B132584

Forward Interpretation of Isle of Rum Gravity Survey

Abstract

This gravity survey uses corrected data from the Isle of Rum to model that island's ultrabasic igneous intrusion as a buried sphere, and thus determine its location and dimensions. Altering the parameters of the buried sphere model and plotting that model against the real data will determine the parameters of the sphere that best fit the data. This investigation found that the best fit for the Isle of Rum data is a sphere centered at 36 $\pm$ 1 eastings, 96 $\pm$ 1 northings, with a depth and radius of 6000 $\pm$ 100 m, and a density contrast to the surrounding rock of 500 $\pm$ 10 kgm^{-3}.

Introduction

Gravity surveys are a common geophysics technique as they are relatively cheap and easy to perform. However, the data they gather requires significant processing in order to be useful. Gravity data must be corrected for the effects of the tides, surrounding terrain, altitude, and varying density of rock.

Interpreting the corrected data is a complicated forward problem, in which the data is compared to successive models that are constrained by the local geology. This investigation models the ultrabasic intrusive body below the Isle of Rum as a sphere of higher mass in order to determine its dimensions and location. This will give geologists greater knowledge of the igneous processes on the island.

Method

This survey used a relative gravity meter to take measurements along two profiles on the Isle of Rum. These raw data were then corrected for tidal drift, terrain, and for elevation via the Free-Air correction. Information about the local geology, the free-air method, and Nettleton's method were used to determine the best density to use for the Bouguer correction, which was then applied to the data. The corrected data were then plotted against horizontal distance for each profile.

A model was then created for a perfect sphere, based on the location of the center of the sphere, its radius, and the density contrast, and graphed on the same axes as the corrected data. Knowledge of the local geology, such as the size of the island, the thickness of the local crust, and the strike of the cone sheets was used to constrain the parameters of the spherical model. These parameters were then altered until the spherical model most closely matched the real data. This was determined by looking at graphs of the two, and computing the root mean square deviation.

Results

The free-air estimation of the density was 4000 $\pm$ 100 kg/m^3.

Graphs of the Bouguer anomaly for densities of 2800 kg/m^3, 3000 kg/m^3, 3200 kg/m^3, 3400 kg/m^3, and 4000 kg/m^3 were plotted against elevation to determine which anomaly has the least correlation to altitude.

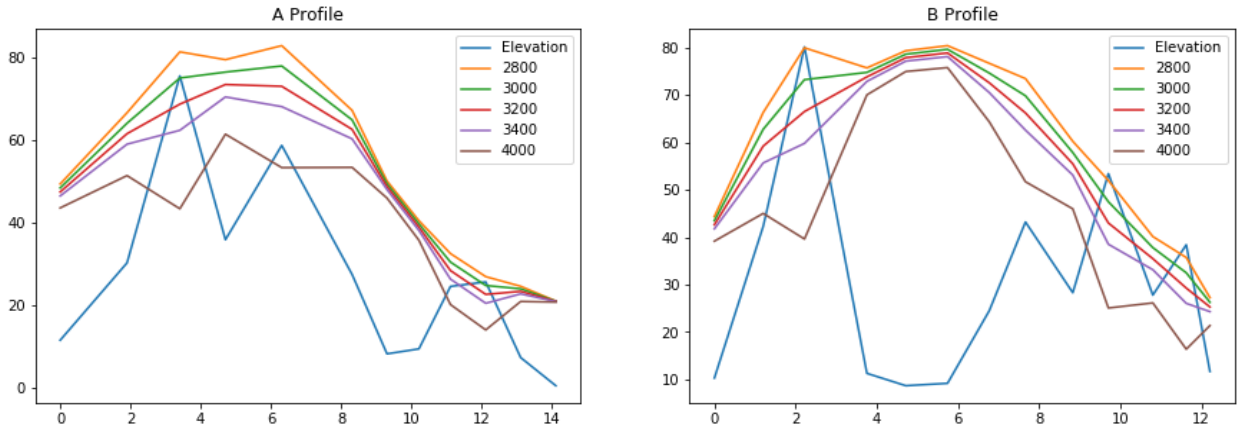


Fig. 1

From Fig. 1 it was determined that the Bouguer anomaly using a density of 3200kgm^{-3} had the least correlation to altitude and therefore should be used. This is supported by the geological evidence that the density of the ultrabasic rock is 3157kgm^{-3} (McCloskey, 2020, p. 1).

Eyeballing the provided geological map (McCloskey, 2020, p. 7) showed that the center of the body is around 36 eastings, 96 northings.

This was used to determine the distance from the center of the sphere for each measurement, and create a model for spheres with different depths and radii, as the density contrast and center were pretty well constrained by the geology- the sphere must have a diameter that is at least the diameter of the island, about 10 km, and the strike of the cone sheets converged around 36 eastings, 96 northings, as indicated on the geological map (McCloskey, 2020, p.7). The depth and radius could also not be so great that the sphere was larger than the thickness of the crust in the area, estimated to be about 30 km. Furthermore, the depth could not be greater than the radius, since the body outcropped on the island. Some of the many different radius and depth models that were tested are displayed below.

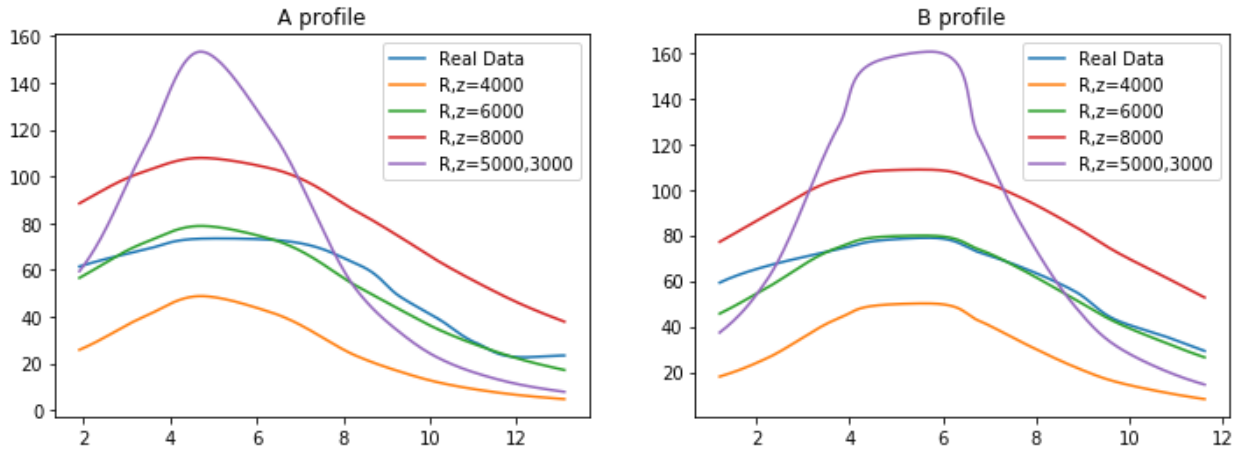


Fig. 2

The closest fitting sphere is one with a radius and depth of 6000 ± 100 m, with a density contrast of 500 ± 10 kgm^{-3}, centered at 36 ± 1 eastings, 96 ± 1 northings. This is confirmed by calculating root mean square deviations for the four above examples as shown in the table below.

	A	B
R,z=4000	28.05215337114036	30.400890321690312
R,z=6000	5.783130090047949	5.3788099000174325
R,z=8000	25.258808748628027	26.8545363189677
R,z=10000	54.58510461587933	57.05391958561995

Table 1

As R,z=6000m has the lowest root mean square deviation, it is the most accurate model.

The Python code used to create the above graphs and calculations is included in an appendix.

Conclusion

The best-fitting model for the Isle of Rum ultrabasic body is a sphere centered at 36 ± 1 eastings, 96 ± 1 northings, with a depth and radius of 6000 ± 100 m.

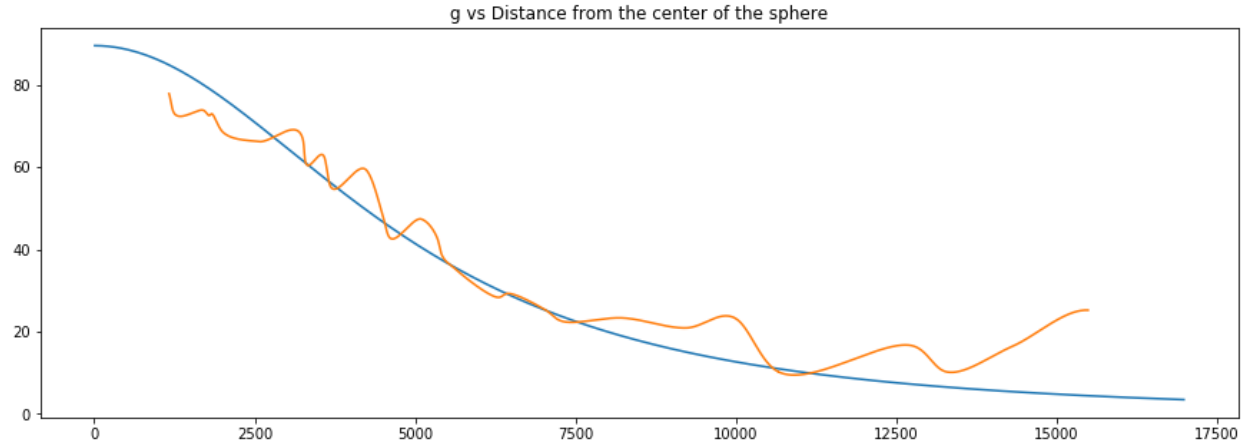
Discussion

Further investigation of this gravity data could include modeling it as other simple shapes, such as a truncated sphere, vertical cylinder, or cone.

There is also the opportunity to use computers to approach this investigation as an inverse problem. It is possible to use a computer to create a complex model that fits the actual shape of the body as closely as possible, and quite simple to determine the parameters for the best fitting sphere. In this case, they are a density contrast of 441 kgm^{-3}, radius of 7059 m, and depth of 7398 m. These numbers are neither unreasonable nor massively different from the values obtained by the forward method, though must be at least slightly incorrect- since the body outcrops on the island, the depth cannot be greater than the radius.

There is, as always, the danger that the computer will adhere too closely to the eccentricities of the data set and create a model that claims to be more accurate than the data itself. Overfitting is especially dangerous in this case as the model is known to be incorrect. It is extremely unlikely that the body is a perfect sphere, and so any spherical model that fits perfectly is necessarily wrong.

Further work could be done on this investigation by working to filter out variations in the data that are of a higher frequency than is relevant to the investigation. There is little variation in the two profiles, but plotting all data based only on its distance from the center of the sphere reveals many such variations, shown in the graph below.



Complicated mathematical techniques can be used to filter out these variations.

As always, other geophysical techniques would reveal more about the underlying geology.

Accurate errors were difficult to estimate in this investigation as the original data had no quoted errors. One can make estimates based on the number of significant figures to which the data is quoted, but this is an order of magnitude estimation only.

Furthermore, the final results were not determined by performing calculations with the collected data, so the original errors had little effect on the final results.

Errors in this investigation were estimated through the same trial-and-error process used to determine the results. The value for radius and depth is quoted as 6000 $\pm$ 100 m as 5900 m and 6100 m were both good fits when plotted against the data, similarly for the density contrast and x,y coordinates of the center of the sphere.

References

"Centripetal Catmull-Rom spline." Wikipedia, available at: https://en.wikipedia.org/wiki/Centripetal_Catmull–Rom_spline

McCloskey, J. (2020) "Interpreting the Gravity Anomaly over the Rum Ultrabasic Intrusion." University of Edinburgh

Appendix

#Free air estimation of density

```
f_air=np.array([31.7858,130.5378,247.1886,34.8718,26.8482,28.3912,75.607,133.3152,87.3338,164.7924,85.7908,118.5024,36.1062,
93.1972,232.6844,110.4788,180.8396,84.5564,25.3052,29.0084,75.607,79.0016,22.5278,1.543])+np.array([24.650805,-14.5840791,
-73.2336817,54.0756479,62.6600121,62.7784036,29.8011835,-9.1709744,6.1009589,-50.1557878,-13.0174326,-37.7075328,4.884895,
27.28272082,8.7508466,-62.9465618,10.8768314,-29.3245162,14.6878542,34.29967,22.42889,-14.403425,-22.07664,10.577755,
20.012175])
elev=[103,423,801,113,87,92,245,432,283,534,278,384,117,115,302,754,358,586,274,82,94,245,256,73,5]
```

```
np.polyfit(elev,f_air,1)[0]/(4.193*10**-5)
```

```
3933.899835618637
```

#Nettleton's Method

```
elev_a=np.array([115,302,754,358,586,274,82,94,245,256,73,5])
raw_a=np.array([27.28272082,8.7508466,-62.9465618,10.8768314,-29.3245162,14.6878542,34.29967,22.42889,-14.403425,-22.07664,
10.577755,20.012175])
elev_b=np.array([103,423,801,113,87,92,245,432,283,534,278,384,117])
raw_b=np.array([24.650805,-14.5840791,-73.2336817,54.0756479,62.6600121,62.7784036,29.8011835,-9.1709744,6.1009589,
-50.1557878,-13.0174326,-37.7075328,4.884895])
```

```
def Bouguer_a(p,label):
    g=raw_a+(.3086*elev_a)-(4.193*(10**-5)*p*elev_a)
    return plt.plot(data['HD_A'][:-1],g,label=label)
def Bouguer_b(p,label):
    g=raw_b+(.3086*elev_b)-(4.193*(10**-5)*p*elev_b)
    return plt.plot(data['HD_B'],g,label=label)
plt.figure(figsize=(15,5))
ax0=plt.subplot(121)
plt.plot(data['HD_A'][:-1],elev_a/10, label='Elevation')
Bouguer_a(p=2800,label='2800')
Bouguer_a(p=3000,label='3000')
Bouguer_a(p=3200,label='3200')
Bouguer_a(p=3400,label='3400')
Bouguer_a(p=4000,label='4000')
plt.title('A Profile')
ax0.legend()
ax1=plt.subplot(122)
line1=plt.plot(data['HD_B'],elev_b/10,label='Elevation')
line2=Bouguer_b(p=2800,label='2800')
line3=Bouguer_b(p=3000,label='3000')
line4=Bouguer_b(p=3200,label='3200')
line5=Bouguer_b(p=3400,label='3400')
line6=Bouguer_b(p=4000,label='4000')
plt.title('B Profile')
ax1.legend()
```

#Spherical Model

```
data=pd.read_excel('Documents/USeful Data.xlsx', header=2)
```

```
#The following code was stolen from the Wikipedia entry on Catmull-Rom splines- this is the spline used by Excel to create
#smooth curves
import numpy
import pylab as plt

def CatmullRomSpline(P0, P1, P2, P3, nPoints=100):
    """
    P0, P1, P2, and P3 should be (x,y) point pairs that define the Catmull-Rom spline.
    nPoints is the number of points to include in this curve segment.
    """
    # Convert the points to numpy so that we can do array multiplication
    P0, P1, P2, P3 = map(numpy.array, [P0, P1, P2, P3])

    # Calculate t0 to t4
    alpha = 0.5
    def tj(ti, Pi, Pj):
        xi, yi = Pi
        xj, yj = Pj
        return (((xj-xi)**2 + (yj-yi)**2)**0.5)**alpha + ti

    t0 = 0
    t1 = tj(t0, P0, P1)
    t2 = tj(t1, P1, P2)
    t3 = tj(t2, P2, P3)

    # Only calculate points between P1 and P2
    t = numpy.linspace(t1, t2, nPoints)
```

```
    # Reshape so that we can multiply by the points P0 to P3
    # and get a point for each value of t.
    t = t.reshape(len(t), 1)
    #print(t)
    A1 = (t1-t)/(t1-t0)*P0 + (t-t0)/(t1-t0)*P1
    A2 = (t2-t)/(t2-t1)*P1 + (t-t1)/(t2-t1)*P2
    A3 = (t3-t)/(t3-t2)*P2 + (t-t2)/(t3-t2)*P3
    #print(A1)
    #print(A2)
    #print(A3)
    B1 = (t2-t)/(t2-t0)*A1 + (t-t0)/(t2-t0)*A2
    B2 = (t3-t)/(t3-t1)*A2 + (t-t1)/(t3-t1)*A3

    C = (t2-t)/(t2-t1)*B1 + (t-t1)/(t2-t1)*B2
    return C

def CatmullRomChain(P):
    """
    Calculate Catmull-Rom for a chain of points and return the combined curve.
    """
    sz = len(P)

    # The curve C will contain an array of (x, y) points.
    C = []
    for i in range(sz-3):
        c = CatmullRomSpline(P[i], P[i+1], P[i+2], P[i+3])
        C.extend(c)

    return C
```

```
def sphere_a(R,p,z,xc,yc,label):
    coords=np.array([data['E_A'],data['N_A']])
    x_a=np.hypot(xc-coords[0],yc-coords[1])
    x_a=np.array(x_a)*1000
    g_a=2.795*10**-5*(p*R**3/z**2)*(1/(1+(x_a/z)**2))**(3/2)
    x1,y1=zip(*CatmullRomChain(np.transpose(np.array([data['HD_A'],g_a]))))
    plt.plot(x1,y1,label=label)

def sphere_b(R,p,z,xc,yc,label):
    coords=np.array([data['E_B'],data['N_B']])
    x_b=np.hypot(xc-coords[0],yc-coords[1])
    x_b=np.array(x_b)*1000
    g_b=2.795*10**-5*(p*R**3/z**2)*(1/(1+(x_b/z)**2))**(3/2)
    x1,y1=zip(*CatmullRomChain(np.transpose(np.array([data['HD_B'],g_b]))))
    plt.plot(x1,y1,label=label)
```

```
plt.figure(figsize=(12,4))
ax0=plt.subplot(121)
x0,y0=zip(*CatmullRomChain(np.transpose(np.array([data['HD_A'],data['g_A']]))))
plt.plot(x0,y0, label='Real Data')
sphere_a(R=4000,p=500,z=4000,xc=36,yc=96,label='R,z=4000')
sphere_a(R=6000,p=500,z=6000,xc=36,yc=96,label='R,z=6000')
sphere_a(R=8000,p=500,z=8000,xc=36,yc=96,label='R,z=8000')
sphere_a(R=5000,p=500,z=3000,xc=36,yc=96,label='R,z=5000,3000')
plt.title("A profile")
ax0.legend()
ax1=plt.subplot(122)
x0,y0=zip(*CatmullRomChain(np.transpose(np.array([data['HD_B'],data['g_B']]))))
plt.plot(x0,y0,label='Real Data')
sphere_b(R=4000,p=500,z=4000,xc=36,yc=96,label='R,z=4000')
sphere_b(R=6000,p=500,z=6000,xc=36,yc=96,label='R,z=6000')
sphere_b(R=8000,p=500,z=8000,xc=36,yc=96,label='R,z=8000')
sphere_b(R=5000,p=500,z=3000,xc=36,yc=96,label='R,z=5000,3000')
plt.title("B profile")
ax1.legend()
```

#Root-Mean-Square Deviation

```
def RMSD_b(R,p,z,xc,yc):
    coords=np.array([data['E_B'],data['N_B']])
    x_b=np.hypot(xc-coords[0],yc-coords[1])
    x_b=np.array(x_b)*1000
    g_b=2.795*10**-5*(p*R**3/z**2)*(1/(1+(x_b/z)**2))**(3/2)
    return np.sqrt(((g_b-data['g_B'])**2).mean())

def RMSD_a(R,p,z,xc,yc):
    coords=np.array([data['E_A'],data['N_A']])
    x_a=np.hypot(xc-coords[0],yc-coords[1])
    x_a=np.array(x_a)*1000
    g_a=2.795*10**-5*(p*R**3/z**2)*(1/(1+(x_a/z)**2))**(3/2)
    return np.sqrt(((g_a-data['g_A'])**2).mean())

print(RMSD_a(R=4000,p=500,z=4000,xc=36,yc=96))
print(RMSD_a(R=6000,p=500,z=6000,xc=36,yc=96))
print(RMSD_a(R=8000,p=500,z=8000,xc=36,yc=96))
print(RMSD_a(R=10000,p=500,z=10000,xc=36,yc=96))

print(RMSD_b(R=4000,p=500,z=4000,xc=36,yc=96))
print(RMSD_b(R=6000,p=500,z=6000,xc=36,yc=96))
print(RMSD_b(R=8000,p=500,z=8000,xc=36,yc=96))
print(RMSD_b(R=10000,p=500,z=10000,xc=36,yc=96))
```

```
28.05215337114036
5.783130090047949
25.258808748628027
54.58510461587933
30.400890321690312
5.3788099000174325
26.8545363189677
57.05391958561995
```

#Spherical parameters via inversion

```
x_data=pd.DataFrame({'all_e':[41,40.1,39.2,37.7,37,36.1,35.2,34.4,33.4,32.8,31.7,31.1,30.6,37,37.1,37.2,37.2,37.3,37.4,37.3,
37.5,37.6,37.5,37.5,29.2,28.6,43.4,47.1,44.9,40.4,30.3],'all_n':[94.1,94.9,95.4,95.9,96.6,97.1,97.6,98,98.6,99.3,99.6,100.2,
100.5,91,92.9,94.4,95.7,97.3,99.3,100.3,101.2,102.1,103.1,104.1,105.1,104.2,89.3,106.4,88.7,84.8,80.2,81.6],'all_g':[
42.616477,59.1972729,66.4799423,73.7855599,77.8349001,78.8254116,72.5350635,66.1801936,55.4629509,42.9866282,35.4724394,
29.2712832,25.292503,47.34148082,61.4268946,68.5691342,73.3206234,72.8879478,62.4800302,48.602438,38.824746,28.330455,
22.575904,23.310707,20.884295,10.3,23.3,16.5,10.2,16.4,16.7,25.2,]})

x_data['all_x']=1000*np.hypot(xc-x_data['all_e'],yc-x_data['all_n'])

x_data=x_data.sort_values(by=['all_x'])
```

```
x_sphere=np.arange(0,17000,10)
g_sphere=2.795*10**-5*(p*R**3/z**2)*(1/(1+(x_sphere/z)**2))**(3/2)
```

```
import scipy.optimize
def bell(x,p,R,z):
    return 2.795*10**-5*(p*R**3/z**2)*(1/(1+(x/z)**2))**(3/2)
fit_params,pcov=scipy.optimize.curve_fit(f=bell, xdata=x_data['all_x'],ydata=x_data['all_g'],bounds=([400,5000,4000],
[600,15000,15000]))
fit_params
```

```
array([ 441.46119749, 7059.49021293, 7397.97009361])
```

#Data plotted against radial distance from the center of the body

```
plt.figure(figsize=(15,5))
x0,y0=zip(*CatmullRomChain(np.transpose(np.array([x_sphere,g_sphere]))))
plt.plot(x0,y0)
x0,y0=zip(*CatmullRomChain(np.transpose(np.array([x_data['all_x'],x_data['all_g']]))))
plt.plot(x0,y0)
plt.title('g vs Distance from the center of the sphere')
```

```
Text(0.5, 1.0, 'g vs Distance from the center of the sphere')
```