Problem 2

Hvad giver foldningen $u(t) * u(t)\sin(t)$?

Sol

$$u(t) * u(t)\sin(t) = \int_{-\infty}^{\infty} u(\tau)\sin(\tau)\cdot u(t-\tau)\, d\tau$$

$$= \int_{0}^{t} \sin(\tau)\, d\tau, \quad t \geq 0$$

$$= \left[-\cos(\tau)\right]_0^t, \quad t \geq 0$$

$$= -\cos(t) - (-\cos(0)), \quad t \geq 0$$

$$= \underline{\underline{(1 - \cos(t))\, u(t)}}$$

Problem 4

Hvad giver foldningen $u(t) * u(t-t_0)$?

Sol

Først udregnes $u(t) * u(t)$.

$$f_1(t) = \int_{-\infty}^{\infty} u(\tau) \cdot u(t-\tau)\, d\tau$$

$$= \int_{0}^{t} 1 \, d\tau \quad , \quad t \geq 0$$

$$= [\tau]_0^t \quad \Rightarrow$$

$$f_1(t) = t\, u(t)$$

Benyt tidsforskydningsteoremet

$$u(t) * u(t-t_0) = f_1(t-t_0) = \underline{\underline{(t-t_0)\, u(t-t_0)}}$$

Problem 5

Hvad giver $u(t-t_0) * tu(t)$?

Sol

Først udregnes foldningen uden tidsforskydning.

$$f_1(t) = u(t) * tu(t) = \int_{-\infty}^{\infty} \tau u(\tau) \cdot u(t-\tau)\, d\tau$$

$$= \int_{0^-}^{t} \tau \, d\tau \quad , \quad t \geq 0$$

$$= \left[\tfrac{1}{2}\tau^2\right]_0^t \quad , \quad t \geq 0$$

$$f_1 = \tfrac{1}{2} t^2 u(t)$$

Benyt tidsforskydningsteoremet

$$u(t-t_0) * tu(t) = f_1(t-t_0) = \underline{\tfrac{1}{2}(t-t_0)^2 u(t-t_0)}$$