameters $\phi_1=-0.9$ and $\Theta_1=0.7$.
4. A $(1,0,0)\times(1,0,0)_{12}$ model with the parameters $\phi_1=0.6$ and $\Phi_1=0.8$.
5. A $(0,0,1)\times(0,0,1)_{12}$ model with the parameters $\theta_1=-0.4$ and $\Theta_1=0.8$.
6. A $(0,0,1)\times(1,0,0)_{12}$ model with the parameters $\theta_1=0.4$ and $\Phi_1=-0.7$.

Based on the ACF and PACF plots which models are seasonal? Explain your reasoning.

Based on the ACF and PACF plots which models are stationary? Explain your reasoning.

Note: `arima.sim` from R does not have a seasonal module, so model formulations as standard ARIMA processes have to be made.