Exercise 3

In [ ]:
```
%load_ext autoreload
%autoreload 2
```

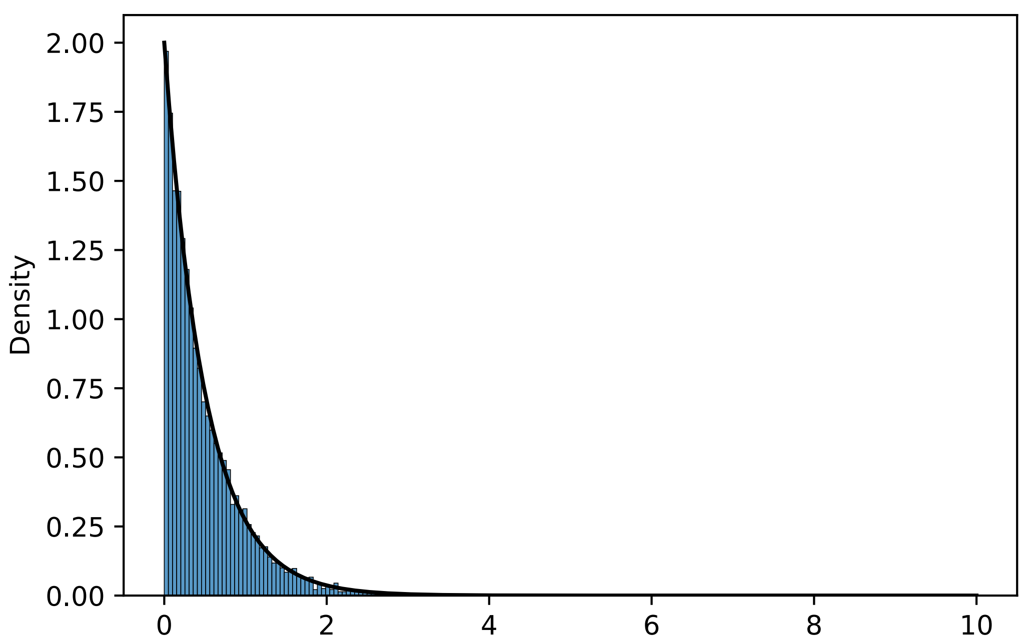
In [ ]:
```
from src.my_random.gen import *
from src.my_random.tests import chi2, kolmogorov, emperical_dist
import matplotlib.pyplot as plt
import seaborn as sns

import scipy.stats as stats
import pandas as pd
```

Exponential Distribution

In [ ]:
```
lmbda = 2
exps = exponential(lmbda, 10_000)
h = sns.histplot(exps, stat='density')
x = np.linspace(0, 10, 1000)
sns.lineplot(x=x, y=stats.expon.pdf(x, scale=1/lmbda), color='k')
kolmogorov(exps, stats.expon())
```

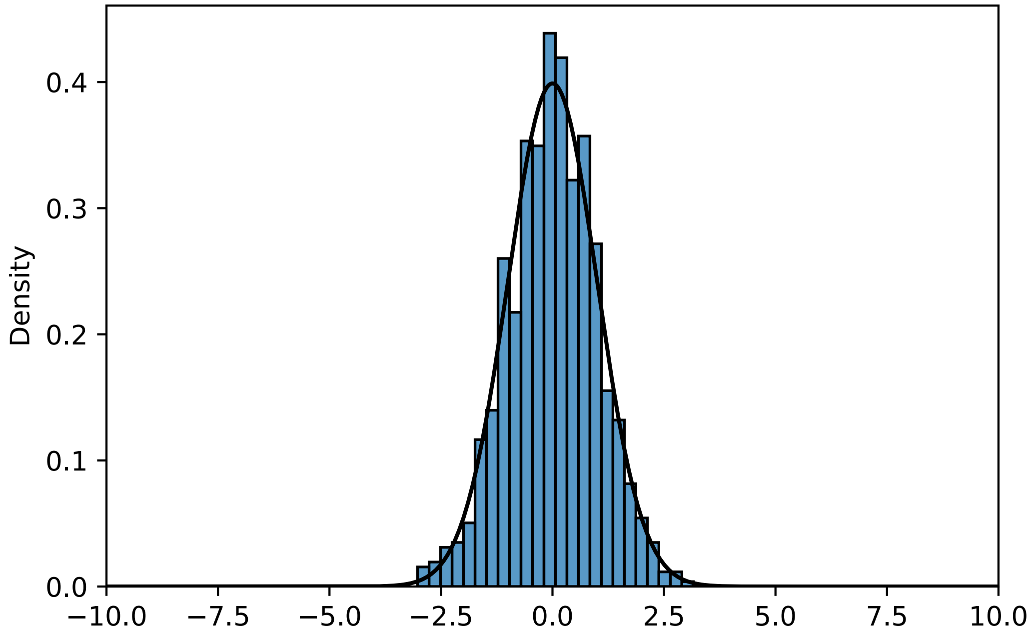
Out[ ]:
```
79.10421538962612
```



Normal Distribution

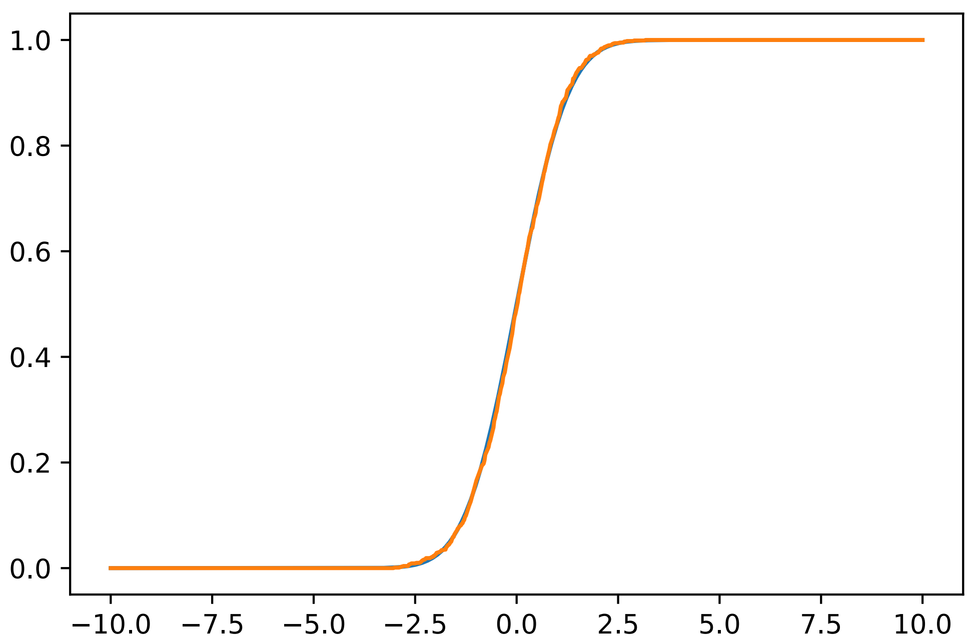
In [ ]:
```
norms = norm_box_mueller(1000)
h = sns.histplot(norms, stat='density')
x=np.linspace(-10,10,1000)
sns.lineplot(x=x, y=stats.norm.pdf(x), color='k')
h.set(xlim=(-10, 10))
```

Out[ ]: [(-10.0, 10.0)]



In [ ]:
```
sns.lineplot(x=x, y=stats.norm.cdf(x))
sns.lineplot(x=x, y=[emperical_dist(i, norms) for i in x])
kolmogorov(norms, stats.norm(), (-10, 10))
```

Out[ ]: (-62.65744413894945, -1.9736956037033961)



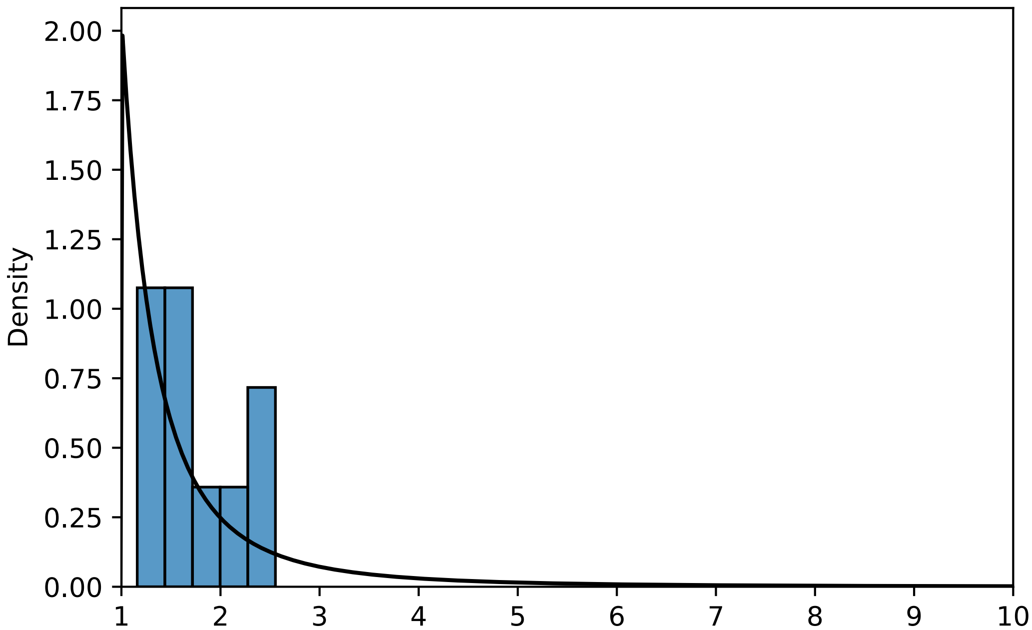
Pareto

In [ ]:
```
paretos = pareto(2.05, 1, 10)
h = sns.histplot(paretos, stat='density')
x=np.linspace(-10,10,1000)
sns.lineplot(x=x, y=stats.pareto.pdf(x, b=2.05, scale=1), color='k')
```

```
h.set(xlim=(1, 10))
kolmogorov(paretos, dist=stats.pareto(b=1, scale=2.05), range=(1,1e4))
```

Out[ ]:

```
6.842882392759879
```

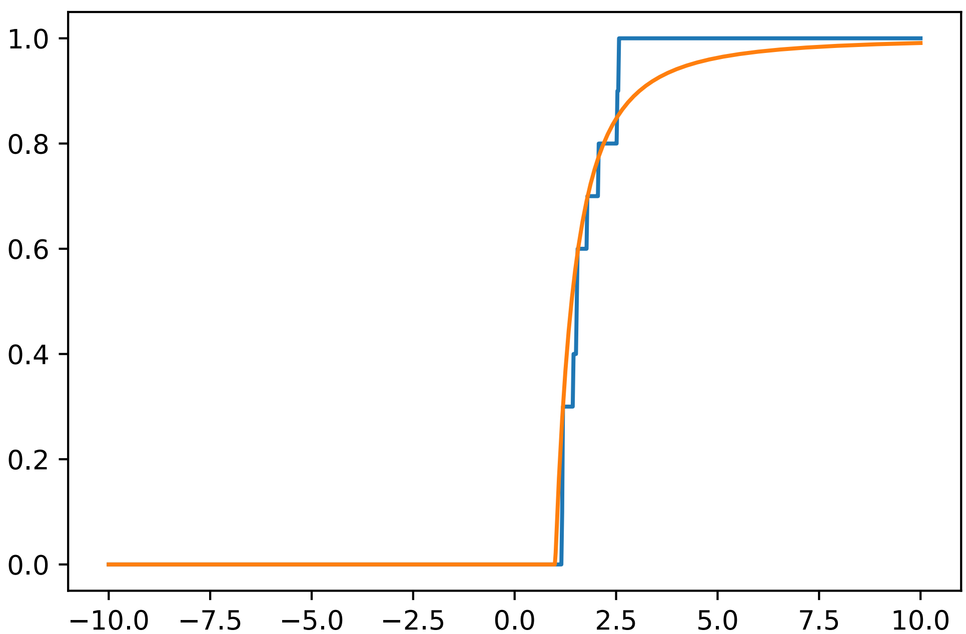


In [ ]:

```
sns.lineplot(x = x, y = [emperical_dist(i, paretos) for i in x] )
sns.lineplot(x=x, y=stats.pareto(b=2.05, scale=1).cdf(x))
```

Out[ ]:

```
<AxesSubplot:>
```



In [ ]:

```
ks = [2.05, 2.5, 3, 4]
df = pd.DataFrame({k: pareto(k, 1, 10) for k in ks})
```

In [ ]:

```
obs_stats =  df.aggregate(['mean', 'var'])
```

In [ ]:

```
def mean_pareto(beta, k):
    return beta*k/(k-1)
```

```python
def var_pareto(beta, k):
    return beta**2*k/((k-1)**2 * (k-2))


true_means = [mean_pareto(1, i) for i in ks]
true_vars = [var_pareto(1, i) for i in ks]
true_stats = pd.DataFrame({'mean': true_means, 'var': true_vars})
true_stats.T
```

Out[ ]:

	0	1	2	3
mean	1.952381	1.666667	1.50	1.333333
var	37.188209	2.222222	0.75	0.222222

In [ ]:
```python
obs_stats
```

Out[ ]:

	2.05	2.50	3.00	4.00
mean	1.797133	1.764837	1.329337	1.285407
var	0.815427	1.253118	0.426290	0.073788

Comparing the means and vars, we see that the estimates are are more off the smaller k are. Furthermore, we notice that the estimates of the means are way better than the variance estimates.

In [ ]:
```python
norms = np.stack([norm_box_mueller(10) for _ in range(100)])
t = np.array(stats.t.interval(.95, 9))
t_confs = np.stack([t*(row.std()/np.sqrt(10)) + row.mean() for row in norms])

chi = np.array(stats.chi2.interval(.95, 9))
chi_confs = np.stack([9*row.var() / chi[::-1] for row in norms])

conf = norms.mean(axis=1) + np.array([-1.96*norms.std(axis=1), 1.96*norms.std(

mean_df = pd.DataFrame({'lwr': t_confs[:,0], 'mean':norms.mean(1), 'upr':t_con
var_df = pd.DataFrame({'lwr': chi_confs[:,0], 'var':norms.var(1), 'upr':chi_co
```

In [ ]:
```python
mean_df.describe()
```

Out[ ]:

	lwr	mean	upr
count	100.000000	100.000000	100.000000
mean	-0.688873	-0.011229	0.666416
std	0.311284	0.261927	0.286284
min	-1.368979	-0.804567	-0.250260
25%	-0.925813	-0.183167	0.469010
50%	-0.687262	-0.048167	0.658813
75%	-0.493565	0.198056	0.850182
max	0.119087	0.665376	1.401509

In [ ]:
```
var_df.describe()
```

Out[ ]:

	lwr	var	upr
count	100.000000	100.000000	100.000000
mean	0.443607	0.937625	3.124967
std	0.187871	0.397091	1.323446
min	0.120146	0.253945	0.846361
25%	0.328122	0.693532	2.311441
50%	0.407786	0.861913	2.872629
75%	0.551490	1.165652	3.884945
max	1.201059	2.538607	8.460802

We see that the confidence intervals vary quite a lot both for the mean and the variance. In the extreme case of the mean, 0 is not even in the confidence interval, which in a lot of experiments would mean we would have concluded a statistical signficant result, even though this is gaussian noise.\ The variance has non-symmetrical confidence intervals. We see that especially the upper bound of the variance has a high standard deviation. This is to be expected, since we only have 10 observations

In [ ]:
```

```