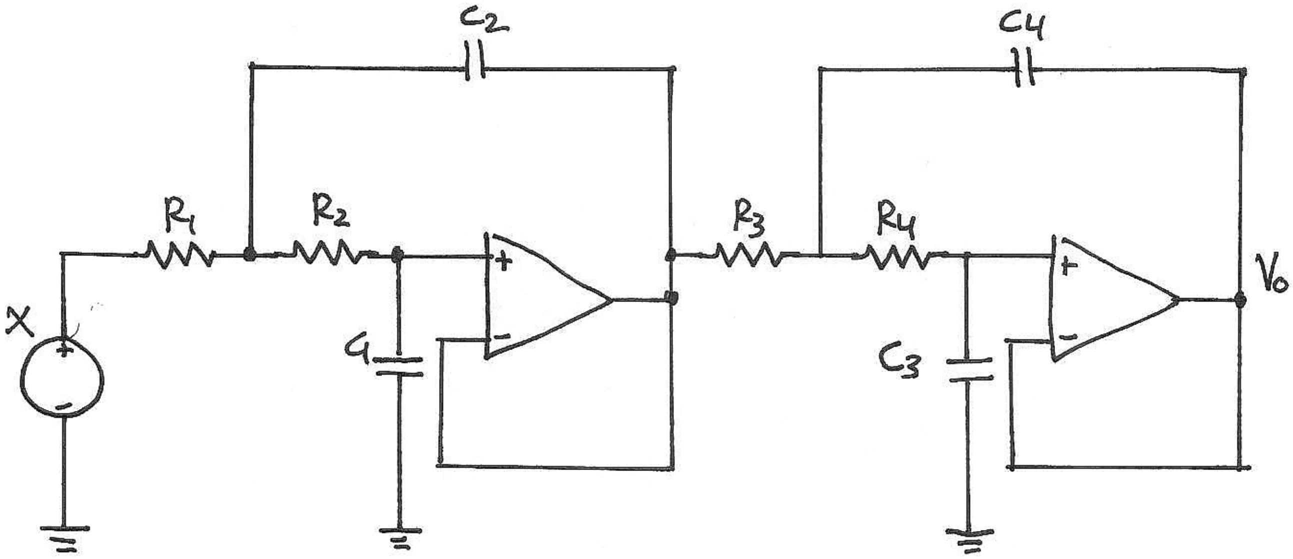
SIGNALS AND SYSTEMS - WEEK 13

Problem 1

Design a 4^{th} order Butterworth lowpass filter with unity gain and cutoff frequency at 500 Hz.
The filter must be insensitive to component drift.



Sol

It's a 4^{th} order filter, so we need 2 stages:

$$H(s) = \frac{1}{s^2 + 0.765367s + 1} \cdot \frac{1}{s^2 + 1.84776s + 1} \quad \text{(Normalized)}$$

For lowpass resistors should be identical to reject drift.

- $\frac{C_2}{C_1} = \frac{4a_0}{a_1^2} = \frac{4 \cdot 1}{(0.765367)^2} = 6.828 \Rightarrow C_1 = 8.2F, \; C_2 = 56F$

- $R_1 = R_2 = \frac{a_1}{2a_0C_1} = \frac{0.765367}{2 \cdot 1 \cdot 8.2} = 0.0466 \, \Omega$

Frequency scale to move f_c from $1 \frac{rad}{s}$ to 500 Hz.

- $K_f = \frac{2\pi \cdot f_{c,new}}{\omega_{c,old}} = \frac{2\pi \cdot 500 \text{ Hz}}{1 \frac{rad}{s}} = 3141.5$

Scale both capacitors with this k_f.

$$C_{1f} = \frac{C_1}{k_f} = \frac{8.2\,F}{3141.5} = 2.610\ mF$$

$$C_{2f} = \frac{C_2}{k_f} = \frac{56\,F}{3141.5} = 17.83\ mF$$

Currently: $C_1 = 2.610\ mF$, $C_2 = 17.83\ mF$, $R_1 = R_2 = 0.0466\ \Omega$.

These values are impractical.

Scale all components with k_z (multiply resistors, divide capacitors).

Impedance scaling:

$$k_z = \frac{C_{1f}}{C_{1,des}} = \frac{2.610\ mF}{10\,nF} = 261014$$

$$C_{1fz} = 10\,nF$$

$$C_{2fz} = \frac{17.83\ mF}{261014} = 68.29\ nF \approx 68\ nF$$

$$R_{1z} = R_{2z} = Rk_z = 0.0466\ \Omega \cdot 261014 = 12181\ \Omega \approx 12\ k\Omega$$

Stage 2

- $\frac{C_4}{C_3} = \frac{4a_0}{a_1^2} = \frac{4 \cdot 1}{(1.84776)^2} = 1.17157 \Rightarrow C_3 = 3.3F,\ C_4 = 3.9F$
- $R_3 = R_4 = \frac{a_1}{2a_0C_3} = 0.2799\ \Omega$

Frequency scaling:

$$k_f = \frac{2\pi \cdot f_{c,new}}{\omega_{c,old}} = \frac{2\pi \cdot 500\ Hz}{1\ \frac{rad}{s}} = 3141.5$$ (same as before)

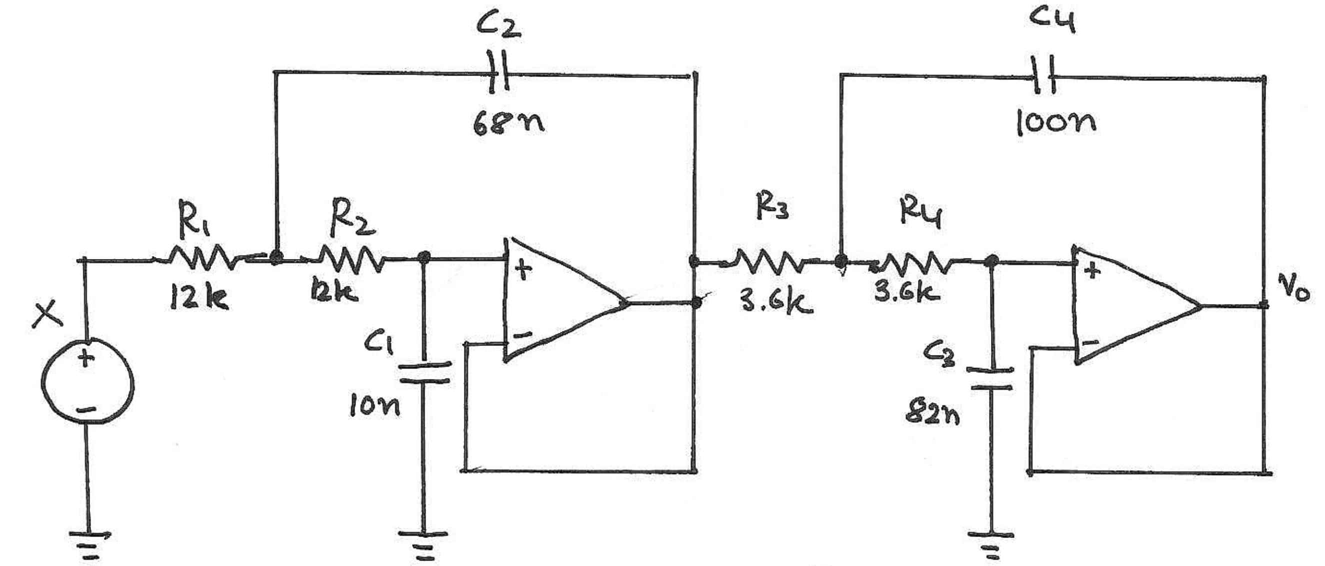
$$C_{3f} = \frac{C_3}{k_f} = \frac{3.3F}{3141.5} = 1.046\ mF$$

$$C_{4f} = \frac{C_4}{k_f} = \frac{3.9\,F}{3141.5} = 1.236\ mF$$

Impedance scaling:

$$k_z = \frac{C_{3f}}{C_{3,des}} = \frac{1.046\ mF}{82\ nF} = 12754$$

- $C_{3fz} = 82\ nF$
- $C_{4fz} = \frac{C_{4f}}{k_z} = \frac{1.236\ mF}{12754} = 96.9\ nF \approx 100\ nF$
- $R_{3z} = R_{4z} = Rk_z = 0.2799\ \Omega \cdot 12754 = 3570\ \Omega = 3.6\ k\Omega$



4^{th} order Butterworth lowpass filter.

Attenuation: $-80\ \frac{dB}{dec}$ after $f_c = 500\ Hz$

Problem 2A

The prescence of EMG-signals in the arms drops by $6\ \frac{dB}{oct} = 20\ \frac{dB}{dec}$ after 500 Hz.

- How much attenuation do we have if we include the 4^{th} order lowpass filter?

Sol

Above 500 Hz: $-20\ \frac{dB}{dec} - 80\ \frac{dB}{dec} = -100\ \frac{dB}{dec}$

$-100\ \frac{dB}{dec} = -30\ \frac{dB}{oct}$

Problem 2B

Assume that the EMG-signal is sampled at $f_s = 4$ kHz by a 12-bit ADC.

- How many bits are toggled at $\frac{f_s}{2}$?
- Do we avoid aliasing?

Sol

A 12-bit ADC has dynamic range: $DR = 12 \text{ bit} \cdot 6 \frac{\text{dB}}{\text{bit}} = 72$ dB.

To avoid aliasing: -72 dB at $\frac{f_s}{2}$.

$\frac{f_s}{2} = \frac{4 \text{ kHz}}{2} = 2$ kHz.

- 1 oct after 500 Hz $= 1000$ Hz : $-30 \frac{\text{dB}}{\text{oct}} \cdot 1 \text{ oct} = -30$ dB
- 2 oct after 500 Hz $= 2$ kHz : $-30 \frac{\text{dB}}{\text{oct}} \cdot 2 \text{ oct} = -60$ dB

We do not avoid aliasing, $-60 \text{ dB} > -72$ dB

1 bit ≈ 6 dB

2 bit ≈ 12 dB

2 bits will be toggled due to aliasing at half the sampling frequency, when $f_s = 4$ kHz.

Solution: Increase f_s or filter order.

Problem 3

Design a 2nd order Butterworth highpass filter, with unity gain and $f_c = 2\ Hz$.

Sol

For Sallen-key highpass filters $C_1 = C_2$ for insensitivity.

- $\frac{R_1}{R_2} = \frac{4a_0}{a_1^2} = \frac{4}{(\sqrt{2})^2} = 2 \Rightarrow R_1 = 68\ \Omega,\ R_2 = 33\ \Omega$

- $C_1 = C_2 = C = \frac{1}{\sqrt{a_0 R_1 R_2}} = \frac{1}{\sqrt{1 \cdot 68 \cdot 33}} = 0.021\ F$

Where we used: $a_1 = \sqrt{2},\ a_0 = 1$ (Butterworth 2nd order)

Frequency scale: $k_f = \frac{2\pi \cdot f_{c,new}}{\omega_{c,old}} = \frac{2\pi \cdot 2\ Hz}{1\ \frac{rad}{s}} = 12.56$

$C_{1f} = C_{2f} = \frac{C}{k_f} = \frac{0.021\ F}{12.56} = 0.00167\ F$

Impedance scale: $C_{1fz} = C_{2fz} = \frac{C_f}{k_z} \Rightarrow k_z = \frac{C_f}{C_{des}} = \frac{0.00167\ F}{1\ \mu F} = 1670$

$C_{1fz} = C_{2fz} = 1\ \mu F$

$R_{12} = R_1 k_z = 68\ \Omega \cdot 1670 = 113560\ \Omega \cong 120\ k\Omega$

$R_{22} = R_2 k_z = 33\ \Omega \cdot 1670 = 55110\ \Omega \cong 56\ k\Omega$