Cheat sheet on trigonometric identities

- Basic relations

$$\sin^2\theta + \cos^2\theta = 1,$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

- Product-to-sum

$$2\cos\theta\cos\varphi = \cos(\theta-\varphi) + \cos(\theta+\varphi)$$

$$2\sin\theta\sin\varphi = \cos(\theta-\varphi) - \cos(\theta+\varphi)$$

$$2\sin\theta\cos\varphi = \sin(\theta+\varphi) + \sin(\theta-\varphi)$$

$$2\cos\theta\sin\varphi = \sin(\theta+\varphi) - \sin(\theta-\varphi)$$

- Sum-to-product

$$\sin\theta \pm \sin\varphi = 2\sin\left(\frac{\theta\pm\varphi}{2}\right)\cos\left(\frac{\theta\mp\varphi}{2}\right)$$

$$\cos\theta + \cos\varphi = 2\cos\left(\frac{\theta+\varphi}{2}\right)\cos\left(\frac{\theta-\varphi}{2}\right)$$

$$\cos\theta - \cos\varphi = -2\sin\left(\frac{\theta+\varphi}{2}\right)\sin\left(\frac{\theta-\varphi}{2}\right)$$

- Lagrange's trigonometric identities

$$\sum_{n=1}^{N}\sin(n\theta) = \frac{1}{2}\cot\frac{\theta}{2} - \frac{\cos\left(\left(N+\frac{1}{2}\right)\theta\right)}{2\sin\left(\frac{\theta}{2}\right)}$$

$$\sum_{n=1}^{N}\cos(n\theta) = -\frac{1}{2} + \frac{\sin\left(\left(N+\frac{1}{2}\right)\theta\right)}{2\sin\left(\frac{\theta}{2}\right)}$$