02418, Assignment 3

Jan Kloppenborg Møller

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Introduction

During the semester you will analyze different cases, in this third assignment you will continue working on the data from Assignments 1 and 2 (only wind and financial data is included here).

The data for the projects can be found together with the assignment on Inside.

It is important to describe your results and conclusions, not only numbers, but also in words interpreting your results.

Project 1: Wind power forecast

In this project you will be analyzing a data set from Tunø Knob wind power plant. Details on data may be found in Assignment 1, and you will continue the analysis from Assignment 2.

In Sssignment 2 you should have estimated a regression model of the form

$$Y^{(\lambda)} = f(w_s) + \varepsilon; \quad \varepsilon \sim N(0, \sigma^2) \tag{1}$$

where $Y^{(\lambda)}$ is defined by one of the transformations given in Assignment 1, and Y is normalised wind power, your model might also include winddirection. If you have chosen a different model you should start by fitting the model

$$Y^{(0.2)} = \beta_0 + \beta_1 w_s + \beta_2 ws^2 + \varepsilon; \quad \varepsilon \sim N(0, \sigma^2) \tag{2}$$

where $Y^{(0.2)}$ is defined by the first transformation for wind power in Assignmet 1. Whether or not you use the λ and transfomation given here or you use the one you found yourself, you should keep the transformation (including λ) fixed throughout this assignment.

Analysis of auto-correlation:

The wind power data is given as a time-series and therefore it is natural to analyse it as such. A simple time series model is the AR(1) model, which we will apply for the residuals, i.e.

$$\varepsilon_i = \phi\varepsilon_{i-1} + u_i; \quad u_i \sim N(0, \sigma_u^2) \quad \text{and} \quad \text{iid} \tag{3}$$

1. Extract the residuals from the linear model given above, and construc the matrix

$$e = \begin{bmatrix} e_1 & e_2 \\ \vdots & \vdots \\ e_{n-1} & e_n \end{bmatrix} \tag{4}$$

2. Fit parameters in the model $[e_i, e_{i+1}]^T \sim N(0, \Sigma)$, with

$$\Sigma = \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \tag{5}$$

   the answer should include

   - Parameter estimates and Wald confidence intervals
   - A contour plot of the likelihood with contour line indicating confidence regions
   - P-values for the likelihood ratio test and wald test for the hypothesis $H_0 : \rho = 0$ against the alternative.

3. Compare the Information matrix calculated by numerical methods with the algebraric form for the Fisher information $I(\hat{\sigma}^2, \hat{\rho})$.

4. Make a plot of the profile liklihood of ρ and compare with the quadratic approximation. Further using the z-transform (see Exersise 3.23) for ρ and the log-transform for σ^2, plot the profile likelihood for z and compare with the quadratic approximation, finally derive Fishers information matrix for the transformed varaibles and compare with the numerical Hessian.

5. Estimate the parameters of the AR(1) model (see Example 11.1), first conditioning on e_1, then full estimation. Compare with the estimation above, is it as expected?

6. Estimate the parameters of the linear model (2) and the parameters of the AR(1) model simultanious, and compare the likelihood of the linear model and the combined model.

7. Discuss the effect fo including the AR(1)-term for short and long term preddictions.

Project 3: Financial data

In this part you should continue the analysis from Assignment 1 on financial data. Background information can be found in that assignment.

1: Mixture model

a) With the same data as in Assignment 1, fit normal mixture models with 2 and 3 components, argue which one is better and compare with the best model from assignment 1.

b) For the best mixture model, report confidence interval for the parameters, and give an interpretation of these intervals.

c) For the two component model make a profile likelihood plot of one of the variance parameters.

d) In the previous question you should see multiple maxima, reprametrize the model such that you only see one maximum.

e) Discuss the interpretaion of the models.

3: Hidden Markov Models

a) Fit two and three state normal Hidden Markov Models to the data and conclude on the best choice

b) For the chosen model report 95% confidence intervals for the working parameters.

c) Report the natural parameters and interpret the result.

d) Compare the following distributions (by plots)

   – The long term distribution of the return.
   – The 1-step ahead distribution given that you known the current state.

e) Report 95% confidence intervals for some (or all) natural parameters (note that the natural paramters include the stationary distribution). Some options for finding these CI's are

   – Formula (3.2) in the (HMM) textbook.
   – The bootstrap method in Section 3.6.2.
   – Profile likelihood.

f) Discuss what would be needed in order to make short term (say 1-2 weeks) predictions.