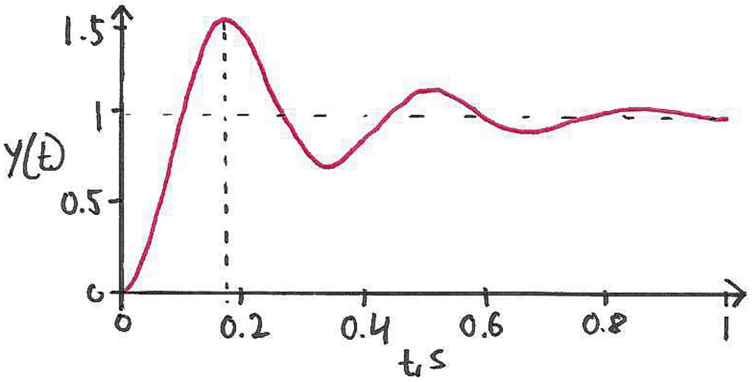
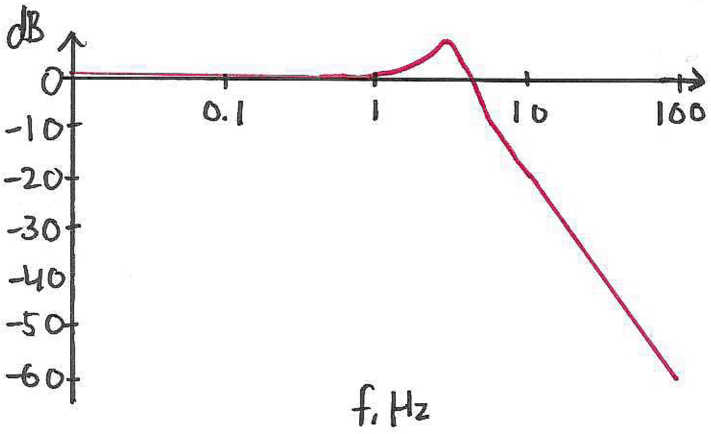
SIGNALS AND SYSTEMS - WEEK 10

Problem 1

Identify the system parameters, write down the differential equation and transfer function for this system.





Sol

- Read PO: PO = 52
- Calculate a: $a = \ln\left(\frac{PO}{100}\right) = -0.654$
- Calculate damping: $\zeta = \frac{|a|}{\sqrt{\pi^2 + a^2}} = \frac{0.654}{\sqrt{\pi^2 + 0.654^2}} = 0.204$
- Calculate time to peak: $t_p = 0.168$ s
- Calculate natural frequency: $\omega_n = \frac{\pi}{t_p\sqrt{1-\zeta^2}} = \frac{\pi}{0.168\sqrt{1-0.204^2}} = 19.1$

We can write the transfer function.

$$H(s) = k \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

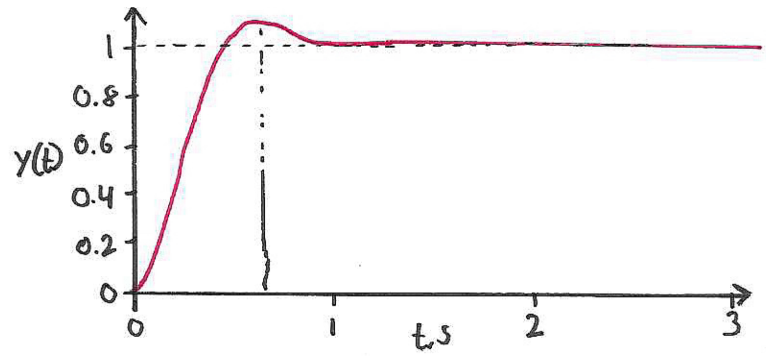
From frequency characteristic we see that $|H(0)| = 0\,dB = 1 \Rightarrow k = 1$.
It also has a "lowpass" characteristic.

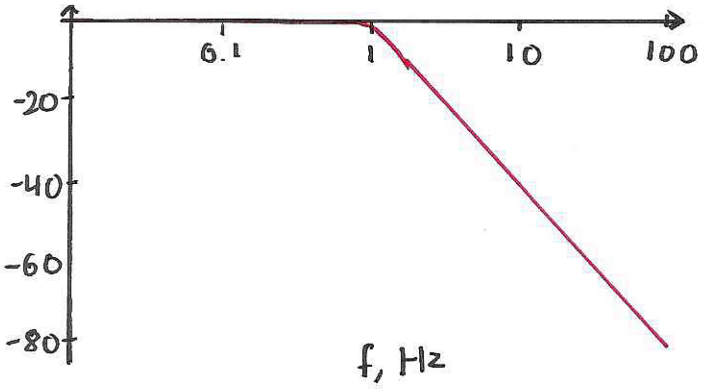
$$H(s) = \frac{19.1^2}{s^2 + 2 \cdot 0.204 \cdot 19.1 \cdot s + 19.1^2} = \frac{364.7}{s^2 + 7.79s + 364.7}$$

$$\ddot{y}(t) + 7.79\dot{y}(t) + 364.7y(t) = 364.7x(t)$$

Problem 2

Identify system parameters, write down the differential equation and transfer function for this system.





Sol

- Read PO: PO = 9.5
- Calculate a: $a = \ln\left(\frac{PO}{100}\right) = -2.354$
- calculate damping: $\zeta = \frac{|a|}{\sqrt{\pi^2 + a^2}} = \frac{2.354}{\sqrt{\pi^2 + 2.354^2}} = 0.599$
- Read peak time: $t_p = 0.615$ s
- Calculate natural frequency: $\omega_n = \frac{\pi}{t_p\sqrt{1-\zeta^2}} = \frac{\pi}{0.615\sqrt{1-0.599^2}} = 6.379$

From $|H(\omega)|$ we see a lowpass characteristic and k=1 (DC-gain)

$$H(s) = k \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{6.379^2}{s^2 + 2 \cdot 0.599 \cdot 6.379 \cdot s + 6.379^2}$$

$$H(s) = \frac{40.69}{s^2 + 7.64s + 40.69}$$

$$\ddot{y}(t) + 7.64\dot{y}(t) + 40.69y(t) = 40.69x(t)$$

Problem 2

A second order LTIC system has poles $p = -1 \pm j2$.
Calculate a_1, a_0 and ζ, ω_n and t_s, t_p, t_r, t_d, -PO.

sol

A system transfer function can be written as

$$H(s) = \frac{N(s)}{D(s)} = \frac{(s-z_1)(s-z_2)\ldots}{(s-P_1)(s-P_2)\ldots}$$

In this problem we are only interested in $D(s)$

$$D(s) = (s-(-1+j2))(s-(-1-j2)) = s^2 + 2s + 5$$

- $a_1 = 2$
- $a_0 = 5$
- $\omega_n = \sqrt{a_0} = 2.23$
- $\zeta = \frac{a_1}{2\sqrt{a_0}} = 0.447$

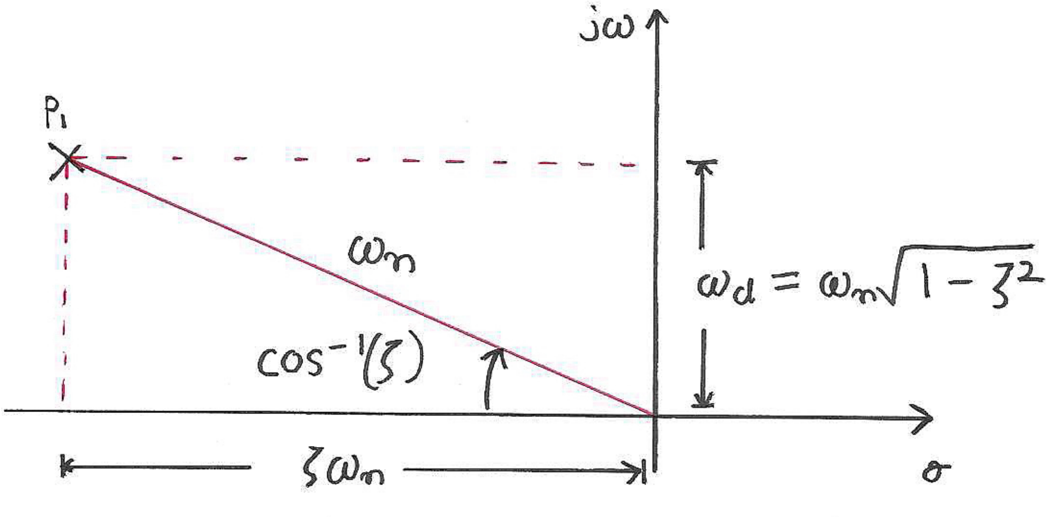
- settling time: $t_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.447 \cdot 2.23} = 4\ s$
- peak time: $t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{2.23\sqrt{1-0.447^2}} = 1.57\ s$
- Rise time: $t_r = \frac{1-0.5167\zeta + 2.917\zeta^2}{\omega_n} = 0.604\ s$
- Delay time: $t_d = \frac{1.1 + 0.125\zeta + 0.469\zeta^2}{\omega_n} = 0.558\ s$
- Percent overshoot: $PO = 100e^{-\zeta \cdot \frac{\pi}{\sqrt{1-\zeta^2}}} = 20.7\ \%$

Maple:
with(DynamicSystems):
sys:=TransferFunction$\left(\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right)$:

Prop:=StepProperties(sys).

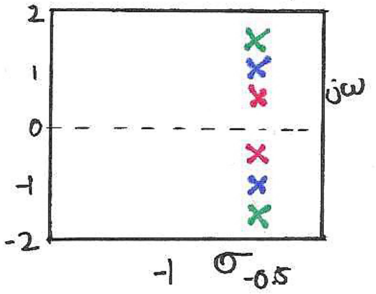
Problem 3

Explain the effect of moving poles.

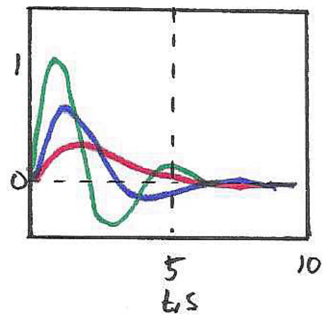


- Vertical distance from σ-axis: ω_d
- Horizontal distance from jω-axis: $\zeta\omega_n$
- Angle between σ-axis and pole-position vector: ζ (cosine)
- Distance from origin to pole: ω_n

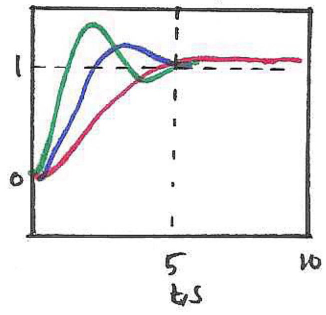
Pole-Zero Map



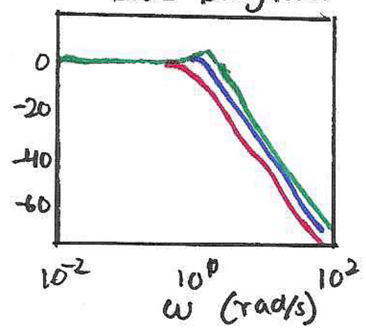
Impulse Response



Step Response



Bode Diagram

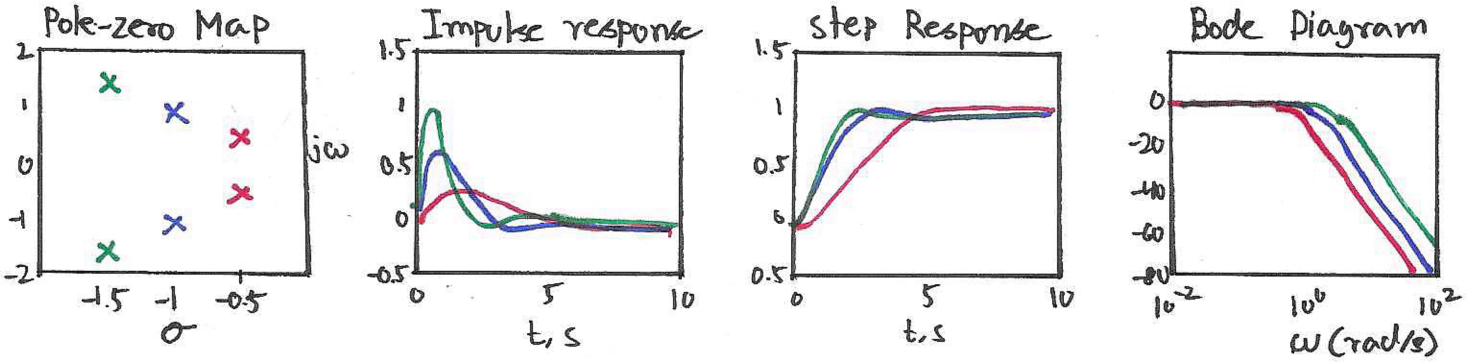


Sol

As the poles move from green to red the angle ~~increases,~~ decreases and ζ increases. This makes sense when looking at the responses, as the oscillations get less severe.

At the same time ω_n is reduced and the rise time $t_r = \frac{1-0.5167\zeta+2.917\zeta^2}{\omega_n}$ increases.

Problem 3B



Sol

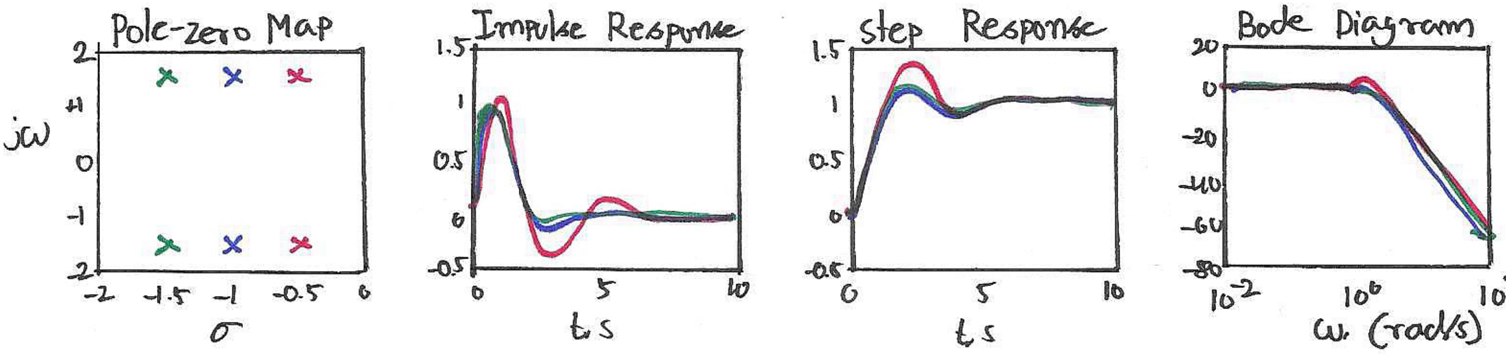
The angle stays the same so ζ = constant.

The distance from origin increases from red to green.

The po stays the same, but the peak time increases from green to red.

Bandwidth increases from red to green.

Problem 3C



The vertical distance to σ-axis stays the same, so ω_d = constant.

The damping decreases from green to red.

The po is highest for the red, but t_p is the same.

The cutoff frequency is almost unchanged.